I Have inspected the Epitome of the *Philosophical Transactions*, since the Year 1700. made by Mr. *Benjamin Motte*; and do believe that it is done by him with due Care and Judgment.

Witness my Hand,

July 26, 1720. *Edm. Halley*, Secr. Reg. Soc.

THE

PHILOSOPHICAL TRANSACTIONS

From the Year MDCC.

(Where Mr. *LOWTHORP* ends)

To the Year MDCCXX.

ABRIDG'D.

AND

Dispos'd under General HEADS.

In Two VOLUMES.

By BENJ. MOTTE.

LONDON:

Printed for R. WILKIN, R. ROBINSON, S. BALLARD, W. and J. INNYS, and J. OSBORN. MDCCXXI.

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VOL. I. Containing

Part I. The MATHEMATICAL Papers.
Part II. The ANATOMICAL and MEDICAL Papers.

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Printed for R. WILKIN, R. ROBINSON, S. BALLARD, W. and J. INNYS, and J. OSBORN. MDCCXXI.

To the MOST NOBLE,

JAMES,

DUKE and BARON of

CHANDOS,

MARQUESS and EARL of

CAERNARVAN;

Viſcount WILTON of *Stukeley-Caſtle* in the County of *Hereford*,

AND

Lord Lieutenant and *Cuſtos Rotulorum* of the Counties of *Hereford* and *Radnor*.

THESE MATHEMATICAL and MEDICAL Papers, Abridg'd and Diſpos'd under General Heads, are, with all Gratitude and Humility, dedicated by

HIS GRACE'S

Obedient, Devoted,

Humble Servant,

BENJ. MOTTE.

THE PREFACE.

AFTER *the Reader has been inform'd, that what now lies before him, is only a Continuation of* Mr. LOWTHORP'*s Abridgment from the Period of his Work to the present Time; and that I have throughout copy'd after his Pattern, so far as my inferiour Judgment will give me leave: it will be unnecessary to spend many Words as to the Method here pursu'd; and sufficient to refer to his Preface for a succinct and particular Account of it. At the same time I look upon it as a Duty incumbent on me ingenuously and with Gratitude to confess, that whatever Advantages may accrue to my self or the Publick by the present Performance, are owing to the ingenious Hint of that judicious Gentleman; and to the Applause and Encouragement which his useful Undertaking was most justly receiv'd with.*

IT *may not however be improper briefly to take notice of the following Particulars.*

1. THE *Anatomical Papers, which are by Mr.* LOWTHORP *made the Third Part, are here plac'd in the Second: which was done*

done only for the Convenience of Binding, and to divide the Two Volumes equally. This Reason, I presume, will be sufficient to justify so minute an Alteration.

2. I *have omitted such Experiments of the late Mr.* Hawksbee, *as he has himself collected and publish'd in his* Physico-Mechanical Experiments *printed in* 1709. *since this Book, in all probability, makes a part of the Library of any Gentleman whose Curiosity and Genius inclines him to Studies of that kind.*

3. Several *Catalogues of Plants, Minerals, Fossils, Insects,* &c. *publish'd by the late inquisitive Naturalist Mr.* James Petiver, *are also omitted; by reason the most material Particulars have been by himself inserted in the several Parts of his* Gazophylacium Naturæ & Artis: *and besides, that those Catalogues are of Bulk sufficient to make a handsome Volume of themselves, if such a Design could meet with suitable Encouragement.*

4. The *Tracts relating to the Disputes between the Mathematicians of our own Country and some Gentlemen abroad, are above the Capacity of the Majority of Readers; and were intended rather for an honest Representation of matter of Fact to Posterity, than for the immediate Instruction of the Studious in those Sciences. For which reason I have contented my self with referring to the Places in the* Transactions, *where those Tracts are to be found, for the Service of such as are desirous to be acquainted with the Merits of that Controversy.*

It *may perhaps be expected that I should say something in this place to excuse what Imperfections may occur in the Course of this*

Work;

Work; and take notice of a Competition wherewith I am threatned. As to the first of these, I have already confest myself indebted to a more able Hand for the Method, and to the Labours of many eminent Men for the Materials, whereof it is compos'd. After this, he must be master of very little Skill and Judgment, who with such Assistances is not able to furnish out a tolerable Performance; and I may without vanity hope, the Reader will find no Errors or Defects, but what a good natured Man may excuse, on account of the Length and Tediousness of the Work, and the number of Papers, and variety of Subjects whereof it consists.

As *to the Competition there is reason to expect, it is a Misfortune which can in no wise be imputed to me. Before I had any Apprehension of a Design of that nature, I was engaged too far to recede with any Conveniency. I notwithstanding endeavour'd an Accommodation, provided it could be concluded upon Terms which implied an Equality; but this readiness of mine to comply was always frustrated by an imagin'd Superiority of Learning, or Interest, or something else on the other side. How reasonable these Presumptions were, must be submitted to the Judgment of the Publick. In the mean time, it would be low and impertinent in this place to descend to the further Particulars of an Affair, which has relation only to two private Persons.*

I *shall conclude this Preface with offering one Consideration to the Thoughts of such Gentlemen as design to communicate their Labours to the World: which is the Convenience that would ensue, if in every Book that is publish'd, a Memorial was inserted of the Day*

and

and Year when it first appear'd in Print, and this Date continu'd in succeeding Impressions. It would give an additional Pleasure to a future Reader, to observe by what Steps and Advances Learning was improv'd; and would enable Posterity to do Justice to the true Author of a fine Thought or useful Discovery; which is no more than necessary in an Age where Plagiary is so common a Vice. Other Reasons might be given to confirm this Caution, but it would be tedious to insist at large on what I only mention as an Hint, in my opinion, worthy Observation; and not improper to be prefixt to a Book which treats of Miscellaneous Subjects.

Sept. 28. 1721.

THE

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For the Information of the Binder, and the Satisfaction of the Gentleman, that his Book is perfect; the following Table is added, being a List of the Plates and the Pages whereunto they belong.

N. B. *The few Figures, included in Crotchets thus* [ ], *refer to the Pages in* Part I. *which come between* Pag. 224. *and* 225. *This Notice concerns both the foregoing and following* Index.

AN
Alphabetical Index
OF THE
Names of the AUTHORS:
WHEREIN

The *Numeral Letters* diſtinguiſh the *Parts*, and the *Figures* ſignifie the *Pages*: The Numbers following immediately after the *Names*, direct to the *Papers Abridg'd* and *Inſerted*; thoſe after *, to the *Titles* of the *Papers Omitted*; and thoſe after **, to the *Titles* of *Books*.

Mr. Caſ-

I

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ERRATA.

PArt II. p. 13. *firft Note in Marg. read* n. 337. p. 281. p. 70. *in Marg. for* Fig. 9. *r.* Fig. 15. p. 101. *l.* 33. *r. Figures.*

THE
Philosophical Transactions
From the Year 1700. to 1720.
Abridg'd and Methodically Digested

PART I.
The *Mathematical* Papers.

CHAP. I.
Geometry, Algebra, Logarithmotechny, Arithmetick.

SIT DE Axis Transversus Ellipseos, AO Axis alter, & C centrum Sectionis. Sit P punctum quodvis in circumferentia ejus: PQ Tangens curvæ ad P, occurrens Axi transverso ad Q; puncta SF Foci; CP, CK Semidiametri conjugatæ; PH Semilatus rectum ad diametrum PC: PG normalis ad Tangentem, cui occurrat HG, perpendicularis ipsi PCH, in puncto G: ut fiat PG radius Curvaturæ Ellipseos in puncto P: sint etiam ST, CR, FV perpendiculares in Tangentem PQ demissæ: Jungatur SO, & demittatur in Axem normalis PL. His positis, dico quod, (I. *Some Properties of the Conick Sections, by* Mr. Abr. de Moivre, *n.* 352. *p.* 622. *Plate* I. *Fig.* 1.)

I. *Rectangulum sub distantiis ab utroque Ellipseos Foco, sive* $SP \times SF$ *æquale est quadrato Semidiametri* CK.

Demonstratio.

$PSq = PCq + CSq - 2CS \times CL$ per 13. II. *Elem.*
$PFq = PCq + CSq + 2CS \times CL$ per 12. II. *Elem.*
Unde $PSq + PFq = 2PCq + 2CSq$.
Jam $PS + PF = DE = 2CD$; ac propterea
$PSq + PFq + 2PS \times PF = 4CDq$.

A   Quare

Quare transponendo, $2PS \times PF = 4CDq - 2PCq - 2CSq$.
Ac Dimidiando $PS \times PF = 2CDq - PCq - CSq$.
Est autem CS *quad.* $= CD$ *quad.* $- CO$ *quad.* atque adeo,
$PS \times PF = CDq + COq - PCq$.
Sed $CDq + COq = PCq + CKq$. *per* 12. VII. *Conic. Apollonii.*
Quare $PS \times PF = CKq$. *Q.E.D.*

II. *Distantia à Foco* SP *est ad perpendicularem in Tangentem demissam, ut Semidiameter Conjugata* CK *ad Semiaxem minorem* CO.

Demonstratio. Ob similia Triangula SPT, FPV, erit $PS : PF :: ST : FV$; ac componendo $PS + PF$ erit ad $ST + FV$, & earundem dimidia CD ad CR, ut PS ad ST. Unde $CD \times CK$ erit ad $CR \times CK$ ut PS ad ST. Sed $CR \times CK$ æquale est rectangulo sub Semiaxibus CD in CO, per 31. VII. *Conic.* Proinde PS est ad ST ut CD in CK ad $CD \times CO$, sive ut CK ad CO. Ac pari argumento demonstrabitur PF esse ad FV in eadem ratione. *Q. E. D.*

III. *In eadem etiam est ratione Semiaxis Transversus* CD *ad normalem è centro* C *ad Tangentem demissam, sive ad* CR.

Etenim cum rectangulum $CR \times CK$ æquale sit rectangulo $CD \times CO$, uti jam dictum est, erit ἀνάλογον CD ad CR ut CK ad CO. *Q. E. D.*

IV. *Semidiameter quævis* PC *est ad distantiam puncti* P *à foco* S, *sive ad* SP, *ut distantia ab altero Foco* FP *ad dimidium lateris recti ad Verticem* P *pertinentis, sive ad* PH.

Hoc autem manifestum est ob *Prop.* I. cum nempe quadratum ex CK æquale sit rectangulo sub $SP \times PF$.

V. *Rectangulum Semiaxium* CD × CO *est ad quadratum semidiametri conjugatæ* CK, *ut* CK *ad Radium Curvaturæ in puncto* P, *sive ad* PG.

Sunt enim Triangula PCR, PGH inter se similia, unde CR est ad PC, ut semilatus rectum PH ad PG: hoc est, per præmissam Proprietatem III, $\frac{CD \times CO}{CK} = CR$ est ad PC ut $\frac{CK^2}{PC} = PH$ ad $\frac{CK^3}{CD \times CO} = PG$. proinde ἀνάλογον $CD \times CO : CK^2 :: CK : PG$. *Q. E. D.*

Theorema Generale I. *Vis centripeta ad idem punctum* S *tendens, in Curvis omnibus, est semper proportionalis Quantitati* $\frac{SP}{PG \times ST^3}$.

Hoc Theorema ante plures annos à me investigatum & cum amicis communicatum, propriis demonstrationibus firmavere Geometræ Clarissimi D. *J. Bernoullius* in *Act. Lipsiæ*; D. *J. Keillius* in harum *Transact.* * N. 317. & D. *Jac. Hermannus* in *Phoronomiâ* sua pag. 70. quos vide.

* *v. infrà, Tract.* XX.

Scribendo autem CK^3 pro PG, per *Propr.* V; & $\frac{SP}{CK}$ juxta *Propr.* II. pro ST, (ob datas scilicet CD, CO) erit Vis centripeta tendens ad focum Ellipseos S, semper ut $\frac{SP \times CK^3}{CK^3 \times SP^3}$, hoc est ut $\frac{SP}{SP^3}$ vel $\frac{1}{SP^2}$, nempe reciproce ut quadratum ex SP. Unde patet quod si Sectio fuerit Ellipsis motu corporis descripta, erit Vis Centripeta ut quadratum distantiæ a

centro

centro Virium reciprocè. Ex his Proprietatibus conſequuntur Corollaria nonnulla notatu non indigna.

Corol. 1. *Velocitas Corporis in Ellipſi revolventis, ad punctum quodlibet* P, *eſt ad Velocitatem revolventis in circulo ad eandem diſtantiam* SP *à centro Virium, in ſubdupla ratione diſtantiæ ab altero foco* PF, *ad Semiaxem transverſum Sectionis, ſive ut media proportionalis inter* PF *&* CD *ad* CD.

Eſt enim velocitas revolventis in Ellipſi ad diſtantiam SP, ad Velocitatem revolventis in Circulo vel Ellipſi ad diſtantiam Semiaxis CD vel SO, ut CO ad ST; hoc eſt per *Propr.* II. ut $\sqrt{PF}$ ad $\sqrt{SP}$. Velocitas autem revolventis in Circulo ad diſtantiam CD eſt ad velocitatem revolventis in Circulo ad diſtantiam SP, ut $\sqrt{SP}$ ad $\sqrt{CD}$. Ex æquo igitur, Velocitas revolventis in Ellipſi ad diſtantiam SP, eſt ad Velocitatem revolventis in Circulo ad eandem diſtantiam ut $\sqrt{PF}$ ad $\sqrt{CD}$.

Coroll. 2. *Ex datis Velocitate in Ellipſi, poſitione Tangentis, & centro Virium ſeu Foco, facile eſt determinare Focum alterum.*

Sit enim Velocitas Data R; ea autem Velocitas qua deſcriberetur Circulus ad datam a centro diſtantiam SP ſit Q; ac per *Coroll.* præcedens, R eſt ad Q ut $\sqrt{PF}$ ad $\sqrt{CD}$, adeoque QQ eſt ad RR ut CD ad PF, & $2QQ - RR$ erit ad RR ut SP ad PF: Datur autem SP; data eſt igitur PF magnitudine. Datur etiam poſitione, ob angulum VPF angulo SPT æqualem. Datur igitur punctum F alter Focorum. Quo invento pronum eſt Sectionem deſcribere.

Si vero $\frac{1}{2}RR$ majus fuerit quadrato ex Q, $2QQ - RR$ fit quantitas Negativa, & loco Ellipſeos Trajectoria deſcribenda in Hyperbolam tranſit. Eritque $RR - 2QQ$ ad RR ut SP ad PF diſtantiam alterius Foci, ad alterum Tangentis latus ponendam, ut habeatur Focus F. Proprietates autem omnes quas in Ellipſi demonſtravimus, mutatis mutandis, etiam Hyperbolæ competunt. *Plate* I. *Fig.* 2.

Quod ſi acciderit QQ æquale eſſe dimidio quadrati ex R; evaneſcente quantitate $2QQ - RR = 0$, quarta proportionalis PF fit infinita: proinde Trajectoria deſcribenda Parabolica eſt, Foco ſcilicet altero in infinitum abeunte. Axis autem Trajectoriæ poſitione datur; eſt enim ipſi PF parallelus, exiſtente ſcilicet angulo FPV angulo dato SPT æquali.

Coroll. 3. *Velocitas revolventis in data Sectione Conica ad diſtantiam* SP *eſt ad Velocitatem ejuſdem ad diſtantiam aliam* SX, *ut media proportionalis inter* FP *&* SX *ad mediam proportionalem inter* SP *&* FX.

Velocitas enim in P eſt ut $\sqrt{\frac{FP}{SP}}$ (per *propr.* II.) & per eandem, Velocitas in X eſt ut $\sqrt{\frac{FX}{SX}}$. Unde manifeſta eſt propoſitio.

Coroll. 4. *Ratio etiam Velocitatum duorum Corporum in eodem Syſtemate, ſed in datis Coniſectionibus diverſis, revolventium, datis utriuſque à communi Orbium Foco diſtantiis, ope Corollarii* 1[mi]. *ſtatim obtinebitur.*

Cum enim Velocitas corporis in P ſit ad Velocitatem in Circulo ad eandem diſtantiam SP, ut $\sqrt{PF}$ ad $\sqrt{CD}$; & in alia ſuppoſita Coniſectione, cujus Semiaxis cd & Foci S, f, ad diſtantiam Sp, Velocitates illæ

ſint ut $\sqrt{pf}$ ad $\sqrt{cd}$: Velocitas autem revolventis in Circulo ad diſtantiam SP ſit ad Velocitatem in Circulo ad diſtantiam Sp ut $\sqrt{Sp}$ ad $\sqrt{SP}$; Compoſitis rationibus, erit Velocitas in P ad Velocitatem in p, ut $\sqrt{PF \times cd \times Sp}$ ad $\sqrt{pf \times CD \times SP}$. Quod ſi Sectio illa altera fuerit Parabola, erunt cd, pf infinitæ, ſed in ratione 1 ad 2; proinde ratio Velocitatum erit ut $\sqrt{PF \times Sp}$ ad $\sqrt{2CD \times SP}$.

Coroll. 5. *Quod ſi in Hyperbola punctum* P *abeat in infinitum, ex præcedentibus manifeſtum eſt, Velocitatem ultimam ac minimam, qua cum corpus in æternum aſcenderet, æqualem eſſe ei qua, ad diſtantiam* CD *Semiaxi tranſverſo æqualem, Circulum deſcriberet.*

Coroll. 6. *Ex data diſtantia à Foco, datur quoque Poſitio Tangentis, ſive angulus* SPT, *ſub diſtantia* SP *&* *Tangente* PT *contentus.*

Eſt enim (per *propr.* II.) PS ad ST ut CK ad CO ſive ut $\sqrt{SP \times PF}$ ad CO, atque ita Radius ad Sinum anguli SPT. At in Ellipſibus Circulis affinibus præſtaret angulum PST, ejuſdem complementum ad quadrantem, inquirere: Hujus autem Sinus eſt ad Radium ut $\sqrt{SP \times PF - COq}$ ad $\sqrt{SP \times PF}$.

Coroll. 7. *Atque hinc conſequuntur Velocitates quibuſcum diſtantiæ* SP *creſcunt vel decreſcunt.*

Nam cum, ex Corollario præcedente, $\sqrt{SP \times PF}$ ſit ad $\sqrt{SP \times PF - COq}$ ut Radius ad ſinum anguli PST, ac in eadem ſit ratione Velocitas Corporis in P ad Velocitatem momenti ipſius SP; Velocitas autem illa in P ſit (per *propr.* II.) ut $\sqrt{\frac{PF}{SP}}$; eliſis ſuperfluis, erit $\sqrt{\frac{SP \times PF - COq}{SP}}$ Velocitati, qua creſcit vel decreſcit diſtantia SP, ſemper proportionalis.

Theorema Generale II. *In omni Trajectoria Curvilinea Velocitates angulares circa centrum Virium ſunt reciprocè proportionales quadratis diſtantiarum à centro.*

Nam ob Sectorum minimorum Areas æquales, arcus angulis minimis ſubtenſi ſive Baſes, ſunt reciproce ut Radii: Anguli autem minimi quibus Baſes æquales ſubtenduntur ſunt etiam reciproce ut Radii. Proinde anguli Sectorum minimorum Area æqualium, ſunt inter ſe reciproce in dupla ratione Radiorum, ſive ut quadrata diſtantiarum.

Coroll. 8. *Hinc Velocitates angulares revolventium in diverſis Ellipſibus datis comparantur inter ſe.*

Velocitates enim angulares quibuſcum ad diſtantias Semiaxibus Tranſverſis æquales circuli deſcriberentur, ſunt reciproce in ratione ſeſquialtera Axium, ſive ut $\frac{1}{CD\sqrt{CD}}$. Velocitates autem angulares has medias habent Corpora revolventia, cum quadrata diſtantiarum æquantur rectangulis ſub ſemiaxibus Ellipſeôn. Ideo (per *Theor.* II.) erit SPq ad

CD×

$CD \times CO$ ut $\frac{1}{CD\sqrt{CD}}$ ad $\frac{CO}{SPq \times \sqrt{CD}}$: quæ quidem Quantitas eſt ut Velocitas anguli ad centrum S, motu rectæ SP, tempore quam minimo dato, deſcripti.

Coroll. 9. *Velocitas angularis qua circumgyratur Tangens* PT, *ſive recta in Tangentem perpendicularis* ST, *eſt ad Velocitatem angularem rectæ* SP, *ut Semiaxis tranſverſus* CD *ad diſtantiam ab altero foco* PF.

Demonſtratio. Sint puncta P, p, quamproxima inter ſe; ductiſque SP, Sp, ſint PT, pt, duæ Tangentes, ad quas demittantur normales ST, St; iiſque parallelæ ducantur radii Curvaturæ PG, pG coeuntes in G: ac deſcribatur, centro S & radio SP, arcus minimus PE occurrens ipſi Sp in E. Manifeſtum eſt angulum PGp æqualem eſſe angulo TSt, ſive angulari Velocitati normalis ST. Eſt autem angulus PSp angularis velocitas rectæ Sp; quare angulus PGp eſt ad angulum PSp ut angularis velocitas ipſius ST ad angularem velocitatem rectæ SP; hoc eſt, ut $\frac{Pp}{PG}$ ad $\frac{PE}{PS}$. Sed $Pp : PE :: SP : ST :: CK : CO$ (per *propr.* II.) Hæ igitur Velocitates ſunt ut $\frac{CK}{PG}$ ad $\frac{CO}{PS}$. Pro PG ſcribe $\frac{CK^3}{CD \times CO}$ (per *propr.* V.) ac $\frac{CK}{PG}$ fiet $\frac{CD \times CO}{CKq} = \frac{CD \times CO}{PS \times PF}$. Hinc $\frac{CD \times CO}{PS \times PF}$ erit ad $\frac{CO}{PS}$ ſive, deletis ſuperfluis, CD ad PF, ut angulus TSt ad angulum PSp, ſive Velocitas angularis Tangentis ad angularem Velocitatem diſtantiæ SP: proinde Velocitas, qua circumgyratur Tangens, ſemper proportionalis eſt quantitati $\frac{CO \times \sqrt{CD}}{PF \times SPq}$.

Plate I. Fig. 3.

II. Sit A Area Curvæ cujus Abſciſſa x, & ordinatim Applicata $x^n\sqrt{dx-xx}$. Sit B Area Curvæ cujus Abſciſſa eadem cum priori, ſed ordinatim Applicata $x^{n-n}\sqrt{dx-xx}$; ponatur $\sqrt{dx-xx}=y$. Erit Area $A=$

A way to ſquare certain kinds of Curves, or to reduce them to more ſimple Ones. by the ſame. n. 278. p. 1113.

$$d\,B \;in\; \frac{2m+1}{2m+4} \;in\; \frac{2m-1}{2m+2} \;in\; \frac{2m-3}{2m} \;in\; \frac{2m-5}{2m-2} \;\&c. = P.$$

$$-\frac{1}{m+2}x^{n-1}y^3 = -Q.$$

$$-\frac{d}{m+1} \;in\; \frac{2m+1}{2m+4}x^{n-2}y^3 = -R.$$

$$-\frac{dd}{m} \;in\; \frac{2m+1}{2m+4} \;in\; \frac{2m-1}{2m+2}x^{n-3}y^3 = -S.$$

$$-\frac{d^3}{m-1} \;in\; \frac{2m+1}{2m+4} \;in\; \frac{2m-1}{2m+2} \;in\; \frac{2m-3}{2m}x^{n-4}y^3 = -T.$$

&c.

Ubi

Ubi notandum 1° quod n ſupponitur numerus integer & affirmativus; 2° Quod Quantitas $d^n B$ in ſerie per P deſignata, multiplicari debet in tot terminos quot ſunt unitates in n; 3° quod tot ſequentes ſeries per $-Q-R,-S,-T$ &c. deſignatæ ſumi debeant, quot ſunt unitates in n; quod ut Exemplo uno vel altero clarius fiat, dico quod ſi $n=1$, tunc A

$$= d^n B \text{ in } \frac{2m+1}{2m+4} - \frac{1}{m+2} x^{m-1} y^3 \text{ \& ſi } n=2,$$

$$A = d^n B \text{ in } \frac{2m+1}{2m+4} \text{ in } \frac{2m-2}{2m+2} - \frac{1}{m+2} x^{m-1} y^3$$

$$- \frac{d}{m+1} \text{ in } \frac{2m+1}{2m+4} \text{ in } x^{m-2} y^3$$

4° quod ſi y ponatur $=\sqrt{d\overline{x-xx}}$ tunc A erit $= Q-R+S-T$ &c. $\pm P$.

Corollarium. Si m ponatur æqualis termino cuivis ſequentis Seriei

$$-\frac{1}{2},\ \frac{1}{2},\ \frac{3}{2},\ \frac{5}{2},\ \frac{7}{2},\ \frac{9}{2}\ \text{\&c.}$$

quadratura Curvæ cujus ordinatim Applicata $x^m\sqrt{d\overline{x-xx}}$, aut $x^m\sqrt{d\overline{x+xx}}$ finita evadit & exhibetur per ſeriem noſtram; quod ut Exemplo illuſtretur, Inquirenda ſit Area Curvæ cujus ordinatim Applicata $x^{-\frac{1}{2}}\sqrt{d\overline{x-xx}}$; fingatur Curvam hanc comparari cum Curva cujus ordinatim Applicata $x^{-\frac{1}{2}}\sqrt{d\overline{x-xx}}$, quoniam hoc in caſu $n=1$, ideo

$$A = d^n B \text{ in } \frac{2m+1}{2m+4} - \frac{1}{m+2} x^{m-1} y^3$$

ſed $m = -\frac{1}{2}$, ergo $2m+1=0$, ideoq;

$$A = -\frac{1}{m+2} x^{m-1} y^3 = -\frac{2y^3}{3\sqrt{x^3}}.$$

Hic Obſervatu dignum eſt quod Area ſic reperta interdum data quantitate deficit a vera Area, aut eandem data quantitate excedit; quo autem exceſſus iſte aut defectus innoteſcat, ſupponatur Area reperta augeri minuive data quantitate q, tuncque poſita $x=0$, ſupponatur Area aucta minutave æqualis nihilo, ſic in præſenti caſu q reperietur $=\frac{2}{3}d\sqrt{d}$, adeoq; $A = \frac{2}{3}d\sqrt{d}\,\frac{2y^3}{3\sqrt{x^3}}$.

Corollarium 2. Si n ponatur æqualis termino cuivis ſequentis ſeriei 3, 4, 5, 6, 7, &c. Quadratura Curvæ cujus ordinatim applicata $x^{-n}\sqrt{d\overline{x-xx}}$ aut $x^{-n}\sqrt{d\overline{x+xx}}$, finita evadit, & exhibetur per ſeriem noſtram;

ſtram; Inquirenda ſit Area Curvæ cujus ordinatim applicata $x^{-3}\sqrt{dx-xx}$, finge eam comparari cum Area Circuli, quæ vocetur A; erit $m=0$, $n=3$, adeoq; $A=P-Q-R-S$. Sed cum quantitas $2m$ infinite parva ſeu potius nulla, in Denominatore termini tertii per quem $d^n B$ multiplicatur, extet; Quantitas deſignata per P infinita eſt; atque ob eandem cauſam, Quantitas deſignata per $-S$ infinita evadit, adeoque Quantitates A, $-Q$, $-R$ evaneſcunt: Igitur $P=S$, diviſaque æquatione per $\frac{2m+1}{2m+4}$ in $\frac{2m-1}{2m+2}$ fit

$$d^n B \text{ in } \frac{2m-3}{2m} = \frac{dd}{m} x^{m-3} y^3 \text{ ſeu } dB \text{ in } \frac{2m-3}{2}$$

$= dd\,x^{m-3}y^3$: ſcriptiſque 0 & 3 pro m & n, prodibit

$$dB \text{ in } -\frac{3}{2} = \frac{y^3}{x^3},\ \text{ſeu } B = -\frac{2y^3}{3x^3}.$$

Corollarium 3. Si m ponatur æqualis termino cuivis ſequentis ſeriei, -2, -1, 0, 1, 2, 3, 4, 5, &c. quadratura Curvæ cujus ordinata $x^m\sqrt{dx-xx}$, pendet a quadratura Circuli: Area vero Curvæ cujus ordinata $x^m\sqrt{dx+xx}$, pendet a quadratura Hyperbolæ, & relatio iſtius Curvæ cum Circulo aut Hyperbola exhibetur per Seriem noſtram in terminis finitis.

Corollarium 4. Si m exponatur per alium quemvis terminum differentem ab iis quas ſupra memoravimus, Curva cujus ordinata $x^m\sqrt{dx-xx}$ aut $x^m\sqrt{dx+xx}$, neque quadratur exacte, nec ab Hyperbola aut Circulo pendet, ſed ad Curvam ſimpliciorem reducitur per ſeriem noſtram.

Theorema II.

Sit A Area Curvæ cujus Abſciſſa x & ordinatim applicata $\frac{x^m}{\sqrt{dx-xx}}$. Sit B area Curvæ cujus Abſciſſa eadem cum priori ſed ordinatim applicata $\frac{x^{m-n}}{\sqrt{dx-xx}}$ ponatur $\sqrt{dx-xx}=y$. Erit $A=$

$$d^n B \text{ in } \frac{2m-1}{2m} \text{ in } \frac{2m-3}{2m-2} \text{ in } \frac{2m-5}{2m-4} \text{ in } \frac{2m-7}{2m-6} \ \&c. = P.$$

$$-\frac{1}{m}x^{m-1}y = -Q.$$

$$-\frac{d}{m-1} \text{ in } \frac{2m-1}{2m} x^{m-2} y = -R.$$

$$-\frac{dd}{m-2} \text{ in } \frac{2m-1}{2m} \text{ in } \frac{2m-3}{2m-2} x^{m-3} y = -S.$$

$$-\frac{d^3}{m-3} \text{ in } \frac{2m-1}{2m} \text{ in } \frac{2m-3}{2m-2} \text{ in } \frac{2m-5}{2m-4} x^{m-4} y = -T, \ \&c.$$

Obſer-

Obſervationes ad primum Theorema, hic & in ſequentibus locum habent.

Corollarium 1. Si m ponatur æqualis Termino cuivis ſequentis ſeriei, $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, $\frac{9}{2}$, &c. quadratura Curvæ cujus ordinatim applicata $\frac{x^m}{\sqrt{dx-xx}}$ aut $\frac{x^m}{\sqrt{dx+xx}}$ finita evadit, & exhibetur per hanc ſeriem.

Corollarium 2. Si n ponatur æqualis Termino cuivis ſequentis ſeriei 1, 2, 3, 4, 5, 6, 7, &c. Curva omnis cujus ordinatim applicata $\frac{x^{-n}}{\sqrt{dx-xx}}$ aut $\frac{x^{-n}}{\sqrt{dx+xx}}$ quadratur per hanc ſeriem in terminis finitis.

Corollarium 3. Si m exponatur per terminum quemlibet ſequentis ſeriei, 0, 1, 2, 3, 4, 5, 6, 7, &c. Curva cujus ordinatim applicata $\frac{x^m}{\sqrt{dx-xx}}$ pendet a Quadratura Circuli. Curva vero cujus ordinatim applicata $\frac{x^m}{\sqrt{dx+xx}}$ a quadratura Hyperbolæ. Etenim ſi Centro C, Diametro $AB=d$ deſcribatur Circulus AEB, ac ſumatur $AD=x$; erecto DE normaliter, junge CE. Sector AEC per $\frac{1}{8}dd$ diviſus æqualis eſt Areæ Curvæ cujus Ordinata $\frac{x^0}{\sqrt{dx-xx}}$. Eodem modo, ſi Centro C, Tranſverſo axi $AB=d$, deſcribatur æquilatera Hyperbola AE, ſumatur $AD=x$, erigatur DE ad angulos rectos, jungatur CE; ſector ACE per $\frac{1}{8}dd$ diviſus æqualis eſt Areæ Curvæ cujus ordinata $\frac{x^0}{\sqrt{dx+xx}}$.

Plate 1. Fig. A. Fig. B.

Corollarium 4. Si m ponatur æqualis Termino cuivis, qui non in limitationes præcedentes cadat, Curva cujus ordinata $\frac{x^m}{\sqrt{dx \pm xx}}$ neque quadratur exacte, nec a Circulo aut Hyperbola pendet, ſed ad Curvam ſimpliciorem reducitur.

Theorema 3.

Sit A Area Curvæ cujus Abſciſſa x, ordinatim applicata $x^m\sqrt{rr-xx}$, ſit B area Curvæ cujus Abſciſſa itidem x, ordinatim applicata $x^{m-2n}\sqrt{rr-xx}$, ponatur $\sqrt{rr-xx}=y$.

Erit

Erit $A =$

$$r^{2n} B \text{ in } \frac{m-1}{m+2} \text{ in } \frac{m-3}{m} \text{ in } \frac{m-5}{m-2} \text{ in } \frac{m-7}{m-4} \text{ \&c.} = P.$$

$$- \frac{1}{m+2} x^{m-1} y^3 = -Q.$$

$$- \frac{rr}{m} \text{ in } \frac{m-1}{m+2} x^{m-3} y^3 = -R.$$

$$- \frac{r^4}{m-2} \text{ in } \frac{m-2}{m+2} \text{ in } \frac{m-3}{m} x^{m-5} y^3 = -S, \text{ \&c.}$$

Corollarium 1. Si m exponatur per terminum quemvis ſequentis ſeriei 1, 3, 5, 7, 9, &c. Quadratura Curvæ cujus ordinata $x^m \sqrt{rr-xx}$ aut $x^m \sqrt{rr+xx}$ finita evadit, & exhibetur per hoc Theorema.

Corollarium 2. Si n exponatur per terminum quemvis ſequentis ſeriei 2, 3, 4, 5, 6, &c. Curva cujus ordinata $x^{-2n} \sqrt{rr-xx}$, aut $x^{-2n} \sqrt{rr+xx}$ quadratur exacte per hoc Theorema.

Corollarium 3. Si m exponatur per Terminum quemvis ſequentis ſeriei -2, 0, 2, 4, 6, 8, &c. Quadratura Curvæ cujus ordinata $x^m \sqrt{rr-xx}$, pendet a Circulo. Quadratura vero Curvæ cujus ordinata $x^m \sqrt{rr+xx}$, pendet ab Hyperbola.

Corollarium 4. Si m exponatur per Terminum quemvis differentem ab illis quos ſupra memoravimus, Curva cujus ordinata $x^m \sqrt{rr-xx}$, aut $x^m \sqrt{rr+xx}$, neque exacte quadratur, nec a Circulo aut Hyperbola pendet, ſed ad ſimpliciorem Curvam reducitur.

Theorema IV.

Sit A Area Curvæ cujus abſciſſa x, ordinatim applicata $\frac{x^m}{\sqrt{rr-xx}}$, Sit B Area Curvæ cujus Abſciſſa itidem x, Ordinatim applicata $\frac{x^{m-2n}}{\sqrt{rr-xx}}$.

Erit $A =$

$$r^{2n} B \text{ in } \frac{m-1}{m} \text{ in } \frac{m-3}{m-2} \text{ in } \frac{m-5}{m-4} \text{ in } \frac{m-7}{m-6} \text{ \&c.} = P.$$

$$- \frac{1}{m} x^{m-1} y = -Q.$$

$$- \frac{rr}{m-2} \text{ in } \frac{m-1}{m} x^{m-3} y = -R.$$

$$- \frac{r^4}{m-4} \text{ in } \frac{m-1}{m} \text{ in } \frac{m-3}{m-2} x^{m-5} y = -S.$$

$$- \frac{r^6}{m-6} \text{ in } \frac{m-1}{m} \text{ in } \frac{m-3}{m-2} \text{ in } \frac{m-5}{m-4} x^{m-7} y = -T, \text{ \&c.}$$

Corollarium 1. Si m exponatur per terminum quemvis ſequentis ſeriei 1, 3, 5, 7, 9, &c. Quadratura Curvæ cujus ordinata $\frac{x^m}{\sqrt{rr-xx}}$ aut $\frac{x^m}{\sqrt{rr+xx}}$, per hoc Theorema habetur in finitis Terminis.

Corollarium 2. Si n exponatur per terminum quemlibet ſequentis ſeriei 1, 2, 3, 4, 5, 6, &c. Curva cujus ordinatim applicata $\frac{x^{-2n}}{\sqrt{rr-xx}}$ aut $\frac{x^{-2n}}{\sqrt{rr+xx}}$ exacte quadratur per hoc Theorema.

Corollarium 3. Si m exponatur per terminum quemvis ſequentis ſeriei 0, 2, 4, 6, 8, 10, &c. Quadratura Curvæ, cujus ordinatim applicata $\frac{x}{\sqrt{rr-xx}}$ pendet a quadratura Circuli. Etenim ſi Centro C radio

Fig. A. $CA = r$ deſcribatur Circulus AEG, ſumatur $CD = x$, erigatur DE normalis ad CD, Jungatur CE: Sector CAE per $\frac{1}{2}rr$ diviſus æqualis eſt Areæ Curvæ cujus ordinatim applicata $\frac{x^0}{\sqrt{rr-xx}}$. Eodem modo ſi

Fig. B. Centro C, Tranſverſo ſemiaxi $CA = r$, deſcribatur æquilatera Hyperbola EAM, ducatur CF ad AC perpendicularis $= x$, ducatur FE axi parallela donec occurrat Hyperbolæ in E, jungatur CE: ſector Hyperbolicus ACE per $\frac{1}{2}rr$ diviſus æqualis eſt Areæ Curvæ cujus ordinatim applicata $\frac{x^0}{\sqrt{rr+xx}}$.

Corollarium 4. Si m exponatur per terminum quemlibet a præcedentibus differentem, Curva cujus ordinata $\frac{x^m}{\sqrt{rr-xx}}$ aut $\frac{x^m}{\sqrt{rr+xx}}$ neque quadratur exacte, nec a Circulo aut Hyperbola pendet, ſed ad Curvam ſimpliciorem reducitur.

Theorema V.

Sit A Area Curvæ cujus abſciſſa x, ordinatim applicata $\frac{x^m}{d-x}$; ſit B Area Curvæ cujus abſciſſa itidem x, ejuſq; ordinatim applicata $\frac{x^{m-n}}{d-x}$

Erit Area

$$A = d^n B - \frac{x^m}{m} - \frac{dx^{m-1}}{m-1} - \frac{ddx^{m-2}}{m-2} \text{ \&c.}$$

Sit ordinatim applicata $\frac{x^m}{d+x}$, tunc Area erit $=$

$$A = \frac{x^m}{m} - \frac{dx^{m-1}}{m-1} + \frac{ddx^{m-2}}{m-2} \text{ \&c.} \pm d^n B.$$

Corol.

Corollarium. Si m exponatur per terminum quemlibet sequentis seriei, 0, 1, 2, 3, 4, 5, 6, &c. Quadratura Curvæ cujus ordinatim applicata $\frac{x^m}{d-x}$, aut $\frac{x^m}{d+x}$ pendet a quadratura Hyperbolæ; Etenim ductis DE, EF ad angulos rectos, sumatur $EG = d$, ducatur GH normalis ad EF & ipsi EG æqualis. Intra Asymptotos DE, EF describatur Hyperbola per H transiens, quo facto sumatur $GK = x$ versus E pro primo casu, at versus F pro secundo; ducatur ordinatim applicata KL: Area $HGKL$ per dd divisa æqualis est Areæ Curvæ cujus ordinatim applicata $\frac{x^0}{d-x}$ aut $\frac{x^0}{d+x}$. Hinc Solidum generatum a portione Cissoidis, dum circa Diametrum circuli genitoris revolvit, in finitis terminis exhibetur, data Hyperbolæ Quadratura. *Vide Fig.* C.

Theorema VI.

Sit A Area Curvæ cujus abscissa x, ordinatim applicata $\frac{x^m}{rr+xx}$; Sit B Area Curvæ cujus abscissa itidem x, ordinatim applicata $\frac{x^{m-2n}}{rr+xx}$, Erit Area $A = \frac{x^{m-1}}{m-1} - \frac{rrx^{m-3}}{m-3} + \frac{r^4x^{m-5}}{m-5}$ &c. $\mp r^{2n}B$.

Corollarium. Si m exponatur per terminum quemlibet sequentis seriei 0, 2, 4, 6, 8, &c. Quadratura Curvæ cujus ordinatim applicata $\frac{x^m}{rr+xx}$ pendet a rectificatione circularis Arcus. Etenim si centro C radio $CA = r$ describatur Circulus AEG, ducatur Tangens $AK = x$ jungatur CK peripheriæ occurrens in E; arcus AE per rr divisus æqualis est Areæ curvæ cujus ordinata $\frac{x^0}{rr+xx}$. *Fig.* B.

Corollarium generale ad hæc sex Theoremata.

Curva omnis mechanica cujus quadratura pendet ab aliqua e Curvis numero infinitis, cujus ordinatæ formas sequentes adipisci possunt $x^m\sqrt{dx \pm xx}$, $\frac{x^m}{\sqrt{dx \pm xx}}$, $x^m\sqrt{rr \pm xx}$, $\frac{x^m}{\sqrt{rr \pm xx}}$, $\frac{x^m}{d \pm x}$, $\frac{x^m}{rr+xx}$, per series has quadrari potest. Hoc Exemplo unico indicare satis erit.

Posito quod Cubus Arcus Circularis Sinui verso correspondentis fiat Ordinata Curvæ, cujus Abscissa sit idem Sinus versus. Inquirenda est Area istius Curvæ.

Sit Abscissa x, arcus circularis v, fluxio Areæ fit $v^3\dot{x}$.

Sit Area $v^3 x - q$. Igitur $v^3 \dot{x} + 3 v^2 \dot{v} x - \dot{q} = v^3 \dot{x}$, unde $\dot{q} = 3 v^2 \dot{v} x$; fed $\dot{v} = \frac{d\dot{x}}{2\sqrt{dx - xx}}$, igitur $\dot{q} = \frac{3 d v^2 x \dot{x}}{2\sqrt{dx-xx}}$ fed per *Theorema* II. $\frac{x\dot{x}}{\sqrt{dx-xx}} = \frac{d\dot{x}}{2\sqrt{dx-xx}} - \dot{y} = \dot{v} - \dot{y}$, adeoq; $\dot{q} = \frac{3}{2} d v^2 \dot{v} - \frac{3}{2} d v^2 \dot{y}$, igitur $q = \frac{1}{2} d v^3 - \frac{3}{2} d v^2 \dot{y}$.

Ergo ad hoc perventum eft ut fluentem quantitatem inveniamus cujus fluxio eft $\frac{3}{2} d v^2 \dot{y}$.

Sit hæc quantitas $\frac{3}{2} d v^2 y - r$.

Igitur $\frac{3}{2} d v^2 \dot{y} + 3 d v \dot{v} y - \dot{r} = \frac{3}{2} d v^2 \dot{y}$.

Adeoque $\dot{r} = 3 d v \dot{v} y = \frac{3}{2} d d v \dot{x}$. Sit $r = \frac{3}{2} d d v x - s$.

Igitur $\frac{3}{2} d d v \dot{x} = \frac{3}{2} d d v \dot{x} + \frac{3}{2} d d x \dot{v} - \dot{s}$.

Adeoque $\dot{s} = \frac{3}{2} d d x \dot{v} = \frac{3 d^3 x \dot{x}}{4\sqrt{dx-xx}} = \frac{3}{4} d^3 \dot{v} - \frac{3}{4} d^3 \dot{y}$, per 2^um *Theorema*.

Igitur $s = \frac{3}{4} d^3 v - \frac{3}{4} d^3 y$. adeoque area quæfita $= v^3 x - \frac{1}{2} d v^3 + \frac{3}{2} d v^2 y - \frac{3}{2} d d v x + \frac{3}{4} d^3 v. - \frac{3}{4} d^3 y$.

Quoniam autem Solida ex rotatione Curvarum genita, Superficies ab eadem rotatione genitæ, Longitudines Curvarum, & Centra Gravitatis horum omnium a Quadratura Curvarum pendent, hæc fi a Curvis fupradictis pendent facillime computantur.

Of deducing the Tangents of Curves, from the Theory of Maxima and Minima, by Mr. H. Ditton. n. 284. p. 1333.

III. Sit Curva AG, cujus vertex A, axis AK, ordinatim applicata FD centrumque (fiquod habet) punctum K. Sumpto puncto L in Axe fit $AL = n$, $AD = x$, $FD = y$, $FL = z$; quarum quantitatum, tres pofteriores funt fluentes, prior vero n permanens ac ftabilis, hæc enim una eademque prioribus variis femper refpondet. Ex Triangulo Rectangulo FDL, hanc habemus Æquationem, $zz = yy + nn - 2nx + xx$; determinandoque z ad extremum, oritur $2y\dot{y} - 2n\dot{x} + 2x\dot{x} = 0$; unde interpretando $2y\dot{y}$ fecundum propriam Curvæ naturam, relinquetur quantitas n expofita in terminis etiam Curvæ propriis.

Plate I. *Fig.* 4.

Cum vero z hoc modo ad valorem extremum determinatam habeamus; hoc eft, linea FL omnium quæ a puncto L ad Curvam duci poffunt vel maxima vel minima fit, indeque ad Curvam in puncto F normalis; ipfam DL effe fubnormalem patet, ex qua fubtangens nullo negotio eruitur.

In exemplum producatur primo Parabola Apolloniana, quam curvam hic delineatam effe fupponemus. Habemus ergo $2y\dot{y} = r\dot{x}$ (pofito Parametro $= r$) unde $r\dot{x} - 2n\dot{x} + 2x\dot{x} = 0$, & $n = \frac{r}{2} + x$, ergoque DL fubnormalis $= \frac{1}{2} r$. (Cujus Theorematis fenfus hic eft, *viz.* Si ultra terminum D abfciffæ AD, defignetur DL femiparametro æqualis, atque a puncto L producatur LF recta ad punctum F; recta fic ducta Parabolæ in puncto F normalis erit, & omnium quæ a puncto L ad Curvam duci poffunt minima. Dico minimam; alicui enim curvæ naturam ac indolem fcienti, apparet Maximam effe non poffe (id quod in fequentibus notatum

tatum velim) sed necessario est vel maxima vel minima, ideoque posterior.) Hæcque pars prior est *Theor.* 5. *Lib.* 7. Conicor. Præclarissimi *de La Hire.*

Ducatur ordinata EB, junganturque puncta EL; fit intercepta $BD = f$, unde $AB = x - f$, & $BL = \frac{r}{2} + f$. Jam $LE^q = \frac{rr}{4} + rx + ff$, & $FL^q = \frac{rr}{4} + rx + ff$, & $FL^q = \frac{rr}{4} + rx$, ergo $LE^q - FL^q = BD^q$; quæ pars posterior est Theorem. 5. ejusdem Lib. Conicor.

Quo propius punctum F, in quo curvam normalis secat, puncto A sive vertici admovetur; eo propius etiam punctum L eidem venit. Ergo quando F cum A coincidit, & sic evanescit ordinata FD, tunc ipsa Minima jacet in Axe AK, & semiparametri quantitatem adæquabit. Hoc est in illo casu $n = \frac{1}{2} r$ tantum; in nihilum abeunte x abscissa ad ordinatam evanescentem pertinente. Si ergo $AL = n = \frac{1}{2} r$, sumpto puncto D inter A & L, fiat $AD = x$; tum oritur $FL^q = \frac{rr}{4} + xx$, ergo $FL^q - AL^q = xx$, hoc est $FL^q - AL^q = AD^q$ semper. Ejusdemque tenoris est Theor. 2. Lib. 7. Conicor. *de La Hire.*

Secundo sit curva quædam ordinis Parabolici superioris, cujus æquatio $r^{p-q} x^q = y^p$.

Tum $yy = r^{\frac{2p-2q}{p}} x^{\frac{2q}{p}}$, adeoque

$2y\dot{y} = \frac{2q}{p} r^{\frac{2p-2q}{p}} x^{\frac{2q-p}{p}} \dot{x}$; substituendoque hunc valorem loco $2y\dot{y}$ in æquatione generali determinante z ad extremum, habemus inde

$n = \frac{q}{p} r^{\frac{2p-2q}{p}} x^{\frac{2q-p}{p}} + x$; & propterea subnormalis

$DL = \frac{q}{p} r^{\frac{2p-2q}{p}} x^{\frac{2q-p}{p}}$.

Hoc vero singulis hisce curvis facillime applicatur, si indices p & q secundum uniuscujusque naturam ac genium debito modo exponantur.

Supponetur tertio Curvam esse Ellipsin cujus $\frac{1}{2}$ Axis Major AK; ex cujus etiam æquatione consequitur $2y\dot{y} = r\dot{x} - \frac{2rx\dot{x}}{q}$. Unde provenit $r\dot{x} - \frac{2rx\dot{x}}{q} - 2n\dot{x} + 2x\dot{x} = 0$, & $n = \frac{r}{2} + x - \frac{rx}{q}$, ac propterea $\frac{r}{2} - \frac{rx}{q}$ subnormali DL æqualis. Si vero ellipseos loco substitueretur Circulus, æquationem eodem modo tractando, inveniemus $DL = r - x$, posito r Circuli Radio æquali.

Sed

Sed ad Ellipſin revertendum, cujus alia proprietas ex hoc fonte deducenda eſt, prout in Parabola factum. Sit $BD = f$, unde $AB = x - f$. Habemus $LE^q (= LB^q + EB^q) = \frac{rr}{4} - \frac{rrx}{q} + \frac{rrxx}{qq} + ff + rx - \frac{rxx}{q} - \frac{rff}{q}$: & $FL^q (= FD^q + LD^q) = rx - \frac{rxx}{q} + \frac{rr}{4} - \frac{rrx}{q} + \frac{rr.xx}{qq}$: Ergo, $LE^q - LF^q = ff - \frac{rff}{q}$; Hoc vero eſt Theor. 6. Lib. 7. Conic. *de La Hire.*

Poſtulat enim Geometra ille ſublimis, ut ſit $q : r :: \frac{q}{2} - x : LD$, cujus valor eſt $\frac{r}{2} - \frac{rx}{q}$ prout ſupra inventum; ideoque quarta proportionalis eſt tribus ante poſitis: Hoc vero ei conceſſo, LF eſſe minimam omnium rectarum quæ a puncto L ad Ellipſin duci poſſunt evidenter demonſtrat. Præterea quoniam eſt $q : q - r :: f : f - \frac{fr}{q}$, Ergo $\square\, ff - \frac{rff}{q}$ ſive $f \times f - \frac{fr}{q}$; idem eſt quod rectangulum apud *D. De La Hire* exemplar vocatum: Hoc vero exemplar ſecundum ejus definitionem, eſt Rectangulum ſimile Rectangulo, differentiam inter Quadratum Axis Tranſverſi & Figuram conſtituenti (hoc eſt Rectangulo $qq - qr$) & præterea ad Rectam BD ſive f applicatum. Et quod Rectangulum $ff - \frac{rff}{q}$ omnes haſce conditiones poſſideat, luce Meridiana clarius eſt.

Notetur, ex valore quantitatis n ſupra invento, plane conſequi $n \sqsupset \frac{r}{2}$. Nam $n = \frac{r}{2} + x - \frac{rx}{q}$, ergo $qn + rx = \frac{qr}{2} + qx$, ſed (propter $q \sqsupset r$) $qx \sqsupset rx$, ergo, $qn \sqsupset \frac{qr}{2}$ & $n \sqsupset \frac{r}{2}$.

Quando (ut in Parabola modo obſervatum) punctum FA in verticem incidit, ipſa Minima in Axe deſignatur; & propter evaneſcentem x, habemus $n = \frac{r}{2}$: Aſſumptoque quovis puncto D inter A & L, ſi $AD =$ alicui x, comparando emergit $FL^q - AL^q = xx - \frac{rxx}{q}$; quod ipſum eſt Theor. 3. Lib. 7. Conic. D. La Hire. Quoniam enim eſt $q : q - r :: x : x - \frac{rx}{q}$, patet $\square\, xx - \frac{rxx}{q}$ eſſe exemplar, ſed applicatum ad abſciſſam x; & præterea hoc eſſe menſuram adequatam defectus quadrati Minimæ a quadrato cujuſvis rectæ alterius, ab eodem puncto ad curvam protenſæ; hæcque demonſtrat ille loco citato.

Theoremata vero ad Axem minorem ſive conjugatum ellipſeos ſpectantia (hactenus enim majore ſive Tranſverſo uſi fuimus) eodem plane modo determinantur. Sit $AK \frac{1}{2}$ Axis Minoris $= \frac{c}{2}$, Parameter $= R$; punctum

punctum L jam ultra centrum, ad alteras partes GK collocari supponitur. Operando ut prius, invenietur AL sive $n = \frac{R}{2} + x - \frac{Rx}{c}$, & subnormalis $DL = \frac{R}{2} - \frac{Rx}{c}$; Hoc est $c : R :: \frac{c}{2} - x : \frac{R}{2} - \frac{Rx}{c}$, adeoque ducta FL omnium quæ a puncto L ad ellipsin duci possunt Maxima, & $LF^q - LE^q = \frac{Rff}{c} - ff =$ Rectangulo exemplar ad BD (sive f) applicato. Quod vero hoc sit exemplar, patet, est enim $c : R - c :: f : \frac{Rf}{c} - f$, adeoque ex definitione, $\frac{Rf}{c} - f \times f =$ Exemplari. Hoc vero Theor. est 7. Lib. 7. Conicor. De La Hire.

Iterum; Puncto F cum A coincidente; propter evanescentem x evanescentis tunc temporis ordinatæ, relinquitur $n = \frac{R}{2}$, & AL omnium quæ a puncto L ad Ellipsin duci possunt Maxima, & $AL^q - FL^q = \frac{Rxx}{c} - xx =$ Exemplari ad AD sive x applicato; eodemque modo sonat Theor. 4. Lib. prædicti Conicorum.

Observandum vero ad casum præcedentem (quod prius ergo notari debuit) ubi invenimus $n = \frac{R}{2} + x - \frac{Rx}{c}$, quod $n \sqsubset \frac{R}{2}$; nam $cn + Rx = \frac{R}{2}c + cx$, & propter $R \sqsupset c$, adeoque $Rx \sqsupset cx$, relinquetur $cn \sqsubset \frac{Rc}{2}$, & $n \sqsubset \frac{R}{2}$.

Jam vero ut res in Ellipsi peracta est, sic eodem prorsus modo in Hyperbola peragenda foret, Minimæque in hac curva lineæ determinandæ: sed talis inter hasce curvas connectio, tam facilisque ab una ad alteram transitus, ut vel Tyronibus ipsis labor inanis videatur. Nil aliud restat, v. gr. ad subnormalem determinandam, quam ut signum $-$ in $+$ mutetur. Nam cum in Hyperbola sit $2y\dot{y} = r\dot{x} + \frac{2rx\dot{x}}{q}$, & $n = \frac{r}{2} + x + \frac{rx}{q}$ (ex æquatione generali) manet $DL = \frac{r}{2} + \frac{rx}{q}$.

Concipietur Quarto Curvam MSN (in altera Fig. parte delin.) Esse unam ex Hyperboloidibus, cujus Asymptoti AK, KH, rectamque SR ad Asymptoton KH ordinatam, SR sit $= y$, $SP = z$, $KR = x$, $KP = n$, quæ hic necessario minor erit quam x, ut consideranti patet. Æquatio curvæ propria est $y^p x^q = r^q s^p$ cujus loco (propter r & s quantitates determinatas) scribi possit $y^p = x^{-q}$, adeoque $y^2 = x^{-\frac{2q}{p}}$, & $2y\dot{y} = -\frac{2q}{p} x^{\frac{-2q-p}{p}} \dot{x}$; hinc cum $zz = yy + xx - 2nx + nn$, pro extremo habemus

mus $2y\dot{y}+2x\dot{x}-2n\dot{x}=0$, hoc eſt $-\frac{2q}{p}\dot{x}x^{\frac{-2q-p}{p}}+2x\dot{x}=2n\dot{x}$, & $n=x-\frac{q}{p}x^{\frac{-2q-p}{p}}$ adeoque ſubnormalis PR $(=x-n)$ $=$ $\frac{q}{p}x^{\frac{-2q-p}{p}}$.

Curvam jam AFG (ultimo loco) Cycloiden primariam concipiamus; ſitque r Radius, c Arcus & y ordinata Circuli genitoris, cujus Diameter per AK repreſentatur, centrumque inter L & K poſitum. Tum vocata FD cycloidis ordinata a, cæteriſque ut prius; curvæ æquatio eſt $aa=yy+2cy+cc$, adeoque zz $(=aa+nn-2nx+xx)=yy+2cy+cc+nn-2nx+xx$, & ($z$ ad extremum determinata) $2y\dot{y}+2c\dot{y}+2y\dot{c}+2c\dot{c}-2n\dot{x}+2x\dot{x}=0$. Eſt vero $\dot{y}=\frac{r\dot{x}-x\dot{x}}{y}$ & $\dot{c}=\frac{r\dot{x}}{y}$, ergo hos valores ſubſtituendo, ac æquationem debite reducendo; habemus $2r-2x+\frac{2rc-2xc}{y}+2r+\frac{2cr}{y}=2n-2x$; ac propterea $2r-x+\frac{2rc-xc}{y}=n-x=DL$ ſubnormali.

Coroll. 1. Exempla hactenus oblata percurrenti, in ſingulis patebit, quod $2y\dot{y}-2n\dot{x}+2x\dot{x}=0$, poſito nempe loco n in hac æquatione, valore ejus ſecundum curvæ naturam. In Hyperboloidibus ergo ex. gr. $\frac{2q}{p}\dot{x}x^{\frac{-2q-p}{p}}-2x\dot{x}+\frac{2q}{p}x\dot{x}^{\frac{-2q-p}{p}}+2x\dot{x}=0$, quod (ipſo oculo judice) manifeſtum eſt; & ſic in aliis (ſine ulla demonſtratione) veritas facile perſpicietur.

Coroll. 2. Ex ſubnormalium inventione, curvarum ordinatas Maximas & Minimas facile determinabimus. Hacque in re dico, ſi ſubnormalis (pro aliquo curvæ puncto) nihilo ponatur æqualis, habemus ordinatam iſtius curvæ ad extremum determinatam; & quidem maximam ſi ad partes curvæ concavas, minimam vero ſi ad convexas applicari intelligatur. Ex. gr. in Circulo (poſita ſubnormali $=l$) eſt $l=r-x$; ſit $r-x=0$; ergo $r=x$, ac inde $y=r$, hoc eſt applicata maxima Radio æqualis. Similiter in Ellipſi, $l=\frac{r}{2}-\frac{rx}{q}$; ſit $\frac{r}{2}-\frac{rx}{q}=0$, tum $rq=2rx$, ac $x=\frac{q}{2}$, ergo $yy=\frac{rq}{4}=4$tæ parti Figuræ (uti vocant) ſive ſemiaxis conjugati quadrato, adeoque maxima $y=$ iſti ſemiaxi. Nec Methodo diſſimili cum aliis curvis operandum foret; inveniatur ſubnormalis ex æquatione data, eaque nihilo æquali poſita, ordinatam curvæ maxi-

maximam vel minimam determinatam habebimus; priorem ad partem curvæ versus axem concavam, posteriorem ad convexam.

P. S. Æque facile hac methodo determinatur Tangens ad partes curvæ convexas operando, ac ad partes concavas uti prius. Sit enim AC Tangens verticalis inque ea ad libitum sumpto puncto C, sit $AC = n$, $CO = z$ (quo etiam charactere omnes lineæ, a puncto C ad curvam convexam AEG ductæ, insigniantur) ergo ducta MO semper ad AC perpendiculari, erit $CM = n - y$, & cum $OM = x$, erit $zz = nn - 2ny + yy + xx$, adeoque (pro extremo ipsius z valore) $2y\dot{y} + 2x\dot{x} - 2n\dot{y} = 0$. In qua æquatione si exponatur $2x\dot{x}$ secundum curvæ naturam, lineam CZ (quæ hoc loco subnormalis vicem subibit) determinatam dabimus. Res clarior est quam quæ exemplis illustrantibus indigeat; quæque jamjam dicta sunt facile hoc opus excusabunt.

Secundo, Sicut Methodo priore, Curvarum Tangentes invenimus, ipsas lineas LE vel CO a puncto dato vel in Axe vel in Tangente verticali sumpto productas, ad extremum determinando; sic etiam considerando lineas QE, &c. a puncto in Axe dato ultra verticem productas, idem (idque Universaliter) perficere possumus. Omnes enim lineæ QE valoris fluentis sunt ac perpetuo mutabilis, sola vero Tangens QF (posito quod QF curvam tangat) stabilis est ac ad unicum valorem determinata. Hoc ergo loco, τοῦ extremi Hypothesi non innitemur, sed quantitatem permanentem tantum speculabimur. Assumantur duo puncta Q L, indeque ad idem curvæ punctum E duæ semper lineæ ducantur LE, QE. Inter punctum F contactus ac verticem, angulus QEL semper erit obtusus, ad alteras vero partes puncti F acutus erit, supposito (quod prius monitum) QF curvam tangere, ac FL ei ad angulos rectos insistere. Sit $QA = p$. $AL = n$. $AB = x$. $BE = y$. $QE = z$. VE (intercepta inter punctum E & V ubi cadit QV perpendicularis ab Q in LE productam) $= v$. Jam propter Triangulum obtusangulum QE habemus hanc æquationem $zz = p^2 + 2pn - y^2 - x^2 + 2nx - \overline{y^2 + n^2 - 2nx + xx}|^{\frac{1}{2}} \times 2v$; sive loco $\overline{y^2 + n^2 - 2nx + xx}|^{\frac{1}{2}}$ scribendo f est $zz = p^2 + 2pn - y^2 - x^2 + 2nx - 2fv$, ideoque $2z\dot{z} = 2y\dot{y} - 2x\dot{x} + 2n\dot{x} - 2f\dot{v} - 2v\dot{f}$. Si z jam fiat quantitas stabilis, quo in Casu QE cum QF tangente coincidet, erit tum $-2y\dot{y} - 2x\dot{x} + 2n\dot{x} = 0$ (rectangulo $2fv$ ejusque adeo fluxione penitus evanescente.) Hæc vero est ipsa æquatio Generalis Methodo superiori determinata, quæque uti videmus non minus facile ac naturaliter ex hoc supposito quantitatis stabilis principio, quam ex illo extremi deducitur.

A Specimen of a general method of determining the Quadratures of Figures, by Mr. Jo. Craig. n. 284. p. 1346.

IV. Cum ex data relatione inter z & y, quæritur $\int: z\,dy$, tum Methodus nostra in determinandis Figurarum Quadraturis, tum Methodus Calculi differentialis inversa postulant, ut z per y & datas exprimatur, quod tamen fieri non potest, quando æquatio relationem illam definiens ultra

ultra Cubicam, vel Biquadraticam ascendit. Porro quamvis Regula innotesceret generalis inveniendi Radices æquationum cujuscunque gradus: huic tamen Methodo inversæ prorsus foret inutilis: Radix enim z surdis tam complicatis involveretur, ut nulla arte (hactenus cognita) a differentiali ad integrale regressus dari posset. Ob has rationes, alias vias, & conatu non prorsus irrito rem sum aggressus.

Sit $z^m + a y^n = b z^e y^r$ æquatio exprimens relationem inter Ordinatam z & abscissam y; in qua Exponentes m, n, e, r, denotant quoslibet Numeros, Integros vel Fractos, Affirmativos vel Negativos. Ponatur $r - n = c$. Erit

$$\text{AREA} = \frac{m}{m+n} z y +$$

$$\frac{mc + ne}{m \times \overline{m+n} \times \overline{c+1} + n \times \overline{m+n} \times \overline{e+1}} \times \frac{b}{a} z^{e+1} y^{c+1} +$$

$$\frac{\overline{m-e} \times \overline{c+1} + r \times \overline{e+1}}{m \times \overline{2c+1} + n \times \overline{2e+1}} \times \frac{bB}{a} z^{2e+1} y^{2c+1} +$$

$$\frac{\overline{m-e} \times \overline{2c+1} + r \times \overline{2e+1}}{m \times \overline{3c+1} + n \times \overline{3e+1}} \times \frac{bC}{a} z^{3e+1} y^{3c+1} +$$

$$\frac{\overline{m-e} \times \overline{3c+1} + r \times \overline{3e+1}}{m \times \overline{4c+1} + n \times \overline{4e+1}} \times \frac{bD}{a} z^{4e+1} y^{4c+1} +$$

$$\frac{\overline{m-e} \times \overline{4c+1} + r \times \overline{4e+1}}{m \times \overline{5c+1} + n \times \overline{5e+1}} \times \frac{bE}{a} z^{5e+1} y^{5c+1} + \&c.$$

De hac Serie hæc sunt notanda: (1.) Quod literæ majusculæ B, C, D, &c. designent coefficientes terminorum ipsis immediate præcedentium: (2.) Quod exhibeat Quadraturas omnium Figurarum Quadrabilium, quarum Curvæ per æquationem trium terminorum definiuntur: (3.) Et quod semper sint Quadrabiles, quando $\frac{m \times r - r}{mn - rm - en}$ est numerus integer & affirmativus, quem vocemus l. (4.) Speciatim $l + 1$ dat numerum Terminorum (ab initio sumptorum) Seriei Aream quæsitam constituentium: (5.) Quod si ponatur $e = 0$, mutabitur hæc Series in Celebre Theorema Newtonianum pro Binomio communi; & proinde hoc Theorema est hujus Seriei casus specialis & simplex: (6.) Cum sit Applicatio hujus Seriei ad Figuram particularem, hæ regulæ sunt observandæ. 1ª Reducatur æquatio Curvam datam definiens ad formam generalem, & ex comparatione particularis cum generali invenientur coefficientes a, b; ut & exponentes m, n, e, r. Secunda, Si exponentes sic determinati non faciant l numerum integrum & affirmativum, (juxta conditionem in Not. 3. assignatam,) tum alius terminus æquationis particularis a quantitate z liberetur; & si nunc exponentibus denuo determinatis non competat illa Quadrabilitatis conditio, tum reliquus terminus a quantitate z libe-

z liberetur: Nam nullo labore quilibet ex tribus terminis æquationem datam conſtituentibus a quantitate z liberari poteſt. Tertia, Si æquationi per Regulam præcedentem tractatæ non conveniat prædicta Quadrabilitatis conditio; tum per Seriem quæratur Areæ complementum $\int: ydz$: quo cognito ſtatim habetur Area quæſita; nam, ut omnibus notum, $zy - \int: ydz = \int: zdy$. Et ut ſine confuſione Complementum per Seriem obtineatur; in æquatione data Curvam particularem definiente pro z ſcribatur Y, & pro y ſcribatur Z: Factaque hac mutatione Ordinatæ in Abſciſſam, & Abſciſſæ in Ordinatam, tractetur æquatio juxta præcepta regulæ ſecundæ; donec illi conveniat Quadrabilitatis conditio, vel eandem ipſi non poſſe convenire pateat.

Exemplum 1. Sit $z^3 + y^3 = bzy$. Quia hic $m = 3$, $n = 3$, $e = 1$, $r = 1$, $a = 1$; ideo $l = 1$, adeoque $l + 1 = 2$. Et proinde (juxta Not. 4.) duo primi Seriei termini dant Aream $= \frac{1}{2}zy - \frac{1}{6}bz^2y^{-1}$.

Exemp. 2. Sit $z^7 + ay^3 = bzy^2$, ubi $m = 7$, $n = 3$, $e = 1$, $r = 2$; qui faciunt $l = 2$; ideo (juxta Not. 4.) tres primi Seriei termini dabunt quæſitam

$$\text{AREAM} = \frac{7}{10}zy - \frac{b}{15a}z^2 - \frac{2b^2}{15a^2}z^3y^{-1}.$$

Exemp. 3. Sit $z^3 + ky^5 = bz^{-2}y^{11}$; ubi $m = 3$, $n = 5$, $e = -2$, $r = 11$; at quia hi non faciunt l numerum integrum & affirmativum; ideo (per Regulam ſecundam) libero terminum $bz^{-2}y^{11}$ a quantitate z; & ſic æquatio fit $z^5 - by^{11} = -kz^2y^5$; ubi $a = -b$, $b = -k$; & $m = 5$, $n = 11$, $e = 2$, $r = e$; qui faciunt $l = 1$: Unde

$$\text{AREA} = \frac{5}{16}zy - \frac{k}{16b}z^3y^{-5}.$$

Exemp. 4. Sit $z^2 - by^2 = -kz^2y^2$; ubi $m = 2$, $n = 2$, $e = 2$, $r = 2$; qui non faciunt l numerum integrum & affirmativum; ideo libero terminum $-kz^2y^2$ a quantitate z; & tum $z^0 + ky^2 = bz^{-2}y^2$; ubi $a = k$, $b = b$; & $m = 0$, $n = 2$, $e = -2$, $r = 2$, qui faciunt $l = 1$, ideo

$$\text{AREA} = \frac{b}{k}z^{-1}y$$

Exemp. 5. Sit $z^2 - \frac{4g^2}{b}y^6 = -\frac{g}{b}z^2y^4$; ubi $m = 2$, $n = 6$, $e = 2$, $r = 4$; qui non faciunt l numerum integrum & affirmativum; idemque contingit liberato (a quantitate z) utrolibet ex reliquis: Ideo juxta regulam Tertiam quæro Complementum; quare (ut jam præmonui) pono $z = Y$; $y = Z$; unde æquatio data erit

$Y^2 - \frac{4g^2}{h}Z^6 = -\frac{g}{b}Z^4Y^2$; quæ (juxta Reg. 1.) reducta ad formam generalem erit hujus modi $Z^6 - \frac{b}{4g^2}Y^2 = \frac{1}{4g}Z^4Y^2$ ubi

ubi $m=6$, $n=2$, $e=4$, $r=2$; qui non faciunt l numerum integrum & affirmativum; ideo (juxta Reg. 2.) libero terminum ultimum a Z; unde $Z^2 - \frac{1}{4g} Y^2 = \frac{b}{4g^2} Z^{-4} Y^2$; ubi $m=2$, $n=2$, $e=-4$, $r=2$; unde $l=r$; & $a=-\frac{1}{4g}$, $b=\frac{b}{4g^2}$; Unde Areæ quæsitæ complementum est $=\frac{1}{2} ZY - \frac{b}{2g} Z^{-3} Y$ seu $\frac{1}{2} zy - \frac{b}{2g} zy^{-3}$; Ergo etiam Area quæsita $\int: z\,dy = \frac{1}{2} zy + \frac{b}{2g} zy^{-3}$.

Sit $z^m + ay^n = bz^{2e} y^{2c+n} + fz^e y^{c+n}$ æquatio exprimens Relationem inter Ordinatam z & Abscissam y. Erit

$$\text{AREA} = Azy + Bz^{e+1}y^{c+1} + Cz^{2e+1}y^{2c+1} + Dz^{3e+1}y^{3c+1} + Ez^{4e+1}y^{4c+1} + Fz^{5e+1}y^{5c+1},\ \&c.$$

Ubi (positis $2c+n=r$, $c+n=s$) $A=\frac{n}{m+n}$;

$$B=\frac{\overline{m-e+s}\times A+e-m}{m\times\overline{c+1}+n\times\overline{e+1}}\times\frac{f}{a}.$$

$$C=\frac{\overline{m-2e+r}\times bA+\overline{\overline{m-e}\times\overline{c+1}+r\times\overline{e+1}}\times fB+2eb-mb}{ma\times\overline{2c+1}+na\times\overline{2e+1}}$$

$$D=\frac{\overline{\overline{m-2e}\times\overline{c+1}+r\times\overline{e+1}}\times bB+\overline{\overline{m-e}\times\overline{2c+1}+s\times\overline{2e+1}}\times fC}{ma\times\overline{3c+1}+na\times\overline{3e+1}}$$

$$E=\frac{\overline{\overline{m-2e}\times\overline{2c+1}+r\times\overline{2e+1}}\times bC+\overline{\overline{m-e}\times\overline{3c+1}+s\times\overline{3e+1}}\times fD}{ma\times\overline{4c+1}+na\times\overline{4e+1}}$$

$$F=\frac{\overline{\overline{m-2e}\times\overline{3c+1}+r\times\overline{3e+1}}\times bD+\overline{\overline{m-e}\times\overline{4c+1}+s\times\overline{4e+1}}\times fF}{ma\times\overline{5c+1}+na\times\overline{5e+1}}$$

De hac Serie (cujus progressio primo fere intuitu est manifesta) hæc sunt notanda. (1.) Quod figuræ (quarum Curvæ prædicta æquatione definiuntur) sunt Quadrabiles, quando Numeri exponentiales m, n, e, c; & coefficientes a, b, f habent relationes modo assignandas; scil: quando $\frac{\overline{2c+m}\times\overline{n-2e}}{-cm-en}$ est numerus integer & affirmativus, quem vocemus l, & (cum l est major quam 2) quando Coefficientium relatio est hæc.

$m-2$

$$\frac{\overline{m-2e}\times\overline{lc-c+1}+r\times\overline{le-e+1}}{\overline{e-m}\times\overline{lc+1}-s\times\overline{le+1}}\times\frac{bU}{f}=$$

$$\frac{\overline{m-2e}\times\overline{lc-2c+1}+r\times\overline{lc-2e+1}}{m\times\overline{lc+1}+n\times\overline{le+1}}\times\frac{bP}{a}+$$

$$\frac{\overline{m-e}\times\overline{lc+c+1}+r\times\overline{le-e+1}}{m\times\overline{lc+1}+n\times\overline{le+1}}\times\frac{fU}{a}.$$

Ubi U & P denotant Coefficientes Terminorum duorum, qui immediate præcedunt ultimo Areæ quæfitæ Termino; fcil: U eft coefficiens termini ad Ultimum propioris, P coefficiens termini ab ultimo remotioris: ut fi $Fz^{5e+1}y^{5c+1}$ effet ultimus Areæ quæfitæ terminus, tum U denotaret E, & P denotaret D. (2.) Ultimus ille Areæ quæfitæ terminus ex valore numeri l cognofcitur; nam hic etiam $l+1$ dat numerum terminorum (ab initio fumptorum) Seriei, qui Aream quæfitam conftituunt. (3.) Si fuerit $l=1$, tum coefficientium relatio debet effe hæc

$$\frac{\overline{2e-m}\times\overline{1-A}+rA}{\overline{e-m}\times\overline{c+1}-s\times\overline{e+1}}\times\frac{b}{f}=\frac{\overline{e-m}\times\overline{1-A}+sA}{m\times\overline{c+1}+n\times\overline{e+1}}\times\frac{f}{a};$$

Si $l=2$; relatio debet effe hæc

$$\frac{\overline{m-2e}\times\overline{c+1}+r\times\overline{e+1}}{\overline{e-m}\times\overline{2c+1}-s\times\overline{2e+1}}\times\frac{bB}{f}=\frac{\overline{2e-m}\times\overline{1-A}+rA}{m\times\overline{2c+1}+n\times\overline{2e+1}}\times\frac{b}{a}+$$

$$\frac{\overline{m-e}\times\overline{c+1}+s\times\overline{e+1}}{m\times\overline{2c+1}+n\times\overline{2e+1}}\times\frac{fB}{a}.$$

Sit $z^m=ay^n+bz^e y^{c+n}+fz^{2e}y^{2c+n}+gz^{3e}y^{3c+n}+hz^{4e}y^{4c+n}+$ *&c.* æquatio exprimens relationem inter ordinatam z & abfciffam y; & conftans terminis quotcunq; Erit

Area $=Azy+Bz^{e+1}y^{c+1}+Cz^{2e+1}y^{2c+1}+Dz^{3e+1}y^{3c+1}+Ez^{4e+1}y^{4c+1}+$ *&c.*

Calculo perfacili inveniuntur A, B, C, D, E, *&c.* ut & Quadrabilitatis conditiones, & quot termini feriei Aream quæfitam conftituant. Crefcit quidem numerus harum conditionum pro multitudine terminorum, ex quibus conftat æquatio relationem inter z & y definiens. Et fpeciatim fi illa terminorum multitudo vocetur N; tum $N-2$ eft numerus conditionum Quadrabilitatis; quarum una Exponentium m, n, e, c relationem refpicit, eftq; hæc; ut $\frac{Nc-2c+2e-Ne+m+n}{-cm-en}$, eft numerus (quem vocemus l) Integer & affirmativus. Reliquæ vero conditiones coefficientium a, b, f, g, h, *&c.* refpiciunt. Ac deniq; $l+1$ dat numerum terminorum (ab initio fumptorum) feriei, qui Aream quæfitam conftituunt.

Corol.

Corol. Ex hac Serie generali deduci poteſt Series, quæ exhibeat Quadraturas Figurarum, quarum Curvæ definiuntur per æquationem conſtantem terminis quibuſvis, qui æquationem Sectionis tertiæ generalem conſtituunt. Nam ad hanc obtinendam opus tantum eſt Seriem computare pro æquatione conſtante tot terminis (ab initio ſumptis) æquationis generalis, quot includent Terminos æquatio Curvas definiens. Tum ex valoribus quantitatum $A, B, C, D,$ &c. Eliminentur illæ coefficientes $b, f, g,$ &c. quæ ad æquationem propoſitam non ſpectant; reliquæ dabunt aream quæſitam. Exemplo res patebit.

Sit $z^m = ay^n + bz^e y^{c+n} + gz^{3e} y^{3c+n}$ Æquatio exprimens relationem inter Z & Y. Jam quia $z^m = ay^n + bz^e y^{c+n} + fz^{2e} y^{2c+n} + gz^{3e} y^{3c+n}$, eſt illa pars æquationis quæ (ſumptis terminis in ordine a principio) includit æquationem datam; quam deinceps (brevitatis cauſa) æquationem completam vocabo; ideo Figurarum (quarum Curvæ definiuntur per æquationem completam.)

Areæ $= Azy + Bz^{e+1} y^{c+1} + Cz^{2e+1} y^{2c+1} + Dz^{3e+1} y^{3c+1} + Ez^{4e+1} y^{4c+1} + Fz^{5e+1} y^{5c+1} +$ &c. & a, b, f, g ingredientur valores quantitatum B, C, D, E, F &c. Si ergo in his valoribus ponatur ubique $f = 0$ (quia $fz^{2e} y^{2c+n}$ æquationem datam non ingreditur) habebis valores quantitatum $A, B, C, D, E,$ &c. qui in Serie ſubſtituti dabunt Areas quæſitas. Et Calculo inito inveni

$$A = \frac{m}{m+n}. \quad B = \frac{e - \overline{m - c - n \times A + m} - e}{m \times \overline{c+1} + n \times \overline{e+1}} \times \frac{b}{a}.$$

$$C = \frac{\overline{c+n} \times \overline{e+1} + \overline{m-e} \times \overline{c+1}}{m \times \overline{2c+1} + n \times \overline{2e+1}} \times \frac{bB}{a}.$$

$$D = \frac{\overline{\overline{m-3e} \times \overline{1-A} + \overline{3c+n}} \times -A \times g}{ma \times \overline{3c+1}}$$

$$\frac{+\overline{\overline{m-e} \times \overline{2c+1} + \overline{c+n} \times \overline{2e+1}} \times -bC}{+na \times \overline{3e+1}}.$$

$$E = \frac{\overline{\overline{m-3e} \times \overline{c+1} + \overline{3c+n} \times \overline{e+1}} \times}{ma \times \overline{4c+1}}$$

$$\frac{-gB + \overline{\overline{m-e} \times \overline{3c+1} + \overline{c+n} \times \overline{3e+1}} \times -bD}{+na \times \overline{4e+1}}.$$

$$F = \frac{\overline{\overline{m-3e} \times \overline{2c+1} + \overline{3c+n} \times \overline{2e+1}} +}{+ma \times \overline{5c+1}}$$

$$\frac{-gC + \overline{\overline{m+e} \times \overline{4c+1} + \overline{c+n} \times \overline{4e+1}} \times -bE}{+na \times \overline{5e+1}}.$$

$G =$

$$G=\frac{\overline{\overline{m-3e}\times\overline{3c+13}+\overline{c+n}\times\overline{3e+1}}\times}{+ma\times\overline{6c+1}}$$

$$\frac{-gD+\overline{\overline{m+e}\times\overline{5c+1}+\overline{c+n}\times\overline{5e+1}}\times-bF}{+na\times\overline{6e+1}}.$$

Ex his patet progreſſio reliquorum in infinitum. Et ſic habetur Series exhibens Quadraturas omnium Figurarum, quarum Curvæ definiuntur per hanc æquationem quatuor terminorum $z^m=ay^n+bz^e y^{c+n}+gz^{3e}y^{3c+n}$. Et notandum quod conditiones Quadrabilitatis & numerus terminorum Seriei, Aream quamlibet quæſitam conſtituentium, eadem ſunt cum conditionibus Quadrabilitatis, & numero Terminorum, quæ conveniunt Figuris, quarum Curvæ per æquationes completas definiuntur.

Corol. Præter has duas ſeries in § 2 & 4 propter Figuras quatuor terminorum, poſſunt eodem modo infinitæ aliæ ſeries computari pro cæteris caſibus Figurarum quatuor terminorum. Quod etiam intelligendum eſt de omnibus aliis Figuris, quarum Curvæ per æquationes quotlibet terminorum numero conſtantes definiuntur.

Non jam vacat ipſam Methodum minutiatim deſcribere per quam ad hujuſmodi Series pervenio; brevem tamen ejus rationem exponere forte non ingratum erit. Aſſumo Seriem ex z pariter ac y compoſitam, ſc. $Azy+Bz^py^q+Cz^ry^s+Dz^ly^k+$ &c. $=\int:zdy$. Cujus ſinguli termini (præter primum) habeant Exponentes incognitos. Tum æquationem inſtituo inter duos valores quantitatis dz, quorum alter ex hac ſerie, alter ex æquatione relationem inter z & y definiente per Methodum Calculi Differentialis directam facile invenitur. Ex terminis hujus æquationis rite reductæ primo determino exponentes incognitos p, q, g, h, l, k &c. Et dein coefficientes A, B, C, &c. Et ſi plures ſint comparationes, quam quæ determinandis his coefficientibus ſufficiunt, tum ex reliquis deduco Quadrabilitatis conditiones. Si recta ineatur via, Calculus eſt longe facillimus; multaſque habeo Regulas huc ſpectantes quas alias forſan tradam; ut & uſum hujus Methodi in inveniendis Quadraturis irrationalibus finitis, quando rationales non dantur: res enim omnino in poteſtate eſt.

V. Having found the Conſtruction of a Curve, from whence (beſides its own *Quadrature* and *Rectification*) the *Quadrature* of the *Hyperbola* is deriv'd, I thought the following Account might not be unacceptable. *Conſtruction and Properties of a Quadratrix to the Hyperbola, by* Mr. — Perks. n. 306. p. 2253

Let AB, CD, be two ſtraight Rulars joyned at B, and there making a right Angle. (Their length according to the largeneſs of the Figure you will deſcribe.) EF is another Rular ſomewhat longer than AB. Near the one end E, let a little *Truckle-wheel* (repreſented edge-wiſe by gh, and made of a thin Plate of Braſs or Iron) be faſtned to the Rular by a Pin (i) thorow its Center, ſo that the Wheel may turn about upon the Pin (i) tight to the Rular without joggling. Fig. 5. Plate I.

On

On the under ſide of this Rular (the ſide from the Eye in the Scheme) let there be pinn'd or glewed a little piece of Wood (in the form of a Quadrant, the part which is ſeen being mark'd kl) whoſe edge (or limb) kl, is an arch of a Circle of Center (i) and *Radius* ih (the ſame with the little Wheel.) The deſign of this piece of Wood is, that in the ſeveral Poſitions of the Rular EF, the circular Limb kl always touching and ſliding by the edge of the Rular AB, the Center of the Wheel may be always in a line (im) parallel to the Rular AB.

In the Rular CD make $MB = ih$ or ik, and at M faſten a little Pin, and another to the Rular EF near the Wheel, as at p. To theſe two Pins let be faſten'd the two ends of a String MR, ſo that its whole length (from Pin to Pin) $+ pi$, be equal to the intended Axis of the Curve TW.

The Inſtrument being thus prepar'd, let a ſtrong Rular SO, be faſtned (or held faſt) upon the Paper or Plain that the Curve is to be drawn upon. Lay the Rular EF from M towards A, and parallel to AB, ſo that the String lye all ſtraight along the edge of the Rular EF, from M to p, the point Sk of the Quadrantal piece of Wood reſting upon the edge of the Rular AB. Then with a ſmall Pin at M, keeping the String cloſe to the edge of the Rular EF, and with your other hand upon the end E, keeping the Wheel tight to the Paper or Plain, move the Pin, String and Rular EF from M towards O, the Rular CD ſliding along by the faſtned Rular SO in a right line, the Wheel gh will by its motion deſcribe the deſired Curve TV.

Note, The Semi-diameter of the little Wheel muſt be about the Sum of the thickneſſes of the two Rulars EF and AB, that it may touch the Paper. Alſo it will be convenient that its edge be thin, and a little rough, that it may not ſlide flat-ways, and that it may leave a viſible impreſſion.

From this Conſtruction the following Properties are demonſtrable.

I. It is evident from the Conſtruction, that the *Sum of the Tangent and Subtangent*, is every where equal to the ſame given Line $= MR + Ri = TW$.) for the String (firſt ſtraight at TW, afterwards making an Angle at R) being every where the ſame, the Line Ri (or $Rp + pi$) is always the Tangent, and the Remainder RM the Subtangent; rhe Contact of the Wheel with the Plain, being the point of the Curve to which they belong.

II. It hence follows, that any aſſignable part of the Curve is *Rectifiable*, or equal to any aſſignable ſtraight Line. Let FAE be a part of the Curve, its Vertex F. HDd is the Line deſcribed by the motion of the Pin R, and may be ſhewn to be Aſymptote to the Curve. FH a perpendicular to HD. Let A be a given point in the Curve, AD the Tangent, and BD the Subtangent to the ſame point A. Let a be another point in the Curve infinitely near to A, to which let ad be the Tangent, and bd the Subtangent. Draw AG, ag perpendicular to FH; and AB, ab, perpendicular to HD. By the Conſtruction $AD + DB =$

$ad +$

$ad + db$. Let $a\delta$ be made equal to aD, and draw $D\delta$. Then because $ad + bd = AD + DB$, subtract bD and aD (or $a\delta$) from both Sums (Equals from Equals) there remains $\delta d + dD = Aa + Bb$ (or Ca). AaC, $Dd\delta$ are like Triangles (or differing infinitely little from such) therefore $Ca\ (Bb) : Aa :: \delta d : Dd$, and compounding $Bb + Aa : Aa :: \delta d + Dd : Dd$. Alternating $Bb + Aa : \delta d + Dd :: Aa : Dd$. But $Bb + Aa = \delta d + Dd$ (as is shewn above) therefore $Aa = Dd$. Aa is the fluxional Particle of the Curve FA, and Dd is the fluxional Particle of the Line HD: These Fluxions or Augments, being equal, their flowing quantities beginning together, are themselves therefore equal, *viz.* $FA = HD$.

Let $FG = x$. $GA\ (= HB) = y$. $AD = t$. $BD = S$. So is the Curve $FA = HD = y + S$: that is, *the Curve from the Vertex to any given point therein, is equal to the Sum of its Ordinate, and Subtangent to the same point which is its second Property.*

III. The next Property (and whereupon I call it the *Hyperbolic Quadratrix)* is this, let FAE be a part of the Curve, (&c. as before) $FIKH$ is a Square upon the Line FH. λIL is an Equilater Hyperbola whose Vertex is I, its Asymptotes HO, HR, its Ax $HI\mu$. From a given point L in the Hyperbola, (below its Vertex I) draw LA parallel to the Asymptote RH, intersecting the Diagonal IH in M, FH in G, and touching the Quadratrix in A. I say, that the Hyperbolic Area ILM is equal to a Rectangle, whose sides are the *Ordinate* GA, and twice FH, the Ax to the Quadratrix, that is, Trilin. $ILM = 2FH \times GA$. Fig. 6. Plate 1.

Let $FH = a$, $FG = x$, $GA = y$. Because of the Hyperbola $GL \times GH\ (LS) = FHq$, therefore $GL = \frac{FHq}{GH}$; and $LM = \frac{FHq}{GH} - GH\ (MG)$ that is, $LM = \frac{aa}{a-x} - a + x = \frac{2ax - xx}{a - x}$, and consequently the fluxion of the Area $ILM = \frac{2ax - xx}{a - x}\dot{x}$.

In the Rectangle triangle ADB, $AB = a - x$, $BD = S$, $AD = t = a - S$; then is $ADq = ABq + BDq$: or $aa - 2aS + SS = aa - 2ax + xx + SS$, which being reduced, gives $S = \frac{2ax - xx}{2a}$.

Let la be a right line supposed infinitely near and parallel to LA, and intersecting AB in C. Because of like triangles ACa, ABD; $AB : BD :: AC : Ca$ that is $a - x : S\ (= \frac{2ax - xx}{2a}) :: \dot{x} : \dot{y}$. therefore $\dot{y} = \frac{2ax - xx}{2aa - 2ax}\dot{x}$. Multiply each by $2a$, and 'tis $2a\dot{y} = \frac{2ax - xx}{a - x}\dot{x}$. The *Flowing quantity* of $2a\dot{y}$ is $2ay$ and the *flowing quantity* of $\frac{2ax - xx}{ax}\dot{x}$ is the Hyperbolic Area ILM (as is shewn before.) These two Area's beginning together at F and I, and having every where equal

equal *Fluxions*, or Augments, are therefore themſelves every where equal.

N. The Quadrature of the Trilinear Figure *ILM* being thus found, any other Area bounded with the Curve Line *IL*, and any other Right Lines, is alſo given.

IV. Suppoſing the ſame things as in the precedent Propoſition, I ſay, that the Area of the Quadratrix *FabHF* is equal to half the ſquare of *Fg*, wanting the Cube of *Fg* divided by ſix *FH*, or $FabHF = \frac{xx}{2} - \frac{xxx}{6a}$. The Fluxion of this Area is the Rectangle $CabB = \overline{a-x} \times \dot{y} = \overline{a-x} \times \frac{2ax - xx}{2aa - 2ax}\dot{x} = x\dot{x} - \frac{xx}{2a}\dot{x}$. The flowing quantity of $x\dot{x}$ is $\frac{1}{2}xx$: And the flowing quantity of $-\frac{xx}{2a}\dot{x}$ is $-\frac{xxx}{6a}$ [as is eaſily ſhewn by bringing back theſe flowing quantities to their reſpective Fluxions.] And hence alſo it follows, that the whole Area continued on infinitely towards *E*, *is one third of the Square FIKH*; or $\frac{1}{3}aa$. For ſuppoſing $x = a$ the Area above becomes $\frac{aa}{2} - \frac{aa}{6} = \frac{aa}{3}$.

V. Suppoſing ſtill the ſame things, I ſay that the Solid made by the converſion of the Area *FabHF* about the Line *Hb* as an Axis, is equal to a Cylinder whoſe Radius is $FH = a$, and height equal to $\frac{xx}{2a} - \frac{x^3}{2aa} + \frac{x^4}{8a^3}$. And the whole Solid made by converſion of the whole Figure infinitely continued, is equal to an eighth part of a Cylinder, whoſe Radius and Height are each equal to *FH* or *a*.

Let $\frac{P}{D}$ expreſs the Proportion of the Periphery and Diameter of a Circle. Then is $\frac{P}{D}\,ab$ quad. the Area of a Circle whoſe Radius is *ab*.

And becauſe $Ca = \dot{y} = \frac{\frac{-xx}{2a}}{a-x}\dot{x}$ the fluxion of the Solid is $\frac{P}{D}\,ab^2 \times \frac{x - \frac{xx}{2a}}{a-x}\dot{x}$, or $\frac{P}{D}\,\overline{a-x}^2 \times \frac{x - \frac{xx}{2a}}{a-x}\dot{x} = \frac{P}{D}\,\overline{ax - \frac{3}{2}xx + \frac{x^3}{2a}} \times \dot{x}$ whoſe flowing quantity is $\frac{P}{D} - \frac{axx - xxx}{2} + \frac{x^4}{8a}$. Which Solid being divided by $\frac{P}{D}\,aa$ (the Area of a Circle whoſe Radius is *a*) gives

xx

$\frac{xx}{2a} - \frac{xxx}{2aa} + \frac{x^4}{8aa}$ for the height of a Cylinder on the ſaid circular Baſe, and equal to the Solid made by converſion of the Area $FabHF$ about the Line Hb as an Axis. When $x = a$ (that is when the whole Figure is turn'd about its Aſymptote) the height $\frac{xx}{2a} - \frac{x^3}{2aa} + \frac{x^4}{8a^3}$ becomes $\frac{1}{8}a$.

VI. The Curve ſurface of the Solid generated by the Converſion of the Figure $FabHF$ about HB, is equal to the Curve ſurface of a Cylinder, whoſe Radius is a, and height equal to $\frac{x}{2} - \frac{xx}{4a} + \frac{xxx}{12aa}$. And the whole Curve Surface of the Solid infinitely continued, is equal to *one third part of the Curve Surface of a Cylinder whoſe Radius and Height are equal to* FH *or* a. Which may be demonſtrated after the manner of the precedent Propoſition.

VII. The Radius of the Curvature of any Particle of the Quadratrix is $\frac{tt}{a-x}$. and this found Geometrically. FAE is the *Quadratrix*, HD the Aſymptote, AD the Tangent, BD the Subtangent to a given point A. Make $BV = AD$. Upon V raiſe the perpendicular VW, from A draw AW perpendicular to the Tangent AD, till it meet VW in W. So is AW the Radius of the Curvature at A. *Fig. 7. Plate I.*

VIII. This Curve may be continued on infinitely above the point F (but by a different and more operoſe way of Conſtruction) whoſe Properties will be theſe, 1. The *Difference* of its Tangent and Subtangent (taking the Subtangent in the Line HS) will be always equal to the ſame given Line FH or a. That is, as $t + s = a$, below F, ſo $t - s = a$ above F. 2. As below F the Curve Line is equal to the *Sum* of its Ordinate and Subtangent, ſo above, it is equal to their *Difference* or $S - y$. 3. As below F, $2ay = ILM$, ſo above, $2ay = I\lambda\mu$. All which (and its other Properties) may be demonſtrated as the Precedent, *mutatis mutandis.*

IX. With a little variation in the precedent Conſtruction may the *Logarithmick Curve* be conſtructed, which is alſo a *Quadratrix* to the Hyperbola. Omitting the String MRP, let the diſtance MR be equal to the *Subtangent* of the intended Logarithmick Curve (which, as 'tis known, is invariable.) Stick a Pin at R in the Ruler CD, to which apply the Ruler EF, ſo that the edge of the little Quadrant kl, reſting upon the Ruler AB, the diſtance Mi be equal to MR. Then keeping the Ruler EF tight to the Pin R and Ruler AB, ſlide the Ruler CD along in a ſtraight Line (by the Ruler or Line SO.) So will the Wheel gh deſcribe a part of the Logarithmick Curve TV, whoſe *Subtangent* is every where MR.

X. Let FAE repreſent the *Logarithmick Curve*, whoſe Subtangent is equal to FH. $LI\lambda$ is an Equilater Hyperbola (*&c.* as before § III.) Let $FG = x$, $Ga = y$. $FH (= BD) = a$. $GH (= LS) = a - x$. $AC = \dot{x}$, $Ca = \dot{y}$. Then $AC : Ca :: AB : BD$, that is $\dot{x} : \dot{y} :: a - x : a :: a : \frac{aa}{a-x}$, therefore $a\dot{y} = \frac{aa}{a-x}\dot{x}$. The *Flowing quantity* of $a\dot{y}$ is ay; and the *Flowing quantity* of $\frac{aa}{a-x}\dot{x}$ is the Hyperbolick Area $FILG$ (for by the nature of the Hyperbola $GL = \frac{aa}{a-x}$) therefore is the Hyperbolick Area $FILG$ equal to ay, a Rectangle whoſe ſides are the Subtangent ($BD = FH$) and Ordinate GA (as here accounted) of a Logarithmick Curve.

Of the Length of Curve Lines. by Mr. Jo. Craig. n. 314. *p.* 64.

VI. Lemma. *Duorum Quadratorum ſummam in alia duo Quadrata dividere.*

Sint dz^2, ds^2, duo Quadrata data, quorum ſumma $dz^2 + ds^2$ dividenda eſt in alia duo Quadrata dx^2, dy^2; ſintque m & n duo quilibet numeri ad arbitrium ſumendi. Jam ex conditione Problematis eſt $dx^2 + dy^2 = dz^2 + ds^2$, unde (ut ex Diophanto conſtat) erit $dx = \frac{\overline{mm - nn} \times dz + 2mnds}{mm + nn}$, $dy = \frac{\overline{nn - mm} \times ds + 2mndz}{mm + nn}$. *Q. E. I.*

Problema. *Curvas innumeras invenire, quæ ſint ejuſdem Longitudinis cum Curva quavis propoſita, ſive Algebraica ſive Tranſcendente.*

Deſignent z, s Coordinatas Curvæ propoſitæ; & x, y Coordinatas Curvæ quæſitæ, quæ ejuſdem ſit longitudinis cum propoſita; Unde ex Curvarum Elementis $dx^2 + dy^2 = dz^2 + ds^2$, Ideoque per Lemma præcedens

$$dx = \frac{\overline{mm - nn} \times dz + 2mnds}{mm + nn},$$

$$dy = \frac{\overline{nn - mm} \times ds + 2mndz}{mm + nn};$$

Quarum integrales ſunt

$$x = \frac{\overline{m^2 - n^2} \times z + 2mns}{mm + nn},$$

$$y = \frac{\overline{n^2 - m^2} \times s + 2mnz}{mm + nn}.$$

Et ſic innoteſcunt Coordinatæ x, y unius ex Curvis quæſitis; ſimiliter ex hac una invenietur ſecunda, ex ſecunda tertia, & ſic porro innumeræ invenientur. *Q. E. I.*

VII. I

Description and Quadrature of a Curve of the Third Order, com. by Mr. de Moivre, *n.* 345. *p.* 329. *Fig.* 8.

VII. I have look'd a little farther into that Curve which fell lately under my consideration. It is not the *Foliate* as I did at first imagine, but I believe it ought not to make a *Species* distinct from it. AEB is the Curve I thus describe. Let AB and BK be perpendicular to each other. From the point A draw AR cutting BK in R, and make $RE = BR$, the point E belongs to the Curve. Draw BC making an Angle of 45 *grad.* with AB, this Line BC touches the Curve in B; from the point E draw ED perpendicular to BC, and calling BD, x; DE, y; AB, a; and making $\sqrt{8aa} = n$, the Equation belonging to that Curve is $x^3 + xxy + xyy + y^3 = nxy$ or $\frac{x^4 - y^4}{x - y} = nxy$. Taking $BG = AB$, and drawing GP perpendicular to BG, PG is an *Asymptote*. In the *Foliate* the Equation is $x^3 + y^3 = \frac{1}{2}nxy$, in which the two Terms $xxy + xyy$ of the former Equation are wanting; and its *Asymptote* is distant from B by $\frac{1}{2}BA$. Again draw EF perpendicular to AB: let BF be called z, and FE, v; the Equation belonging to the Curve AEB is $vv = \frac{azz - z^3}{a + z}$. In the *Foliate* the Equation is $vv = \frac{azz - z^3}{a + 3z}$. From these two last Equations it seems that these Curves differ no more from one another than the *Circle* from the *Ellipsis*.

The Quadrature of the Curve here described hath something of Simplicity with which I was well pleased. With the Radius BA and Center B describe a Circle AKG, let the Square $HPST$ circumscribe it, so that HP be parallel to AG · prolong FE till it meet the Circumference of the Circle in M, and through M draw LMQ parallel to HP. The Area BFE is equal to the Area $KHLM$, comprehended by KH, HL, LM and the Arc KM. And the Area Bfe is equal to the Area $KmLH$ or $KMPQ$. Therefore if BF and Bf are equal, the two Areas BFE, Bfe taken together are equal to the Rectangle HQ, and therefore the whole Space comprehended by $BEAXBeYGZ$ supposing T and Z to be at an infinite Distance) is equal to the circumscribed Square HS.

'*N.B.* This Quadrature is easily demonstrated from the Equation: for 'by it $a + z : a - z :: zz : vv$, that is $AF : EF :: MF : FB$, and so φF 'the *Fluxion* of AF to Ll the *Fluxion* of MF. Hence the *Areola* $EF\varphi e$ 'will be always equal to the *Areola* $MLl\mu$, and therefore the Area 'AEF always equal to the Area MAL.

'Hence it appears that this Curve requires the Quadrature of the Circle to square it; whereas the *Foliate* is exactly quadrable, the whole '*Leaf* thereof being but one third of the Square of AB, which in this is 'above three sevenths of the same. Again in our Curve, the greatest 'Breadth is when the Point F divides the Line AB in extreme and mean 'Proportion, whereas in the *Foliate* it is when AB is triple in power to 'BF.

'*BF.* And the greateſt *EF* or Ordinate in the *Foliate* is to that of our
'Curve nearly as 3 to 4, or exactly as $\sqrt{\frac{2}{3}\sqrt{\frac{1}{3}}-\frac{1}{3}}$ to $\sqrt{5\sqrt{\frac{5}{4}}-5\frac{1}{2}}$.

But ſtill theſe Differences are not enough to make them two diſtinct Species, they being both defin'd by a like Equation, if the Aſymptote *SGP* be taken for the Diameter. And they are both comprehended under the 40*th* kind of Curves of the 3*d* Order, as they ſtand enumerated by Sir *Iſaac Newton* in his incomparable Treatiſe on that Subject.

The Construction and Meaſure of Curves, by Mr. Maclaurin, *n.* 356. *p.* 803.

VIII. § 1. Sint L & l puncta quamproxima in Curva BlL; ſit lo arcus centro S deſcriptus perpendicularis in SL; & erit Ll ut momentum Curvæ & Lo momentum Radii SL: Ac ſi detur ratio Ll ad Lo, vel ad lo in diſtantia SL, dabitur æquatio Curvæ ad centrum S. Sint LP, lp Tangentes Curvæ in punctis L & l, in quas ex S demittantur normales SP, Sp iis occurrentes in punctis P & p; ſimiliter in omnes Curvæ Tangentes demittantur perpendiculares ex dato puncto S, & conſtruetur Curva tranſiens per omnes Tangentium & perpendiculorum interſectiones. Hujus triangulum elementare Pnp ſimile erit triangulo Lol, quæ proinde dabitur ex data Curva BlL. Quippe ob æquales Snp, PnL, & rectos Spn, SPL æquiangula erunt triangula Spn, PnL, & proinde $Pn : pn :: Ln : Sn :: Lo : lo$, adeoque ob angulos Pnp, SnL, Lol æquales, erunt triangula Pnp, SnL, Lol ſimilia. Cum igitur eadem ſit ratio Ll ad lo quæ Pp ad pn, & SL ad SP, manifeſtum eſt data ratione Ll ad lo, & recta SL, dari rationem Pp ad pn & rectam SP, adeoque Curvam DPp. Eadem ratione ex DP conſtrui poteſt Tertia, & ex ea dein Quarta, & progrediendo prodibit ſeries Curvarum infinita, quæ omnes ex uno dato innoteſcunt. Quod ſi erigantur LN & ln perpendiculares in radios SL, Sl, ſibi mutuo occurrentes in n; & per omnia ſimiliter definita perpendicularium concurſuum puncta deſcribatur Curva EN: ea ipſa erit Curva ex qua deduci poteſt BL, eadem ratione qua conſtruximus DP ex BL. Ex EN ſimiliter conſtrui poteſt alia Curva, atque ex hac quoque parte Series infinita Curvarum conſtrui poterit.

Fig. 9.

§ 2. Curvarum vero hac ratione conſideratarum ſimpliciſſimæ ſunt quarum Ll eſt ad Lo in ratione poteſtatis alicujus Radii, ita ut, ſi a ſit data quantitas, r denotet Radium Curvæ, n numerum quemcunque, ſit Ll ad lo ut a^n ad r^n æquatio earum generalis. Omnes vero hæ Apſidem habent cum $r=a$, quoniam in eo caſu $Ll=lo$. Ut inveſtigem æquationem Curvæ DP, cum in BL eſt ut Ll ad lo ita a^n ad r^n, ita r ad $SP = \frac{r^{n+1}}{a^n}$, ita $a^{\frac{n}{n+1}} \times SP^{\frac{1}{n+1}}$ ad SP, ita $a^{\frac{n}{n+1}}$ ad $SP^{\frac{n}{n+1}}$, ita Pp ad pn. Proinde ſi $\dot{z}$ repreſentet momentum Curvæ, $\dot{y}$ arcum circularem radio deſcriptum a centro S, & r radium correſpondentem, quæcunque ſit Curva cujus Æquatio inveſtigatur, erit Æquatio Curvæ BL, $\dot{z} : \dot{y} :: a^n : r^n$; Æquatio vero Curvæ DP, $\dot{z} : \dot{y} :: a^{\frac{n}{n+1}} : r^{\frac{n}{n+1}}$. Angulus autem PSp erit

ad

ad Angulum LSl ut $\frac{pn}{SP}$ ad $\frac{lo}{SL}$, sive ut $\frac{Pn}{SP}$ ad $\frac{Lo}{SL}$, vel (si SP dicatur x, & SL, r) ut $\frac{\dot{x}}{x}$ ad $\frac{\dot{r}}{r}$, hoc est, (ob $x = \frac{r^{n+1}}{a^n}$) ut $\frac{n+1}{r}\dot{r}$ ad $\frac{\dot{r}}{r}$, sive ut $n+1$ ad 1. Hinc BSP est ad BSL ut $n+1$ ad 1; unde facilius absque Tangentium ope duci potest Curva BP. Si sumatur angulus BSP ad BSL in ratione $n+1$ ad 1, & in SP demittatur perpendicularis ex L, erit occursus perpendiculi cum SP, in Curva BP prius Tangentium ope descripta. *Fig.* 10.

§ 3. Ostendimus quo pacto ex una series Curvarum infinita deducitur; quo vero pacto singularum longitudines ex illius & unius alterius longitudinibus datis innotescant pergo demonstrare. Cum angulus $SPp = SLl$, atque LSl sit ad PSp ut 1 ad $n+1$, erit Ll ad Pp ut SL ad $\overline{n+1}\,SP$, sive (ob $SL : SP :: Ll : lo$) ut Ll ad $\overline{n+1}\,lo$, ac proinde $Pp = \overline{n+1}\,lo$: sed $lo = ln - on = ln - LN + Nn$; ergo $Pp = \overline{n+1} \times \overline{ln - LN + Nn}$. Sed $ln - LN$ est momentum rectæ LN normalis in SL, Pp momentum Curvæ BP, & Nn momentum Curvæ BN: Cumque BP, BN, BL simul evanescant in B, erunt in ratione momentorum, adeoque $BP = \overline{n+1} \times \overline{BN + \text{vel} - LN}$. Unde Curva BP est ad summam vel differentiam Curvæ penultimæ in Serie ejusque Tangentis ab intermedia interceptæ, ut $n+1$ ad 1; sive, si m sit Index æquationis Curvæ BP (quoniam $m = \frac{n}{n+1}$) ut 1 ad $1-m$. *Fig.* 10.

Hinc 1^mo^ in serie Curvarum infinita supra descripta, si dentur Longitudines duarum proximarum, dabuntur longitudines omnium; quippe mensura cujusvis pendet a mensura penultimæ semper in serie, & proinde unum par omnibus mensurandis sufficiet: Si una Curva sit rectis commensurabilis vel incommensurabilis, erit integræ seriei pars dimidia rectis commensurabilis vel incommensurabilis. Hinc 2^do^. Licet Curvæ BP & BN essent rectis incommensurabiles, differentia tamen Curvæ BP ab $n+1$ Curvæ BN esset æqualis assignabili rectæ. 3^tio^. Si Curva transit per S, recta LN evanescente in S, erit $BPS = \frac{BNS}{1-m}$.

§ 4. Curvarum de quibus egimus, quarum nimirum $\dot{s} : \dot{y} :: a^n : r^n$, maxime insignis est Circulus, existente S in circumferentia, cujus æquatio est $\dot{s} : \dot{y} :: a : r$, ut ex similitudine triangulorum Lol, BLS manifestum est, adeoque $n = 1$; & proinde $m = \frac{n}{n+1} = \frac{1}{2}$, & æquatio Curvæ BP erit $\dot{s} : \dot{y} :: a^{\frac{1}{2}} : r^{\frac{1}{2}}$, quæ ipsa est æquatio *Epicycloidis* revolutione Circuli super basim sibi æqualem revolventis descripti, ad punctum ubi punctum describens tangit basim, quæ D^no^ *Paschal* dicitur *la Limaçon de M. Roberval*, quamque *M. De la Hire* considerat ut *Conchoidem* Basis Circularis, in Actis Academiæ Parisiensis Anni 1708. Perpendiculares omnes LN, ln concurrunt in puncto B, adeoque $BN = 0$: unde $BP =$ *Fig.* 11.

$\frac{BN+NL}{1-m} = 2BL$: Hinc Curva tota $BPS = 2BS$, ac longitudo *Epicycloidis* ſemper dupla eſt chordæ arcus in circulo correſpondentis. 2^{do}. Ex *Epicycloide* deſcribatur Curva $B\pi S$, eadem ratione qua *Epicycloidem* ex Circulo deſcripſimus: In hoc caſu $n=\frac{1}{2}$, & $m = \frac{n}{n+1} = \frac{\frac{1}{2}}{\frac{1}{2}+1} = \frac{1}{3}$, ac proinde æquatio Curvæ $B\pi S$ erit $\dot{s} : \dot{y} :: a^{\frac{1}{3}} : r^{\frac{1}{3}}$. Longitudo Curvæ erit $\frac{BL+LP}{1-m} = \frac{3}{2}\,\overline{BL+LP} = \frac{3}{2}\,\overline{BL+LG}$, & proinde $B\pi$ eſt ſeſquiplus ſummæ Arcus circularis ejuſque Sinus recti. Quod ſi ſumatur $CD = BD$, & radio SD centro S deſcribatur Circulus occurrens rectæ SP in H, & ſit HK perpendicularis in BS; quoniam $DH = \frac{3}{2}BL$, erit $B\pi = DH + HK$. Hinc arcus $B\pi$ neque ſunt rectis neque arcubus circularibus commenſurabiles, differentia tamen arcuum $B\pi$ & DH eſt recta HK. In puncto S evaneſcit LG, adeoque $B\pi S = \frac{3}{2}BLS$, unde tota Curva eſt ſeſcupla ſemicirculi. Nulla vero pars hujus Curvæ aſſignabilis commenſurari poteſt toti, nec integra Curva in data quavis ratione ſecabilis eſt, ita ut portiones rationem aſſignabilem habeant ad ſe mutuo aut ad totam. Si hæc curva in data aliqua ratione Geometrice ſecari poſſet, conſtaret Quadratura Circuli, nam ſi *e. gr.* eſſet $B\pi$ ad $B\pi S$ ut 1 ad m, & BL ad BLS ut 1 ad n, eſſet $B\pi = \frac{B\pi S}{m} = \frac{3BLS}{2m} = \frac{3nBL}{2m} = \frac{3}{2}\,\overline{BL+LG}$, unde eſſet $BL = \frac{mLG}{n-m}$ & $BLS = \frac{nm}{n-m}LG$. 3^{tio} Ex $B\pi S$ conſtruatur explicata methodo Curva BR, & quoniam $n=\frac{1}{3}$ erit $m = \frac{n}{n+1} = \frac{1}{4}$, atque æquatio Curvæ BR erit $\dot{s} : \dot{y} :: a^{\frac{1}{4}} : r^{\frac{1}{4}}$. Hinc longitudo Curvæ fiet $\frac{4}{3}\,\overline{2BL+P\pi}$, totalis vero Longitudo Curvæ $BRS = \frac{8}{3}$ diametri SB. Si harum Curvarum Conſtructiones continuentur, prodibit hujuſmodi ſeries Æquationum quæ facile producitur ad libitum.

Æquatio Circuli	1.	$\dot{s} : \dot{y} :: a : r$
Epicycloidis	2.	$\dot{s} : \dot{y} :: a^{\frac{1}{2}} : r^{\frac{1}{2}}$
Secundi	3.	$\dot{s} : \dot{y} :: a^{\frac{1}{3}} : r^{\frac{1}{3}}$
Tertii	4.	$\dot{s} : \dot{y} :: a^{\frac{1}{4}} : r^{\frac{1}{4}}$
Cujuſvis	n.	$\dot{s} : \dot{y} :: a^{\frac{1}{n}} : r^{\frac{1}{n}}$, &c.

Obſervare licet in genere, omnes quarum Indicum denominatores ſunt Numeri pares, perfectæ rectificationis eſſe capaces; cumque quævis ſit ad penultimam ut 1 ad $1-m$, perpendenti manifeſtum erit Curvæ cujuſvis longitudinem fore $= \frac{1}{1-m} \times \frac{1-2m}{1-3m} \times \frac{1-4m}{1-5m} \times \frac{1-6m}{1-7m}$, &c. $\times SB$

conti-

continuando seriem donec ad nihilum reducatur Fractio. Quod si Indicis denominator sit Numerus impar, Curvæ erunt perfectæ rectificationis incapaces, & earum arcus quicunque erunt sibi mutuo, ipsis totis, rectis quibusvis & arcubus Circularibus incommensurabiles: exprimi vero possunt omnes arcubus circularibus & rectis: At Curvæ cujusvis totalis Longitudo erit ad Semicirculum ut $\frac{1}{1-m} \times \frac{1-2m}{1-3m} \times \frac{1-4m}{1-5m}$, &c. ad unitatem. Denique si Areola a Corpore in harum quavis revolvente sumatur constans, hoc est si $r\dot{y} = 1$, subtensa anguli contactus, cui semper (ob datum data area tempus) proportionalis est *Vis Centripeta* tendens ad S, erit reciproce ut potestas distantiæ cujus Index est $2m+3$; atque hoc est non contemnendum harum Curvarum privilegium, quod in iis omnibus Vis centripeta tendens ad S sit ut aliqua reciproca distantiæ dignitas, quæ simplicissima est, & utilissima, in Naturæ indagine, Virium Centripetarum lex.

§ 5. Curvarum quarum $\dot{z} : \dot{y} :: a^n : r^n$ proxime consideranda venit (quæ Curva quidem improprie dicitur) ipsa Linea recta, existente S extra rectam. In hac linea, ob similia triangula Ppn, PBS erit (si $BS = a$ & $SP = r$) $\dot{z} : \dot{y} :: r : a$. Ex linea recta methodo directa nihil nisi punctum *Fig.* 12.
B construi potest, Methodo vero inversa, perpendicularium nimirum PL, pl concursu, construi potest Curva, cujus Index (si m sit Index Curvæ BP) æqualis erit $\frac{m}{1-m}$; nam si Index Curvæ BL sit n, erit $m = \frac{n}{n+1}$, ac proinde $n = \frac{m}{1-m}$. Unde in hoc casu, cum $m = -1$ erit $n = \frac{-1}{2}$, & æquatio Curvæ BL erit $\dot{z} : \dot{y} :: r^{\frac{1}{2}} : a^{\frac{1}{2}}$, quæ æquatio est Parabolæ ad Focum. Ex hac construe aliam, constituendo angulum $LSN = LSB$ & erigendo LN normalem in SL occurrentem ipsi SN in N. Quoniam vero $m = \frac{-1}{2}$ erit $n = \frac{-1}{3}$, & æquatio Curvæ $\dot{z} : \dot{y} :: r^{\frac{1}{3}} : a^{\frac{1}{3}}$ & $BP = \frac{BN-LN}{1-m} = \frac{1}{2}\overline{BN-LN}$, ergo $BN = 2BP + LN$; proinde hæc Curva est rectificabilis. Si Series continuetur, prodibunt ut prius æquationes in hoc ordine.

Æquatio	Rectæ	$\dot{z} : \dot{y} :: r : a$
	Parabolæ	$\dot{z} : \dot{y} :: r^{\frac{1}{2}} : a^{\frac{1}{2}}$
	Secundæ	$\dot{z} : \dot{y} :: r^{\frac{1}{3}} : a^{\frac{1}{3}}$
	Tertiæ	$\dot{z} : \dot{y} :: r^{\frac{1}{4}} : a^{\frac{1}{4}}$
	Cujusvis	$\dot{z} : \dot{y} :: r^{\frac{1}{n}} : a^{\frac{1}{n}}$

In hac Serie primæ ſunt Recta & Parabola, unde patet dimidiam hujus ſimiliter ac prioris Seriei eſſe rectis menſurabilem: alia vero dimidia pars in rectis & arcubus Parabolicis exhiberi poteſt. In his omnibus Vis centripeta ad S eſt reciproce ut poteſtas diſtantiæ cujus Index $3-2m$, ac proinde ſemper inter duplicatam & triplicatam rationem diſtantiæ reciproce.

§ 6. Æquatio Hyperbolæ æquilateræ ad centrum eſt $\dot{s} : \dot{y} :: r^2 : a^2$, ex qua deducitur methodo directa Series hujuſmodi,

$$1.\ \dot{s} : \dot{y} :: r^2 : a^2$$
$$2.\ \dot{s} : \dot{y} :: a^2 : r^2$$
$$3.\ \dot{s} : \dot{y} :: a^{\frac{2}{3}} : r^{\frac{2}{3}}$$
$$4.\ \dot{s} : \dot{y} :: a^{\frac{2}{5}} : r^{\frac{2}{5}}$$
$$5.\ \dot{s} : \dot{y} :: a^{\frac{2}{2n-1}} : r^{\frac{2}{2n-1}}$$

Ex his Curvæ, quarum Indicum denominatores ſunt in progreſſione $-1, 3, 7, 11,$ &c. exhiberi poſſunt in rectis & arcubus Hyperbolicis; reliquæ vero in rectis & arcubus Curvæ, cujus æquatio ad axem SB (ſi x Fig. 11. ſit abſciſſa, y vero Ordinata) eſt $\overline{xx+yy}|^2 = a^2x^2 - a^2y^2$, quæque conſtruitur biſecando angulum BSL & ſumendo SN mediam proportionalem inter SB & SL.

Curvæ quæ ex Hyperbola methodo inverſa conſtrui poſſunt progrediuntur in hac Serie,

$$\text{Hyperbolæ } 1.\ \dot{s} : \dot{y} :: r^2 : a^2$$
$$2.\ \dot{s} : \dot{y} :: r^{\frac{2}{3}} : a^{\frac{2}{3}}$$
$$3.\ \dot{s} : \dot{y} :: r^{\frac{2}{5}} : a^{\frac{2}{5}},\ \&c.$$

Ubi Curvæ quarum Indicum denominatores ſunt in progreſſione 1, 5, 9, 13, &c. exprimi poſſunt in rectis & arcubus Hyperbolicis; reliquæ vero in rectis & arcubus Curvæ modo explicatæ.

Si aliæ Curvæ deſiderentur quæ alias exhiberent Series, id facillime fieri poteſt ope vel Circuli vel Rectæ: quippe ex earum aliqua omnes, in quibus $\dot{s} : \dot{y} :: a^n : r^n$, conſtrui poſſunt; ſumendo, ſi ope Circuli Problema Fig. 13. ſit ſolvendum, BSR ad BSL ut 1 ad n, & SN in ipſa $SR = a^{\frac{n-1}{n}} \times SL^{\frac{1}{m}}$; quippe Curvæ per omnia puncta N ductæ æquatio erit $\dot{s} : \dot{y} :: a^n : r^n$. Similiter ope Rectæ conſtrui poſſunt quarum æquatio eſt $\dot{s} : \dot{y} :: r^n : a^n$.

Duas exhibuimus Series infinitas Curvarum rectis commenſurabilium; aliam arcubus circularibus, aliam Parabolicis, aliam Hyperbolicis una cum rectis menſurabiles demonſtravimus; eæ vero ad rectarum menſuram arte ſola infinita reduci poſſe videntur, ſicut æquatione ſola infinita in rectis exprimuntur.

IX. Inter

IX. Inter innumera sublimiaque Magni *Newtoni* inventa, quibus Geometria amplissime ditata in immensam excrevit luculentissimæ Cognitionis molem, Constructionem exhibuit Curvarum Mechanicam, post Enumerationem Linearum Tertii Ordinis, ad finem *Opticæ* editam, arduo summi Viri ingenio dignam; qua simpliciorem & simul adeo Universalem aliam exhibuit Nemo. Methodum vero suam ad Curvas Tertii Ordinis puncto duplice carentes, aut eas altioris Ordinis puncto multiplice destitutas, non extendit; earumque descriptionem Problematibus Geometriæ difficilioribus annumerandam pronuntiat. Atque hinc in spem venio Methodum sequentem, qua Curvæ Geometricæ cujuscunque Ordinis, licet puncto duplice aut multiplice quovis destitutæ, construuntur, non fore Geometris ingratam.

To describe all sorts of Curves mechanically with the help only of Angles and right Lines, by the same, n. 359. p. 939.

1. Lineæ primi Ordinis ipsæ sunt Rectæ; quæ in uno solo puncto sibi mutuo occurrere possunt. Lineæ secundi Ordinis sunt Sectiones Conicæ, quæ in pluribus punctis quam duobus a recta quavis secari non possunt. Eæ vero omnes secundum Lemma 21. Lib. I. *Princip.* D. *Newtoni* sic construi possunt, Circa data duo puncta C & S moveantur Anguli dati MCR, LSN; ita ut Crurum CM SL concursus semper ducatur per rectam indefinitam positione datam AE; tunc crurum aliorum CR & SN concursus in P describet Lineam secundi Ordinis seu Sectionem Conicam. *Fig.* 14.

2. Moveatur ut prius Angulus MCR circa datum punctum C; Angulus vero datus LNQ semper percurrat Angulari suo puncto N rectam datam AE, ita ut crus NQ semper transeat per datum punctum S. 1. Si concursus crurum CR & SN, tum punctum Q ducatur per rectam infinitam AB, concursus crurum CM & NL describet Curvam lineam Tertii Ordinis punctum duplex habentem in C. 2. Reliquis manentibus, si crurum CM & NL concursus ducatur per rectam indefinitam AB: concursus crurum CR & SN in P describet Curvam Tertii Ordinis punctum duplex habentem in S. *Fig.* 15. *Fig.* 16.

Exemplum Casus 1. Sint anguli MCR, LNS recti, & AE, DB, CS parallelæ; sint quoque SA & SD normales respective in rectas AE & DB; sitque $SD = 2SA$. Hisce positis, si SD sit minor recta CS, Curva secundum regulam Casus primi descripta, erit Parabola Nodata cum Ovali, Speciei 68*væ* Curvarum D. *Newtoni*: Quod si $SD = CS$, Ovalis evanescit & nodus evadit Cuspis, atque Curva descripta erit Parabola *Neiliana* seu semicubica; Si vero sit SD major quam CS, erit Curva Parabola punctata Campaniformis Speciei 69*næ*. *Fig.* 17.

3. Moveantur Anguli dati RMT, KNL, ita ut puncta M & N percurrant rectas indefinitas BM, DN respective; & crura RM, KN semper transeant per data puncta C & S. Si primo Crurum MT & NL concursus Q ducatur per rectam indefinitam AQ; tunc concursus crurum MR & NR in P describet lineam Quarti Ordinis puncta duo duplicia habentem, alterum in C, alterum vero in S. Sed secundo si crurum MR & NK concursus ducatur per rectam indefinitam AQ; tunc concursus cru- *Fig.* 18. *Fig.* 19.

rum MT & NL deſcribet Lineam Quarti Ordinis puncto duplice carentem.

4. Quod ſi in primo caſu hujus Conſtructionis rectæ CMR, SNK, una coincidant cum CS; tunc puncta C & S evadunt ſimplicia, & Curva erit Tertii Ordinis abſque puncto duplice. *Exemplum.* Sint rectæ BM, AQ, DN, ſibi mutuo parallelæ atque omnes perpendiculares in CS. Sint quoque Anguli RMT, KNL recti, & ſi ſecundum regulam primi Caſus deſcribatur Curva, Crura CMR, SNK una coincident cum CS; & hac conſtructione deſcribi poſſunt Curvæ D. *Newtoni* 10, 11, 20, 21, 40, ſecundum varias poſitiones punctorum C & S reſpectu trium rectarum BM, AQ, DN; Omnes vero hæ Species puncto duplice carent.

Fig. 18.

Fig. 20.

5. Lineæ vero Quarti Ordinis quæ punctum triplex habent ſic conſtrui poſſunt. Sint tres rectæ AQ, BN, DM poſitione datæ; ſint etiam Anguli QCT, SNM & NML dati & invariabiles; percurrant puncta N & M rectas BN & DM, ita ut crus NQ ſemper tranſeat per datum punctum S: Revolvatur QCT circa C ita ut concurſus crurum CK, SN percurrat tertiam rectam AQ; tunc concurſus crurum CT, ML deſcribet Lineam Quarti ordinis punctum triplex habentem in C.

Fig. 21.

6. Oſtendi quo pacto Lineæ Quarti Ordinis deſcribi poſſunt, quæ punctum triplex habent aut duo duplicia; Aliæ quæ unicum habent punctum duplex ſic commode deſcribuntur. Sint tres rectæ ut prius poſitione datæ, AQ, BN, DM, dentur etiam Anguli SNK, SML, RCT; ſint puncta N, M & S ſemper in eadem recta linea; Moveantur puncta N & M ut prius per rectas BN, DM; Si concurſus crurum CR, NK ducatur per rectam indefinitam AQ, tunc concurſus crurum CT, ML deſcribet Lineam Quarti Ordinis habentem punctum duplex unicum in C. Hæ vero duæ ultimæ Propoſitiones novas Methodus ſuppeditant lineas Tertii Ordinis deſcribendi, tum quæ puncta duplicia habent, tum quæ iis deſtituuntur; Eæ vero in brevi hoc Methodus noſtræ ſpecimine ſunt omittendæ.

Fig. 22.

7. Maneant Anguli atque rectæ ut in *Prop.* III. Concurſus vero nunc rectarum MT, NK ducatur per indefinitam rectam AQ; & Concurſus crurum MR & NL deſcribet Lineam Quinti Ordinis punctum qnadruplex habentem in S. Habeo etiam alias Methodus curvas deſcribendi Quinti Ordinis, quæ punctum habent triplex, duplex, aut duo duplicia, vel nulla niſi puncta ſimplicia; ſed hæc ſufficiant ad ſimplicitatem & univerſalitatem Methodus demonſtrandam. Notandum vero in ſpecialibus ſimplicioribus Angulorum & Rectarum circumſtantiis, Lineam aliquando migrare in Curvam ordinis inferioris quam in *Prop.* explicatur; Imo ſingulæ Propoſitiones Methodus ſuppeditant particulares, curvas aliqnas ordinis cujuſcunque inferioris deſcribendi.

Fig. 23.

8. *Propoſitio Generalis.* Sumantur ad libitum Rectæ in eodem plano ubicunque poſitæ, quarum ſit numerus (n) ut BN, ER, FT. Sumantur etiam ad libitum aliæ rectæ ut DM, GL, & HK &c. quarum ſit numerus (m). Sint Anguli CNR, NRT, RTQ, &c. atque anguli SML, MLK, LKQ, &c. invariati, dum puncta angularia N, R, T, M, L, K, per-

Fig. 24.

percurrant rectas indefinitas BN, ER, FT, DM, GL, HK; Ducatur concursus crurum TQ & KQ per rectam indefinitam AQ; Invenire ordinem curvæ quam concursus cruris SM cum aliqua rectarum CN, NR, RT, TQ, &c. *ex. gr.* cum RT, perpetuo tanget.

In Serie rectarum CN, NR, RT, TQ, &c. denotet s numerum rectæ RT, cujus concursu cum SM Curva est describenda, a CN inclusive; qui in hoc casu est ternarius: erit Curva ordinis quem exprimit numerus $sm+s+m+1$; unde in Casu quem Figura designat, cum $s=m=n=3$, erit Curva ordinis $16^{tæ}$.

In his descriptionibus Rectas solummodo atque Angulos dari postulavimus: Sed facilius sæpe simpliciorum Curvarum ope complexiores describuntur; atque Propositiones his non minus Universales huic pertinentes investigavi: Eas vero cum harum demonstrationibus utpote prolixis impræsentiarum omitto; easdem postea publici juris facturus.

X. 1. *Invenire Curvam quam Corpus descendens brevissimo tempore describeret; urgente Vi Centripeta ad datum punctum tendente, quæ crescat vel decrescat juxta quamvis Potentiam distantiæ a Centro; dato nempe imo Curvæ puncto & altitudine in principio Casus.* *The Curve of quickest Descent, by Mr. Machin, n. 358. p. 860.*

Sit centrum Virium C, quo centro ad distantiam CB æqualem altitudini unde Corpus casurum est, describatur Circulus BEG, & fiat angulus BCG rectus. Ponatur A punctum Curvæ infimum, ubi axi CB occurrit ad datam distantiam CA. Oportet invenire punctum Q, ubi Curva celerrimi descensus EQA occurrit circulo QF, ad datam aliam distantiam CF. Problema hoc duos habet Casus, quorum alter pendet ab Hyperbola & Circulo, alter ab Ellipsi & Circulo. *Fig. 25.*

Cas. 1. Si fuerit Vis centripeta reciproce ut distantia a Centro. Sit KLM Hyperbola quævis rectangula centro C & Asymptoto CB descripta, quæ occurrat normalibus BK, AM, super ipsam BC ad puncta B, A erectis, in K & M; ordinatæ vero cuilibet intermediæ FL ad punctum F erectæ, in L. Fiat CD ad CG ut $\sqrt{AFLM}$ ad $\sqrt{ABKM}$, & sit DH normalis super CG: dein capiatur Sector RCB ad Aream $HDCB$ ut data Area Hyperbolica $ABKM$ ad datum Rectangulum $CA \times AM$. Tum recta RC occurret circulo FQ in puncto Q, quod quidem est ad Curvam celerrimi descensus EQA.

Habebitur autem punctum E, a quo inciperet Corporis casus, capiendo Sectorem BCE ad Aream Quadrantis BCG, in eadem ratione Areæ Hyperbolicæ $ABKM$ ad rectangulum sub CA & AM contentum.

Coroll. Hinc si recta RC, circa centrum C revoluta, faciat Sectores RCB proportionales Areis $HDCB$, in quibus quadrata Basium CD sumuntur in progressione Arithmetica: tum rectæ CR intersecabunt Curvam EQA ad distantias a centro CQ, quæ decrescant in progressione Geometrica.

Cas. 2. Si vero Vis centripeta fuerit reciproce ut alia quævis Potestas distantiæ a centro; sit $n+1$ Index istius Potestatis (ubi n potest esse Numerus quilibet integer vel fractus, affirmativus vel negativus) sitque $H=CB$ altitudo maxima Curvæ quæsitæ EQA, $b=CA$ altitudo minima ejusdem, & $A=CF$ altitudo alia quævis intermedia. *Fig. 26.*

In

In recta CG capiatur CD ad CB ut $\sqrt{h^n}$ ad $\sqrt{H^n}$, atque etiam CH ad CD ut $\sqrt{A^n - h^n}$ ad $\sqrt{H^n - h^n}$. Centro C, semiaxibus CD, CB, describatur Ellipsis BLD, cui occurrat ordinatim applicata HL in puncto L; & ducatur recta LK, quæ Ellipsin tangat in L, & Axi minori CD producto conveniat in K: dein Tangenti KL parallela ducatur NM, circulum $BEMG$ tangens in M & ipsi CD occurrens in N. Denique capiatur Sector RCB, qui sit ad Aream $NMBLKN$, inter Circulum & Ellipsin & utriusque Tangentes rectamque NK comprehensam, in ratione Numeri binarii ad Numerum n. Tum recta RC intersecabit Circulum FQ in puncto Q, quod erit ad Curvam celerrimi Descensus EQA.

Quod si fiat Sector BCE ad aream BDG, inter Ellipseos & Circuli Quadrantes interceptam, in ratione dicta Binarii ad Numerum n, coeuntibus scilicet punctis L, D & M, G; (ob $A^n = H^n$) erit punctum E unde inchoaretur Casus Corporis brevissimo tempore descendentis ad A, descensuque suo Curvam EQA describentis, quam tangit recta CE in E, quamque ad angulos rectos secat CB in A.

Harum Constructionum Demonstrationes e Celeberrimi D. *Newtoni Quadraturis*, ejusdemque *Philos. Nat. Principiis* (Prop. XXXIX. & sequentibus aliquibus) petitæ, alia data occasione ostendentur. Problema autem est alterius generis, Describere Curvas per quas Corpora, de puncto summo E, seu principio casus, demissa, celerrimo descensu ad inferiora data puncta Q, urgente qualibet Vi centripeta, ferrentur; cujus quidem solutio in potestate est. In præsentia sufficiat generalem hujusmodi Curvarum tradidisse Ideam, earumque ad Circuli & Hyperbolæ Quadraturas relationes indicasse, absque quibus easdem Geometrice construere haud adeo proclive est.

The same, by Mr. Craig. n. 268. p. 750.

2. *Problema. Invenire Lineam Celerrimi Descensus.*

Sint BC, CD duæ particulæ infinite parvæ in curva quæsita. Jam Curva illa debet esse talis ut transitus a B ad D post casum ab horizontali AQ fiat in tempore minimo; quærendum itaque est in linea RS (ita ad AQ parallela ut differentiæ ordinatarum GC, DE sint æquales) tale punctum C ut hoc contingat.

Jam velocitas ejus in puncto C est $\sqrt{LC}$ & velocitas in puncto D est $\sqrt{QD}$; Ergo $\frac{BC}{\sqrt{LC}}$ est tempus descensus per BC, & etiam $\frac{CD}{\sqrt{QD}}$ est tempus descensus per CD (per *Prop.* liv. *pag.* 158. *Newtoni*) Ergo punctum C debet esse tale ut $\frac{BC}{\sqrt{LC}} + \frac{CD}{\sqrt{QD}} =$ minimo. Supponendo B & D esse

Fig. 27.

fixa, sint constantes $GC = DE = m$, $LC = b$, $QD = p$; indeterminatæ $BG = u$, $CE = z$; unde $\frac{\sqrt{m^2+u^2}}{\sqrt{b}} + \frac{\sqrt{m^2+z^2}}{\sqrt{p}} =$ minimo; Ergo $\frac{u\,du}{b^{\frac{1}{2}}\sqrt{m^2+u^2}} + \frac{z\,dz}{p^{\frac{1}{2}}\sqrt{m^2+z^2}} = 0$. sed $du = -dz$ (quia $u + z$ = constanti)

Ergo

Ergo $\frac{u}{b^{\frac{1}{2}}\sqrt{m^2+u^2}} = \frac{z}{p^{\frac{1}{2}}\sqrt{m^2+z^2}}$; unde patet $\frac{u}{b^{\frac{1}{2}}\sqrt{m^2+u^2}} =$ constanti; sit jam Abscissa $AL = x$, ordinata $LC = y$; adeoque $BG = dx$, $GC = dy$, $BC = \sqrt{dx^2+dy^2}$ sitque a linea quævis constans. Erit $\frac{dx}{y^{\frac{1}{2}}\sqrt{dx^2+dy^2}} = \frac{1}{\sqrt{a}}$, unde $dx\sqrt{a} = \sqrt{y}\sqrt{dx^2+dy^2}$. Sed in omni Curva dx est ad $\sqrt{dx^2+dy^2}$ ut Subtangens ad Tangentem: Ergo talis est Natura Curvæ quæsitæ, ut ejus Subtangens sit ad Tangentem ut $\sqrt{a}$ ad $\sqrt{y}$. Quam utique Cycloidis proprietatem esse sciunt omnes, quibus notum est Tangentem Cycloidis esse parallelam Chordæ arcus contermini in Circulo genitore, cujus Diameter est a, & cujus vertex deorsum spectat.

XI. *Lemma.* Invenire rationem inter resistentiam, quam patitur Triangulum rectangulum AIG, & resistentiam quam patitur rectangulum circumscriptum $AIGg$, dum utrumque in fluido movetur juxta directionem Lineæ IA, ab I versus X. *The Solid of least Resistance, by the same.* ibid.

A puncto quovis B ducatur BC normalis ad AG; & Bb parallela ad AI, item BM normalis ad AI. Tum in bB capiantur $bH = \frac{CM^2}{BC}$ & $bE = BC$; & per puncta H, E ducantur rectæ HA, EA, quæ productæ secent Gg in K & F: Dico Resistentiam Trianguli AIG esse ad resistentiam Rectanguli $AIGg$ ut Area trianguli AKG, ad Aream Trianguli AFG. Imo & resistentiam in partem quamlibet lineæ AG ad resistentiam in partem correspondentem lineæ Ag, *exem. gra.* in AB & Ab ut Area AHB ad Aream AEB. Demonstratio pendet a Theoremate generali, quod facillime deduxi ex *Prop.* xxxv. *Newtoni, p.* 324. *Fig.* 28.

Coroll. 1. Sint jam BG, bg partes infinite parvæ linearum AG, Ag, & producatur bB ad L; dico resistentiam in BG (quam vocemus e) esse ad resistentiam in bg (quam vocemus E) ut GL^2 ad GB^2.

Nam $e:E :: KHbg:FEbg$, id est $e:E :: bg \times bH : bg \times bE$ (per Lemma præcedens) Ergo $e:E :: bH:bE$, id est $e:E :: \frac{CM^2}{BC} : BC$ (per constructionem superioris Lemmatis) Ergo $e:E :: CM^2 : BC^2$. Sed $CM^2 : BC^2 :: GL^2 : GB^2$. (ob similia Triangula BMC, GLB) Ergo $e:E :: GL^2 : GB^2$. *Q.E.D.*

Corol. 2. Resistentia in partem infinite parvam GB est æqualis Cubo lineæ GL diviso per Quadratum lineæ GB. Nam si omnes partes infinite parvæ in linea Ag ut bg supponantur æquales, tum Resistentia in bg per ipsam bg exprimi possit, id est, $E = bg$, adeoque $E = GL$. Ergo per Corollarium primum $e : GL :: GL^2 : GB^2$; unde $e = \frac{GL^3}{GB^2}$ *Q. E. D.*

Corol. 3. Sit r radius & c circumferentia cujusvis circuli, dico resistentiam in conicam superficiem genitam a rotatione lineolæ GB circa AI esse æqualem producto ex $\frac{c \times BM}{r}$ in $\frac{GL^3}{GB^2}$. Nam resistentia in Conicam illam

illam superficiem est æqualis omnibus resistentiis in lineolam GB, id est omnibus e; id est æqualis circumferentiæ circuli cujus radius est BM in e multiplicatæ; id est, resistentia in Conicam illam superficiem est æqualis $\frac{c \times BM}{r} \times e$; adeoque per *Corol.* 2. æqualis $\frac{c \times BM}{r} \times \frac{GL^3}{GB^2}$ *Q. e. D.*

Fig. 29. *Problema* Invenire Lineam curvam cujus rotatione producatur Solidum rotundum, quod (dum in medio fluido secundum axis sui directionem movetur) minimam patiatur Resistentiam.

Sint OG, GB duæ particulæ infinite parvæ in Curva quæsita, quæ circa AQ rotata producat Solidum rotundum minimæ Resistentiæ. Ducantur BM, GP normales ad AQ, item BL, GN ad AQ, & ON ad BM parallelæ. Jam $\frac{c \times BM \times GL^3}{r \times GB^2}$ est resistentia in superficiem genitam a rotatione lineolæ GB circa AQ, & $\frac{c \times GP \times ON^3}{r \times OG^2}$ est resistentia in superficiem genitam similiter ab OG per *Cor.* 3. Jam utraque hæc Resistentia simul sumpta debet esse minima scil. $\frac{c \times BM \times GL^3}{r \times GB^2} + \frac{c \times GP \times ON^3}{r \times OG^2} =$ minimæ. Adeoque in linea RS ita ad AQ parallela ut ON sit $= GL$, quærendum est punctum G ut hoc contingat; quod supponendo puncta O & B esse fixa facile invenietur per notissimam Maximorum & Minimorum Methodum. Calculum prosequendo devenietur tandem ad $\frac{BM \times BL}{BG^4} = \frac{GP \times NG}{OG^4}$; unde patet $\frac{BM \times BL}{BG^4} =$ constanti; sic si abscissa AM vocetur x, & ordinata BM, y, erit $BL = dx$, $LG = dy$ (quam constantem in toto hoc calculo supposui) adeoque $BG^2 = dx^2 + dy^2$, unde $\frac{y\,dx}{dx\,dx + dy\,dy^2} =$ constanti; Sit a linea quælibet constans, & proinde, ut observetur *Lex* homogeneorum erit $\frac{y\,dx}{dx\,dx + dy\,dy^2} = \frac{a}{dy^3}$ ut ab Illustriss. *Hospitalio* & celeber. *Jo. Bernoullio* inventum est.

The same, by Mr. Facio. n. 337. p. 172. 2. In adjuncta Figura sit C Centrum Circuli Osculantis AEF, qui cum Sectione Solidi quæsiti cujus Axis sit ST quam intime coincidat in A. Eritque hujus Circuli Radius CA vel $u = \frac{3psx}{tt} - \frac{px}{s}$: quæ erat Solutio nostra, in Tractatu hac de re *Londini* impresso.

Fig. 30. Fiat, ut prius, AS ad Solidi Axem TS perpendicularis $= x$; cujus fiat Fluxio invariatæ Magnitudinis $AB = \dot{x}$: Sitque BE ad Axem parallela $= \dot{y}$: Rursusque erigatur $EG = \dot{x}$; Et erit GF ad Axem parallela $= \dot{y} + \ddot{y}$.

Erit autem p ad t ut u seu $\frac{3psx}{tt} - \frac{px}{s}$ ad $\frac{3sx}{t} - \frac{tx}{s}$ sive $\frac{3\dot{y}x}{\dot{x}} -$

$-\frac{x\dot{x}}{\dot{y}}$; quod æquabitur ipſi n ſeu AD ad Axem parallelæ, poſita ſcilicet CD ad eundem Axem perpendiculari: quæ CD vocetur m.

Rurſus erit p ad s ut n ſeu $\frac{3psx}{tt} - \frac{px}{s}$ ad $\frac{3ssx}{tt} - x$, ſive $\frac{3\dot{y}\dot{y}x}{\dot{x}\dot{x}} - x$; quod æquabitur ipſi m ſeu CD. Cujus Valorem dediſſe otioſum quidem hic eſt, ſed uſum habebit in ſequentibus.

Jam vero ex Oſculantis Circuli AEF Proprietate habebitur, productis ipſis BA, BE ad alteram uſque Circumferentiæ partem, $\dot{x} \times \overline{2m + \dot{x}} = \dot{y} \times \overline{2n - \dot{y}}$.

Rurſuſque ex ejuſdem Circuli Proprietate habebitur, productis ipſis GE, GF ad alteram uſque Circumferentiæ Partem, $\dot{x} \times \overline{2m + 3\dot{x}} = \overline{\dot{y} + \ddot{y}} \times \overline{2n - 3\dot{y} - \ddot{y}}$.

Ergo, ſubducta priori hac Æquatione a poſteriori, erit $2\dot{x}\dot{x} = -2\dot{y}\dot{y} - \dot{y}\dot{y} + 2n\ddot{y} - 3\dot{y}\ddot{y} - \ddot{y}\ddot{y}$.

Deletiſque Terminis infinite minoribus quam ſint reliqui, erit $2\dot{x}\dot{x} = -2\dot{y}\dot{y} + 2n\ddot{y}$.

Subſtitutoque ipſius n Valore, erit $\dot{x}\dot{x} = -\dot{y}\dot{y} + \frac{3x\dot{y}\ddot{y}}{\dot{x}} - \frac{x\dot{x}\ddot{y}}{\dot{y}}$: Id eſt $\underset{*}{\dot{x}^3\dot{y}} + \underset{**}{\dot{x}\dot{y}^3} - \underset{**}{3x\dot{y}\dot{y}}\ddot{y} + \underset{*}{x\dot{x}\dot{x}}\ddot{y} = 0$.

Componitur hæc Æquatio ex ſolis Indeterminatis x, $\dot{y}$, earumque Fluxionibus $\dot{x}$, $\ddot{y}$, invariabilique Quantitate $\dot{x}$, Quantitatibuſve Coefficientibus datis. Bina autem ſunt Paria Terminorum in quibus occurrunt eædem utrinque literæ, literarumque Poteſtates, niſi quatenus Quantitas Fluens per Literam unam expreſſa in Fluxionem convertitur, vel Fluxio in Fluentem. Quæ Paria Terminorum ſunt $\dot{x}^3\dot{y} + x\dot{x}\dot{x}\ddot{y}$, & $\dot{x}\dot{y}^3 - 3x\dot{y}\dot{y}\ddot{y}$; ex Terminis utique Generatoribus duobus duntaxat orta. In tota enim Æquatione nihil obſtat quominus ipſa transformetur ſcilicet Multiplicatione facta in $x^{\kappa}\dot{y}^{\lambda}$, determinatis rite ipſis Indicibus κ & λ, ut ea ratione nova Æquatio proveniens tractabilis evadat.

Ergo juxta noſtram iſtarum Transformationum Theoriam, in Generatore ex quo exoritur Terminorum Par primum unico Aſteriſco notatum, erit Numerus Dimenſionum Indeterminatæ x ad Numerum Dimenſionum Indeterminatæ $\dot{y}$, id eſt, Erit $1 + \kappa$ ad $1 + \lambda$, ut Coefficiens 1 in Termino $\dot{x}^3\dot{y}$ ad Coefficientem 1 in Termino $x\dot{x}\dot{x}\ddot{y}$. Rurſus in Generatore, ex quo exoritur Terminorum Par alterum Aſteriſco duplici notatum, Erit Numerus Dimenſionum Indeterminatæ x ad Numerum Dimenſionum Indeterminatæ $\dot{y}$, id eſt, Erit $1 + \kappa$ ad $1 + \lambda$, ut Coefficiens 1 in Termino $\dot{x}\dot{y}^3$ ad Coefficientem -3 in Termino $-3x\dot{y}\dot{y}\ddot{y}$; unde fit $\kappa = -\frac{3}{2}$, & $\lambda = -\frac{3}{2}$; ac proinde Multiplicator $x^{\kappa}\dot{y}^{\lambda} = x^{-\frac{3}{2}} \times \dot{y}^{-\frac{3}{2}}$.

Erit igitur $-x^{-\frac{1}{2}}\dot{x}^2\dot{y}^{-\frac{1}{2}} - x^{-\frac{1}{2}}\dot{y}^{\frac{3}{2}} = \pm q$ Æquatio ſuperioris Æquationis per $x^{-\frac{3}{2}}\dot{y}^{-\frac{3}{2}}$ multiplicatæ Generatrix: Eſt autem q Quantitas determinata. Quam Æquationem Generatricem, (Fluentem autem

vocant alii) si quadraveris, ut tollantur Radices, proveniet $x^{-1}\dot{x}^4\dot{y}^{-1} + 2x^{-1}\dot{x}^2\dot{y} + x^{-1}\dot{y}^3 = qq$; id est $\frac{\dot{x}^4 + 2\dot{x}^2\dot{y}^2 + \dot{y}^4}{x\dot{y}} = qq$. Quæ ipsa est Æquatio *Newtoniana*, quam & *Joh. Bernoullius* invenit, & ipse ego antehac erui, facillima omnium quæ sperari possint ejusdem Æquationis Investigatione. Determinatur autem Quantitas qq, vel datis Positione Axe indefinito *TS*, Puncto *A*, & Solidi Tangente in *A*; vel datis Positione Puncto *A*, Centro Osculantis circuli *C*, & *AD* ad Axem Solidi parallela.

A Problem propos'd in the Leipsick Acts, solv'd generally. by ---. n. 347. p. 399.

XII. Problema. *Quæritur Methodus generalis inveniendi Seriem Curvarum, quæ Curvas in serie alia quacumque data constitutas, ad angulum vel datum vel data lege variabilem secabunt.*

Solutio. Natura Curvarum secandarum dat Tangentes earundem ad intersectionum puncta quæcumque; & anguli intersectionum dant perpendicula Curvarum secantium; & perpendicula duo coeuntia, per concursum suum ultimum, dant centrum Curvaminis Curvæ secantis ad punctum intersectionis cujuscumque. Ducatur Abscissa in situ quocumque commodo, & sit ejus Fluxio Unitas; & positio perpendiculi dabit Fluxionem primam Ordinatæ ad Curvam quæsitam pertinentis; & Curvamen hujus Curvæ dabit Fluxionem secundam ejusdem Ordinatæ. Et sic Problema semper deducetur ad æquationes. Quod erat faciendum.

Scholium. Non hujus sed alius est methodi æquationes reducere, & indeterminatas separare, absolute si fieri possit, sin minus per Series infinitas.

Another, solv'd, by Dr. Taylor. n. 354. p. 696.

Problema supra scriptum ediderunt Cl. V. *Leibnitius* & *Bernoulli*, quo vires Geometrarum Anglorum experirentur in Methodo Fluxionum, cujus Inventionem sibi appropriare conatus est ipse *Leibnitius*. Solutum est ab Anonymo nostrate methodo generali supra exposita. Illa vero minime contenti fuerunt *Leibnitius* & Fautores ejus, quin illam derisui habuere quasi qui illam excogitaverat non potuisset eam ad casum specialem applicare. *Si non viderint quomodo ex illa æquationes sunt deducendæ*, dicit Cl. *Taylor*, *id profecto illorum imperitiæ tribuendum erit.* Paulo ante *Leibnitii* obitum prodiit Problema sequens Problematis illius generalis Casus particularis.

Fig. 31.

XIII. Problema. *Super recta* A G *tanquam axe, ex puncto* A *educere infinitas Curvas, qualis est* A B D, *ejus naturæ, ut radii Osculi, in singulis punctis* B *& ubique ducti,* B O *secentur ab axe* A G *in* C, *in data ratione, ut nempe sit* B O *ad* B C *ut* 1 *ad* n.

Deinde construendæ sunt Trajectoriæ E B F *primas Curvas* A B D *normaliter secantes.*

Inven-

Inventio Curvarum secandarum ABD. *Solutionis pars prima.*

1. Ducta ordinata BH ad axem AG normali, sint, Abscissa $AH = z$, Ordinata $HB = x$, Curva $AB = v$. Tum per Methodum Fluxionum directam erit $BC = \frac{\dot{v}}{\dot{z}} x$, & fluente uniformiter v, $BO = \frac{\dot{v}\dot{x}}{\ddot{z}}$. Unde per conditionem Problematis fit $BO\left(\frac{\dot{v}\dot{x}}{\ddot{z}}\right) : BC\left(\frac{\dot{v}}{\dot{z}} x\right) :: 1 : n$; adeoque $\dot{z}\dot{x} - n\ddot{z}x = 0$.

2. Collata hac æquatione cum formula Fluxionum secunda, in calce *Prop.* 6. Methodi Incrementorum, invenitur $\dot{z}x^{-n} = \dot{v}\alpha^{-n}$; existente α linea data, per cujus valorem potest Curva ABD accommodari conditioni alicui Problemati annexæ.

3. Pro $\dot{v}$ scripto ipsius valore $\sqrt{\dot{x}^2 + \dot{z}^2}$, migrat æquatio $\dot{z}x^{-n} = \dot{v}\alpha^{-n}$ in hanc $\dot{z} = \frac{\dot{x}x^n}{\sqrt{\alpha^{2n} - x^{2n}}}$. Unde datur z ex data x, per quadraturam Curvæ cujus abscissa existente x est ordinata $\frac{x^n}{\sqrt{\alpha^{2n} - x^{2n}}}$.

4. Sint σ & τ numeri integri, vel affirmativi vel negativi, tales ut sit Curvarum isto modo provenientium simplicissima, ea cujus est Abscissa y, & Ordinata $y^{\frac{1-n+2\sigma n}{2n}} \times \overline{\alpha - y}|^{\tau - \frac{1}{2}}$; tum erit ea omnium Curvarum simplicissima, per quarum Quadraturam datur Abscissa z ex data Ordinata x.

5. Est Curva ABD Geometrica, quoties pro n sumitur reciprocum numeri cujusvis imparis.

6. In prædictis Curvam ABD consideravimus ut versus axem AG concavam, quo in casu maxima ordinata x æqualis est lineæ datæ α, quam Parametrum Curvæ commode vocare licet. Et in hoc casu Curva actu occurret Axi. Unde fluente ipsius $\frac{\dot{x}x^n}{\sqrt{\alpha^{2n} - x^{2n}}}$ debite sumpta, hoc est, ita ut simul evanescant z & x, transibit Curva per punctum datum A, sicut postulat Problema.

7. Sed si quæratur Curva ABD, quæ sit versus axem convexa, ad eundem modum pervenietur ad æquationem $\dot{z} = \frac{\alpha^n \dot{x}}{\sqrt{x^{2n} - \alpha^{2n}}}$; quæ etiam ex æquatione priori derivari potest mutando signum ipsius n. Et in hoc casu est curva ABD Geometrica, quoties pro n sumitur reciprocum cujusvis numeri paris. In hoc vero casu Ordinata omnium minima x æqualis est Parametro α; adeoque Curva nusquam occurrit Axi. Quare limitatur Problema ad casum priorem.

8. Ex præmissis facile colligitur Curvas omnes ABD esse inter se similes, & circa punctum datum A similiter positas, lateribus earum homologis existentibus proportionalibus Parametris α.

Inventio Curvæ secantis. *Solutionis pars altera.*

9. Ex § 2. fit $\dot{v} : \dot{z} :: \alpha^n : x^n$. Sed est $BC : BH :: \dot{v} : \dot{z}$, Unde fit $BC : BH :: \alpha^n : x^n$. Ex conditione vero Problematis est BC tangens

 Curvæ

Curvæ quæſitæ EBF. Quare ſi jam ſumantur $AH\ (z)$ & $BH\ (x)$ pro coordinatis Curvæ EBF, Curva ipſa EB exiſtente r, erit, per Meth. Flux. direct. $\dot{r} : -\dot{x} :: (BC : BH ::)\ a^n : x^n$. Unde fit $\frac{x^n}{a^n} = \frac{-\dot{x}}{\dot{r}}$.

10. In Curva ABD finge æquationem $\dot{z} = \frac{\dot{x}x^n}{\sqrt{a^{2n} - x^{2n}}}$ transformari in æquationem ſignis radicalibus non affectam $\dot{z} = A\dot{x}\frac{x^n}{a^n} + B\dot{x}\frac{x^{3n}}{a^{3n}} +$ &c. Tum regrediendo ad Fluentes fiet $z = \frac{1}{n+1}A\frac{x^{n+1}}{a^n} + \frac{1}{3n+1}B\frac{x^{3n+1}}{a^{3n}}$ $+$, &c. coefficiente nova introducta nulla, quoniam per conditionem Problematis debent ſimul naſci z & x. Hinc vice $\frac{x^n}{a^2}$ ſubſtituto ipſius valore $\frac{-\dot{x}}{\dot{r}}$ in § 9 invento, fit $z = \frac{1}{n+1}Ax\frac{-\dot{x}}{\dot{r}} + \frac{1}{3n+1}Bx\frac{-\dot{x}^3}{\dot{r}^3} +$ &c. quæ æquatio fluxionalis eſt primi gradus ad Curvam quæſitam EBF. Revocatur autem ad formulam ſimpliciorem in terminis numero finitis, modo ſequenti.

11. Fluat uniformiter r, & exiſtente a quantitate non fluente, ſit $\frac{-\dot{x}}{\dot{r}} = \frac{s^n}{a^n}$. Subſtituto hoc valore ipſius $\frac{-\dot{x}}{\dot{r}}$ in æquatione noviſſime inventa, atque ducta æquatione in $\frac{s}{x}$, transformatur ea in hanc $\frac{zs}{x} =$ $\frac{1}{n+1}A\frac{s^{n+1}}{a^n} + \frac{1}{3n+1} \times B\frac{s^{3n+1}}{a^{3n}} +$ &c. Unde capiendo Fluxiones fit $\frac{\dot{s}zx + s\dot{z}x - sz\dot{x}}{x^2} = A\dot{s}\frac{s^n}{a^n} + B\dot{s}\frac{s^{3n}}{a^{3n}} = \frac{\dot{s}s^n}{\sqrt{a^{2n} - s^{2n}}}$. Quod ultimum conſtat ex Analogia Serierum $A\dot{x}\frac{x^n}{a^n} +$ &c. & $A\dot{s}\frac{s^n}{a^n} +$ &c.

Hinc pro s & $\dot{s}$ ſubſtitutis eorum valoribus ex æquatione $\frac{-\dot{x}}{\dot{r}} = \frac{s^n}{a^n}$ collectis, elicitur æquatio $n\dot{x}^2\dot{z}z - \ddot{x}x\dot{z}z - n\dot{x}x\dot{z}^2 - \ddot{x}\dot{x}x^2 = 0$. Quæ ad Fluxiones primas revocatur modo ſequenti.

12. In termino ultimo $-\ddot{x}\dot{x}x^2$ vice $\ddot{x}\dot{x}$ ſcripto ipſius valore $-\ddot{x}\dot{z}$, & æquatione deinde applicata ad $\dot{z}$, fit $n\dot{x}^2z - \ddot{x}xz - nx\dot{x}\dot{z} + xx\ddot{z} = 0$. Quæ æquatio in x^{-n-1} ducta eſt Fluxio æquationis $-\dot{x}x^{-n}z + x^{1-n}\dot{z} = a^{1-n}\dot{r}$; exiſtentibus a & $\dot{r}$ non fluentibus. Eſt ergo $-\dot{x}x^{-n}z + x^{1-n}\dot{z} = a^{1-n}\dot{r}$, ſeu $\overline{\dot{z}x - z\dot{x}} \times a^{n-1} = \dot{x}x^n$, æquatio fluxionalis primi gradus ad Curvam quæſitam EBF.

13. In iſta autem æquatione eſt a valor Ordinatæ BH, quando incidit punctum H in punctum A.

14. Haud proclive eſt æquationem $\overline{\dot{z}x - z\dot{x}} \times a^{n-1} = \dot{r}x^n$, manente n in terminis generalibus, revocare ad æquationem Fluentes tantum in-

vol-

volventem, vel ad quadraturam Curvarum. Sed puncta curvæ EBF poſſunt commode inveniri per deſcriptionem Curvæ ABD, & Curvæ cujuſdam Geometricæ. Per Geometricam hic intelligo Curvam, cujus æquationem non ingrediuntur Fluxiones, nec fluentes in Indicibus dignitatum. Secetur enim Curva ABD, cujus Parameter ſit α, in B, a Curva geometrica cujus æquatio eſt $a\,\alpha^n x^n - z\,a^n x^n = x\,a^n \sqrt{a^{2n}-x^{2n}}$; atque erit punctum illud interſectionis B ad unam ex Trajectoriis quæſitis, nempe quæ tranſit per punctum E, exiſtente $AE = a$ & normali ipſi AG.

15. Hinc ſi ABD ſit Curva Geometrica, erit etiam EBF geometrica.

Scholium. Poteſt & alio modo inveniri æquatio $\overline{\dot z x - z \dot x} \times a^{n-1} = \dot r x^n$. Nam certa quadam Analyſi quam nunc celare ſtatuo, inveni æquationem $\frac{\dot\alpha}{\alpha} = \frac{\dot r\,\dot r}{\dot z\,\dot z + \dot x\,\dot x}$. Qua comparata cum æquatione $\frac{x^n}{\alpha^n} = \frac{-\dot x}{\dot r}$ (§ 9.) eliminando α & $\dot\alpha$, tandem pervenitur ad prædictam æquationem $\overline{\dot z x - z \dot x} \times a^{n-1} = \dot r x^n$.

Exemplum. Ad demonſtrationem Solutionis noſtræ ſuffecerit exemplum ſimpliciſſimum. Sit itaque $n = 1$; quo in Caſu eſt ABD ſemicirculus diametro AG deſcriptus, atque eſt EBF item ſemicirculus deſcriptus diametro AE. Eſt autem in hoc Caſu $\frac{\dot x x^n}{\sqrt{\alpha^{2n}-x^{2n}}} = \frac{\dot x x}{\sqrt{\alpha^2-x^2}}$. Unde in § 3. fit $\dot z = \frac{\dot x x}{\sqrt{\alpha^2-x^2}}$; adeoque $z = \alpha - \sqrt{\alpha^2-x^2}$, quæ æquatio eſt ad Circulum diametro $AG = \alpha$ deſcriptum, ut fieri debuit. Item pro n ſcripto 1, æquatio $\overline{\dot z x - z \dot x} \times \alpha^{n-1} = \dot r x^n$ (§ 12.) migrat in hanc $\dot z x - z \dot x = \dot r x$. Unde exterminando $\dot r$ ope æquationis $\dot r\,\dot r = \dot x\,\dot x + \dot z\,\dot z$, fit $\frac{2\dot z z x - \dot x z^2}{x^2} = -\dot x$; adeoque regrediendo ad Fluentes $\frac{zz}{x} = -x + a$, quæ æquatio eſt ad Circulum diametro $AE = a$ deſcriptum, ut etiam fieri debuit.

XIV. *Suppono Materiam omnem diviſibilem eſſe in infinitum, eamque poſſe formam quamcunque ſeu figuram induere, & ad quamcunque tenuitatem ſeu craſſitiem quamcunque exiguam reduci.* *Of the infinite Diviſibility of Matter, by Dr.* John Keill, *n.* 339. *p.* 82.

Lemma. Data quavis materiæ quantitate, ex ea, vel ex quavis ejus parte, formari poteſt ſphera concava, cujus ſemidiameter ſit datæ rectæ æqualis.

Sit materiæ particula a^3 & data recta ſit b. Ratio peripheriæ circuli ad Radium ſit p ad r, dicatur ſemidiameter concavitatis x, & craſſities pelliculæ concavitatem ſphæræ ambientis, erit $b - x$, & Cylindrus ſphæræ circumſcriptus cujus radius eſt b erit $\frac{p \times b^3}{r}$, unde ſphæra cylindro inſcripta erit $\frac{2 \times pb^3}{3r}$, Eadem ratione ſphæra cujus radius eſt x erit $\frac{2 \times px^3}{3r}$ qua-

rum

rum differentia $\frac{2p}{3\times r} \times b^3 - x^3$ ponenda est sphæricæ lamellæ æqualis, seu materiæ particulæ datæ; hoc est erit $\frac{2p}{3r} \times \overline{b^3 - x^3} = a^3$ seu $b^3 - x^3 = \frac{3ra^3}{2p}$ unde $x^3 = b^3 - \frac{3ra^3}{2p}$ & $x = \sqrt[3]{b^3 - \frac{3ra^3}{2p}}$ adeoque crassities lamellæ sphæricæ seu $b - x$ erit $= b - \sqrt[3]{b^3 - \frac{3ra^3}{2p}}$.

Eadem ratione fieri possunt ex data materiæ quantitate Cubi *concavi*, Cylindri *concavi, vel corpora etiam alterius cujusvis figuræ concavæ, quorum latera sunt datæ rectæ æqualia.*

Theorema Primum. *Data quavis materiæ quantitate quantumvis exigua, & dato spatio quovis finito utcunque amplo; quod* v. gr. *sit cubus, qui sphæram Saturni circumscriberet; Possibile est ut materia istius Arenulæ per totum illud spatium diffundatur, atque ipsum ita adimpleat, ut nullus sit in eo porus cujus diameter datam superet lineam.*

Sit datum spatium Cubus cujus latus sit recta *AB*, diametro scil. orbitæ Saturni æqualis, deturque materiæ particula cujus quantitas sit b^3, & data recta (qua pororum diametri non majores esse debent) sit d. Dividi concipiatur recta AB in partes æquales rectæ d, quarum numerus finitus erit, cum nec recta AB ponitur infinite magna, nec recta d infinite parva: sit numerus ille n, hoc est sit $nd = AB$, adeoque erit $n^3 d^3$ æqualis cubo rectæ AB. Concipiatur item spatium datum dividi in cubos quorum singulorum latera sunt æqualia rectæ d, eritque cuborum numerus n^3, & hi cubi per spatia $efgh$ in figura representententur. Dividi porro supponatur particula b^3 in partes quarum numerus sit n^3, & in unoquoque spatio cubico ponatur una harum particularum, & hac ratione materia b^3 per omne illud spatium diffundetur. Potest præterea unaquæque ipsius b^3 particula in sua quasi cella locata in sphæram concavam formari, cujus diameter sit æqualis datæ rectæ d; unde fiet, ut sphæra quælibet proximam quamque tangat, & data materiæ particula utcunque exigua b^3 spatium datum ita adimplebit, ut nullus fiet in eo porus cujus diameter datam rectam d superat. *Q. E. D.*

Fig. 32.

Cor. Hinc dari potest corpus, cujus materia si in spatium absolute plenum redigatur; spatium illud fieri potest prioris magnitudinis pars quælibet data.

Theorema Secundum. *Possunt esse duo corpora mole æqualia, quorum materiæ quantitates sint utcunque inæquales, & datam quamvis ad se invicem obtineant rationem, pororum tamen summæ, seu spatia vacua inter corpora, ad rationem æqualitatis fere accedant.* Vel in stilo Cartesiano: *Spatium omne, quod a materia subtili intra unius corporis poros occupatur, posset esse fere æquale spatio quod a simili materia intra alterum corpus tenetur. Licet materia propria unius corporis decies millies vel centies millies superat materiam propriam alterius Corporis, & Corpora sint mole æqualia.*

Ex.

Ex. gr. Sit Digitus cubicus *Auri* & Digitus cubicus *Aeris* vulgaris non condensati. Certum est quantitatem materiæ in *Auro* vicies millies circiter superare materiam aeris, attamen fieri potest, ut spatia in *auro* vel absolute vacua, vel materia subtili repleta, sint fere æqualia spatiis in aere, vel vacuis, vel materia tantum subtili repletis.

Sint *A* & *B* corpora duo, magnitudine æqualia: utrumque *v. gr.* sit cubus unius digiti. Et corpus *A* decies millies sit gravius corpore *B*, unde & corpus *A* quantitate materiæ decies millies superabit corpus *B*. Ponamus jam materiæ quantitatem in *A* redigi in spatium absolute plenum, quod sit digiti cubici pars centies millesima; (liquet enim ex corollario præcedentis Theorematis id fieri posse). Unde cum materia in *A* decies millies superat materiam in *B*, materia illa in *B*, si in spatium absolute plenum compingatur, occupabit tantum digiti cubici partem $\frac{1}{1000000000}$ seu millies decies centies millesimam; Adeoque partes reliquæ 999999999 vel erunt absolute vacuæ, vel materia aliqua subtili, qualis supponitur Cartesiana, tantum repletæ. Porro, cum materiæ quantitas in *A* impleat tantum digiti partem centies millesimam, erunt in corpore *A* partes 99999 centies millesimæ, vel vacuæ, vel materia subtili repletæ, hoc est reducendo fractionem ad denominatorem prioris fractionis, erunt in *A* partes vacuæ 999990000 millies decies centies millesimæ. Adeoque vacuitates in *A* erunt ad vacuitates in *B*, ut numerus 999990000 ad numerum 999999999, qui numeri sunt ad se invicem fere in ratione æqualitatis, nam eorum differentia parvam admodum ad ipsos numeros obtinet rationem. Adeoque spatia vacua, vel materia subtili tantum repleta, quæ sunt in duobus corporibus *A* & *B*, eandem cum ipsis numeris ad se invicem rationem obtinentes, sunt etiam fere in ratione æqualitatis. *Q. E. D.*

Corpora autem omnia esse rarissima, hoc est pro mole sua parvam admodum continere materiæ quantitatem, ex diaphanorum proprietatibus certissime constat; nam *Radii Lucis* inter vitrum vel aquam non secus ac in aere per rectas lineas diffunduntur; quæcunque luci exposita sit corporis Diaphani facies; Adeoque a minima quavis assignabili Diaphani parte, ad aliam quamvis ejusdem partem, semper extenditur in his corporibus porus rectilineus, per quem transiverit lux, atque hoc fieri non potest nisi Materia Diaphani ad ejus molem parvam admodum obtineat rationem, nec fortasse materiæ quantitas in vitro ad ejus magnitudinem majorem habet rationem, quam magnitudo unius Arenulæ ad totam Terreni orbis molem: Hoc autem non esse impossibile, superius ostensum est. Unde cum Aurum non sit octuplo densius Vitro; ejus quoque materia, ad propriam molem, exiguam admodum obtinebit rationem.

Hinc ratio reddi potest, cur effluvia magnetica eadem fere facilitate densum Aurum & tenuem aerem pervadunt.

Ex his etiam propositionibus, & ex maxima lucis celeritate, ratio reddi potest, cur *Lucis* radii ex pluribus objectis prodeuntes & per tenue foramen transmissi, se mutuo non impediunt, sed per eandem rectam in

motu

motu suo perseverant. Quod per motum seu impulsum fluidi, plenum efficientis, vix explicari potest; *corpus enim omne a pluribus potentiis, secundum diversas directiones, simul impulsum, unam tantum & determinatam directionem accipit ex omnibus compositam.*

To find the Center of Oscillation, by Dr. Taylor. n. 337. p. 11.

XV. Definitio. *Est Centrum Oscillationis punctum quoddam in corpore pendulo, cujus vibrationes singulæ eodem modo atque eodem tempore peraguntur, ac si illud solum ad eandem distantiam a puncto suspensionis filo suspenderetur.*

Per se vix satis manifestum est in corpore aliquo dari hujusmodi punctum: utpote cujus acceleratio debeat, (*per hanc def.*) in omnibus inclinationibus corporis penduli ad Horizontem, perinde esse, ac si a propria tantum gravitate urgeatur; reliquis particulis totius corporis ejus motum proprium haud perturbantibus. Itaque in ordine ad inventionem hujus Centri, præmittenda est una atque altera propositio, unde constet tale punctum dari.

Prop. I. Prob. I. *In corporis Oscillantis data quavis inclinatione ad Horizontem, invenire punctum cujus acceleratio perinde sit, ac si ab ipsius propria tantum gravitate urgeatur.*

Fig. 33. Sit ABD corporis propositi sectio in plano ad Horizontem perpendiculari, in quo movetur centrum gravitatis G, centro suspensionis existente C. Distinguatur corpus in elementa prismatica plano ABD perpendicularia, adeoque Horizonti semper parallela; ut patebit ex motu centri gravitatis G in plano illo ABD. Atque ob hujusmodi situm, tale elementum quodvis spectari potest tanquam punctum Physicum p in plano eodem ABD ad punctum z locatum. Reducatur itaque corpus propositum in planum Physicum ABD constans ex hujusmodi particulis p.

In hoc plano ut inveniatur punctum O, cujus acceleratio propria non mutatur ab actionibus particularum reliquarum, attendendum est ad vires particulæ cujusvis singularis p in puncto z sitæ. Nam ex hisce viribus conjunctis oritur plani totius motus absolutus; cujus ope datur motus puncti cujusvis propositi; unde vicissim invenitur punctum cujus motus est datus.

At urgetur particula p a vi propriæ gravitatis; quæ si partium cohæsio dissolveretur, in dato tempore minimo, datam produceret accelerationem motus in perpendiculari ad Horizontem zy. Ad Cz duc normalem yx, & resolvetur acceleratio zy in partes zx & xy. Ob corporis rigiditatem, tollitur vis zx per resistentiam puncti C. At vi reliqua xy trahitur spatium ABD in gyrum circa punctum C; & ducta horizontali Co & perpendiculari zs, erit ea ut $\frac{Cs}{Cz}$: Nempe ob gravitatis vim datam, & similia triangula xyz & sCz. Ergo vis particulæ p ad movendum spatium ABD est ut $\frac{Cs}{Cz}\times p$.

Ad has vires in unum colligendas, sit O punctum invariabile, in linea ad libitum ducta & ad distantiam adhuc incognitam CO. Tum erit vis particulæ

particulæ p ad movendum punctum O, ut $\frac{Cz}{CO} \times \frac{Cs}{Cz} \times p$, hoc est ut $\frac{Cs}{CO} \times p$. Acceleratio autem, quam tribuit p eidem puncto O, erit ut $\frac{CO}{Cz} \times \frac{Cs}{Cz}$. Itaque applicata vi illa $\frac{Cs}{CO} \times p$ ad hanc accelerationem $\frac{CO \times Cs}{Czq:}$, erit quotiens $\frac{Czq:}{COq:} \times p$ particula, quæ, si in ipso puncto O fingatur moveri cum eadem acceleratione $\frac{CO \times Cs}{Czq:}$, eundem omnino produceret motum, quem in eodem puncto O producit particula p. Hinc demum reducitur Problema ad motuum Theorema notissimum: Applicata enim summa virium $\frac{Cs}{CO} \times p$ ad summam particularum $\frac{Czq:}{COq:} \times p$, erit quotiens acceleratio absoluta puncti O. Dein ducta perpendiculari $O\,o$, & posita hac acceleratione æquali datæ accelerationi $\frac{Co}{CO}$ ipsius puncti O, dabitur distantia CO. Sit enim $\frac{Co}{CO} = d$, & (juxta methodum *Fluxionum*) $Cs \times p = \dot{M}$, & $Czq: \times p = \dot{C}$. Tum ob CO invariabilem erit summa omnium virium $\frac{Cs}{CO} \times p = \frac{M}{CO}$, & summa omnium particularum $\frac{Czq:}{COq:} \times p = \frac{C}{COq:}$. Unde, applicata summa momentorum ad summam corporum, erit $\frac{M}{C} \times CO = d$ adeoque $CO = \frac{dC}{M}$. Inventis igitur C & M, per *Fluxionum* methodum inversam, dabitur CO. *Q. E. I.*

Cor. A Centro gravitatis G ad horizontalem Co duc perpendicularem Gg, & sit corpus ipsum $ABC = A$. Tum ex notissima indole centri gravitatis erit $M = Cg \times A$. Unde est $CO = \frac{dC}{Cg \times A}$.

Prop. 2. Theor. 1. *Iisdem positis, quæratur punctum* O *in recta* CG *transeunte per centrum gravitatis* G. *Tum erit* O *centrum Oscillationis corporis* A.

Etenim in hoc casu fit $\frac{Co}{CO} = \frac{Cg}{CG} = d$; Unde $CO = \left(\frac{dC}{Cg \times A}\right.$, *per Cor. Prop.* 1. $\left.\right) \frac{C}{CG \times A}$. At datur A, & dato puncto C, dantur CG & quantitas C. Unde datur CO, qualiscunque sit corporis Oscillantis in- Fig. 34.

clinatio ad Horizontem. Ideoque *per Def. & Prob.* 1. eſt O centrum Oſcillationis corporis A. *Q. E. D.*

Prop. 3. Theor. 2. *Iiſdem poſitis, ſit* D *aggregatum omnium* $Gz^2 \times p$; *Tum erit* $CO = CG + \frac{D}{CG \times A}$.

Ad CG duc normalem zF, atque erit $Czq := CGq :+ Gzq :- 2CG \times GF$. nempe cadente F intra C & G. At ubi F cadit in CG producta, erit $Czq := CGq :+ Gzq :+ 2CG \times Gf$: Eſt ergo $C =$ (aggregato omnium $Czq : \times p =$) aggregato omnium $CGq : \times p + Gzq : \times p - 2CG \times GF \times p + 2CG \times Gf \times p$. At ob centrum gravitatis G, eſt aggregatum omnium $2CG \times GF \times p =$ aggregato omnium $2CG \times Gf \times p$. Quare eſt $C =$ aggregato omnium $CGq : \times p + Gzq : \times p = CGq : \times A + D$. At enim *per Theor.* 1. eſt $CO = \frac{C}{CG \times A}$. Ergo $CO = CG + \frac{D}{CG \times A}$. *Q. E. D.*

Fig. 35.

Cor. Hinc datur parallelogrammum $CG \times GO$. Eſt enim $GO = \frac{D}{CG \times A}$. At dantur A & D. Quare datur $CG \times GO = \frac{D}{A}$.

Prop. 4. Theor. 3. *Iiſdem poſitis, ſi in puncto* O *conſtituatur particula phyſica* $\frac{CG \times A}{CO}$, *quæ propria gravitate agitata Oſcillet circa punctum* C, *ſpatii* ABC *motus perinde omnino erit, ac ſi agitaretur ab Oſcillatione ipſius corporis* A.

Conſtat tam ex Natura centri gravitatis, quam *per Prob.* 1. Eſt enim $\frac{CG \times A}{CO}$ aggregatum omnium $\frac{Czq : \times p}{COq :} = \frac{C}{COq :}$.

Prop. 5. Prob. 2. *Datis corporis cujuſvis magnitudine* A, *centro gravitatis* G, *& puncto ſuſpenſionis* C: *Invenire ejuſdem centrum Oſcillationis* O.

Fit per *Theor.* 1. inveniendo quantitatem C; vel per *Theor.* 2. quærendo quantitatem D.

Scholium. Ad inſtituendum calculum in caſu particulari, eligenda eſt quantitas C vel D, prout ſuggerit natura figuræ propoſitæ. Dein data earum alterutra, altera item dabitur per æquationem (*Prop.* 3.) $C = CGq : \times A + D$. Unde etiam dabitur pgr. $CG \times GO = \frac{D}{A}$ (*Cor. Prop.* 3.) $= \frac{C}{A} - CGq :$. Cujus ope, ex datis centro gravitatis & puncto ſuſpenſionis, datur centrum Oſcillationis per ſolam diviſionem. Quare in quolibet exemplo ſemper commodiſſimum erit hoc parallelogrammum primum eruere, vel per computum ipſius D, vel per quantitatem C, ex idonea aſſumptione centri ſuſpenſionis.

Super-

Superest, ut hæc exemplis aliquot illustremus.

Ex. 1. Sit figura proposita Pyramis ADC, cujus basis est pgr. AD, sitque motus centri gravitatis in plano transeunte per verticem C & diametrum basis EF lateri AB parallelam. *Fig.* 36.

Ad calculum commodissime instituendum, sit ipse vertex C centrum suspensionis. Tum ad modum *Prob.* 1. reducatur figura ad planum physicum trianguli Isoscelis CEF, in quo ef parallela ipsi EF repræsentat lineam physicam ex particulis p compositam. Sit $CH = a$, $HF = b$, & $Cb = x$. Tum ex natura figuræ erit $eb = \frac{bx}{a}$, & particula p sita ad punctum z erit ut x; vel potius, facto $bz = v$, erit $\dot{v}\,\dot{x}$ elementi prismatici basis, & p erit ut $\dot{v}\,\dot{x}\,x$. Unde erit $\dot{c} = Czq : \times \dot{v}\,\dot{x}\,x = \dot{v}\,\dot{x}\,x^3 + \dot{x}\,\dot{v}\,v^2\,x$. Ideoque summa omnium $Czq : \times p$ in linea bz erit $v\,\dot{x}\,x^3 + \frac{\dot{x}\,x\,v^3}{3}$; & in linea ef (pro v ponendo $\frac{bx}{a}$) erit summa illa $\frac{6ba^2 + 2b^3}{3a^3} \times \dot{x}\,x^4$. Unde iterum capiendo fluentem, & pro x scribendo a, erit $C = \frac{6ba^2 + 2b^3}{15} \times a^2$. Est autem pyramis ipsa $A = \frac{2baa}{3}$, & distantia centri gravitatis G a vertice C est $CG = \frac{3}{4}a$. Unde $\frac{C}{A} - CGq : = \frac{D}{A} = CG \times GO = \frac{3a^2 + 16b^2}{80}$. *Fig.* 37.

Ex. 2. Sit figura proposita Conus rectus descriptus rotatione trianguli isoscelis ECF circa perpendiculum CH.

Hic iterum sumpto vertice C pro centro suspensionis, & factis $CH = a$, $HE = b$, $Cb = x$, $bz = v$, ut supra; erit $p = 2\,\dot{x}\,\dot{v} \times \sqrt{\frac{bb}{aa}xx - vv}$; unde $\dot{c} = 2\,\dot{v}\,\dot{x} \times \overline{xx + vv} \times \sqrt{\frac{bb}{aa}xx - vv}$. Sit B segmentum circuli diametro ef descripti, quod adjacet Abscissæ $bz = v$, & Ordinatæ $\sqrt{\frac{bb}{aa}xx - vv}$; tum erit summa omnium $Czq : \times p$ in recta $bz = 2\,\dot{x} \times \frac{4a^2 + b^2}{4a^2}x^2 B - \frac{1}{2}\,\dot{x}\,v \times \overline{\frac{b^2}{a^2}x^2 - v^2}\Big|^{\frac{3}{2}}$. Et quando $v = eb$, erit hæc summa $2\,\dot{x} \times \frac{4a^2 + b^2}{4a^2}x^2 B$; cujus duplum $\frac{4a^2 + b^2}{a^2}\,\dot{x}\,x^2 B$ est pars ipsius C in recta ef. Est autem area B ut x^2; sit ergo $B = cx^2$; atque pars illa ipsius C erit $\frac{4a^2 + b^2}{a^2} \times c\,\dot{x}\,x^4$. Unde capiendo fluentem erit $C = \frac{4a^2 + b^2}{5} \times ca^3$. Est autem conus ipse $A = \frac{4}{3}ca^3$, & $CG = \frac{3}{4}a$. Unde $\frac{C}{A} - CGq : = \frac{D}{A} = \frac{3a^2 + 12b^2}{80}$.

 Atque

Atque ad hunc modum procedit calculus in aliis figuris, ubi rationes Cb ad be, & bz ad p sunt magis compositæ.

Ex. 3. Ut pateat ratio calculi quantitatis D, sit figura proposita parallelepipedon, cujus facies Horizonti perpendicularis, & parallela plano motus centri gravitatis est ABD. Duc diametros EF & HI, & sit altitudo elementorum p, & sit tr parallela HI; & $GF = a$, $GH = b$, $Gs = x$, & $sz = v$. Tum erit $\dot{D} = \dot{v}\dot{x}xx + \dot{x}\dot{v}vv$. Unde ipsius D pars in recta tr erit $2b\dot{x}x^2 + 2b^3\dot{x}$: atque iterum sumendo fluentis duplum, erit $D = \frac{4ba^3 + 4b^3a}{3}$. Atqui est $A = 4ab$; unde est $\frac{D}{A} = \frac{aa+bb}{3}$ $= \frac{1}{12}$ DB quad.

Fig. 38.

Ex. 4 Sit ultimum exemplum in Sphæra, cujus circulus maximus Btr, diameter AB, & centrum G. Tum ductis lineis ut in Schemate satis patent, erit $\dot{D} = Gsq : \times p + Gmq : \times p$. At summa omnium $Gsq : \times p$ in recta tr est Gsq: ductum in aream circuli diametro tr descripti. Item summa omnium $GMq : \times p$ in recta ki est $Gmq : \times$ aream circuli diametro ki descripti. Unde statim constat esse $D =$ quater fluenti ipsius Gsq: in aream circuli cujus diameter est tr. Sit ergo c area circuli cujus radii quadratum est 1, & sit $GA = a$, & $Gs = x$. Tum erit $\dot{D} = 4\dot{x}x\,x \times caa - cxx = 4ca^2\dot{x}x^2 - 4c\dot{x}x^4$. Unde sumendo fluentem, & faciendo $x = a$, erit $D = \frac{8}{15}ca^5$. Est autem $A = \frac{4}{3}ca^3$. Unde $\frac{D}{A} = \frac{2}{5}aa$.

Fig. 39.

Ob affinitatem solutionis libet his subjungere Problema de inventione Centri Percussionis.

Prop. 6. Prob. 3. *Corporis cujusvis circa datum punctum rotati, invenire Centrum Percussionis; punctum scilicet tale, ut Corpus in illud impingens, & eadem opera solutum a puncto suspensionis, neque huc neque illuc inclinet.*

Primum constat hoc punctum quæri debere in plano motus centri gravitatis. Si enim corpus resolvatur in elementa prismatica plano isti normalia, ferentur ea motu sibi parallelo; unde momenta ex utraque parte istius plani erunt æqualia; adeoque per resistentiam factam in hoc plano, corporis punctum nullum de eo pelletur. Sit ergo planum illud AB, ad quod reducetur corpus per contractionem elementorum prismaticorum in particulas p ad puncta z sitas, ut in *Prob.* 1. In hoc plano sit C centrum rotationis; aut saltem ejus projectio, facta per lineam perpendicularem in hoc planum demissam; & sit Q punctum quæsitum. Per C duc ad libitum $C\xi$, in qua sume puncta duo z & ξ, ita ut ductis zQ, & ξQ, sit angulus CzQ obtusus, & angulus $C\xi Q$ acutus: atque in punctis z & ξ sint particulæ p & π. Tum ad $C\xi$ ductis normalibus zr & ξr, quæ sint ad invicem ut Cz ad $C\xi$, iis repræsentabuntur velocitates absolutæ particularum p & π. At harum velocitatum partes

Fig. 40.

quæ

quæ ſunt in directionibus $z\mathcal{Q}$ & $\xi\mathcal{Q}$, tolluntur per reſiſtentiam puncti $\mathcal{Q}$. Ad $\mathcal{Q}z$ & $\mathcal{Q}\xi$ duc normales CD & Cd; & ob angulos æquales $zCD = rz\mathcal{Q}$, & $\xi Cd = r\xi\mathcal{Q}$, velocitatum partes reliquæ, in directionibus ipſis $\mathcal{Q}z$ & $\mathcal{Q}\xi$ perpendicularibus, erunt ut zD & ξd. Unde habita ratione diſtantiarum $\mathcal{Q}z$ & $\mathcal{Q}\xi$, erunt vires particularum p & π ad movendum ſpatium AB in partes contrarias, ut $Dz \times z\mathcal{Q} \times p$, & $d\xi \times \xi\mathcal{Q} \times p$. At per conditiones Problematis debent ſummæ hujuſmodi contrariarum virium eſſe inter ſe æquales.

Ob angulos ad D & d rectos, ſunt puncta D & d ad circumferentiam circuli diametro $C\mathcal{Q}$ deſcripti. Sit iſtius circuli centrum E. Tum ductis Ez & $E\xi$ circulo occurrentibus in F & I, f & i, erit $Dz \times z\mathcal{Q} = Fz \times zI = EFq: - Ezq: = E\mathcal{Q}q: - Ezq:$, & $d\xi \times \xi\mathcal{Q} = E\xi q: - E\mathcal{Q}q:$ Quare erit ſumma omnium $E\mathcal{Q}q: \times p - Ezq: \times p =$ ſummæ omnium $E\xi q: \times \pi - E\mathcal{Q}q: \times \pi$; & terminis tranſpoſitis, ſumma omnium $E\mathcal{Q}q: \times \overline{p + \pi}: =$ ſummæ omnium $Ezq: \times p + E\xi q: \times \pi$, hoc eſt, ſi p ponatur tam pro particula p intra circulum, quam pro particula π extra circulum, erit ſumma omnium $E\mathcal{Q}q: \times p =$ ſummæ omnium $Ezq: \times p$. Ad $C\mathcal{Q}$ duc normalem zs. Tum erit $Ezq: = Czq: + ECq: - \mathcal{Q}C \times Cs$. Quo valore ipſius $Ezq:$ ei ſubſtituto, & æquatione debite tractata, tandem invenies ſummam omnium $C\mathcal{Q} \times Cs \times p =$ ſummæ omnium $Czq: \times p$. Unde eſt $C\mathcal{Q} = \dfrac{\text{ſummæ omnium } Czq: \times p}{\text{ſumm: omnium } Cs \times p}$. At enim eſt ſumma omnium $Czq: \times p$ ipſa quantitas C in calculo centri Oſcillationis: & ſi centrum gravitatis ſit G, & ad $C\mathcal{Q}$ ducatur normalis Gg, & corpus ipſum dicatur A, erit ſumma omnium $Cs \times p = Cg \times A$. Unde eſt $C\mathcal{Q} = \dfrac{C}{Cg \times A}$. Sit centrum Oſcillationis O; tum per *Theor.* 1. erit $CO = \dfrac{C}{CG \times A}$. Unde eſt $Cg : CG :: CO : C\mathcal{Q}$. Quare per O ducta ad CO perpendicularis tranſibit per punctum $\mathcal{Q}$. $\mathcal{Q}.E.I.$

XVI. Lemma I. *Sint* ADFB, *&* A $\Delta\Phi$ B *Curvæ duæ, quarum relatio inter ſe hæc eſt, ut, ductis ad libitum ordinatis* C Δ D, E Φ F, *ſit* C Δ : CD :: E Φ : E F. *Tum ordinatis in infinitum imminutis, adeo ut coincidant Curvæ cum axe* AB; *dico quod ſit ultima ratio curvaturæ in* Δ *ad curvaturam in* D, *ut* C Δ *ad* C D. *Of the Motion of a ſtretcht String, by the ſame. n.* 337. *p.* 26. *Fig.* 41.

Demonſtratio. Duc ordinatam $c\delta d$ ipſi CD proximam; & ad D & Δ duc tangentes Dt & $\Delta\theta$, ordinatæ cd occurrentes in t & θ. Tum ob $c\delta : cd :: C\Delta : CD$ (per Hypotheſin) tangentes productæ ſibi invicem & axi occurrent in eodem puncto P. Unde ob triangula ſimilia CDP & ctP, $C\Delta P$ & $c\theta P$, erit $c\theta : ct :: C\Delta : CD$ ($:: c\delta : cd$, per Hyp.) $:: \delta\theta$ ($= c\theta - c\delta$) ad dt ($= ct - cd$.) Atqui ſunt curvaturæ in Δ & D, ut anguli contactus $\theta\Delta\delta$ & tDd; & ob $\delta\Delta$ & dD coincidentes cum cC, anguli iſti ſunt ut eorum ſubtenſæ $\delta\theta$ & dt, hoc eſt (per analogiam ſupra inventam) ut $C\Delta$ & CD. Quare, *&c.* $\mathcal{Q}.E.D.$

Lemma

Fig. 42. Lemma 2. *In aliquo articulo vibrationis ſuæ induat Nervus tenſus, inter puncta A & B, formam curvæ cujuſvis $Ap\pi B$. Tum dico quod ſit incrementum velocitatis puncti alicujus* P, *ſeu acceleratio oriunda a vi tenſionis Nervi, ut curvatura Nervi in eodem puncto.*

Demonſtratio. Finge Nervum conſtare ex particulis rigidis æqualibus infinite parvis pP & $P\pi$, *&c.* & ad punctum P erige perpendicularem PR = radio curvaturæ in P, cui occurrant tangentes pt & πt in t, iis parallelæ πs & ps in s, & chorda $p\pi$ in c. Tum, per Principia Mechanicæ, vis abſoluta, qua urgentur particulæ ambæ pP & $P\pi$ verſus R, erit ad vim tenſionis fili, ut st ad pt; & hujus vis dimidium, quo urgetur particula una pP, erit ad Nervi tenſionem, ut ct ad tp, hoc eſt, (ob triangula ſimilia ctp, tpR) ut tp vel Pp ad Rt vel PR. Quare, ob tenſionis vim datam, erit vis acceleratrix abſoluta ut $\frac{Pp}{PR}$. Sed eſt acceleratio genita in ratione compoſita ex rationibus vis abſolutæ directe & materiæ movendæ inverſe; atque eſt materia movenda ipſa particula Pp. Quare eſt acceleratio ut $\frac{1}{PR}$, hoc eſt ut Curvatura in P. Eſt enim Curvatura reciproce ut radius circuli oſculatorii. *Q. E. D.*

Prob. 1. *Definire motum Nervi tenſi.*

In hoc Problemate & ſequentibus pono Nervum moveri per ſpatium minimum ab axe motus; ut incrementum tenſionis ex aucta longitudine, item obliquitas radiorum curvaturæ poſſint tuto negligi.

Fig. 43. Itaque extendatur Nervus inter puncta A & B; & plectro deducatur punctum z ad diſtantiam Cz ab axe AB. Tum amoto plectro, ob flexuram in puncto ſolo C, illud primum incipiet moveri (*per Lemma* 2.) At ſtatim inflexo Nervo in punctis proximis Φ & d, incipient hæc puncta etiam moveri; & deinde E & e, & ſic deinceps. Item ob magnam flexuram in C, illud punctum primo velociſſime movebitur; & exinde aucta curvatura in punctis proximis D, E, *&c.* ea continuo velocius accelerabuntur; &, eadem opera, imminuta curvatura in C, id punctum viciſſim tardius accelerabitur. Et univerſaliter, punctis juſto tardioribus magis & velocioribus minus acceleratis, tandem fiet ut viribus inter ſe rite temperatis, motus omnes conſpirent, punctis omnibus ad axem ſimul euntibus & ſimul redeuntibus, vicibus alternis ad infinitum.

Sed ut hoc fiat debet Nervus ſemper induere formam curvæ $ACDEB$, cujus curvatura in quovis puncto E eſt ut ejuſdem diſtantia ab axe $E\eta$; velocitatibus etiam punctorum C, D, E, *&c.* conſtitutis inter ſe in ratione diſtantiarum ab axe Cz, $D\vartheta$, $E\eta$, *&c.* Etenim in hoc caſu, ſpatia Cx, $D\delta$, $E\varepsilon$, *&c.* eodem tempore minimo percurſa, erunt inter ſe ut velocitates, hoc eſt ut ſpatia percurrenda Cz, $D\vartheta$, *&c.* Unde erunt ſpatia reſidua xz, $\delta\vartheta$, $\varepsilon\eta$, *&c.* inter ſe in eadem ratione. Item (*per Lemma* 2.) erunt accelerationes inter ſe in eadem ratione. Quo pacto, ſemper ſervata ratione velocitatum inter ſe eadem ac ſpatiorum percurrendorum, puncta omnia ſimul pervenient ad axem & ſimul redibunt: adeoque recte definitur curva $ACDEB$. *Q. E. D.*

Præ-

Præterea, comparatis inter ſe duabus curvis $ACDEB$, & $A x \delta \epsilon B$, *per Lemma* 1. erunt curvaturæ in D & δ, ut diſtantiæ ab axe $D\vartheta$ & $\delta\vartheta$: adeoque *per Lemma* 2. acceleratio dati cujuſvis puncti in Nervo erit ut ejuſdem diſtantia ab axe. Unde (*per Phil. Nat. Princip. Math. Sect.* X. *Prop.* 51.) vibrationes omnes, tam maximæ quam minimæ, peragentur in eodem tempore periodico, & puncti cujuſvis motus ſimilis erit oſcillationi corporis Funipenduli in Cycloide. *Q. E. I.*

Cor. Sunt Curvaturæ reciproce ut radii circulorum oſculantium. Sit ergo a linea data, atque erit radius curvaturæ in $E = \frac{aa}{En}$.

Prob. 2. *Datis longitudine & pondere Nervi, una cum pondere tendente; invenire tempus unius vibrationis.*

Extendatur nervus inter puncta A & B per vim ponderis P, & ſit nervi ipſius pondus N, & longitudo L. Item conſtituatur nervus in poſitione $AFpCB$, & ad punctum medium C erige normalem CS = radio curvaturæ in C, & occurrentem axi AB in D; & ſumpto puncto p ipſi C proximo, duc normalem pc & tangentem pt. Fig. 44.

Ergo, ut in *Lemmate* 2, conſtat vim abſolutam qua acceleratur particula pC, eſſe ad vim ponderis P, ut ct ad pt, *i. e.* ut pC ad CS. Sed eſt pondus P ad pondus ipſius particulæ pC, in ratione compoſita ex rationibus P ad N, & N ad pondus particulæ pC, vel L ad pC, hoc eſt, ut $P \times L$ ad $N \times pC$. Quare compoſitis his rationibus, eſt vis acceleratrix ad vim gravitatis ut $P \times L$ ad $N \times CS$. Conſtituatur itaque pendulum longitudine CD: tum (per *Princip. Math. Sect.* X. *Prob.* 52.) erit tempus periodicum Nervi ad tempus periodicum iſtius penduli, ut $\sqrt{N} \times CS$ ad $\sqrt{P} \times L$. At (per eandem Propoſit.) data vi gravitatis longitudines pendulorum ſunt in duplicata ratione temporum periodicorum; unde erit $\frac{N \times CS \times CD}{P \times L}$, vel (pro CS ſcripto $\frac{aa}{CD}$, *per Cor. Prob.* 1.) $\frac{N \times aa}{P \times L}$ longitudo penduli cujus vibrationes ſunt iſochronæ vibrationibus Nervi.

Ad inveniendam lineam a, ſit Curvæ abſciſſa $AE = z$, & ordinata $EF = x$, & ipſa Curva $AF = v$, & $CD = b$. Tum (*per Cor. Prob.* 1.) erit radius curvaturæ in $F = \frac{aa}{x}$. At dato $\dot{v}$ eſt radius curvaturæ $\frac{\dot{v}\dot{x}}{\ddot{z}}$. Unde $\frac{aa}{x} = \frac{\dot{v}\dot{x}}{\ddot{z}}$; adeoque $aa\ddot{z} = \dot{v}x\dot{x}$: & ſumptis fluentibus $aa\dot{z} = \frac{\dot{v}xx}{2} - \frac{\dot{v}bb}{2} + \dot{v}aa$ (ubi additur data quantitas $-\frac{\dot{v}bb}{2} + \dot{v}aa$, ut fiat $\dot{z} = \dot{v}$ in puncto medio C.) Et hinc peracto calculo erit $\dot{z} = \frac{a^2\dot{x} - \frac{1}{2}b^2\dot{x} + \frac{1}{2}x^2\dot{x}}{\sqrt{a^2b^2 - a^2x^2 - \frac{1}{4}x^4 - \frac{1}{4}b^4 + \frac{1}{2}b^2x^2}}$. Evaneſcant jam b & x reſpectu a, ut coincidat curva cum axe, & fiet $\dot{z} = \frac{a\dot{x}}{\sqrt{bb - xx}}$. At centro C

&

& radio $CD = b$ deſcripto quadrante circulari DPE, & facto $CQ = x$, & erecta normali QP, atque arcu DP exiſtente y, erit $\dot{y} = \sqrt{\frac{bx}{bb - xx}} = \frac{b}{a}\dot{z}$.

Fig. 45.

Unde $y = \frac{b}{a}z$, & $z = \frac{a}{b}y$. Et facto $x = b = CD$, (quo caſu etiam fit $y =$ arcui quadrantali DPE, & $z = AD = \frac{1}{2}L$) erit $\frac{1}{2}L = a \times \frac{DE}{CD}$, atque $a = L \times \frac{CD}{2DE}$. Sit ergo CD ad $2DE$ (ut diameter circuli ad circumferentiam) ut d ad c; atque erit $aa = LL \times \frac{dd}{cc}$. Subſtituto itaque hoc valore pro aa, erit $\frac{N}{P} \times L \times \frac{dd}{cc}$ longitudo penduli iſochroni ipſi Nervo. Sit ergo D longitudo cujus tempus periodicum eſt 1, atque erit $\frac{d}{c}\sqrt{\frac{N}{P} \times \frac{L}{D}}$ tempus periodicum Nervi. *Q. E. I.* Sunt enim pendulorum tempora periodica in dimidiata ratione longitudinum.

Cor. 1. Numerus vibrationum Nervi in tempore unius vibrationis penduli D eſt $\frac{c}{d} \times \sqrt{\frac{P}{N} \times \frac{D}{L}}$.

Cor. 2. Ob datum $\frac{d}{c} \times \sqrt{\frac{1}{D}}$; tempus periodicum Nervi eſt ut $\sqrt{\frac{N}{P} \times L}$. Et dato pondere P eſt tempus ut $\sqrt{N \times L}$. Item conſtitutis Nervis ex eodem filo, quo caſu fit N ut L, eſt tempus ut L.

The Proportion of Mathematical Points to each other, by Fra. Robarts, *Eſq. n.* 334. *p.* 470.

XVII. It has heretofore paſs'd for a current Maxim, that all Infinites are equal. Divines and Metaphyſicians have not ſcrupled to ground many of their Arguments on that Foundation. The Poſition nevertheleſs is certainly erroneous, as * Dr. *Halley* abundantly has ſhewn, and has given divers Inſtances of infinite quantities which are in a determinate finite proportion one to another, and ſome infinitely greater one than another.

* *n.* 195. *p.* 556. Abr. *Vol.* I. *p.* 10.

The like may be obſerv'd of infinitely ſmall quantities (*viz.*) Mathematical Points,) as the following Propoſitions will make appear.

Prop. 1. *The Points of contact between Circles and their Tangents are in Subduplicate proportion to the Diameters of the Circles.*

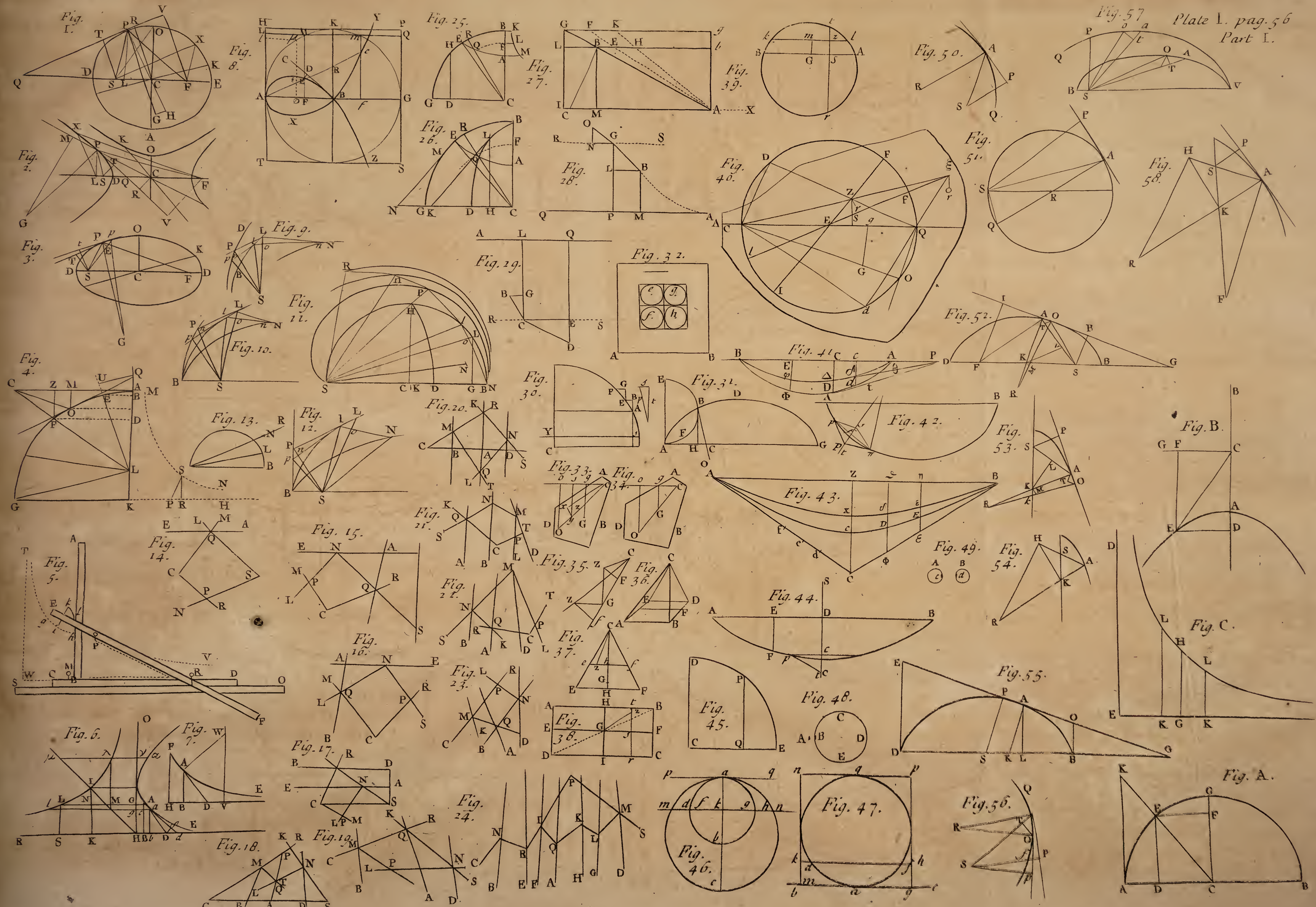
Let two Circles $adcb$, $afbg$, touch one another from within at the point a. Draw the Tangent paq, and parallel to it the line mn. From the point a draw the Diameter ac.

Fig. 46.

Let ac the Diameter of the greater Circle be equal to R, and ab the Diameter of the leſſer Circle be equal to S.

Let

Plate I. pag. 56
Part I.



Let db the Chord of the Arch dab be equal to z, and fg the Chord of the Arch fag be equal to y, and let the Abſciſs ak be equal to x.

If the Line mn be ſuppoſed to move till it becomes coincident with the Tangent paq, the nature of a Circle will always give the following Æquations.

$$zz = 4Rx - 4xx.$$
$$yy = 4Sx - 4xx.$$

When the Line is arrived at the Tangent, z and y will become the two Points of Contact, and then $zz = 4Rx$ and $yy = 4Sx$. ($4xx$ being laid aſide as Heterogeneous to the reſt of the Æquation, by reaſon of x being become infinitely little) Therefore

$$zz : yy :: 4Rx : 4Sx :: R.S.$$
Therefore $z : y :: \sqrt{R} : \sqrt{S}$. *Q. E. D.*

Prop. II. *The Point of Contact between a Sphere and a Plane is infinitely greater than that between a Circle and a Tangent.* Fig. 47.

Let a be the Point of Contact between the Sphere $adqf$ and the Plane bc. About the Sphere deſcribe the Cylinder $npgm$.

Draw kh to repreſent a Circle parallel to the Plane. Let the Circle be ſuppos'd to move, till it becomes coincident with the Plane. The Cylindrical Surface $khgm$ will always be equal (according to *Archimedes*) to the Spherical Surface daf.

Now when theſe Surfaces become infinitely ſmall, one terminates in the Point of Contact, and the other in the Periphery of the Baſe of the Cylinder. Therefore the Point of Contact is equal to the Periphery of the Baſe of the Cylinder (equal to a Periphery which has the ſame Diameter as the Sphere) and by conſequence is infinitely greater than any point of Contact between a Circle and a Tangent. *Q. E. D.*

Prop. III. *The Points of Contact by Spheres of different Magnitude are to one another as the Diameters of the Spheres.*

For by the ſecond Propoſition the Points of Contact are equal to the Peripheries of ſuch Diameters, whoſe proportion is the ſame as the Diameters. *Q. E. D.*

XVIII. Ponenda ſunt fundamenti loco hæc tria, quibus omnis Phyſice innititur, principia. 1. Spatium inane. 2. Quantitatis in infinitum diviſibilitas. 3. Materiæ vis Attractrix. Dari ſpatium inane conſtat ex motu corporum. Quantitatis in infinitum diviſibilitatem ex continuæ quantitatis natura demonſtrant Geometræ. Materiæ ineſſe vim attractricem confirmat experientia. Ex duobus primis principiis ſequitur. *Of the Laws of Attraction and other Phyſical Principles, by Dr.* John Keill. n. 315. p. 97.

Theorema I. *Materiæ exigua qualibet particula poteſt ita ſpatium quantumvis largum occupare, ut pororum ſeu omnium meatuum diametri ſint data recta minores, vel ut particulæ omnes ſint a ſe invicem remotæ intervallo data recta minore.*

Theorema II. *Dari possunt duo corpora mole æqualia, at pondere seu densitate (id est quantitate materiæ) utcunque inæqualia, in quibus erunt meatuum seu pororum summæ fere æquales.*

Sit *v. g.* digitus cubicus alter auri, alter aeris: quamvis materia in cubo aureo vicesies millies superat materiam in cubo aereo, fieri tamen potest ut spatia vacua in digito cubico auri sint fere æqualia spatiis vacuis in digito cubico aeris, scil. ut auri vacuitates sint ad vacuitates aeris ut 999999 ad 1000000.

Theor. III. *Particulæ quæ aquam vel aerem vel alia ejusmodi fluida constituunt (si modo se tangant) non sunt absolute solidæ, sed ex aliis compositæ particulis multos meatus & poros intra se continentibus.*

Particulæ corporum minimæ & absolute solidæ, hoc est vacui omnino expertes, vocentur primæ compositionis; Moleculæ ex pluribus hisce particulis coalescentibus ortæ vocentur particulæ secundæ compositionis; Moles ex pluribus moleculis coeuntibus conflatæ, vocentur particulæ tertiæ compositionis; & sic deinceps, donec tandem perventum fuerit ad particulas, e quibus corporum fit ultima compositio, & in quas eorundem fit prima resolutio.

Theor. IV. *Præter vim illam Attractricem, qua Planetarum Cometarumque corpora in propriis orbitis retinentur, alia etiam inest materiæ potentia, qua singulæ, ex quibus illa constat, particulæ se invicem attrahunt, & reciproce a se invicem attrahuntur: quæ vis decrescit in majore quam duplicata ratione distantiæ augescentis.*

Theorema hoc multis potest probari experimentis; at ratio qua minuitur vis illa, dum a se invicem recedunt particulæ, num scilicet sit triplicata, quadruplicata, vel alia quævis distantiarum augescentium ratio, quæ major sit duplicata, nondum æque per experimenta patet; erit fortasse aliquando tempus, cum accuratiore adhibita diligentia innotescet.

Theor. V. *Si corpus constet ex particulis, quarum singulæ vi pollent attractrice, in triplicata vel plusquam triplicata ratione distantiarum decrescente; erit vis qua ab eo corpore urgetur corpusculum, in ipso contactu, vel intervallo a contactu infinite exiguo, infinite major, quam si corpusculum illud ad datam a dicto corpore distantiam locaretur.* *

* *Vide Prop.* 80. & 91. *Princip. Newtoni.*

Theor. VI. *Iisdem positis si vis illa attractiva in assignabili distantia, ad Gravitatem obtineat rationem finitam; eadem in ipso contactu, vel in distantia infinite parva, vi Gravitatis erit infinite major.*

Theor. VII. *Si vero in ipso contactu, vis corporum Attractiva ad Gravitatem obtineat rationem finitam, eadem in omni distantia assignabili est vi gravitatis infinite minor, adeoque evanescit.*

Theor. VIII. *Vis Attractiva, qua pollent singulæ materiæ particulæ in ipso contactu, vim gravitatis prope in immensum superat; non tamen est vi Gravitatis infinite major; adeoque in data distantia vis illa evanescet.*

Vis

Vis igitur hæc materiæ ſuperaddita, non niſi per ſpatiola admodum perexigua diffunditur; in majoribus diſtantiis prorſus nulla eſt; unde motus corporum Cœleſtium (quæ longis intervallis a ſe invicem disjuncta ſunt) per vim hanc Attractivam nulla ratione turbari poſſunt, ſed eadem ratione continuo peraguntur, ac ſi vis illa a corporibus iis prorſus abeſſet.

Theor. IX. *Si corpuſculum aliquod corpus tangat, vis qua urgetur illud corpuſculum, hoc eſt, vis qua cum eo corpore cohæret, erit quantitati contactus proportionalis; nam partes a contactu remotiores nihil conferunt ad cohærentiam.*

Adeoque pro vario particularum contactu varii orientur cohærentiæ gradus; omnium autem maximæ ſunt vires cohærentiæ, quando ſuperficies, in quibus ſe invicem tangunt corpora, planæ exiſtunt; quo in caſu, cæteris paribus, vis qua corpuſculum cum aliis cohæret, erit ut ſuperficierum partes ſeſe tangentes.

Hinc patet ratio, cur duo marmora exactiſſime polita, & ſeſe ſecundum ſuperficies planas tangentia, a ſe invicem divelli non poſſunt, niſi a pondere, quod Gravitatem Aeris incumbentis multum ſuperat.

Hinc etiam decantatiſſimi iſtius Problematis, de cohærentia materiæ, ſolutio elici poteſt.

Theor. X. *Ea corpuſcula facillime a ſe invicem ſeparantur, quarum contactus cum aliis ſunt pauciſſimi, & minimi; quales contingere ſolent in corpuſculis Sphæricis infinite exiguis.*

Hinc fluiditatis ratio redditur.

Theor. XI. *Vis qua corpuſculum aliquod ad aliud corpus maxime propinquum attrahitur, quantitatem ſuam non mutat, ſive augeatur corporis attrahentis materia, ſive minuatur, eadem manente corporis denſitate, & corpuſculi diſtantia.*

Nam cum vires particularum Attractrices per minima tantum diffundantur ſpatia; liquet partes remotiores ad *C*, *D* & *E*, nihil conferre ad attrahendum corpuſculum *A*. Adeoque eadem vi verſus *B* trahetur corpuſculum ſive adſint hæ partes, ſive amoveantur, ſive denique aliæ ipſis conjungantur. Fig. 48.

Theor. XII. *Si ea ſit corporis alicujus textura, ut particulæ ultimæ compoſitionis, per vim quandam externam (qualis eſt pondus eas comprimens, vel ab altero corpore proveniens ictus) a primigeniis ſuis contactibus paululum dimoveantur, nec interim in novos contactus commigrent; particulæ, per vim attractivam ſeſe mutuo petentes, ad contactus primigenios cito redibunt: iiſdem vero redeuntibus particularum corpus quodvis componentium contactibus & poſitionibus, eadem quoque redibit corporis figura; adeoque per vim attractivam corpora priſtinas quas amiſerunt figuras poſſunt denuo recuperare.*

Hinc Elaſticitatis ratio reddi poteſt. Cum autem per vim Elaſticam corpora, in ſe invicem impingentia, a ſe mutuo reſiliant (ut demonſtratum

tum est in Lectionibus nostris Physicis) a vi attractiva corporum oriri etiam debet eorundem a se invicem discessus.

Theor. XIII. *Quod si ea sit corporis textura, ut particulæ a prioribus contactibus per vim impressam dimotæ, in alios qui ejusdem sunt gradus immediate deveniant, corpus illud in pristinam figuram non se restituet.*

Hinc qualis sit textura, in qua corporum mollities consistit, intelligi potest.

Theor. XIV. *Particulæ materiæ pro diversa ipsarum structura & compositione diversis pollebunt viribus attractivis, puta non erit æque fortis attractio, cum particula datæ magnitudinis pluribus perforata sit meatibus, ac si omnino solida & vacui expers esset.*

Theor. XV. *Particularum perfecte solidarum vires attractivæ ex figuris ipsarum multum pendent*, Nam si parva aliqua materiæ particula in laminam circularem indefinite exiguæ crassitudinis formetur, & corpusculum in recta per centrum transeunte & ad planum circuli Normali locetur; sitque distantia corpusculi æqualis decimæ parti semidiametri circuli: vis qua urgetur corpusculum tricesies minor erit, quam si materia attrahens coalesceret in Sphæram, & virtus totius particulæ ex uno quasi puncto Physico diffunderetur. Quin etiam eadem circularis lamella fortius ad se trahit corpusculum, quam alia ejusdem ponderis particula, quæ in tenuem & longum formatur Cylindrum.

Theor. XVI. *Sales sunt corpora, quorum particulæ ultimæ compositionis magna vi attractiva pollent, inter quas tamen particulas plurimi interjacent meatus, particulis, quas habet aqua, ultimæ compositionis pervii: quæ igitur a salinis particulis fortiter attractæ, in eas cum impetu ruunt, & a mutuo contactu eas disjungunt, cohærentiamque salium dissolvunt.*

Theor. XVII. *Si corpuscula duo viribus attractivis decrescentibus in triplicata aut plusquam triplicata ratione distantiarum se mutuo petunt; erit velocitas in se invicem impingentium infinite major quam in dato intervallo.* Vide Prop. 39. Princip. Newtoni.

Theor. XVIII. *Corporis aqua gravioris eo usque diminui potest magnitudo, ut tandem in aqua suspensum maneat, nec vi propriæ Gravitatis descendat.*

Hinc patet ratio, cur particulæ Salinæ, Metallicæ, & aliæ ejusmodi, in minima redactæ, in suis menstruis suspensæ hæreant.

Theor. XIX. *Corpora majora minore velocitate ad se invicem accedunt, quam minora.*

Fig. 49. Vis enim, qua se mutuo petunt corpora *A* & *B*, particulis maxime propinquis tantum inest; remotiorum quippe vires nullæ sunt. Non igitur major vis adhibetur ad movenda corpora *A* & *B* quam ad particulas *c* & *d* movendas, sed corporum eadem vi motorum velocitates sunt corporibus reciproce proportionales: unde erit velocitas qua corpus *A* tendit versus

versus *B*, ad velocitatem, qua particula *c*, a corpore soluta, versus idem *B* tenderet, ut particula *c* ad corpus *A*. Multo igitur minor est velocitas corporis *A*, quam foret velocitas particulæ *c* a corpore solutæ.

Hinc fit, ut corporum majorum motus sua natura adeo languidus & lentus sit, ut ab ambiente fluido & aliis circumjacentibus corporibus plerumque impediatur. In minimis vero corpusculis viget virtus, & ab iis perplurimi producuntur effectus: tanto plus Energiæ minoribus inest corporibus, quam majoribus.

Hinc patet ratio istius Axiomatis Chymici, Sales non agunt nisi soluti.

Theor. XX. *Duo corpuscula sese non contingentia, adeo sibi vicina locari possunt, ut vis, qua se mutuo petunt, vim Gravitatis multum superet.*

Theor. XXI. *Si corpusculum in fluido locatum a particulis ambientibus undique æqualiter trahatur, nullus exinde orietur corpusculi motus; quod si ab aliis particulis magis, ab aliis minus urgeatur; ad eam partem tendet corpusculum, ubi major est attractio: & motus productus inæqualitati attractionis respondebit, scilicet in majori inæqualitate major erit motus, in minore minor.*

Theor. XXII. *Corpuscula in fluido natantia & magis se invicem trahentia quam fluidi particulas interjectas, depulsis fluidi particulis ad se invicem accedent ea vi, qua ipsorum attractio mutua superat attractionem particularum fluidi.*

Theor. XXIII. *Si corpus aliquod in fluido locetur, cujus partes fluidi particulas magis ad se trahunt, quam fluidi particulæ a se invicem trahuntur; sintque in corpore meatus plurimi particulis fluidi pervii, per hos meatus fluidum illud cito se diffundet; & si partium in corpore connexio non tam firma sit, quin ab impetu irruentium particularum superari possit, orietur exinde corporis immersi dissolutio.*

Hinc ut menstruum dato corpori dissolvendo sit idoneum, tria requiruntur, 1. Ut partes corporis particulas menstrui magis ad se trahant, quam eæ a se invicem trahuntur. 2. Ut corpus habeat meatus particulis menstrui patentes, & pervios. 3. Ut cohærentia particularum corpus constituentium tanta non sit, quin ab impetu irruentium particularum menstrui divelli possit. Hinc quoque constat particulas Spiritum Vini constituentes, magis a se invicem trahi, quam a particulis corporis salini in Spiritu Vini demersi.

Theor. XXIV. *Si corpuscula in fluido natantia, & se invicem petentia, Elastica sint, post congressum, a se mutuo resilient; & inde in alia corpuscula rursus impingentia, denuo reflectentur: ex quo fient innumeri alii cum aliis corpusculis conflictus continuæque resilitiones. Per vim autem attractivam, continuo augebitur corpusculorum velocitas, & sensui patebit partium motus Intestinus; sed prout fortius aut imbecillius se invicem trahunt corpuscula, & pro varia, qua pollent Elasticitate, varii erunt hi motus, & diversis gradibus atque temporibus, fient sensibiles.*

Theor.

Theor. XXV. *Si corpuscula se invicem trahentia, se mutuo contingant, nullus orietur motus; propius enim accedere nequeunt: Si ad exiguum admodum a se invicem seponantur spatium, orietur motus, sed si longius distent, non majore vi se invicem trahent, quam fluidi particulas interjectas; adeoque nullus producetur motus.*

Ex hisce principiis pendent omnia Fermentationis & Effervescentiæ Phænomena. Hinc patet ratio cur oleum Vitrioli, cui paululum aquæ immittitur, effervescit atque ebullit: corpuscula enim salina infusa aqua a mutuo contactu paululum dimoventur; unde cum magis se invicem trahant quam aquæ particulas, & cum undique æqualiter non trahuntur, motum exinde oriri necesse est.

Hinc etiam liquet ratio, cur tanta cietur ebullitio, cum limatura Chalybis mixturæ supradictæ injicitur: particulæ enim chalybis magna pollent Elasticitate, unde valida oritur reflectio. Hinc etiam videre est, cur menstrua quædam fortiori vi agunt, citiusque corpus aliquod dissolvunt, si aqua dilutiora fiant.

Theor. XXVI. *Si corpuscula se mutuo attrahentia vi Elastica careant, a se invicem non reflectuntur; sed congeries seu moleculas particularum efficient, unde fiet* Coagulum: *& si particularum sic coacervatarum Gravitas superet Gravitatem fluidi, succedet quoque* Præcipitatio. *Oriri quoque potest præcipitatio ex aucta vel diminuta Gravitate menstrui, in quo natant corpuscula.*

Theor. XXVII. *Si corpusculorum sese invicem attrahentium, & in fluido natantium, ea sit figura, ut in datis quibusdam ipsorum partibus, majore vi attractiva polleant, quam in aliis, & major sit in iisdem contactus; corpuscula illa coibunt in corpora datas figuras habentia, & inde emergent Chrystallisationes; corpusculorumque componentium figuræ ex data figura Chrystalli per Geometriam determinari possunt.*

Theor. XXVIII. *Si corpuscula magis trahantur a fluidi particulis, quam a se invicem; fiet ut quasi se mutuo fugientes, a se invicem recedant, & per omne fluidum cito diffundentur.*

Theor. XXIX. *Si inter duas fluidi particulas aliquod intercedat corpusculum, cujus binæ oppositæ facies maximis pollent viribus attractivis, hoc interjectum corpusculum particulas fluidi sibi agglutinabit; & plura istiusmodi corpuscula per fluidum diffusa ejus particulas omnes in corpus firmum compingent, fluidumque in* Glaciem *reducent.*

Theor. XXX. *Si corpus aliquod maximam emittat effluviorum copiam, quorum vires attractrices sunt fortissimæ; cum effluvia hæc corpori alicui leviusculo appropinquent, ipsorum vires attractrices Gravitatem corporis levioris tandem superabunt; & effluvia corpus illud ad se sursum trahent; cumque multo magis conferta sunt Effluvia, in minoribus ab emittente corpore distantiis, quam in majoribus; corpus leve versus densiora Effluvia semper urgebi-*

urgebitur, donec tandem ipsi corpori effluvia emittenti adhæreat. Hinc plurima Electricitatis *Phænomena explicari possunt.*

Contra nostram hanc de viribus attractricibus doctrinam fortasse objiciet aliquis; Si vis hæc attractrix omni inesset materiæ; corpora ponderosiora & plus materiæ in dato spatio habentia, plus debere attrahere, quam corpora minus Gravia, quod experientiæ repugnat. Sed huic objectioni facile respondetur. Particulæ scilicet ultimæ compositionis (quibus solis tribuitur vis attractrix) confertim juxta se invicem locatæ, possunt corpus ponderosum constituere, etiamsi ipsæ in se sint rariores, quam eæ quæ corpus leve constituunt ultimæ compositionis particulæ, a se invicem remotiores, & plures & patentiores meatus inter se habentes.

Alia multa sunt Naturæ Phænomena, quæ mihi videntur iisdem principiis explicari posse, uti ascensus succi in Plantis & Arboribus, foliorum & florum determinatæ & constantes figuræ, eorumque virtutes specificæ, *&c.* Multa quoque quæ in corpore animali quotidie occurrunt, præcipue quæ ad fluidorum cursus Secretionesque spectant, ab iisdem materiæ qualitatibus pendent, & hinc morborum *Theoria* & medicamentorum effectus optime eruuntur.

XIX. Theorema. *Si corpus urgente vi Centripeta in curva aliqua moveatur; Erit vis illa in quovis Curvæ puncto, in ratione composita ex directa ratione distantiæ corporis a centro virium, & reciproca ratione Cubi perpendicularis a Centro in rectam in eodem puncto Curvam Tangentem demissæ, ducti in Radium Curvaturæ quem ibi obtinet curva.* *Of the Laws of Centripetal Force. by the same. n. 317. p. 175.*

Sit *QAO* Curva quælibet a mobili urgente vi centripeta ad punctum *S* tendente descripta. Sitque *AO* arcus in minimo quovis tempore percursus, *Pm* ejus tangens, *AR* Radius circuli æquicurvi, hoc est cujus Peripheriæ pars minima cum Arcu *AO* coincidat. Et sit *SP* recta a puncto *S* in tangentem perpendiculariter demissa; Ducantur *Om* ad *SA* & *On* ad *SP* Parallelæ. Et exponat *Om* vim qua mobile in *A* urgetur versus *S*. Vis qua perpendiculariter a tangente recedit corpus, erit ut *On*, id est vis tendens versus *R*, & faciens ut mobile, eadem qua prius velocitate latum, describet circulum æquicurvum arcui *AO*, erit ad vim tendentem versus *S*, qua corpus in curva *AO* movetur, ut *On* ad *Om*, vel ob æquiangula triangula ut *SP* ad *SA*. Sed corporum in circulis latorum vires centripetæ sunt ut quadrata velocitatum applicata ad Radios; per *Corol. Theorem.* 4. *Princip. Newtoni.* Est vero velocitas reciproce ut *SP*, sive directe ut $\frac{1}{SP}$ adeoque quadratum velocitatum erit ut $\frac{1}{SP^2}$: vis igitur ut *On*, sive vis qua in circulo æquicurvo moveri potest corpus, erit ut $\frac{1}{SP^2 \times AR}$: Ostensum autem est, esse *SP* ad *SA* ut vis tendens versus *R*, qua corpus in circulo æquicurvo moveri potest, ad vim tendentem versus *S*: sed est vis tendens versus *R* ut $\frac{1}{SP^2 \times AR}$, adeoque cum sit *SP* : *SA* :: *Fig.* 53.

$:: \frac{1}{SP^2 \times AR} : \frac{SA}{SP^3 \times AR}$ erit vis tendens versus S, ut $\frac{SA}{SP^3 \times AR}$. *Q. E. D.*

Cor. Si curva QAO sit circulus, erit vis centripeta tendens versus S,
Fig. 51. ut $\frac{SA}{SP^3}$. Adeoque si vis centripeta tendat ad S punctum in circumferentia situm, erit [per 32 tertii] ang. $PAS =$ ang. AQS; adeoque ob similia triangula ASP, ASQ, erit $AQ : AS :: AS : SP$: unde $SP = \frac{AS^2}{AQ}$ & $SP^3 = \frac{AS^6}{AQ^3}$ unde $\frac{SA}{SP^3} = \frac{SA \times AQ^3}{AS^6} = \frac{AQ^3}{AS^5}$, hoc est, ob datum AQ, erit vis reciproce ut AS^5.

Sit DAB, Ellipsis cujus Axis DB, foci F & S, AR, OR duæ perpen-
Fig. 52. diculares in curvam sibi proximæ: ducantur KL, OT in SA, & KM in
(a) Prop. 3. El. 6ti. OR perpendiculares. Quia $SA : SK :: (a)\ FA + SA : FS$, hoc est data ratione, erunt rectarum SA, SK Fluxiones AT, Kk ipsis SA, SK pro-
(b) Prop. 6. partis 4tæ Sect. Con. Milnij. portionales; & est $AL = (b)\ \frac{1}{2}$ lateris Recti $= \frac{1}{2} L$. Porro ob KA ad SP parallelam, est angulus $ASP = KAL = TOA$ ob ang. TAO utrinque complementum ad rectum: quare $KA : AL :: SA : SP$, unde $SP = \frac{L}{2} \times \frac{SA}{KA}$ & $KA = \frac{L}{2} \times \frac{SA}{SP}$. Porro ob æquiangula triang. KMk, GPS & OTA, SPA.

Est $KM : Kk :: GP : GS :: AP : SK$
Item $Kk : AT \quad :: \quad SK : SA$
Item $AT : AO \quad :: \quad AP : SA$

Erit $KM : AO :: AP^2 : SA^2 :: SA^2 - SP^2 : SA^2 :: SA^2 - \frac{L^2 \times SA^2}{4AK^2} : SA^2 :: 4AK^2 - L^2 : 4AK^2$, unde $L^2 : 4AK^2 :: (AO - KM : AO ::)\ AK : AR$ ac proinde $AR = \frac{4AK^3}{L^2}$. Eodem prorsus ratiocinio invenietur Radius Curvaturæ in Hyperbola æqualis $\frac{4AK^3}{L^2} = \frac{L \times SA^3}{2SP^3}$.

In Parabola vero facilior est calculus. Nam ob datam subnormalem,
Fig. 53. est Kk semper $= AT =$ Fluxioni Axis; & triangula KkM, ATO, SPA, AKL, æquiangula, unde $KM : Kk :: AP : SA$, item est AT vel Kk : $AO :: AP : SA$, unde $KM : AO :: AP^2 : SA^2 :: SA^2 - SP^2 : SA^2 ::$ unde erit $SP^2 : SA^2 :: AO - KM : AO :: AK : AR$, ac proinde $AR = \frac{SA^2 \times AK}{SP^2}$ sed est $AL = \frac{1}{2}$ lateris Recti $= \frac{1}{2} L$, & $AK : AL :: SA : SP$, quare erit $\frac{L}{2} \times \frac{SA}{AK} = SP$, & $SP^2 = \frac{L^2 \times SA^2}{4AK^2}$; quare erit $AR = \frac{4AK^3}{L^2}$, vel quoniam est, $AK = \frac{L \times SA}{2SP}$, erit $AR = \frac{L \times SA^3}{2SP^3}$.

Atque ex his facillima oritur constructio, pro determinando Radio curvaturæ in quavis Sectione Conica. Sit enim AK perpendicularis in Sectionem occurrens Axi in K, ex K super AK erigatur perpendicularis HK,

HK, cum AS producta concurrens in H. Ex H erigatur super AH perpendicularis HR, erit AR radius curvaturæ. In Parabola paulo simplicior adhuc evadit constructio. Nam quoniam ex natura Parabolæ est $SA = SK$, & ang. AKH rectus, erit S centrum circuli per AKH transeuntis, unde invenitur Radius curvaturæ producendo SA in H, ut $SH = SA$, & in H erigendo perpendicularem HR; Et R erit centrum circuli osculantis Parabolam in A. Fig. 54.

Vis Centripeta tendens ad focum Sectionis Conicæ in qua corpus movetur, est reciproce proportionalis quadrato distantiæ. Nam quoniam $AR = \frac{L \times SA^3}{2SP^3}$ erit $\frac{SA}{SP^3 \times AR} = \frac{SA \times 2SP^3}{SP^3 \times L \times SA^3} = \frac{2}{L \times SA^2}$ hoc est ob datam $\frac{2}{L}$ erit vis centripeta ut $\frac{1}{SA}$.

Sit Ellipsis BAD quam tangit in A recta GE. Sintque SP per centrum Ellipsis & KA per contactum, transeuntes, perpendiculares in tangentem. Erit $SP \times KA =$ quartæ parti figuræ Axis seu $=$ quadrato semiaxis minoris $= BO \times DE$. Nam ob æquiangula triang. GBO, GLA, GAK, GPS & GDE, Fig. 55.

$$SP : SG :: BO : GO$$
$$SG : DG :: BG : LG :: GO : GA$$
$$DG : DE :: GA : AK,$$

unde $SP : DE :: BO : AK$; & $SP \times AK = DE \times BO = \frac{1}{2} L \times SB$.

Hinc si Mobile moveatur in Ellipsi, vi centripeta tendente ad centrum Ellipsis, erit vis illa directe ut distantia; Nam est $\frac{SP^3 \times 4 AK^3}{L^2} =$ datæ quantitati. Quia est $SP \times AK$ quantitas data. Vis igitur, ut $\frac{SA}{SP^3 \times AR}$, erit ut SA distantia.

In figura 52 demissa ab altero umbilico F: in Tangentem Perpendiculari FI. Ob æquiangula Triangula SAP, FAI, erit $SA : SP :: FA : FI = \frac{SP \times FA}{SA}$ unde erit $SP \times FI = \frac{SP^2 \times FA}{SA} =$ quadrato semiaxis minoris: unde si Axis major vocetur b, minor autem $2d$, erit $SP^2 = \frac{d^2 SA}{b - SA}$ & $SP = \frac{d SA^{\frac{1}{2}}}{\sqrt{b - SA}}$.

In Hyperbola autem est $SP = \frac{d SA^{\frac{1}{2}}}{\sqrt{b + SA}}$.

In Parabola est $SP = \sqrt{d\, SA}$, posito ejus latere recto $= 4d$.

Quoniam est $TA^2 : TO^2 :: AP^2 : SP^2 :: SA^2 - SP^2 : SP^2 :: SA^2 - \frac{d^2 SA}{b - SA} : \frac{d^2 SA}{b - SA} :: SA - \frac{d^2}{b - SA}\ \frac{d^2}{b - SA} :: bSA - SA^2 - d^2 : d^2$, erit $\sqrt{bSA - SA^2 - d^2} : d :: TA : TO$; cumque sit $TA = SA$, erit $TO = \frac{d S A}{\sqrt{bSA - SA^2 - d^2}}$.

Sit jam QAO quælibet curva, cujus arcus minimus ſit AO, tangentes in punctis A & O, AP, Op. Radius Curvaturæ AR, Perpendiculares in tangentes ſint SP, Sp, erit $\frac{SA\times TA}{fP}=AR$. Nam ob æquiangula triangula eſt $fP:AO::PA:RA$ & $AO:TA::SA:PA$; unde ex æquo erit $fP:TA$ vel $S\dot{A}::SA:RA$, eſt vero $fP=S\dot{P}$, quare erit $RA=\frac{SA\times S\dot{A}}{S\dot{P}}$.

Fig. 56.

Hinc ſi diſtantia SA, in ſuam Fluxionem ducatur, & dividatur per Fluxionem perpendicularis, habebitur radius Curvaturæ; Quo Theoremate facile determinatur Curvatura in Radialibus curvis. Exempli Gratia, Sit AQ, Spiralis Nautica; quoniam angulus SAP datur, ratio quoque SA ad SP dabitur; ſit illa ratio a ad b, erit $SP=\frac{bSA}{a}$ & $S\dot{P}=\frac{bS\dot{A}}{a}$ & $AR=\frac{SAS\dot{A}}{S\dot{P}}=\frac{aSA}{b}$; unde facile conſtabit, Spiralis Nauticæ Evolutam eſſe eandem Spiralem, in alia poſitione.

Quoniam $AR=\frac{SAS\dot{A}}{S\dot{P}}$, erit $\frac{SA}{SP^3\times AR}=\frac{S\dot{P}}{SP^3\times S\dot{A}}$. Atque hinc rurſus, ex data relatione SA ad SP, facile invenietur lex vis centripetæ.

Exemplum. Sit VAB Ellipſis cujus focus S, Axis major $VB=b$, Axis minor $=2d$, latus Rectum $=2R$. Sitque VaQ alia curva, ita ad hanc relata, ut ſit perpetuo angulus VSA angulo VSa proportionalis, & ſit $Sa=SA$. Quæritur lex vis centripetæ tendentis ad S, qua corpus in curva VaQ moveri poteſt.

Fig. 57.

Quoniam ang. VSA eſt ad VSa, in data ratione; horum angulorum incrementa erunt in eadem ratione, ſitque ea ratio m ad n; unde erit $ot=\frac{n\times OT}{m}$. Eſt autem $OT=\frac{dS\dot{A}}{\sqrt{bSA-SA^2-d^2}}$ unde erit $ot=\frac{ndS\dot{A}}{m\sqrt{bSA-SA^2-d^2}}$.

Quoniam autem eſt $SA^2+SP^2:SP^2::ta^2+ot^2:ot^2::S\dot{A}^2+\frac{n^2d^2S\dot{A}^2}{m^2\,bSA-SA^2-d^2}:\frac{n^2d^2S^2}{m^2\,bSA-SA^2-d^2}::1+\frac{n^2d^2}{m^2\times\overline{bSA-SA^2-d^2}}:\frac{n^2d^2}{m^2\times\overline{bSA-SA^2-d^2}}::m^2bSA-m^2SA^2-m^2d^2+n^2d^2:n^2d^2$, unde erit $\sqrt{m^2bSA-m^2SA^2-m^2d^2+n^2d^2}:nd::SA:SP$, & $SP=\frac{ndSA}{\sqrt{m^2bSA-m^2SA^2-m^2d^2+n^2d^2}}$. Cujus ut habeatur fluxio pro $m^2bSA-m^2SA^2-m^2d^2+n^2d^2$, ſcribatur x; & erit $SP=\frac{ndSA}{\sqrt{x}}$, & $SP^3=\frac{n^3d^3SA^3}{x^{\frac{3}{2}}}$; & eſt $\dot{x}=m^2bS\dot{A}-2m^2SAS\dot{A}$, & $S\dot{P}=ndS\dot{A}\times x^{-\frac{1}{2}}$

$-\frac{1}{2}$

$-\frac{1}{2}\frac{nASA\dot{x}}{x^{\frac{3}{2}}}$, & reducendo partes ad eundem denominatorem; erit $S\dot{P} = \frac{nd S\dot{A}\ x - \frac{1}{2} nd SA\dot{x}}{x^{\frac{3}{2}}}$. Et in numeratore, loco x & $\dot{x}$, ponendo ipsorum valores, & ordinando fit $S\dot{P} = \frac{nd S\dot{A} \times \frac{1}{2}m^2 bSA - m^2d^2 + n^2d^2}{x^{\frac{3}{2}}}$, unde erit $\frac{S\dot{P}}{SP^3 \times S\dot{A}} = \frac{\frac{1}{2}m^2 bSA - m^2d^2 + n^2d^2}{n^2d^2SA^3}$. Sed est $\frac{S\dot{P}}{SP^3 \times S\dot{A}}$, ut vis centripeta, quare erit vis, ut $\frac{\frac{1}{2}m^2 bSA - m^2d^2 + n^2d^2}{n^2d^2SA^3}$ vel ob datam n^2d^2 in denominatore erit vis, ut $\frac{\frac{1}{2}m^2 bSA - m^2d^2 + n^2d^2}{SA^3}$, vel loco d^2 ponendo $\frac{bR}{2}$ erit vis ut $\frac{\frac{1}{2}m^2 bSA - \frac{1}{2}m^2 bR + \frac{1}{2}n^2 bR}{SA^3}$, seu ob datam $\frac{b}{2}$, ut $\frac{m^2SA - Rm^2 + Rn^2}{SA^3} = \frac{m^2}{SA^2} + \frac{Rn^2 - Rm^2}{SA^3}$. Quæ omnia exacte coincidunt cum iis quæ a Domino *Newtono* de vi centripeta corporis in eadem curva moti, traduntur, in *Prop.* 44. *Princip.*

Quoniam vis Centripeta tendens ad punctum S, qua urgente corpus in curva moveri potest, est semper ut $\frac{S\dot{P}}{SP^3 \times S\dot{A}}$; hinc ex data lege vis Centripetæ, Inveniri potest relatio SA ad SP, ac proinde per methodum Tangentium Inversam, exhiberi potest Curva quæ data vi Centripeta describi possit.

Sit verbi gratia Vis reciproce ut distantiæ Dignitas quælibet m, hoc est, sit $\frac{S\dot{P}}{SP^3 \times S\dot{A}} = \frac{b}{a^2 \overline{SA}^m}$, erit $\frac{S\dot{P}}{SP^3} = \frac{bS\dot{A}}{a^2\overline{SA}^m}$, & capiendo harum fluxionum fluentes; erit $\frac{1}{2}\ SP^{-2} = \frac{bSA^{1-m} \mp e}{m - 1 \times a^2}$, unde erit $\frac{\frac{m-1}{2} \times a^2}{bSA^{1-m} \mp e} = SP^2$, & multiplicando tam numeratorem, quam denominatorem fractionis, per SA^{m-1}; & loco $\frac{m-1}{2}a^2$ ponendo d^2, fit $\frac{d^2 SA^{m-1}}{b \mp e\overline{SA}^{m-1}} = SP^2$; quare erit $SP = \frac{d\sqrt{\overline{SA}^{m-1}}}{\sqrt{b \mp e SA^{m-1}}}$.

Quod si quantitas constans e sit nihilo æqualis erit $SP \frac{\sqrt{SA^{m-1}}}{\sqrt{b}}$.

Adeoque si vis reciproce ut distantiæ quadratum, poni potest $SP = \frac{\sqrt{d^2 SA}}{\sqrt{b}}$, & curva erit Parabola cujus latus rectum est $\frac{4d^2}{b}$, vel potest esse $SP = d \times \frac{\sqrt{SA}}{\sqrt{b - SA}}$, & curva erit Ellipsis, vel denique potest esse $SP = d \times \frac{\sqrt{SA}}{\sqrt{b \times SA}}$, & curva evadit Hyperbola.

Si vis ſit reciproce ut diſtantiæ cubus, ſupponi poteſt, ut SP ſit $= \frac{dSA}{b}$ & curva ſit ſpiralis Nautica, vel fieri poteſt ut ſit $SP = \frac{dSA}{\sqrt{b - eSA^2}}$, & Curva erit eadem cum ea cujus conſtructionem a ſectore hyperbolæ petit Dominus *Newtonus*; vel poteſt eſſe $SP = \frac{dSA}{\sqrt{b + eSA^2}}$, & ejus Curvæ conſtructionem per Sectores Ellipticos tradit idem *Newtonus*, *Cor.* 3. *Prop.* I. *lib.* I. *Princip.*

Si vis centripeta ſit reciproce ut diſtantia; relatio inter SA & SP, æquatione Algebraica definiri nequit, Curva tamen per Logarithmicam vel per quadraturam Hyperbolæ conſtruitur, ſit enim $SP = \frac{d}{\sqrt{b - L.SA}}$, ubi $L.SA$ deſignat Logarithmum ipſius SA.

Fig. 50. Moveatur jam corpus in Curva QAO, urgente vi centripeta tendente ad S; & Celeritas corporis in A dicatur C; celeritas autem qua corpus urgente eadem vi centripeta, in eadem diſtantia, in circulo moveri poteſt, dicatur c. Conſtat ex Theoremate primo, quod ſi SA exponat vim Centripetam tendentem ad S; vis Centripeta tendens ad R, qua urgente, corpus cum celeritate C, circulum cujus radius eſt AR deſcribet; per SP exponetur. Corporum autem circulos deſcribentium vires Centripetæ ſunt ut velocitatum quadrata ad circulorum radios applicata, quare erit $SP : SA :: \frac{C^2}{AR} : \frac{c^2}{SA}$, unde erit $SP \times AR : SA^2 :: C^2 : c^2$ & $C : c :: \sqrt{SP \times AR} : SA$.

Si SP cum SA coincidat, ut ſit in figurarum verticibus, erit $C : c :: \sqrt{AR} : \sqrt{SA}$. Quod ſi curva ſit Sectio Conica AR, radius curvaturæ in ejus vertice eſt æqualis dimidio lateris recti $= \frac{1}{2} L$, ac proinde erit velocitas corporis in vertice Sectionis, ad velocitatem corporis in eadem diſtantia circulum deſcribentis, in dimidiata ratione lateris recti, ad diſtantiam illam duplicatam.

Quoniam eſt $AR = \frac{SA \times S\dot{A}}{S\dot{P}}$, erit $C^2 : c^2 :: \frac{SP \times SA \times S\dot{A}}{S\dot{P}} : SA^2 :: \frac{SP \times S\dot{A}}{S\dot{P}} : SA :: SP \times S\dot{A} : SA \times S\dot{P}$, adeoque ex data relatione SP ad SA, dabitur ratio C ad c. Exempli Gratia, Si vis ſit reciproce ut diſtantiæ dignitas m, hoc eſt ſit $\frac{S\dot{P}}{SP^3 \times S\dot{A}} = \frac{b}{a^2 SA^m}$ & erit $S\dot{P} : \frac{bSP^3 \times S\dot{A}}{a^2 SA^m}$, adeoque erit $C^2 : c^2 :: SP \times S\dot{A} : \frac{bSP^3 \times SA \times S\dot{A}}{a^2 SA^m} :: a^2 SA^{m-1} : bSP^2$.

Unde ſi ponatur $SP^2 = \frac{d^2 SA^{m-1}}{b} = \frac{m-1}{2} \frac{a^2 SA^{m-1}}{b}$, erit $C^2 : c^2 :: a^2 SA^{m-1} : \frac{m-1}{2} a^2 SA^{m-1} :: m - 1 : 2$ ac proinde erit $C : c :: \sqrt{2} : \sqrt{m-1}$.

Quod

Quod si ponatur $SP^2 = \frac{d^2\, SA^{m-1}}{b - e\, SA^{m-1}} = \frac{m-1}{2} \frac{a^2\, SA^{m-1}}{b - e\, SA^{m-1}}$ fiet C^2 ad c^2, ut $a^2\, SA^{m-1}$ ad $\frac{\frac{m-1}{2}\, a^2\, bSA^{m-1}}{b - eSA^{m-1}}$, hoc est ut $b - e\, SA^{m-1}$ ad $\frac{m-1}{2}\, b$, sed est ratio $b - e\, SA^{m-1}$, ad $\frac{m-1}{2} \times b$, minor ratione b ad $\frac{m-1}{2}\, b$, seu ratione 2 ad $m-1$, unde erit C ad c in minore ratione quam est $\sqrt{2}$ ad $\sqrt{m-1}$.

Similiter, si capiatur $SP = \frac{d^2\, SA^{m-1}}{b + e\, SA^{m-1}}$, invenietur esse C ad c in majore ratione quam est $\sqrt{2}$ ad $\sqrt{m-1}$.

Cor. Si corpus in Parabola moveatur, & vis Centripeta tendat ad focum S, erit velocitas corporis ad velocitatem corporis in eadem distantia circulum describentis ubique ut $\sqrt{2}$ ad 1, nam in eo casu est $m = 2$ & $m - 1 = 1$. Velocitas corporis in Ellipsi est ad velocitatem corporis in circulo ad eandem distantiam moti, in minore ratione quam $\sqrt{2}$ ad 1. Velocitas in Hyperbola est ad velocitatem in circulo in majore ratione, quam $\sqrt{2}$ ad 1.

Si Corpus in Spirali Nautica deferatur, est ejus velocitas ubique æqualis velocitati corporis in eadem distantia circulum describentis, nam in eo casu est $m = 3$ & $m - 1 = 2$.

Problema. *Posito quod vis Centripeta (cujus quantitas absoluta nota est,) sit reciproce ut distantiæ quadratum, & projiciatur corpus secundum datam rectam cum data velocitate. Invenire curvam in qua movetur corpus.* Fig. 56.

Projiciatur Corpus secundum datam rectam AB, cum data velocitate C. Et quoniam quantitas absoluta vis centripetæ nota est, dabitur inde velocitas qua corpus possit circulum ad distantiam SA describere urgente eadem vi; est enim æqualis ei quæ acquiritur, dum corpus vi illa uniformiter applicata urgente, cadit per $\frac{1}{2}\, SA$. Sit illa velocitas c. Ex A in AB, erigatur perpendicularis AK, & in ea capiatur AR, quarta proportionalis ipsis c^2 C^2 & $\frac{SA^2}{SP}$, & erit AR radius curvaturæ in A. Ex R in AS demittatur perpendicularis RH & ex H in AR perpendicularis HK, & ducta recta SK, dabit axis positionem; Fiat angulus FAK = angulo SAK. Et si FA sit ad SK Parallela, figura in qua movetur corpus erit Parabola. Si autem Axi SK occurrat in F; & puncta S & F cadant ad eandem partem puncti K, figura erit Hyperbola; sin ad contrarias partes cadant puncta S & K, erit figura Ellipsis, unde focis S & F & Axe $= SA \pm FA$ describetur sectio, in qua corpus movebitur.

XX. *Cum* D. J. Bernoulli *objectiones quasdam edidisset contra Solutionem* Newtonianam *hujusce Problematis, adjecta Solutione a seipso inventa; Cl.* Keillius, *Objectionibus ejus dilutis, Solutionem aliam edit, repetens pauca quæ supra tradita sunt.* *A Solution of the inverse Problem of Centripetal Forces, by Dr.* Keill. N. 340. p. 91.

Sit *VIL* Curva quævis, quam corpus urgente vi centripeta ad centrum C tendente describit: hanc Curvam in duobus punctis infinite vicinis I & K tangant rectæ IP, Kp, ad quas e centro demittantur perpendiculares CP, Cf; centro item C describantur KE, ID, & ducatur CI. Erit

Erit vis centripeta ut Quantitas $\frac{Pp}{PC^3 \times IN}$. Quod Theorema licet in prædicto loco demonſtravimus, ecce aliam ejus demonſtrationem. Ex K ducantur Km ad CP & Kn ad CI parallelæ. Et ob æquiangula triangula ICP, IKN, nKM. Itemque ob IKm & IpP æquiangula, Erit,

$$Ip \text{ vel } IP : IK :: pP : Km$$
$$PC : IP :: Km : mn$$
$$IN : IK :: mn : nK \quad \text{unde ex æquo fiet}$$
$$PC \times IN : IK^2 :: pP : nK, \text{ \& erit } nK = \frac{pP \times IK^2}{PC \times IN}.$$

Præterea tempus quo deſcribitur arcus IK eſt ut Area ſeu triangulum ICK, vel ejus duplum $PC \times IK$; adeoque ſi tempus detur erit $PC \times IK$ quantitas conſtans. Dato autem tempore, vis centripeta eſt ut lineola Kn quæ ſub urgente vi illa deſcribitur, adeoque vis centripeta eſt ut lineola illa Kn ducta in quantitatem conſtantem $\frac{1}{PC^2 \times IK^2}$, hoc eſt, erit vis centripeta ut $\frac{1}{PC^2 \times IK^2} \times \frac{Pp \times IK^2}{PC \times IN}$ ſeu ut quantitas $\frac{Pp}{PC^3 \times IN}$. Quod erat demonſtrandum.

Velocitas corporis in quovis loco eſt ut via in minimo quovis tempore percurſa directe & ut tempus illud inverſe; adeoque & ut $IK \times \frac{1}{PC \times IK}$, hoc eſt, velocitas erit reciproce ut Perpendicularis e centro in Tangentem.

Si diſtantia corporis a centro dicatur x, & Perpendicularis in tangentem p, erit $IN = \dot{x}$ & $Pp = \dot{p}$ & vis centripeta exponi poteſt per quantitatem $\frac{f^4 \dot{p}}{p^3 \dot{x}}$, aſſumendo quantitatem quamlibet pro f^4.

* Comm. Phyſ. Mathem. Par. An. 1710.

Adeoque ſi cum * Domino *Bernoullio* vim centripetam nominemus φ, erit $\frac{f^4 \dot{p}}{p^3 \dot{x}} = \varphi$ & $\frac{f^4 \dot{p}}{p^3} = \dot{x}\varphi$; & capiendo harum quantitatum fluentes erit $\frac{f^4}{2p^2} =$ Fluenti quantitatis $\dot{x}\varphi$.

At cum velocitas corporis ſit reciproce ut perpendicularis p, ejus quadratum exponi poteſt per $\frac{f^4}{2p^2}$. Si itaque velocitas dicatur v, erit $v^2 = \frac{f^4}{2p^2} =$ Fluenti quantitatis $\dot{x}\varphi$. Quod ſi A ſit locus de quo cadere debet corpus ut acquirat in D vel I velocitatem v, deque loco corporis D erigatur perpendicularis $DF = \varphi$ erit rectangulum $DE \times DF = \dot{x}\varphi$. Sit jam BFG linea curva cujus ordinatæ exponant vires centripetas, ſeu quantitates φ. Fluens quantitatis $\dot{x}\varphi$ erit Area curvilinea $ABFD = v^2 = \frac{f^4}{2p^2}$, adeoque erit v ut Areæ $ABFD$ latus quadratum. Quod ſi velocitas ea ſit

quæ

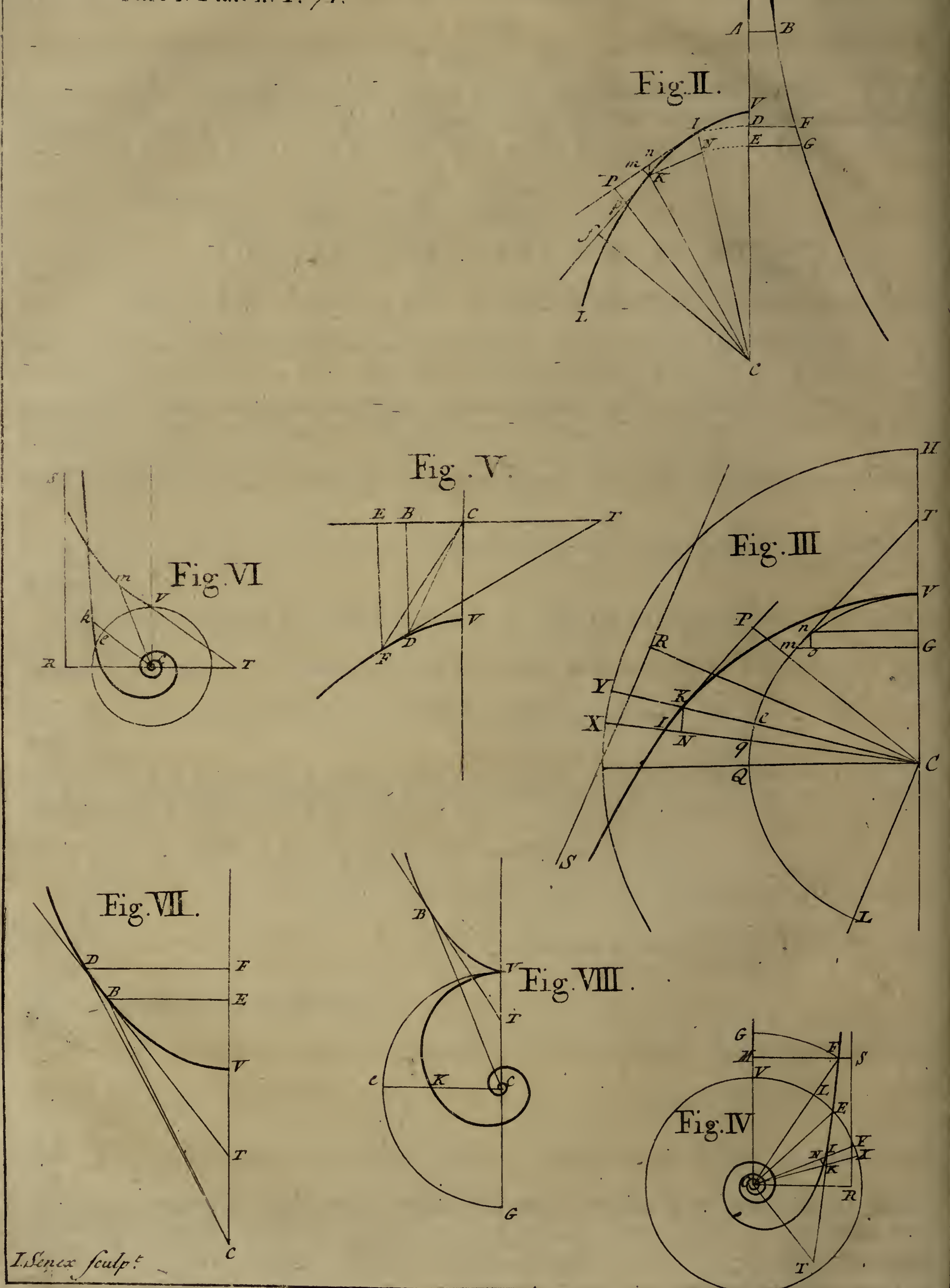


Fig. II.
Fig. III
Fig. IV
Fig. V.
Fig. VI
Fig. VII.
Fig. VIII.
I. Senex sculp.

quæ ab infinita diſtantia cadendo acquiritur, erit v^2 ſeu fluens ipſius $\dot{x}\,\phi$ æquale areæ $ODFO$ indefinite protenſæ.

Hinc ſemper dabitur quantitas p in terminis finitis, quando Area illa curvilinea terminis finitis exponi poteſt. Sit, verbi gratia, vis centripeta reciproce ut diſtantiæ dignitas m, hoc eſt, ſit $\dot{x}\,\phi = \frac{g\dot{x}}{x^m}$. Si velocitas corporis ſit ea quæ acquiritur cadendo ab infinita diſtantia, erit $v^2 = \frac{g}{\overline{m-1} \times x^{m-1}} = \frac{f^4}{2p^2}$ & in hiſce omnibus caſibus Area indefinite protenſa eſt quantitas finita. Poteſt autem corpus in trajectoria revolvi velocitate cujus quadratum vel majus fieri poteſt, vel minus quantitate $\frac{g}{\overline{m-1} \times x^{m-1}}$, vel huic æquale. Adeoque erit $v^2 = \frac{f^4}{2p^2} = \frac{g}{\overline{m-1}\, x^{m-1}} \pm e^2$.

Hinc urgentibus his viribus tria Curvarum genera deſcribi poſſunt; prout e^2 eſt quantitas poſitiva vel negativa vel nulla.

V.G. Si Velocitas major ſit ea quæ acquiritur ab infinita diſtantia cadendo, ſit $\frac{f^4}{2p^2} = \frac{g}{\overline{m-1}\, x^{m-1}} + e^2$: ſi velocitas ſit minor, erit $\frac{f^4}{2p^2} = \frac{g}{\overline{m-1}\, x^{m-1}} - e^2$: ſi æqualis, erit $\frac{f^4}{2p^2} = \frac{g}{\overline{m-1}\, x^{m-1}}$.

Sit $\frac{1}{2} f^4 = a^2 e^2$ & $\frac{1}{m-1} \times g = b^2 e^2$. Et ſi velocitas corporis ſit ea quæ ab infinito cadendo acquiritur, erit $p^2 = \frac{a^2 x^{m-1}}{b^2}$ ſeu $p = \frac{a x^{\frac{m-1}{2}}}{b}$.

At ſi velocitas major ſit aut minor hac velocitate, fiet uti oſtenſum eſt $\frac{f^4}{2p^2} = \frac{g}{\overline{m-1}\, x^{m-1}} \pm e^2 = \frac{\frac{1}{m-1} g \pm e^2 x^{m-1}}{x^{m-1}}$. Unde pro $\frac{1}{2} f^4$ & $\frac{g}{m-1}$ ponendo earum valores $a^2 e^2$ & $b^2 e^2$, erit $\frac{a^2 e^2}{p^2} = \frac{b^2 e^2 \pm e^2 x^{m-1}}{x^{m-1}}$ ſeu $\frac{a^2}{p^2} = \frac{b^2 \pm x^{m-1}}{x^{m-1}}$, & fiet $p^2 = \frac{a^2 x^{m-1}}{b^2 \pm x^{m-1}}$.

Adeoque ſi Vis centripeta ſit reciproce ut cubus diſtantiæ, hoc eſt, ſi ſit $m = 3$ & $m - 1 = 2$. Erit $p^2 = \frac{a^2 x^2}{b^2}$, vel $p^2 = \frac{a^2 x^2}{b^2 + x^2}$, vel denique $p^2 = \frac{a^2 x^2}{b^2 - x^2}$.

In primo caſu conſtat Curvam eſſe Spiralem Logarithmicam: nam ſit $p = \frac{ax}{b}$, & $b : a :: x : p$. adeoque ob conſtantem rationem b ad a, erit angulus CIP ubique conſtans.

Pona-

Ponamus jam eſſe $p^2 = \frac{a^2 x^2}{b^2 + x^2}$ & ex hac ſuppoſitione tres oriuntur diverſæ Curvarum ſpecies, prout a major eſt quam b, aut ei æqualis, aut minor.

Fig. 3. Et primo ſit a major quam b. Centro C & ad diſtantiam quamvis datam deſcribatur circulus HYX, cui rectæ CK, CI productæ occurrant in Y & X. Et eſt $IN^2 : KN^2 :: IP^2 : PC^2$ & ita $CI^2 - PC^2 : PC^2 :: x^2 - p^2 : p^2 :: x^2 - \frac{a^2 x^2}{b^2 + x^2} : \frac{a^2 x^2}{b^2 + x^2} :: 1 - \frac{a^2}{b^2 + x^2} : \frac{a^2}{b^2 + x^2} :: b^2 + x^2 - a^2 : a^2$. Quare erit $\sqrt{x^2 + b^2 - a^2} : a :: IN : KN : \dot{x} : \frac{a\dot{x}}{\sqrt{x^2 + b^2 - a^2}} = KN$. Et quoniam eſt a major quam b, erit $b^2 - a^2$ quantitas negativa. Sit illa $-c^2$, unde ſit $KN = \frac{a\dot{x}}{\sqrt{x^2 - c^2}}$. Dicatur radius circuli HYb, & eſt $CK : KN :: CY : YX$ hoc eſt $x : \frac{a\dot{x}}{\sqrt{x^2 - c^2}} :: b : \frac{ba\dot{x}}{x\sqrt{x^2 - c^2}} = YX = \dot{y}$, ſi arcus HY vocetur y.

Sit $x = \frac{c^2}{z}$ unde $\dot{x} = -\frac{c^2 \dot{z}}{z^2}$ & $\frac{\dot{x}}{x} = -\frac{\dot{z}}{z}$. Item erit $x^2 - c^2 = \frac{c^4}{z^2} - c^2 = \frac{c^4 - c^2 z^2}{z^2} = \frac{c^2}{z^2} \times c^2 - z^2$: unde $\sqrt{x^2 - c^2} = \frac{c}{z} \times \sqrt{c^2 - z^2}$: quibus valoribus ſubſtitutis, erit $\frac{ba\dot{x}}{x\sqrt{x^2 - c^2}} = \frac{-ba\dot{z}}{c\sqrt{c^2 - z^2}}$. Sit $a : c :: n : 1$. hoc eſt, ſit $a = nc$, & fiet XY ſeu $\dot{y} = -\frac{nb\dot{z}}{\sqrt{c^2 - z^2}}$. Eſt vero $\frac{nb\dot{z}}{\sqrt{c^2 - z^2}}$ ad $\frac{c\dot{z}}{\sqrt{c^2 - z^2}}$ ut nb ad c; hoc eſt in ratione data: adeoque eorum fluentes, ſi ſimul incipiunt, erunt in eadem ratione, hoc eſt erit HY ſeu y ad fluentem quantitatis $\frac{c\dot{z}}{\sqrt{c^2 - z^2}}$ ut nb ad c.

Quod ſi centro C radio $CV = c$ deſcribatur circulus VL, & CG ſit $= z$, & $no = \dot{z}$, fiet arcus $mn = \frac{c\dot{z}}{\sqrt{c^2 - z^2}} =$ fluxioni arcus Qm, quando fluxio eſt quantitas poſitiva: ſed quando eſt negativa, ejus fluens eſt arcus Vm prioris complementum. Arcus enim ejuſque complementum eandem habent quantitatem fluxionem denotantem, diverſis tantum ſignis affectam; quia creſcente uno decreſcit alter.

Hinc eſt HY ad Vm ut nb ad c: ſed eſt CV ad CH ut $Ve : HY$, hoc eſt $c : b :: Ve : \frac{b \times Ve}{c} = HY$, quare erit $\frac{b \times Ve}{c} : Vm :: nb : c$, unde $Ve : Vm :: n : 1$.

Præterea ex natura circuli erit $CG : CV :: CV : CT$, quando mT circulum tangit: hoc eſt erit $z : c :: c : \frac{c^2}{z} = CT = x$. Hinc ſi capiatur angulus

gulus VCe ad angulum VCm ut n ad 1, & producatur Ce ad K ut ſit CK = ſecanti CT, erit K punctum in Curva quæſita.

Hic obiter notandum eſt, ſi n ſit numerus, hoc eſt ſi ſit a ad c vel a ad $\sqrt{a^2 - b^2}$ ut numerus ad numerum, Curva VI fiet Algebraica: nam in hoc caſu relatio mG ad ſinum anguli VCe æquatione definitur, & inde habebitur relatio ſinus anguli VCe ad CT vel CK per æquationem determinatam, & inde demum dabitur æquatio quæ exprimet relationem inter ordinatam & interceptam a puncto C incipientem. Harum Curvarum ordines & gradus in Scala æquationum Algebraica diverſi erunt pro magnitudine numeri n. In his omnibus Curvis ſic deſcriptis Aſymptoti poſitio hac ratione determinatur: Fiat angulus VCL ad rectum angulum ut n ad 1. In eo angulo diſtantia corporis a centro evadit infinita. Jam quad. perpendicularis in Tangentem $PC = \frac{a^2 x^2}{b^2 + x^2}$, ubi x eſt infinita, fit $PC^2 = \frac{a^2 x^2}{x^2}$, ſeu $PC = a$. Ducatur itaque CR ad CL perpendicularis & æqualis rectæ a, & ſi per R ducatur RS rectæ CL parallela, hæc Curvam tanget ad infinitam diſtantiam, ſeu erit Curvæ Aſymptotos.

Si corpus in quavis harum Curvarum deſcendendo, ad Apſidem imam pervenerit; Hinc rurſus aſcendet in infinitum, & aliam Curvam priori ſimilem, ſeu potius ejuſdem Curvæ ſimilem portionem aſcendendo deſcribet.

Curvæ hæ poſſunt pluribus revolutionibus circa centrum torqueri, priuſquam ad Aſymptoton convergere incipiant, & motus angularis rectæ CK erit æqualis totidem rectis quot numerus n conſtat Unitatibus. *v. g.* ſi n ſit 100, perficientur viginti quinque integræ revolutiones priuſquam diſtantia a centro evadat infinita.

Aucto numero n, eadem manente a, minuitur c: eſt enim $\frac{a}{n} = c$ & $\frac{a^2}{n^2} = c^2 = a^2 - b^2$, unde fiet $\overline{n^2 - 1} \times a^2 = n^2 b^2$. Et proinde fiet $a^2 : b^2 :: n^2 : n^2 - 1$; adeoque ſi b^2 ad æqualitatem accedat ipſius a^2, perveniet quoque $n^2 - 1$ ad rationem æqualitatis cum n^2, & proinde augebitur n & in eadem ratione minuetur c. Ponatur itaque eſſe b^2 fere æquale ipſi a^2; adeo ut cum differentia ſit infinite parva, fiat n numerus infinite magnus, & radius circuli c fiet infinite parvus, ſeu circulus in ſuum centrum contrahetur. At ſic evaneſcente c, non pariter evaneſcit CT, ſi angulus VCM ſit propemodum rectus: nam in omni circulo, etiam minimo, ſecans anguli recti eſt quantitas infinita. Curva itaque hæc, ob n numerum infinitum, infinitis numero revolutionibus centrum ambibit, priuſquam ad Aſymptoton convergere incipiet.

Evaneſcente autem c fit $b = a$ & $p = \frac{ax}{\sqrt{x^2 + a^2}}$. Et quoniam in omni caſu eſt $\dot{y} = \frac{ha\dot{x}}{x\sqrt{x^2 + c^2}}$, evaneſcente c fiet $\dot{y} = \frac{ha\dot{x}}{x^2}$, unde capiendo Fluentes fiet $y = \frac{ha}{x}$ ſeu $xy = ha =$ datæ quantitati. *Fig.* 4.

 Hæc

Hæc Curva eſt Spiralis Hyperbolica, quæ plures habet notabiles proprietates. Si ducatur radius quilibet CIY Curvæ occurrens in I, & peripheriæ circuli in Y, & ex C ad CI excitetur perpendicularis CT, atque IT tangat Curvam in I, & rectæ CT occurrat in T: erit CT conſtans recta, æqualis ſcil. arcui VE; qua proprietate Logarithmicam æmulatur, cum CT Curvæ Subtangens dici poſſit. Sit enim Radius circuli $CE = b$, arcus $VE = a$, dicatur CI x, & VY ſit y. Quia eſt $ba = x \times y$ erit $\frac{ba}{x} = y$ & $\frac{ba\dot{x}}{x^2} = \dot{y}$. Porro eſt $CY : CI :: YX : NK$ hoc eſt $b : x :: \frac{ba\dot{x}}{x^2} : NK$: quæ proinde eſt $\frac{a\dot{x}}{x}$. Et quoniam eſt $IN : NK :: CI : CT$, hoc eſt $\dot{x} : \frac{a\dot{x}}{x} :: x : CT$, erit $CT = a$.

Si centro C, intervallo quovis CG, deſcribatur circuli arcus GF, hic arcus inter rectam CV & curvam interceptus erit ſemper æqualis conſtanti rectæ CT vel a. Nam quoniam eſt $VL \times CF = CV \times VE$, erit $VL : VE :: CV : CF :: VL : GF$ unde æquantur VE & GF. Si ad CG ex C excitetur normalis $CR = VE$ vel FG vel a, & per R agatur RS rectæ CV parallela, erit RS Curvæ Aſymptotos. Nam eſt recta MS æqualis arcui GF, & proinde FS diſtantia Curvæ ab RS eſt ſemper æqualis exceſſui quo arcus ſuperat ſuum ſinum: at cum diſtantia creſcat in infinitum, exceſſus ille minuetur in infinitum, & fiet tandem data quavis recta minor, & proinde RS erit Curvæ Aſymptotos.

Sit jam b major quam a; & ſimiliter, ut in priore caſu, invenietur $KN = \frac{a\dot{x}}{\sqrt{x^2 + b^2 - a^2}}$: at quoniam b ſuperat a, erit $c^2 = b^2 - a^2$ quantitas poſitiva, & KN fiet $= \frac{a\dot{x}}{\sqrt{x^2 + c^2}}$ & ponendo radium circuli $HY = b$, invenietur $XY = \frac{ba\dot{x}}{x\sqrt{x^2 + c^2}}$. Ponatur $x = \frac{c^2}{z}$, & erit $\dot{x} = -\frac{c^2\dot{z}}{z^2}$ & $\frac{\dot{x}}{x} = -\frac{\dot{z}}{z}$. Erit quoque $x^2 = \frac{c^4}{z^2}$ & $x^2 + c^2 = \frac{c^4}{z^2} + c^2 = \frac{c^4 + c^2z^2}{z^2} = \frac{c^2}{z^2} \times \overline{c^2 + z^2}$: unde $\sqrt{x^2 + c^2} = \frac{c}{z} \times \sqrt{c^2 + z^2}$. His itaque valoribus ſubſtitutis fit $\frac{ba\dot{x}}{x\sqrt{x^2 + c^2}} = -\frac{ba\dot{z}}{c\sqrt{c^2 + z^2}} = -\dot{y}$. Nam tale ſumi poteſt initium arcus HY, ut ſimul cum Fluente quantitatis $\frac{-ba\dot{z}}{c\sqrt{c^2 + z^2}}$ creſcat & decreſcat. Fiat $nc = a$ & erit $\frac{nb\dot{z}}{\sqrt{c^2 + z^2}} = \dot{y}$, & $\frac{\frac{1}{2}nb^2\dot{z}}{\sqrt{c^2 + z^2}} = \frac{1}{2}b\dot{y} =$ ſectori CXY.

Eſt

Eſt autem $\frac{\frac{1}{2}nb^2\dot{z}}{\sqrt{c^2+z^2}} : \frac{\frac{1}{2}c^2\dot{z}}{\sqrt{c^2+z^2}} :: nb^2 : c^2$, hoc eſt in data ratione. Adeoque erit ſector CXT ad $\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{c^2+z^2}}$ ſemper in data ratione. Harum itaque quantitatum fluentes erunt in eadem ratione, cum ſimul incipere ponantur. Fluens autem ſectoris CXT eſt ſector CVT, & fluens quantitatis $\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{c^2+z^2}}$ eſt ſector Hyperbolæ, quod ſic oſtenditur.

Centro C ſemiaxe tranſverſo $CV=c$ deſcribatur Hyperbola æquilatera, & ex duobus punctis vicinis D & F ordinentur ad axem conjugatum rectæ DB, EF; ducantur item CD, CF. Et incrementum ſeu fluxio trianguli BCD æquale erit $BE\times BD$ — ſectore DCF: unde ſector DCF (qui eſt Fluxio ſectoris CVD) æqualis erit $BE\times BD$ — incremento trianguli BCD. Et ſi BC dicatur z, ob Hyperbolam, eſt $BD^2=BC^2+CV^2=z^2+c^2$: unde $BD=\sqrt{c^2+z^2}$, & $BE\times BD=\dot{z}\times\sqrt{c^2+z^2}$. Triangulum autem BCD eſt $\frac{1}{2}z\times\sqrt{c^2+z^2}$, cujus fluxio eſt $\frac{1}{2}\dot{z}\times\sqrt{c^2+z^2}+\frac{\frac{1}{2}\dot{z}\times z^2}{\sqrt{c^2+z^2}}$. Subtrahatur hæc quantitas ab $\dot{z}\times\sqrt{c^2+z^2}$, & reſtabit ſector Hyperbolæ minimus $CDF=\frac{1}{2}\dot{z}\times\sqrt{c^2+z^2}-\frac{\frac{1}{2}\dot{z}\times z^2}{\sqrt{c^2+z^2}}=\frac{\frac{1}{2}\dot{z}\times\overline{c^2+z^2}-\frac{1}{2}\dot{z}\times z^2}{\sqrt{c^2+z^2}}$ $=\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{c^2+z^2}}$. Adeoque fluens ſectoris CDF eſt æqualis fluenti quantitatis $\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{c^2+z^2}}$. Proinde erit ſector CVD fluens quantitatis $\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{c^2+z^2}}$. Præterea DT recta tangat Hyperbolam & occurrat Axi conjugato in T. Eſt ex natura Hyperbolæ $BC:CV::CV:CT$, hoc eſt $z:c::c:\frac{c^2}{z}=CT=x$. Atque hinc oritur conſtructio quæ ſequitur. Fig. 5.

Centro C ſemiaxe tranſverſo CV, deſcribatur Hyperbola æquilatera Vm, item circulus Ve. Capiatur ſector circularis CVe ad ſectorem Hyperbolicum CVm ut n ad 1; tangat Hyperbolam in m recta Tm, occurrens Axi conjugato in T; producatur Ce ad k ut ſit $Ck=CT$, & punctum k erit in Curva quæſita. Nempe talis eſt ea Curva, ut ſi Ck dicatur x, Perpendicularis a C in tangentem ejus demiſſa erit ſemper æqualis $\frac{ax}{\sqrt{b^2+x^2}}$. Quando x eſt infinita evaneſcit b^2, & perpendicularis fit $=a$, & tunc coincidit CR cum CV. Si itaque capiatur in axe conjugato $CR=a$, & ducatur RS ipſi CV parallela, erit hæc Curvæ Aſymptotos. Fig. 6.

Si eo uſque augeatur a ut fiat quantitas b^2-a^2 infinite parva, tunc evaneſcet c^2, & quantitas $\frac{ha\dot{x}}{x\sqrt{x^2+c^2}}$ fit $\frac{ha\dot{x}}{x^2}=\dot{y}$. Unde ſi capiantur harum quantitatum fluentes, habebimus $\frac{ha}{x}=y$, & $ha=xy$, hoc eſt rectangulum ſub arcu circulari & diſtantia Curvæ a centro erit ſemper

data quantitas; atque hac ratione migrabit curva in spiralem Hyperbolicam. Est itaque spiralis Hyperbolica Curva media seu quasi limes, inter eas Curvas quæ construuntur per sectores circulares & eas quæ construuntur per sectores Hyperbolicos. Itaque spiralis illa Hyperbolica concipi potest formari vel per sectorem Circuli aut Ellipsis, vel per sectorem Hyperbolæ, cujus Axis transversus minuitur in infinitum, & in eadem ratione augetur numerus n.

Ad eum jam devenimus casum ubi velocitas corporis minor est ea quæ acquiritur cadendo ab infinita distantia, & ubi $p^2 = \frac{a^2x^2}{b^2 - x^2}$. Et hic, simili ratiocinio ac in priori casu, invenietur $KN = \frac{a\dot{x}}{\sqrt{b^2 - a^2 - x^2}}$, ubi necesse est ut sit b^2 majus quam a^2. Hinc si $b^2 - a^2$ dicatur c^2, fit $KN = \frac{a\dot{x}}{\sqrt{c^2 - x^2}}$; & proinde XY seu $\dot{y} = \frac{ha\dot{x}}{x\sqrt{c^2 - x^2}}$.

Sit jam $x = \frac{c^2}{z}$ & fiet $\frac{\dot{x}}{x} = -\frac{\dot{z}}{z}$ seu $\frac{ha\dot{x}}{x} = -\frac{ha\dot{z}}{z}$ & $c^2 - x^2$, erit $= \frac{c^2}{z^2} \times \overline{z^2 - c^2}$, quibus valoribus substitutis fit $\frac{-ha\dot{z}}{c\sqrt{z^2 - c^2}} = \frac{ha\dot{x}}{x\sqrt{x^2 - c^2}} = -\dot{y}$. Nam tale ponendum est initium arcus VX, ut simul cum fluente quantitatis $\frac{ha\dot{z}}{c\sqrt{z^2 - c^2}}$ incipiat: unde erit $\frac{\frac{1}{2}h^2a\dot{z}}{c\sqrt{z^2 - c^2}} = \frac{1}{2}h\dot{y} =$ sectori $CXY = \frac{\frac{1}{2}nh^2\dot{z}}{\sqrt{z^2 - c^2}}$, ponendo $nc = a$. Est vero $\frac{\frac{1}{2}nh^2\dot{z}}{\sqrt{z^2 - c^2}}$ ad $\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{z^2 - c^2}}$ ut nh^2 ad c^2, hoc est in ratione constanti: Quare harum quantitatum Fluentes sunt in eadem ratione, hoc est Fluens quantitatis $\frac{1}{2}h\dot{y}$ seu $\frac{\frac{1}{2}nh^2\dot{z}}{\sqrt{z^2 - c^2}}$ erit ad fluentem quantitatis $\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{z^2 - c^2}}$ ut nh^2 ad c^2. Est autem fluens quantitatis $\frac{1}{2}h\dot{y} =$ sectori CVX, & Fluens quantitatis $\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{z^2 - c^2}}$ est sector Hyperbolæ, quod sic ostenditur.

Fig. 7. Centro C semiaxe transverso $CV = c$ describatur Hyperbola æquilatera, & ex duobus punctis infinite vicinis B & D ad axem ordinentur duæ rectæ BE, DF; ducantur item CB, CD. Et erit Fluxio seu incrementum trianguli $CBE =$ triangulo $CBD + BE \times EF$; unde triangulum CBD, seu sector minimus CBD, erit $=$ incremento trianguli $CBE - BE \times EF$. Dicatur CE, z, & erit $BE = \sqrt{z^2 - c^2}$, & $BE \times EF = \dot{z}\sqrt{z^2 - c^2}$. Est quoque triangulum $CBE = \frac{1}{2}z\sqrt{z^2 - c^2}$, cujus Fluxio est $\frac{1}{2}\dot{z} \times \sqrt{z^2 - c^2} + \frac{\frac{1}{2}\dot{z} \times z^2}{\sqrt{z^2 - c^2}}$; a quo si subtrahatur quantitas $\dot{z} \times \sqrt{z^2 - c^2}$, fit sector minimus $CBD = \frac{\frac{1}{2}\dot{z} \times z^2}{\sqrt{z^2 - c^2}} - \frac{1}{2}\dot{z} \times \sqrt{z^2 - c^2} = \frac{\frac{1}{2}\dot{z} \times z^2 - \frac{1}{2}\dot{z} \times \overline{z^2 - c^2}}{\sqrt{z^2 - c^2}} =$

$\frac{1}{2}c^2$

$\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{z^2-c^2}}$ :: unde conſtat ſectorem CBE eſſe fluentem quantitatis $\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{z^2-c^2}}$. Præterea ſi BT tangens Hyperbolam Axi tranſverſo occurrat in T, ex natura Hyperbolæ fit $CE : CV :: CV : CT$, hoc eſt $z : c :: c : \frac{c^2}{z} = CT = x$.

Hinc deducimus ſequentem conſtructionem. Centro C, ſemiaxe tranſverſo $CV = c$, deſcribatur Hyperbola æquilatera VB, & circulus CeG ex centro C. Ad hyperbolam ducatur recta CB, & hyperbolæ Tangens BT axi tranſverſo occurrat in T. Capiatur circuli ſector CVe, qui ſit ad ſectorem Hyperbolicum CVB ut n ad 1. In Ce capiatur $CK = CT$, & erit K punctum in Curva quæſita, cujus perpendiculum e centro C ad Tangentem in K demiſſum, ſi CK dicatur x, eſt æquale $\frac{ax}{\sqrt{b^2-x^2}}$. Fig 8.

Et in hac Curva, urgente vi centripeta quæ ſit reciproce ut cubus diſtantiæ, movebitur corpus, ſi ſecundum directionem Tangentis cum juſta velocitate exeat. Qualis autem debet eſſe velocitas quæ faciat ut corpus harum Curvarum quamvis deſcribat, ſic invenietur.

Cum velocitas qua corpus in trajectoria quacunque movetur ſit reciproce ut quantitas p, aſſumendo conſtantem quamvis a, ea ſemper exponi poteſt per $\frac{a}{p}$. Et ſi ad Axem CV ordinentur rectæ quæ ſint reciproce ut cubi diſtantiarum a centro, ſeu ut vires centripetæ, & hac ratione formetur Figura curvilinea, ejus Area indefinite extenſa ſemper exponi poteſt per $\frac{b^2}{x^2}$, ut ex Quadraturis conſtat. At Area illa eſt ut quadratum velocitatis quæ acquiritur ab infinita diſtantia cadendo, adeoque velocitas hoc caſu acquiſita erit ut $\frac{b}{x}$. Hinc ſi velocitas illa dicatur y, & velocitas qua corpus in Trajectoria movetur dicatur v, taleſque aſſumantur quantitates a & b, ut in una aliqua a centro diſtantia ſit $y : v :: \frac{b}{x} : \frac{a}{p}$, erit ubique in omnibus diſtantiis $y : v :: \frac{b}{x} : \frac{a}{p} :: p : \frac{ax}{b}$. Unde ſi $y = v$, erit $p = \frac{ax}{b}$, & Curva hac velocitate deſcripta erit Spiralis Nautica; vel Circulus exiſtente $p = x$ & $a = b$.

Si y ſit major quam v, tunc p major erit quam $\frac{ax}{b}$; eritque illa, ut ex præcedentibus conſtat, $= \frac{ax}{\sqrt{b^2-x^2}}$. Curva autem conſtruetur per ſectorem Hyperbolicum, ut in ultimo caſu oſtenſum fuit, ubi diſtantia corporis a centro per concurſum Tangentis Hyperbolæ cum Axe tranſverſo deter-

determinatur. Si y ſit minor quam v, at in tantilla ratione ut maneat b major quam a, Curva formabitur per eundem ſectorem hyperbolicum. At diſtantia corporis a centro deſumitur ex concurſu Tangentis cum Axe conjugato.

Si ſit $y : v :: p : x$, erit in eo caſu $a = b$, & Curva evadit Spiralis Hyperbolica, ubi eſt $p = \frac{ax}{\sqrt{a^2 + x^2}}$. Hinc ſi de loco quovis projiciatur corpus ſecundum datam rectam, cum ea velocitate quæ ſit ad velocitatem ab infinito cadendo acquiſitam, ut diſtantia corporis a centro ad perpendicularem e centro ad lineam directionis demiſſam, movebitur illud corpus in Spirali Hyperbolica. Si denique ſit v tanto major quam y, ut ſit etiam a major quam b, Curva conſtruetur per Sectores Circulares. Atque hac ratione data velocitate ſemper determinari poſſit relatio quantitatum a & b, ac proinde Curva deſcribetur in qua corpus cum illa velocitate movebitur: & viciſſim data Curva, ſeu datis quantitatibus a & b, invenietur velocitas qua Curva illa deſcribitur,

Omnium Curvarum Areæ (ſi circulum excipias) quæ urgente hac vi centripeta deſcribi poſſunt, ſunt perfecte quadrabiles. Nam primo, in ſpirali Logarithmica, quia eſt $p = \frac{ax}{b}$, erit $KN = \frac{a\dot{x}}{\sqrt{b^2 - a^2}} = \frac{a\dot{x}}{c}$, ponen-

Fig. 2. do $b^2 - a^2 = c^2$: *vid. Fig.* II. adeoque erit triangulum $CKI = \frac{\frac{1}{2}ax\dot{x}}{c}$, cujus Fluens eſt $\frac{ax^2}{4c}$ = Areæ Curvæ.

Si p ſit $\frac{ax}{\sqrt{b^2 + x^2}}$, & a major quam b, oſtenſum eſt eſſe $KN = \frac{a\dot{x}}{\sqrt{x^2 - c^2}}$, unde $KN \times \frac{1}{2}CI = \frac{\frac{1}{2}ax\dot{x}}{\sqrt{x^2 - c^2}}$, cujus Fluens eſt $\frac{1}{2}a \times \sqrt{x^2 - c^2}$ = Areæ Curvæ. At ſi a minor ſit quam b, ſit $KN = \frac{a\dot{x}}{\sqrt{x^2 + c^2}}$, & $KN \times \frac{1}{2}CI = \frac{\frac{1}{2}ax\dot{x}}{\sqrt{x^2 + c^2}}$, cujus Fluens eſt $\frac{1}{2}a\sqrt{x^2 + c^2} - Q$ = Areæ Curvæ. Ponatur $x = 0$, & fiet $\frac{1}{2}ac - Q = 0$, unde $Q = \frac{1}{2}ac$, & Area Curvæ fit $= \frac{1}{2}a\sqrt{x^2 + c^2} - \frac{1}{2}ac$.

In Spirali Hyperbolica evaneſcit quantitas c, & Area Curvæ fit $\frac{1}{2}ax$.

Si p ſit $= \frac{ax}{\sqrt{b^2 - x^2}}$, oſtenſum eſt eſſe $KN = \frac{a\dot{x}}{\sqrt{c^2 - x^2}}$, unde $\frac{1}{2}CI \times KN = \frac{\frac{1}{2}ax\dot{x}}{\sqrt{c^2 - x^2}}$, cujus fluens eſt $Q - \frac{1}{2}a\sqrt{c^2 - x^2}$ = Areæ. Fiat $x = 0$, & erit $Q - \frac{1}{2}ac = 0$, ſeu $Q = \frac{1}{2}ac$; unde erit Area Curvæ ſemper æqualis $\frac{1}{2}ac - \frac{1}{2}a\sqrt{c^2 - x^2}$. Fiat $c^2 - x^2 = 0$ ſeu $c = x$, & Area curvæ fit $\frac{1}{2}ac$. Unde ſi initium Areæ non capiatur ab initio ipſius x, ſeu ubi x eſt $= 0$,

Fig. 7. ſed ubi $x = c$ eſt maxima, hoc eſt ſi Area ab V incipiat, (*vid. Fig.* 7.) erit Area ſemper æqualis $\frac{1}{2}a\sqrt{c^2 - x^2}$.

De Areis quas deſcribunt corpora radiis ad centrum ductis, urgente vi centripeta

centripeta quæ ſit reciproce ut diſtantiarum cubi, ſequentia adnotavit Collega meus peritiſſimus Geometriæ Profeſſor *Halleius.* Nempe ſi corpora diverſos circulos vel diverſas Spirales Hyperbolicas hac lege deſcribunt; erunt areæ ſectorum, tam in Circulis quam in Spiralibus illis omnibus, æqualibus temporibus deſcriptæ, ſemper æquales: Nam velocitates corporum in circulis motorum ſecundum hanc legem, debent eſſe radiis ſeu diſtantiis reciproce proportionales, adeoque arcus ſimul percurſi erunt quoque in eadem radiorum reciproca ratione, unde ſtatim patebit ſectores ſimul deſcriptos eſſe æquales.

In reliquis omnibus Curvis cum ſit velocitas ad velocitatem corporis in eadem diſtantia in circulo moti ut $\frac{a}{b}\times x$ ad p, ſeu ut $\frac{a}{x}\times IK$ ad KN; interea dum corpus in Trajectoria percurrit Lineolam IK, corpus aliud in Circulo in eadem diſtantia motum percurret arcum $\frac{b}{a}\times KN$; & Area Sectoris Circuli & Trajectoriæ ſimul deſcriptæ erunt $\frac{b}{a}\times KN\times\frac{1}{2}CN$ & $KN\times\frac{1}{2}CN$, quæ duæ Areæ ſunt in ratione data, ſcil. ut b ad a. Adeoque ubi eſt $a=b$, uti fit in ſpirali Hyperbolica, Area ſic deſcripta erit ſemper æqualis Areæ Sectoris circularis in æquali tempore deſcriptæ. v. Fig. 3.

XX. Sit n numerus quicunque, y quantitas incognita, ſive Æquationis Radix quæſita, ſitque a quantitas quævis omnino cognita, ſive ut vocant Homogeneum Comparationis: Atque horum inter ſe relatio exprimatur per Æquationem $ny+\frac{nn-1}{2\times3}ny^3+\frac{nn-1}{2\times3}\times\frac{nn-9}{4\times5}ny^5+\frac{nn-1}{2\times3}\times\frac{nn-9}{4\times5}\times\frac{nn-25}{6\times7}ny^7$, &c. $=a$.

Analytical Solution of certain infiniteſimal Equations, by Mr. De Moivre. *n.* 309. *p.* 2368.

Ex hujus ſeriei natura manifeſtum eſt, quod ſi n ſumatur numerus aliquis impar (integer ſcilicet, nec refert utrum ſit affirmativus vel negativus) tunc ſeries ſponte ſua terminabitur, & Æquatio fit una ex ſupra præfinitis, cujus Radix eſt

$$(1)\ y=\tfrac{1}{2}\sqrt[n]{\sqrt{1+aa}+a}-\frac{\frac{1}{2}}{\sqrt[n]{\sqrt{1+aa}+a}}.$$

$$\text{vel }(2)\ y=\tfrac{1}{2}\sqrt[n]{\sqrt{1+aa}+a}-\tfrac{1}{2}\sqrt[n]{\sqrt{1+aa}-a}$$

$$\text{vel }(3)\ y=\frac{\frac{1}{2}}{\sqrt[n]{\sqrt{1+aa}-a}}-\tfrac{1}{2}\sqrt[n]{\sqrt{1+aa}-a}$$

$$\text{vel }(4)\ y=\frac{\frac{1}{2}}{\sqrt[n]{\sqrt{1+aa}-a}}-\frac{\frac{1}{2}}{\sqrt[n]{\sqrt{1+aa}-a}}.$$

Exempli gratia, ſit hujus Æquationis poteſtatis quintæ $5y+20y^3+16y^5=4$ Radix invenienda, quo in caſu erit $n=5$ & $a=4$. Radix juxta

juxta formam primam erit $y = \frac{1}{2}\sqrt[5]{\sqrt{17}+4} - \frac{\frac{1}{2}}{\sqrt[5]{\sqrt{17}+4}}$, quæ in numeris vulgaribus expeditissime explicari potest ad hunc modum. Est $\sqrt{17}+4 = 8.1231$, cujus Logarithmus 0.9097164, & hujus pars quinta 0.1819433, huic respondens numerus est $1.5203 = \sqrt[5]{\sqrt{17}+4}$. Ipsius vero 0.1819433 Complementum Arithmeticum est 9.8180567, cui respondet numerus 0.6577 $= \frac{1}{\sqrt[5]{\sqrt{17}+4}}$. Igitur horum numerorum semidifferentia $0.4313 = y$.

Hic venit Observandum quod loco Radicis generalis, non incommode sumeretur $y = \frac{1}{2}\sqrt[n]{2a} - \frac{\frac{1}{2}}{\sqrt[n]{2a}}$, si quando numerus a respectu unitatis, sit satis magnus, ut si Æquatio fuerit $5y + 20y^3 + 16y^5 = 682$, erit Log. $2a = 3.1348143$, cujus pars quinta 0.6269628, & huic respondens numerus 4.236. Complementi autem Arithmetici 9.3730372 numerus est 0.236 & horum numerorum semidifferentia $2 = y$.

Atqui præterea, si in Æquatione præcedenti signa alternatim sint affirmantia & negantia, vel quod eodem redit, si series obvenerit hujus modi $ny + \frac{1-nn}{2\times3}ny^3 + \frac{1-nn}{2\times3}\times\frac{9-nn}{4\times5}ny^5 + \frac{1-nn}{2\times3}\times\frac{9-nn}{4\times5}\times\frac{25-nn}{6\times7}ny^7$, &c. $= a$. Erit hujus Radix

$$(1)\ y = \tfrac{1}{2}\sqrt[n]{a+\sqrt{aa-1}} + \frac{\frac{1}{2}}{\sqrt[n]{a+\sqrt{aa-1}}}.$$

$$\text{vel}(2)\ y = \tfrac{1}{2}\sqrt[n]{a+\sqrt{aa-1}} + \tfrac{1}{2}\sqrt[n]{a-\sqrt{aa-1}}.$$

$$\text{vel}(3)\ y = \frac{\frac{1}{2}}{\sqrt[n]{a-\sqrt{aa-1}}} + \tfrac{1}{2}\sqrt[n]{a-\sqrt{aa-1}}$$

$$\text{vel}(4)\ y = \frac{\frac{1}{2}}{\sqrt[n]{a-\sqrt{aa-1}}} + \frac{\frac{1}{2}}{\sqrt[n]{a+\sqrt{aa-1}}}.$$

Hic autem Notandum, quod si $\frac{n-1}{2}$ numerus extiterit impar, Radicis inventæ signum in ei contrarium permutandum est.

Proponatur Æquatio $5y - 20y^3 + 16y^5 = 6$, unde $n = 5$ & $a = 6$. Erit Radix $= \frac{1}{2}\sqrt[5]{6+\sqrt{35}} + \frac{\frac{1}{2}}{\sqrt[5]{6+\sqrt{35}}}$. Vel quoniam $6+\sqrt{35} = 11.916$, erit hujus logarithmus 1.0761304 & ejus pars quinta 0,2152561, Complementum vero Arithmeticum 9.7847439. Horum Logarithmorum numeri sunt 1.6415 & 0.6091 respective, quorum semisumma $1.1253 = y$.

Verum si acciderit ut a sit minor unitate, tunc Radicis forma secunda, ut quæ proposito est magis conveniens, præ reliquis seligenda est. Sic si

si Æquatio fuerit $5y - 20y^3 + 16y^5 = \frac{61}{64}$, erit $y = \frac{1}{2}\sqrt[5]{\frac{61}{64} + \sqrt{\frac{-375}{4096}}}$ $+ \frac{1}{2}\sqrt[5]{\frac{61}{64} - \sqrt{\frac{-375}{4096}}}$. Et quidem si Binomialium Radix quintana ullo pacto extrahi queat, prodibit Radix proba & possibilis, etsi expressio ipsa impossibilitatem mentiatur. Binomialis vero $\frac{61}{64} + \sqrt{\frac{-375}{4096}}$ Radix quintana est $\frac{1}{4} + \frac{1}{4}\sqrt{-15}$, & Binomialis $\frac{61}{64} - \sqrt{\frac{-375}{4096}}$ Radix itidem quintana est $\frac{1}{4} - \frac{1}{4}\sqrt{-15}$; quorum Binomialium semisumma $= \frac{1}{4} = y$.

Si autem extractio ista vel non peragi posset, vel etiam difficilior videretur, res ubique confici potest per Tabulam sinuum naturalium ad modum sequentem.

Ad Radium 1 sit $a = \frac{61}{64} = 0.95112$ sinus arcus cujusdam, qui proinde erit $72°:23'$ cujus pars quinta (eo quod $n = 5$) est $14°:28'$; hujus sinus $0.24981 = \frac{1}{4}$ proxime. Nec secus procedendum in Æquationibus graduum superiorum.

XXI. In *Phil. Transf.* * N°. 210. Dr. *Halley* has publish'd a compendious and useful Method of extracting the Roots of adfected Æquations of the common Form, in Numbers. This Method proceeds by assuming the Root desir'd, nearly true, to one or two Places in Decimals (which is done by Geometrical Construction, or some other convenient way) and correcting the Assumption, by comparing the Difference between the true Root and the assum'd, by means of a new Equation, whose Root is the Difference, and which he shews how to form from the Equation propos'd, by substituting the Value of the Root sought, partly in known, and partly in unknown Terms.

Abridg. Vol. I. p. 81.

An Attempt to improve the Method of Approximating in extracting the Roots of Equations in Numbers, by Dr. Taylor. *n.* 352. *p.* 610.

In doing this, he makes use of a Table of Products, (which he calls *Speculum Analyticum*) by which he computes the Co-Efficients in the new Equation for finding the Difference mention'd. This Table, I observ'd, was form'd in the same manner from the Equation propos'd, as the Fluxions are, taking the Root sought for the only flowing Quantity, its Fluxion for Unity, and after every Operation dividing the Product successively by the Numbers, 1, 2, 3, 4, *&c.* Hence I soon found that this Method might easily and naturally be drawn from *Cor.* 2. *Prop.* 7. of my *Methodus Incrementorum*, and that it was capable of a further degree of Generality; it being applicable, not only to Equations of the common Form, (*viz.* such as consist of Terms wherein the Powers of the Root sought are positive and integral, without any Radical Sign) but also to all Expressions in general, wherein any thing is proposed as given which by any known Method might be computed; if *vice versa*, the Root were consider'd as given: such as are all Radical Expressions of Binomials, Trinomials, or of any other Nomial, which may be computed by the Root given at least by Logarithms, whatever be the Index of the Power of that Nomial, as likewise Expres-

 sions

ſions of Logarithms, of Arches by the Sines or Tangents, of Areas of Curves by the *Abſciſſa*'s or any other Fluents, or Roots of Fluxional Equations, *&c.*

For the ſake of this great Generality, it may not be improper to ſhew how this Method is derived from the foreſaid *Corollary*. Therefore z and x being two flowing quantities (whoſe Relation to one another may be expreſt by any Equation whatſoever) by this *Corollary*, while z by flowing uniformly becomes $z + v$, x will become $x + \frac{\dot{x}}{1 \,.\, \dot{z}} v + \frac{\ddot{x}}{1 \,.\, 2 \dot{z}^2} v + \frac{\dddot{x}}{1 \,.\, 2 \,.\, 3 \dot{z}^3} v^3 +$ *&c.* or $x + \frac{\dot{x} v}{1} + \frac{\ddot{x} v^2}{1 \times 2} + \frac{\dddot{x} v^3}{1 \,.\, 2 \,.\, 3} +$ *&c.* for $\dot{z}$ putting 1.

Hence if y be the Root of any Expreſſion formed of y and known Quantities, and ſuppoſed equal to nothing, and z be a part of y, and x be formed of z and the known Quantities, in the ſame manner as the Expreſſion made equal to nothing is formed of y; and let y be equal to $z + v$: the difference v will be found by Extracting the Root of this expreſſion $x + \frac{\dot{x} v}{1} + \frac{\ddot{x} v^2}{1 \,.\, 2} + \frac{\dddot{x} v^3}{1 \,.\, 2 \,.\, 3} +$ *&c.* $= 0$. For in this Caſe z being become $z + v = y$, x which is now become $x + \dot{x} v + \frac{\ddot{x} v}{2} +$ *&c.* muſt become equal to nothing.

The Root v in the Equation $x + \frac{\dot{x} v}{1} + \frac{\ddot{x} v^2}{1 \,.\, 2} + \frac{\dddot{x} v^3}{1 \,.\, 2 \,.\, 3} +$ *&c.* $= 0$, is to be found upon the Suppoſition of its being very ſmall with reſpect to z, (as it muſt be, if z be taken tolerably exact) by which means the Terms $\frac{\dddot{x} v^3}{1 \,.\, 2 \,.\, 3} + \frac{\ddddot{x} v^4}{1 \,.\, 2 \,.\, 3 \,.\, 4} +$ *&c.* may be neglected, upon account of their ſmallneſs with reſpect to the other Terms, ſo as to leave the Equation $x + \frac{\dot{x} v}{1} + \frac{\ddot{x} v^2}{1 \,.\, 2} = 0$, for finding the firſt approximation of v.

By extracting the Root of this Equation, we have $v = \sqrt{\frac{\dot{x}^2}{\ddot{x}^2} - \frac{2x}{\ddot{x}}} - \frac{\dot{x}}{\ddot{x}}$. That is,

Firſt, $\sqrt{\frac{\dot{x}^2}{\ddot{x}^2} - \frac{2x}{\ddot{x}}} - \frac{\dot{x}}{\ddot{x}}$, if $x + \dot{x} v + \frac{\ddot{x} v^2}{2} = 0$.

Sec. $\sqrt{\frac{\dot{x}^2}{\ddot{x}^2} + \frac{2x}{\ddot{x}}} - \frac{\dot{x}}{\ddot{x}}$, if $-x + \dot{x} v + \frac{\ddot{x} v^2}{2} = 0$.

Third. $\sqrt{\frac{\dot{x}}{\ddot{x}}} - \sqrt{\frac{\dot{x}^2}{\ddot{x}^2} - \frac{2x}{\ddot{x}}}$, if $x - \dot{x} v + \frac{\ddot{x} v^2}{2}$, *&c.* $= 0$.

Fourth.

Fourth. $\sqrt{\frac{\dot{x}}{\ddot{x}}} - \sqrt{\frac{\dot{x}^2}{\ddot{x}^2} + \frac{2x}{\ddot{x}}}$, if $-x - \dot{x}v + \frac{\ddot{x}v^2}{2}$, &c. $= 0$.

This approximation gives v exact to twice as many places as there are true Figures in z, and therefore trebles the number of true Figures in the Expression of y by $z + v$, which may be taken for a new Value of z, for computing a second v, seeking other Values of x, $\dot{x}$, $\ddot{x}$, &c. Tho' when z is tolerably exact (which it may be esteem'd when it contains two or three or more true Figures in the Value of y, according to the Number of Figures the Root is propofed to be computed to,) the Calculation may be restor'd without so much trouble, only by taking $\sqrt{\frac{\dot{x}^2}{\ddot{x}^2} \pm \frac{2x}{\ddot{x}} - \frac{2\dddot{x}}{2 . 3\, \ddot{x}}v^3 - \frac{2\ddddot{x}}{1 . 2 . 3 . 4\, \ddot{x}}v^4 \text{ \&c.}}$ instead of $\sqrt{\frac{\dot{x}^2}{\ddot{x}^2} \pm \frac{2x}{\ddot{x}}}$ taking every time for v its Value last computed.

From the same Equation $x + \dot{x}v + \frac{\ddot{x}v^2}{2} + \frac{\dddot{x}v^3}{1 . 2 . 3} +$ &c. $= 0$, may be gather'd also a rational Form, *viz.* $v = \frac{-x}{\dot{x} - \frac{x\ddot{x}}{2\dot{x}}}$. For neglecting the Terms $\frac{\dddot{x}v^3}{1 . 2 . 3}$, &c. we have $v = \frac{-x}{\dot{x} + \frac{\ddot{x}}{2}v}$ which is nearly $= \frac{-x}{\dot{x}}$. Therefore in the Divisor instead of v writing $\frac{-x}{\dot{x}}$ we have more exactly $v = \frac{-x}{\dot{x} - \frac{\ddot{x}x}{2\dot{x}}}$, that is

1. $\frac{-x}{\dot{x} - \frac{\ddot{x}x}{2\dot{x}}}$, when $x + \dot{x}v + \frac{\ddot{x}v^2}{2}$ &c. $= 0$.

2. $\frac{x}{\dot{x} + \frac{\ddot{x}x}{2\dot{x}}}$, when $-x + \dot{x}v + \frac{\ddot{x}v^2}{2}$ &c. $= 0$.

3. $\frac{x}{\dot{x} - \frac{\ddot{x}x}{2\dot{x}}}$, when $x - \dot{x}v + \frac{\ddot{x}v^2}{2}$ &c. $= 0$.

4. $\frac{-x}{\dot{x} + \frac{\ddot{x}x}{2\dot{x}}}$, when $-x - \dot{x}v + \frac{\ddot{x}v^2}{2}$ &c. $= 0$.

This *Formula* will also triplicate the number of true Figures in z. And the Calculation may be repeated, after every Operation, taking for

for a Divisor $\dot{x} \pm \frac{\ddot{x}}{2}v + \frac{\dddot{x}v^2}{1.2.3} + \frac{\ddddot{x}v^3}{1.2.3.4} +$ &c. instead of $x + \frac{\dddot{x}x}{2\dot{x}}$.

Dr. *Halley* has fully explain'd the manner of using both these *Formula*'s in Æquations of the common Form; wherefore I shall be the shorter in explaining two or three Examples of another sort.

Ex. 1. Let it be proposed to find the Root of this Equation $\overline{y^2+1}|^{\sqrt{2}} + y - 16 = 0$. In this Case for y writing z, and for o writing x, we have $\overline{z^2+1}|^{\sqrt{2}} + z - 16 = x$. Whence by taking the Fluxions, we have $\dot{x} = 2\sqrt{2} \times z \times \overline{z^2+1}|^{\sqrt{2}-1} + 1$, and $\ddot{x} = 2\sqrt{2} \times \overline{8 - 4\sqrt{2}\, z^2} \times \overline{z^2+1}|^{\sqrt{2}-2}$. For finding the first Figures of the Root y, for $\sqrt{2}$ take $\frac{3}{2}$, and we have the Equation $\overline{y^2+1}|^{\frac{3}{2}} + y - 16 = 0$, which being expanded gives $y^6 + 3y^4 + 2y^2 + 32y - 255 = 0$.

By this Equation I find that for the first supposition we may take $z = 2$. Therefore in order to find v, let us now make $\sqrt{2} = \frac{7}{5}$, (which is nearer than before) and we have $x = \overline{z^2+1}|^{\frac{7}{5}} + z - 16 = \overline{2^2+1}|^{\frac{7}{5}} - 14 = 5^{\frac{7}{5}} - 14 = -4,48$; $\dot{x} = 10,66$; $\ddot{x} = 4,72$. Whence by the second rational Form $v = \frac{4,48}{10,66 + \frac{4,72 \times 4,48}{2 \times 10,66}} = 0,38$; which must be too big, because $\frac{7}{5} \sqsubset \sqrt{2}$, and therefore will require a larger Value of y to exhaust the Equation, than where $\sqrt{2}$ is exact. For the second supposition therefore, let us take $z = 2,3$, and make $\sqrt{2} = 1,4142136$, and by help of the Logarithms we shall have $\overline{z^2+1}|^{\sqrt{2}} = 13,47294$, whence $x = -0,22706$; $\dot{x} = 14,93429$, and $\ddot{x} = 5,18419$. Hence by the 2*d* irrational *Formula* $v = \sqrt{\frac{14,93429^2}{5,18419^2} + \frac{0,45412}{5,18419}} - \frac{14,93429}{5,18419} = 0,01516$, which gives $y = z + v = 2,31516$, which is true to six Places. If you desire it more exact than to the extent of the Tables of Logarithms, taking $z = 2,31516$ for the next Supposition, the Calculation must be repeated by computing of $\overline{zz+1}|^{\sqrt{2}}$ to a sufficient number of Places; which must be done by the Binomial Series, or by making a Logarithm on purpose, true to as many places as are necessary.

Ex. 2. For another Example, let it be required to find the Number whose Logarithm is 0,29, supposing we had no other Table of Logarithms but Mr. *Sharp*'s of 200 Logarithms to a great many places. This amounts to the resolving this Equation $ly = 0,29$, or $ly - 0,29 = 0$. Hence therefore we have $x = lx - 0,29$, $\dot{x} = \frac{a}{z}$ (a being the *Modulus* belonging to the Table we use, *viz.* 0,434294481 9, &c.) $\ddot{x} = \frac{-a}{z^2}$, $\dddot{x} = \frac{2a}{z^3}$, $\ddddot{x} =$

$-6a$

$\frac{-6a}{z^4}$ &c. In this Case because $\ddot{x}$ has a negative Sign, changing the Signs of all the Coefficients, the Canon for v will be found in the fourth Case, which in the irrational Form gives $v = \frac{\dot{x}}{\ddot{x}} - \sqrt{\frac{\dot{x}^2}{\ddot{x}^2} + \frac{2x}{\ddot{x}} - \frac{2\dddot{x}}{2\cdot 3\ddot{x}}v^3 - \frac{2\ddddot{x}}{2\cdot 3\cdot 4\ddot{x}}v^4 \&c.} = z - \sqrt{z^2 + \frac{2lz - 0{,}58}{a} \times z^2 + \frac{2v^3}{3z} - \frac{2v^4}{4z^2} + \frac{2v^5}{5z^3} \&c.}$ In this Case to avoid often dividing by z, it will be most convenient to compute $\frac{v}{z}$, which is got from this Equation $\frac{v}{z} = 1 - \sqrt{1 + \frac{2lz - 0{,}58}{a} + \frac{2v^3}{3z^3} - \frac{2v^4}{4z^4} + \frac{2v^5}{5z^5}}$, &c. The nearest Logarithm, in the Tables proposed, to the proposed Logarithm 0,29 is 0,2900346114, its Number being 1,95. Therefore for the first supposition taking $z = 1{,}95$, we have $x\ (= lz - 0{,}29 = 0{,}2900346114 - 0{,}29) = 0{,}0000346114$, and $\frac{2lz - 0{,}58}{a} = \frac{0{,}0000692228}{0{,}4342944819} = 0{,}00015939139$, and $1 + \frac{2lz - 0{,}58}{a} = 1{,}00015939139$. Whence for the first approximation we have $\frac{v}{z} = 1 - \sqrt{1{,}00015939139} = -0{,}00007969247$; and $v = -0{,}00015540032$, and $y = z + v = 1{,}94984459968$. Which is true to eleven places, and may easily be corrected by the Terms $\frac{2v^3}{3z}$ &c. which I leave to the Readers curiosity.

Being upon the Subject of Approximations, it may not be amiss to set down here two Approximations I have formerly hit upon. The one is a Series of Terms for expressing the Root of any Quadratick Equation: and the other is a particular Method of Approximating in the invention of Logarithms, which has no occasion for any of the Transcendental Methods, and is expeditious enough for making the Tables without much trouble.

A general Series for expressing the Root of any Quadratick Equation.

Any Quadratick Equation being reduc'd to this Form $xx - mqx + my = 0$, the Root x will be exprest by this Series of Terms.

$$x = \frac{y}{q} + A \times \frac{1}{\frac{mq^2}{y} - 2} + B \times \frac{1}{a^2 - 2} + C \times \frac{1}{b^2 - 2} + D \times \frac{1}{c^2 - 2}$$

&c. Which must be thus interpreted.

1. The Capital Letters A, B, C, &c. stand for the whole Terms with their Signs, preceding those wherein they are found, as $B = A \times \frac{1}{\frac{mq^2}{y} - 2}$.

2. The

2. The little Letters a, b, c, &c. in the Divisors, are equal to the whole Divisors of the Fraction in the Terms immediately preceding; thus $b = a^2 - 2$.

For an Example of this, let it be required to find $\sqrt{2}$. Putting $\sqrt{2} = x + 1$, we have $x^2 + 2x - 1 = 0$. which being compared with the general *Formula*, gives $mq = -2$, and $my = -1$: therefore for m taking -1, we have $q = 2$, and $y = 1$, which Values substituted in the Series give $x = \frac{1}{2} - \frac{1}{2 \times 6} - \frac{1}{2 \times 6 \times 34} - \frac{1}{2 \times 6 \times 34 \times 1154} - \frac{1}{2 \times 6 \times 34 \times 1154 \times 1331714}$ &c. The Fractions here wrote down giving the Root true to twenty three Places.

A new Method of computing Logarithms.

This Method is founded upon these Considerations.

1. That the Sum of the Logarithms of any two Numbers is the Logarithm of the Product of those two Numbers Multiplied together.

2. That the Logarithm of Unite is nothing; and consequently that the nearer any Number is to Unite, the nearer will its Logarithm be to 0. 3*dly*. That the Product by Multiplication of two Numbers, whereof one is bigger, and the other less than Unite, is nearer to Unite than that of the two Numbers which is on the same side of Unite with its self; for Example the two Numbers being $\frac{2}{1}$ and $\frac{4}{5}$, the Product $\frac{8}{5}$ is less than Unite, but nearer to it than $\frac{4}{5}$, which is also less than Unite. Upon these Considerations, I found the present Approximation; which will be best explain'd by an Example. Let it therefore be proposed to find the Relation of the Logarithms of 2 and of 10. In order to this, I take two Fractions $\frac{128}{100}$ and $\frac{8}{10}$, *viz.* $\frac{2|^7}{10|^2}$ and $\frac{2|^3}{10|^1}$ whose Numerators are Powers of 2, and their Denominators Powers of 10; one of them being bigger, and the other less than 1. Having set these down in Decimal Fractions in the first Column of the Table annext, against them in the second Column I set A and B for their Logarithms, expressing by an Equation the manner how they are Compounded of the Logarithms of 2 and 10, for which I write $l\,2$ and $l\,10$. Then Multiplying the two Numbers in the first Column together, I have a third Number 1, 024, against which I write C for its Logarithm, expressing likewise by an Equation in what manner C is formed of the foregoing Logarithms A and B. And in the same manner the Calculation is continued; only observing this *Compendium*, that before I Multiply the two last Numbers already got in the Table, I consider what Power of one of them must be used to bring the Product the nearest to Unite that can be. This is found, after we have gone a little way in the Table, only by dividing the Differences of the Numbers from Unite one by the other, and taking the Quotient with the nearest, for the Index of the Power wanted. Thus the two last Numbers

bers in the Table being 0,8 and 1,024, their Differences from Unit are 0,200 and 0,024; therefore $\frac{0,200}{0,024}$ gives 9 for the Index; wherefore Multiplying the ninth Power of 1,024 by 0,8, I have the next Number 0,990352031429, whose Logarithm is $D = 9\,C + B$. In seeking the Index in this manner by Division of the Differences, the Quotient ought generally to be taken with the least: but in the present Case it happens to be the most, because instead of the Difference between 0,8 and 1, we ought strictly to have taken the Difference between the reciprocal 1,25 and 1, which would have given the Index 10; and that would be too big, because the Product by that means would have been bigger than 1, as 1,024 is. Whereas this Approximation requires that the Numbers in the first Column be alternately greater and less than 1, as may be seen in the Table.

1, 28000000000	$A = 7\,l\,2 - 2\,l\,10$	$l\,2$	$> 0,28$
0, 800000000000	$B = 3\,l\,2 - l\,10$		$< 0,33$
1, 02400000000	$C = B + A = 10\,l\,2 - 3\,l\,10$		$> 0,300$
0, 990352031429	$D = 9\,C + B = 93\,l\,2 - 28\,l\,10$		$< 0,30107$
1, 004336277664	$E = 2\,D + C = 196\,l\,2 - 59\,l\,10$		$> 0,301020$
0, 998959536107	$F = 2\,E + D = 485\,l\,2 - 146\,l\,10$		$< 0,3010309$
1, 000162894165	$G = 4\,F + E = 2136\,l\,2 - 643\,l\,10$		$> 0,30102996$
0, 999936281874	$H = 6\,G + F = 13301\,l\,2 - 4004\,l\,10$		$< 0,301029997$
1, 000035441215	$I = 2\,H + G = 28738\,l\,2 - 8651\,l\,10$		$> 0,3010299951$
0, 999971720830	$K = I + H = 42039\,l\,2 - 12655\,l\,10$		$< 0,3010299959$
1, 000007161046	$L = K + I = 70777\,l\,2 - 21306\,l\,10$		$> 0,30102999562$
0, 999993203514	$M = 3\,L + K = 254370\,l\,2 - 76573\,l\,20$		$< 0,30102999567$
1, 000000364511	$N = M + L = 325147\,l\,2 - 97879\,l\,10$		$> 0,301029995 6635$
0, 999999764687	$O = 18\,N + M = 6107016\,l\,2 - 1838395\,l\,10$		$< 0,3010299956640$
Com. Ar. 235313			

$0 = 364511\,O + 235313\,N = 2302585825187\,l\,2 - 693147400972\,l\,10$ $\quad > 0,301029995663987$

When

When I have in this manner continued the Calculation, till I have got the Numbers ſmall enough, I ſuppoſe the laſt Logarithm to be equal to nothing. Which gives me an Equation, from which having got away the Letters by means of the foregoing Equations, I have the relation of the Logarithms propoſed. In this manner if I ſuppoſe $G = 0$, I have $2136\, l\, 2 - 643\, l\, 10 = 0$. Which gives the Logarithm of 2 true in ſeven Figures, and too big in the Eighth; which happens becauſe the Number correſponding with G is bigger than Unite.

There is another Expedient which renders this Calculation ſtill ſhorter. It is founded upon this Conſideration, that when x is very ſmall $\overline{1+x}|^n$ is very nearly $1 + nx$. Hence if $1 + x$, and $1 - z$ are the two laſt Numbers already got in the firſt Column of the Table, and their Powers $\overline{1+x}^m$ and $\overline{1-z}|^n$ are ſuch as will make the Product $\overline{1+x}|^m \times \overline{1+z}^n$ very near to Unite, m and n may be found thus: $\overline{1+x}|^m = 1 + mx$, and $\overline{1-z}|^n = 1 - nz$, and conſequently $\overline{1+x}|^m \times \overline{1-z}|^n = 1 + mx - nz - mnzx$, or (neglecting $mnzx$) $1 + mx - nz$. Make this equal to 1, and we have $m : n :: z : x :: l\,\overline{1-z} : l\,\overline{1+x}$. Whence $x\, l\,\overline{1-z} + z\, l\,\overline{1+x} = 0$. To give an Example of the Application of this, let 1,024 and 0,990352 be the laſt Numbers in the Table, their Logarithms being C and D. Then we have $1{,}024 = 1 + x$, and $0{,}990352 = 1 - z$, and conſequently $x = 0{,}024$, and $z = 0{,}009648$. Whence the Ratio $\frac{z}{x}$ in the leaſt Numbers is $\frac{201}{500}$. So that for finding the Logarithms propoſed we may have $500\, D + 201\, C = 48510\, l\, 2 - 14603\, l\, 10 = 0$, which gives $l\, 2 = 0{,}3010307$, which is too big in the laſt Figure; but it is nearer the truth, than what is got from the Logarithm F ſuppoſed equal to nothing. So that by this means we have ſaved four Multiplications, which were neceſſary to find the Number 9989595 &c. correſpondent to F, and which muſt have been had if we would make the Logarithm true to the ſame Number of places without this *Compendium*.

Of infinite Series's, by Mr. Monmort. *n. 353. p. 633.*

XXI. *Prop.* I. *Prob.* Invenire ſummam terminorum quot libuerit Seriei hujus $a \times \overline{a+n} \times \overline{a+2n} \times$ &c. $\times \overline{a+p-1n} + \overline{a+n} \times \overline{a+2n} \times \overline{a+3n} \times$ &c. $\times \overline{a+pn} + \overline{a+2n} \times \overline{a+3n} \times \overline{a+4n} \times$ &c. $\times \overline{a+p+1n} + a + 3n \times$ &c. Ubi eſt n differentia data, tam inter Factores continuos, a, $a + 2n$, &c. ejuſdem cujuſvis termini, quam inter Factores homologos terminorum diverſorum in Serie continuata; atque deſignat p numerum factorum hujuſmodi in quovis termino.

Solutio. Per x deſignetur primus Factorum in ultimo terminorum quorum ſumma requiritur, atque ſumma illa erit

$$\frac{x \times \overline{x+n} \times \text{\&c.} \times \overline{x+pn} - \overline{a-n} \times a \times \text{\&c.} \times \overline{a+p-1n}}{\overline{p \quad 1n}} \quad Q.E.I.$$

Ex. 1. Proponatur Series numerorum naturalium $1+2+3+4+$ &c. & invenienda ſit ſumma tot terminorum quot ſunt unitates in numero

mero z, qui in hoc casu est etiam ultimus terminorum quorum summa requiritur. In hoc itaque casu sunt $a=1$, $n=1$, $p=1$, & $x=z$. Unde fit $x \times \overline{x+n} \times$ &c. $\times \overline{x+pn} = z \times \overline{z+1}$, $\overline{a-n} \times a \times$ &c. $\times \overline{a+\overline{p-1}\,n} = 0 \times 1$, atque $\overline{p+1}\,n = 2 \times 1$; adeoque summa quæsita est $\frac{z \times \overline{z+1}}{2}$.

Ex. 2. Invenienda sit summa tot terminorum, quot sunt unitates in numero z, Seriei $1+3+6+10+$ &c. Numerorum Triangularium. Numeri 1, 3, 6, 10, &c. in hac Serie sic scribi possunt $\frac{1\times 2}{2}$, $\frac{2\times 3}{2}$, $\frac{3\times 4}{2}$, $\frac{4\times 5}{2}$, &c. Hoc pacto, seposito divisore dato 2, Series revocatur ad formam Propositionis, existentibus $a=1$, $n=1$, & $p=2$, $x=z$. Unde summa Seriei duplicata est $\frac{x \times \overline{x+1} \times \overline{x+2} - 0 \times 1 \times 2}{3} = \frac{x \times \overline{x+1} \times \overline{x+2}}{3}$; adeoque habita ratione divisoris 2, Summa Seriei ipsius est $\frac{x \times \overline{x+1} \times \overline{x+2}}{2 \times 3}$, vel $\frac{z \times \overline{z+1} \times \overline{z+2}}{2 \times 3}$, in hoc casu existente x eodem ac z. Ad eundem modum inveniuntur summæ cæterorum numerorum figuratorum, quorum formulæ jam vulgo innotescunt.

Ex. 3. Sint $a=1$, $n=2$, $p=3$, ut sit Series proposita $1\times 3\times 5+3\times 5\times 7+5\times 7\times 9+$ &c. In hoc itaque casu formula summæ fit $\frac{x \times \overline{x+2} \times \overline{x+4} \times \overline{x+6} - \overline{1-2} \times 1 \times 3 \times 5}{4 \times 2} = \frac{x \times \overline{x+2} \times \overline{x+4} \times \overline{x+6} + 15}{8}$. Verbi gratia si quæratur summa decem terminorum, fit $x=19$ (nempe terminus decimus in Serie Arithmetice proportionalium, 1, 3, 5, 7, &c.) adeoque summa est $\frac{19 \times 21 \times 23 \times 25 + 15}{8} = 28680$. Propositio vero sic demonstratur.

Demonstratio. Sit Series quantitatum A, B, C, D, E, &c. quarum differentiæ constituant Seriem a, b, c, d, &c. (nempe ut sint $a=B-A$, $b=C-B$, $c=D-C$, &c) Hinc statim colligitur esse $a+b=C-A$, $a+b+c=D-A$, $a+b+c+d=E-A$: & in genere aggregatum quotlibet terminorum Seriei a, b, c, d, &c. æquale est termino proxime insequenti Seriei A, B, C, D, E, &c. mulctato termino primo A. Pro A, B, C, &c. sume terminos $\frac{\overline{a-n} \times a \times \text{\&c.} \times \overline{a+\overline{p-1}\,n}}{\overline{p+1}\,n}$, $\frac{a \times \overline{a+n} \times \text{\&c.} \times \overline{a+pn}}{\overline{p+1}\,n}$, $\frac{\overline{a+n} \times \overline{a+2n} \times \text{\&c.} \times \overline{a+\overline{p+1}\,n}}{\overline{p+1}\,n}$, &c. hoc est, valores successivos ipsius $\frac{x \times \overline{x+n} \times \text{\&c.} \times \overline{x+pn}}{\overline{p+1}\,n}$; & eorum differentiæ, pro a, b, c, d, &c. sumendæ, erunt $a \times \overline{a+n} \times$ &c. $\times \overline{a+\overline{p-1}\,n}$, $\overline{a+n} \times \overline{a+2n} \times$ &c. $\times \overline{a+pn}$, &c. qui sunt ipsissimi termini Seriei propositæ. Sed comparando has

 Series,

Series, si terminus aliquis Seriei posterioris sit $x \times \overline{x+n} \times \&c. \times \overline{x+\overline{p-1}\,n}$, constat terminum uno ulteriorem in Serie priori fore $\frac{x \times \overline{x+n} \times \&c. \times \overline{x+pn}}{\overline{p+1}\,n}$. Summa itaque Seriei posterioris usque terminum $x \times \overline{x+n} \times \&c. \times \overline{x+\overline{p-1}\,n}$ inclusive est

$$\frac{x \times \overline{x+n} \times \&c. \times \overline{x+pn} - \overline{a-n} \times a \times \&c. \times \overline{a+\overline{p-1}\,n}}{\overline{p+1}\,n} \quad Q.\ E.\ D.$$

Scholium. 1. In hac propositione continetur particula quædam Methodi incrementorum, de qua ante biennium librum edidit D. *Brook Taylor Soc. Reg. Lond. Secr.* mihi amicitia conjunctissimus. Librum ipsum adeat qui de ea methodo plura scire velit: ad institutum nostrum sufficit observare quanta intersit affinitas inter Methodum hanc & Methodum Fluxionum seu differentialem. Nam ut in Methodo differentiali, ad inveniendum differentiale ipsius x dignitatis x^m, unum latus x convertendum est in differentiam dx; & ortum ducendum est in dignitatis Indicem m, ut sit $m\,dx\,x^{m-1}$ differentiale quæsitum; sic in Methodo Incrementorum *Ad inveniendum Incrementum facti hujusmodi* $x \times \overline{x+n} \times \overline{x+2n}$, *(ubi factores* $x, x+n, x+2n$, *sunt in progressione Arithmetica, cujus differentia communis est ipsius* x *Incrementum datum* n,*) Factorum minimus* x *convertendus est in Incrementum, & ortum ducendum est in numerum Factorum, ut sit* $3n \times \overline{x+n} \times \overline{x+2n}$ *Incrementum quæsitum, numero Factorum in casu exposito existente* 3. Sic etiam ipsius $x \times \overline{x+n}$ Incrementum sit $2n \times \overline{x+n}$.

2. Incrementa etiam Reciprocorum hujusmodi Factorum inveniuntur per eandem regulam; hoc nempe observato, quod cum sit Divisio contrarium Multiplicationis, vice ablationis minimi Factorum, sit jam addendus alius factor adhuc uno Incremento major; item quod Factorum numerus sit scribendus cum signo negativo. Hoc pacto ipsius $\frac{1}{x}$ Incrementum fit $\frac{-1 \times n}{x \times \overline{x+n}}$; ipsius $\frac{1}{x \times \overline{x+n}}$ Incrementum fit $\frac{-2 \times n}{x \times \overline{x+n} \times \overline{x+2n}}$, & sic de aliis hujusmodi. Hoc facile probatur sumendo differentias inter Integralium valores duos continuos.

3. Insistendo vestigiis Methodi directæ, hinc colliguntur præcepta Methodi inversæ, quibus inveniuntur Integralia Incrementorum oblatorum. *Applicetur* enim *Incrementum oblatum ad lateris Incrementum datum; addatur Factor adhuc uno Incremento minor, & applicetur ortum ad numerum Factorum sic auctorum.* Sic e. g. oblato Incremento $n \times x \times \overline{x+n} \times \overline{x+2n}$, fit primo $x \times \overline{x+n} \times \overline{x+2n}$, deinde $\overline{x-n} \times x \times \overline{x+n} \times \overline{x+2n}$, addito Factore $\overline{x-n}$; denique $\frac{\overline{x-n} \times x \times \overline{x+n} \times \overline{x+2n}}{4}$, quod est Integrale quæsitum. Hoc quidem ubi Factores sunt Multiplicantes; Ubi vero Factores occupant locum divisoris, mutatis mutandis, regula hæc est, *Applicetur Incrementum oblatum ad lateris incrementum datum; rejiciatur Factorum maximus, & applicetur ortum ad numerum Factorum relictorum cum signo negativo.* Exempli gratia oblato Incremento

n

$\frac{n}{x \times \overline{x+n} \times \overline{x+2n}}$, fit primo $\frac{1}{x \times \overline{x+n} \times \overline{x+2n}}$, deinde $\frac{1}{x \times \overline{x+n}}$, denique $\frac{1}{-2 \times x \times \overline{x+n}}$, ſeu $\frac{-1}{2x \times \overline{x+n}}$, quod eſt Integrale quæſitum.

4. In caſu hoc noviſſimo Integrale inventum, cum ſigno contrario, æquale eſt ſummæ omnium Incrementorum in Serie in infinitum continuata; *v. g.* eſt $\frac{1}{2x \times \overline{x+n}} = \frac{n}{x \times \overline{x+n} \times \overline{x+2n}} + \frac{n}{\overline{x+n} \times \overline{x+2n} \times \overline{x+3n}} + \frac{n}{\overline{x+2n} \times \overline{x+3n} \times \overline{x+4n}} +$ *&c.* Nam in hoc caſu, facto x tandem infinito, evaneſcit $\frac{1}{2x \times \overline{x+n}}$, hoc eſt, ultimus terminorum $A, B, C,$ *&c.* fit nihil; & ob contrarietatem ſignorum Integralis & Incrementi, vice $-A$ exprimitur aggregatum per $+A$.

Lemma 1. Per X deſignetur terminus quilibet in Serie quavis numerorum $M, N, O, P,$ *&c*; per x deſignetur locus termini iſtius X in Serie illa (*v. g.* ut ſit $x=1$, quando deſignat X terminum primum M; ſit $x=2$, quando deſignat X terminum ſecundum N, & ſic de cæteris) & ſint terminorum M, N, O, P prima differentiarum primarum b, c prima differentiarum ſecundarum, d prima tertiarum, e prima quartarum, & ſic porro. Tum erit $X = M + b \times \frac{x-1}{1} + c \times \frac{x-1}{1} \times \frac{x-2}{2} + d \times \frac{x-1}{1} \times \frac{x-2}{2} \times \frac{x-3}{3} + e \times \frac{x-1}{1} \times \frac{x-2}{2} \times \frac{x-3}{3} \times \frac{x-4}{4} +$ *&c.* Sequitur hoc ex tabula æquationum *pag.* 66. tractatus noſtri *Eſſay d' Analyſe, &c.*

Lemma 2. Iiſdem poſitis, per z deſignetur terminus quilibet in Serie Arithmetice proportionalium $a, \overline{a+n}, \overline{a+2n},$ *&c.* & ſit jam $X = A + Bz + Cz \times \overline{z+n} + Dz \times \overline{z+n} \times \overline{z+2n} + Ez \times \overline{z+n} \times \overline{z+2n} \times \overline{z+3n} +$ *&c.* Tum ipſorum $A, B, C, D, E,$ *&c.* valores erunt

$$A = M + b \times \frac{-a}{n} + c \times \frac{-a}{n} \times \frac{-a-n}{2n} + d \times \frac{-a}{n} \times \frac{-a-n}{2n} \times \frac{-a-2n}{3n} + e \times \frac{-a}{n} \times \frac{-a-n}{2n} \times \frac{-a-2n}{3n} \times \frac{-a-3n}{4n} + \&c.$$

$$B = \frac{1}{n} \times b + c \times \frac{-a-n}{n} + d \times \frac{-a-n}{n} \times \frac{-a-2n}{2n} + e \times \frac{-a-n}{n} \times \frac{-a-2n}{2n} \times \frac{-a-3n}{3n} \&c.$$

$$C = \frac{1}{n} \times \frac{1}{2n} \times c + d \times \frac{-a-2n}{n} + e \times \frac{-a-2n}{n} \times \frac{-a-3n}{2n} + \&c.$$

$$D = \frac{1}{n} \times \frac{1}{2n} \times \frac{1}{3n} d + e \times \frac{-a-3n}{n} + \&c.$$

E =

$$E = \frac{1}{n} \times \frac{1}{2n} \times \frac{1}{3n} \times \frac{1}{4n} e + \&c.$$

Ordo formandi coefficientes ipſorum b, c, d, e, &c. in his valoribus, per ſe eſt ſatis manifeſtus.

Demonſtratio. Quoniam per x & z deſignantur termini correſpondentes progreſſionum Arithmeticarum $1, 2, 3, 4$, &c. & a, $a+n$, $a+2n$, $a+3n$, &c. indicabit $x-1$ numerum differentiarum n qui in z continetur, ut ſit $z = a + \overline{x-1}\, n$. Hinc fit $x - 1 = \frac{z-a}{n}$, $x - 2 = \frac{z-n-a}{n}$, $x - 3 = \frac{z-2n-a}{n}$, &c. Subſtituendo itaque hos valores $x-1$, $x-2$, $x-3$, &c. in Serie Lemmatis præcedentis, & terminis in ordinem redactis, prodeunt ipſorum A, B, C, &c. valores exhibiti.

Cor. Ubi $a = n$, prodeunt A, B, C, D, &c. per formulas ſimpliciores, nempe

$$A = M - b + c - d + e \ \&c.$$

$$B = \frac{1}{n} \times \overline{b - 2c + 3d - 4e \ \&c.}$$

$$C = \frac{1}{n} \times \frac{1}{2n} \times \overline{c - 3d + 6e \ \&c.}$$

$$D = \frac{1}{n} \times \frac{1}{2n} \times \frac{1}{3n} \times \overline{d + 4e \ \&c.}$$

Lemma 3. Symbolis X & x eodem modo interpretatis ac in Lemmate primo, ſint q, r, s, t, u, &c. generatores Trianguli Arithmetici, cujus lineam tranſverſam occupat Series M, N, O, P, Q, &c. in ordine nempe inverſo, ut ſit $q\ (= M)$ generator ultimus, r penultimus, s antepenultimus, & ſic porro. Tum erit

$$X = q + r \times \frac{x-1}{1} + s \times \frac{x-1}{1} \times \frac{x}{2} + t \times \frac{x-1}{1} \times \frac{x}{2} \times \frac{x+1}{3} + \&c.$$

Conſtat ex contemplatione ipſius Trianguli Arithmetici, quam exhibuimus *pag.* 63. tractatus *Eſſay d' Analyſe*, &c. ubi idem fuſius explicatur.

Lemma 4. Iiſdem poſitis, & Symbolo z eodem modo interpretato ac in *Lem.* 2. ſi ſit $X = A + Bz + Cz \times \overline{z+n} + \&c.$ ut in *Lem.* 2. erunt coefficientium A, B, C, D, &c. valores.

$$A = q + r \times \frac{-a}{n} + s \times \frac{-a}{n} \times \frac{-a+n}{2n} + t \times \frac{-a}{n} \times \frac{-a+n}{2n} \times \frac{-a+2n}{3n} + \&c.$$

$$B = \frac{1}{n} \times r + s \times \frac{-a}{n} + t \times \frac{-a}{n} \times \frac{-a+n}{2n} + \&c.$$

C =

Plate V. Part II. P. 92.

$$C = \frac{1}{n}\times\frac{1}{2n}\times s + t\times\frac{\overline{-a}}{n} + \&c.$$

$$D = \frac{1}{n}\times\frac{1}{2n}\times\frac{1}{3n}\times t + \&c.$$

Ordo coefficientium in his valoribus eſt manifeſtus, & demonſtratur Lemma ad modum *Lemmatis* 2.

Cor. 1. Ubi $a = n$, coefficientes $A, B, C, D,$ *&c.* prodeunt per formulas ſimpliciores, nempe

$$A = q - r, \qquad C = \frac{1}{n}\times\frac{1}{2n}\times\overline{s-t}$$

$$B = \frac{1}{n}\times\overline{r-s}, \qquad D = \frac{1}{n}\times\frac{1}{2n}\times\frac{1}{3n}\,\overline{t-u}, \ \&c.$$

Cor. 2. Unde ſi generatorum $q, r, s, t, u,$ *&c.* aliquot ſint inter ſe æquales, exhibebitur X per formulam ſimpliciorem, evaneſcentibus aliquot coefficientium $A, B, C, D,$ *&c.*

Sic exempli gratia, propoſita Serie numerorum 4, 69, 530, 2676, 10350, *&c.* qui conſtituunt lineam decimam tranſverſam in Triangulo Arithmetico cujus generatores tres priores ſunt 54, −18, 5, & ſeptem poſteriores ſunt æquales 4; exiſtente $a = 1 = n$, Terminus X exhibetur per formulam quatuor tantum terminorum, $-\frac{z}{1}\cdot\frac{z+1}{2}\cdot\frac{z+2}{3}\ \&c.\times\frac{z+6}{7} + 23\,\frac{z}{1}\cdot\frac{z+1}{2}\ \&c.\times\frac{z+6}{7} - 72\,\frac{z}{1}\cdot\frac{z+1}{2}\ \&c.\times\frac{z+7}{8} + 54\,\frac{z}{1}\cdot\frac{z+1}{2}\ \&c.\times\frac{z+8}{9}$, evaneſcentibus coefficientibus ſex primis A, B, C, D, E, F.

Prop. II. *Prob.* Invenire ſummam quotlibet terminorum Seriei $\frac{M}{a\times\overline{a+n}\times\&c.\times\overline{a+\overline{p-1}\,n}} + \frac{N}{\overline{a+n}\times\&c.\times\overline{a+pn}} + \frac{O}{\overline{a+2n}\times\&c.\times\overline{a+\overline{p+1}\,n}} + \&c.$ ubi numeratores $M, N, O,$ *&c.* conſtituunt Seriem quamlibet terminorum, quorum differentiæ, vel primæ, vel ſecundæ, vel aliæ quædam dantur; vel quod perinde eſt, qui conſtituunt lineam quamvis tranſverſam in dato quovis triangulo Arithmetico; Denominatores autem conſtituunt Seriem in *Prop.* I. exhibitam.

Solutio. Per X deſignetur primus factorum $a, a+n, a+2n,$ *&c.* in denominatore ejuſdem termini, ut ſint X & z iidem ac in *Lemm.* præmiſſis, adeoque deſignetur terminus quilibet Seriei per $\frac{X}{z\times\overline{z+n}\times\&c.\times\overline{z+p-n}}$

Per *Lem.* 2, vel per *Lem.* 4. (prout magis commodum videatur vel differentias, vel generatores trianguli Arithmetici adhibere,) reſolvatur X in Multinomium $A + B\times z + Cz\times\overline{z+n} + Dz\times\overline{z+n}\times\overline{z+2n} + \&c.$

Hoc

Hoc pacto (terminis multinomii ad denominatorem $z \times \overline{z+n} \times \&c. \times \overline{z+p-n}$, applicatis) terminus quilibet Seriei revocabitur ad formulam $\frac{A}{z \times \overline{z+n} \times \&c. \times \overline{z+p-1n}} + \frac{B}{\overline{z+n} \times \&c. \times \overline{z+p-1n}} + \frac{C}{\overline{z+2n} \times \&c. \times \overline{z+p-1n}} + \&c.$

Unde (per Scholium 4 *Prop.* I.) aggregatum totius Seriei, a termino $\frac{X}{z \times \overline{z+n} \times \&c. \times \overline{z+p-1n}}$ inclusive in infinitum continuatæ, est $\frac{A}{\overline{p-1} \times n \times z \times \overline{z+n} \times \&c. \times \overline{z+p-2n}} + \frac{B}{\overline{p-2} \times n \times \overline{z+n} \times \&c. \times \overline{z+p-2n}} + \frac{C}{\overline{p-3} \times n \times \overline{z+2n} \times \&c. \times \overline{z+p-2n}} + \&c.$ quare si dematur hoc aggregatum ab ejusdem aggregati valore quando $z=a$, residuum erit summa omnium terminorum ante terminum $\frac{X}{z+\&c.}$, hoc est, tot terminorum quot sunt unitates in $\frac{z-a}{n}$. *Q. E. I.*

Ex. 1. Sit primum exemplum in Serie $\frac{5}{3.5.7.9.11.13} + \frac{41}{5.7.9.11.13.15} + \frac{131}{7.9.11.13.15.17} + \frac{275}{9.11.13.15.17.19} + \frac{473}{11.13.15.17.19.21} + \&c.$ Sunt hic $a=3$, $n=2$, $p=6$, $M=5$, & capiendo differentias numeratorum inveniuntur $b=36$, $c=54$, $d=0=e=\&c.$ Hinc in Lemmate secundo sunt $A=5+36\times\frac{-3}{2}+54\times\frac{-3}{2}\times\frac{-5}{4}=\frac{209}{4}$, $B=\frac{-1}{2}\times 36+54\times\frac{-5}{2}=\frac{-99}{2}$, $C=\frac{1}{2}\times\frac{1}{4}\times 54=\frac{27}{4}$, $D=0=E=\&c.$ Summa itaque totius Seriei est $\frac{209}{4\times5\times2\times3.5.7.9.11.} + \frac{-99}{2\times4\times2\times5.7.9.11} + \frac{27}{4\times3\times2\times7.9.11} = \frac{283}{80\times3.5.7.9.11}$ atque summa terminorum numero $\frac{z-3}{2}(=\frac{z-a}{n})$ est $\frac{283}{80\times3.5.7.9.11} - \frac{209}{40\times z.\overline{z+2}.\overline{z+4}.\overline{z+6}.\overline{z+8}} + \frac{99}{16\times\overline{z+2}\times\overline{z+4}.\overline{z+6}.\overline{z+8}} - \frac{27}{24\times\overline{z+4}.\overline{z+6}.\overline{z+8}}$ Quærantur *v. g.* octo termini; tum existente $\frac{z-3}{2}=8$ fit $z=19$, quo valore in formula adhibito, prodit summa $\frac{155891}{2.3.3.3.3.5.5.5.7.11.19.23}$.

Iidem

Iidem Numeratores occupant lineam tertiam transversam in Triangulo Arithmetico

$$54 \cdot 54 \cdot 54 \cdot 54 \cdot 54 \cdot 54 \cdot \&c.$$
$$-18 \cdot 36 \cdot 90 \cdot 144 \cdot 198 \cdot \&c.$$
$$5 \cdot 41 \cdot 131 \cdot 275 \cdot \&c.$$

Unde in formula *Lem.* 4. sunt generatores $q=5$, $r=-18$; $s=54$, $t=0=\&c.$ & prodeunt coefficientes $A=5-18\times\frac{-3}{2}+54\times\frac{-3}{2}\times\frac{-3+2}{4}=\frac{209}{4}$, $B=\frac{1}{2}\times-18+54\times\frac{-3}{2}=\frac{-99}{2}$, $C=\frac{1}{2}\times\frac{1}{4}\times 54=\frac{27}{4}$, $D=0=E=\&c.$ iidem ac supra.

Ex. 2. Sit Series $\frac{4}{1.2.3.4.5.6.7.8.9.10.11}+\frac{69}{2.3.\&c.\,12}+\frac{530}{3.4.\&c.\,13}+\frac{2676}{4.5.\&c.\,14}+\frac{10350}{5.6.\&c.\,15}+\&c.$ Ubi sunt $a=1$, $n=1$, $p=11$, atque Numeratores constituant Seriem in *Corol.* 20. *Lem.* 4. exhibitam. Applicando itaque valorem X in *Corol.* illo ad denominatorem $z\times\overline{z+1}\times\&c.\times\overline{z+10}$, fit Seriei propositæ Terminus

$$\frac{-1}{1.2.3.4.5.6\times\overline{z+6}.\overline{z+7}.\overline{z+8}.\overline{z+9}.\overline{z+10}}$$
$$+\frac{23}{1.2.3.4.5.6.7.\times.\overline{z+7}.\overline{z+8}.\overline{z+9}.\overline{z+10}}$$
$$-\frac{72}{1.2.3.4.5.6.7.8\times\overline{z+8}.\overline{z+9}.\overline{z+10}}$$
$$+\frac{54}{1.2.3.4.5.6.7.8.9.\times\overline{z+9}\times\overline{z+10}}.$$

Adeoque per hanc *Prop.* summa Seriei a termino illo in infinitum continuatæ est

$$\frac{-1}{4\times 1.2.3.4.5.6\times\overline{z+6}.\overline{z+7}.\overline{z+8}.\overline{z+9}}$$
$$+\frac{23}{3\times 1.2.3.4.5.6.7.\times\overline{z+7}.\overline{z+8}.\overline{z+9}}$$
$$-\frac{72}{2\times 1.2.3.4.5.6.7.8.\times\overline{z+8}.\overline{z+9}}$$
$$+\frac{54}{1\times 1.2.3.4.5.6.7.8.9\times\overline{z+9}}.$$

Itaque pro z sumpto 1, fit summa totius Seriei $\frac{305}{12\times 1.2.3.4.5.6.7.8.9.10}$. Et in genere summa terminorum numero $\frac{z-1}{1}$, est $\frac{305}{12\times 1.2.3.4.5.6.7.8.9.10}$

$$+\frac{1}{4\times 1.2.3.4.5.6\times\overline{z+6}.\overline{z+7}.\overline{z+8}.\overline{z+9}}$$

$$-\frac{23}{3\times 1\,.\,2\,.\,3\,.\,4\,.\,5\,.\,6\,.\,7\,.\times z\,\overline{+7}\,.\,z\,\overline{+8}\,.\,z\,\overline{+9}}$$

$$+\frac{72}{2\times 1\,.\,2\,.\,3\,.\,4\,.\,5\,.\,6\,.\,7\,.\,8\times z\,\overline{+8}\times z\,\overline{+9}}$$

$$-\frac{54}{1\times 1\,.\,2\,.\,3\,.\,4\,.\,5\,.\,6\,.\,7\,.\,8\,.\,9\times z\,\overline{+9}}$$

Scholium 1. In computandis ſummis hujuſmodi Serierum, calculus plerumque levior eſt adhibitis generatoribus trianguli Arithmetici, quam ſi adhibeantur differentiæ. Libet itaque hac occaſione oſtendere quomodo ex datis differentiis inveniri poſſunt generatores Trianguli Arithmetici.

Sunto itaque ω primus Seriei terminus, a differentia ultima data, b prima differentiarum penultimarum, c prima antepenultimarum, & ſic porro d, e, &c. atque ſint t, u, x, y, &c. generatores quæſiti Trianguli Arithmetici, cujus lineam tranſverſam ordine p occupet Series propoſita. Tum (quod ex contemplatione Trianguli Arithmetici facile conſtat) ſunt.

$$a=t$$

$$b=\frac{p-1}{1}t+u$$

$$c=\frac{p-1}{1}\times\frac{p-2}{2}t+\frac{p-2}{1}u+x$$

$$d=\frac{p-1}{1}\times\frac{p-2}{2}\times\frac{p-3}{3}t+\frac{p-2}{1}\times\frac{p-3}{2}u$$
$$+\frac{p-3}{1}x+y \text{ \&c.}$$

Unde colliguntur generatorum valores

$$t=a$$

$$u=b-\frac{p-1}{1}t$$

$$x=c-\frac{p-1}{1}\times\frac{p-2}{2}t-\frac{p-2}{1}u$$

$$y=d-\frac{p-1}{1}\times\frac{p-2}{2}\times\frac{p-3}{3}t-\frac{p-2}{1}\times\frac{p-3}{2}u$$
$$-\frac{p-3}{1}x \text{ \&c.}$$

Ultimus autem generator æqualis eſt Seriei termino primo ω.

2. D^nus^ de *Monſoury Abbas Orbacenſis* mihi amiciſſimus, & ruri vicinus, poſtquam cum eo hæc communicaveram, aliam invenit hujus Problematis Solutionem, cujus formulam ob ejus miram ſimplicitatem hic referre juvat. Itaque in Serie numeratorum ſint ω terminus primus, b prima differentiarum primarum, c prima ſecundarum, d prima tertiarum, & ſic porro; atque ſit termini primi Denominator $z\times\overline{z+n}\times$ &c. $\times\ \overline{z+\overline{p-1}\,n}$; Tum ſumma totius Seriei in infinitum continuatæ exhibebitur per formulam $\frac{\omega}{n\times\overline{p-1}\times z\times\overline{z+n}\times\text{\&c.}\times\overline{z+\overline{p-2}\,n}}$

+

$$+\frac{b}{n^2\times\overline{p-1}\times\overline{p-2}\times\overline{z+n}\times\&c.\times\overline{z+p-2n}}+$$
$$\frac{c}{n^3\times\overline{p-1}\times\overline{p-2}\times\overline{p-3}\times\overline{z+2n}\times\&c.\times\overline{z+p-2n}}+\&c.$$

Sit exemplum in Serie $\frac{5}{3\,.\,5\,.\,\&c.\,13}+\frac{41}{5\,.\,7\,.\,\&c.\,15}+\frac{131}{7\,.\,9\,.\,\&c.\,17}+\frac{275}{9\,.\,11\,.\,\&c.\,19}+\&c.$ cujus ſummam jam exhibuimus. In hoc caſu ſunt $\omega=5$, $b=36$, $c=54$, $d=0=e=\&c.$ Unde per formulam ſumma Seriei integræ fit $\frac{5}{2\,.\,5\times3\,.\,5\,\ldots\,11}+\frac{36}{4\,.\,5\,.\,4\times5\,\ldots\,11}+\frac{54}{8\,.\,5\,.\,4\,.\,3\times7\,\ldots\,11}=\frac{283}{80\times3\,.\,5\,\ldots\,11}$, ut per formulam noſtram exhibetur. Si quæratur ſumma ejuſdem Seriei incipientis a termino decimo $\frac{2273}{21\,\ldots\,31}$, in eo caſu $\omega=2273$, $b=522$, $c=54$, & ſumma eſſet $\frac{2273}{2\,.\,5\times21\,\ldots\,29}+\frac{522}{4\,.\,5\,.\,4\times23\,\ldots\,29}+\frac{54}{8\,.\,5\,.\,4\,.\,3\times25\,\ldots\,29}$.

Hæc formula eſt commodiſſima, & ſummam exhibet nullo fere negotio, quoties quæritur ſumma Seriei integræ, & differentiæ non ſunt nimis multæ. Sed ubi plures ſunt differentiæ, & quæritur non Series integra, ſed termini tantum initiales aliquammulti, formulæ noſtræ ſunt commodiores.

3. Quando Serierum termini formantur tantum per Multiplicationem, nec afficiuntur diviſoribus variabilibus, ſummæ ſemper exhiberi poſſunt per Methodum in *Prop.* I. traditam, ſint licet formulæ quantumlibet compoſitæ. Nam poſſunt ſemper revocari ad terminos in forma quam poſtulat Propoſitio illa. Sic ſi differentiæ ipſorum z & x ſint m & n, & deſignetur terminus Seriei per $z\,x$; hic terminus revocabitur ad formam $\overline{a-n}\,z+\frac{n}{m}\,z\times\overline{z+m}$; cujus Integrale datur *per Prop.* I; nempe quoniam $d\,x=n$, & $d\,z=m$, eſt $d\,x=d\,z\times\frac{n}{m}$; unde regrediendo ad integralia fit $x=\frac{n}{m}\,z+a$ (adjecto invariabili a, ut habeatur ratio relationis inter z & x in Seriei termino primo,) quod ſic ſcribi poteſt $\overline{a-n}+\frac{n}{m}\times\overline{z+m}$, ut deinde in z ductum induat formam requiſitam. Et ad eundem modum procedere licet in aliis caſibus ejuſmodi. Sed ubi formulæ oblatæ diviſoribus afficiuntur, eædem ac in Calculo integrali, ut vocant, difficultates occurrunt, eadem induſtria ſuperandæ. Nec tamen ſemper ſuperari poſſunt. Nam præterquam quod vix certo ſciri poſſit quæ debeat relatio intercedere inter Numeratorem fractionis & Denominatorem, ut formula oblata ad Integrale revocari poſſit; ſæpe etiam difficillimum eſt

 explo-

explorare an adsit jam talis relatio in formula ista, aut si desit, an introduci possit. Quicquid ego in hac materia potissimum inveni, continetur in tribus sequentibus propositionibus.

Prop. III. *Prob.* Crescentibus, z, u, y, x, &c. per differentias datas n, m, l, o, &c. invenire valorem numeratoris integri N, ut existente Denominatore $z.\overline{z+n}.$ &c. $\overline{z+pn}\times u.\overline{u+m}.$ &c. $\overline{u+qm}\times y.\overline{y+l}.$ &c. $\overline{y+rl}\times x.\overline{x+o}$ &c. $\overline{x+so}$. &c. Fractio ad Integrale revocari possit.

Solutio. Fiat $N=\overline{z+pn}\times\overline{u+qm}\times\overline{y+rl}\times\overline{x+so}\times$ &c. $-\,zuyx$ &c. atque Integrale erit fractio, cujus Denominator $z.\overline{z+n}.$ &c. $\overline{z+p-1\,n}\times u.\overline{u+m}.$ &c. $\overline{u+q-1\,m}\times y.\overline{y+l}.$ &c. $\overline{y+r-1\,l}\times x.\overline{x+o}.$ &c. $\overline{x+s-1\,o}\times$ &c. existente 1 Numeratore.

Differentia enim hujus fractionis est fractio cujus numerator est ipsius N valor exhibitus, & denominator idem est ac denominator propositus, ut fieri debuit.

Ex. 1. Sit denominator propositus $z\times\overline{z+2}\times u\times\overline{u+3}$. In hoc casu sunt $n=2$, $m=3$, $p=1$, $q=1$; adeoque est $N=\overline{z+2}\times\overline{u+3}-zu$ $=3z+2u+6$, & per $\frac{3z+2u+6}{z.\overline{z+2}\times u.\overline{u+3}}$ representatur terminus Seriei summabilis, cujus nempe in infinitum continuatæ summa exhibetur per $\frac{1}{zu}$. Sint verbi gratia, ipsorum z & u primus valor communis 1, atque Series summabilis erit $\frac{11}{1.3\times1.4}+\frac{23}{3.5\times4.7}+\frac{35}{5.7\times7.10}$ + &c. quippe cujus totius summa est 1. Per p designetur ordo termini cujusvis in hac Serie, erit $p=\frac{z-1+2}{2}=\frac{u-1+3}{3}$, adeoque $z=2p-1$, & $u=3p-2$; quibus valoribus pro z & u scriptis, designabitur terminus per formulam $\frac{12p-1}{\overline{2p-1}\times\overline{2p+1}\times\overline{3p-2}\times\overline{3p+1}}$. Summa autem terminorum omnium ante terminum illum, hoc est terminorum initialium numero $\frac{z-1}{2}=p-1$, est $1-\frac{1}{zu}$, $=\frac{zu-1}{zu}$, hoc est $\frac{6pp-7p+1}{\overline{2p-1}\times\overline{3p-2}}$. Quare pro p scripto $p+1$, erit $\frac{p\times\overline{6p+5}}{\overline{2p+1}\times\overline{3p+1}}$ aggregatum tot terminorum initialium quot sunt unitates in p.

Ex. 2. Iisdem manentibus z, u, n, m, sit denominator $z.\overline{z+2}.\overline{z+4}\times u.\overline{u+3}$. Tum per formulam numerator erit $\overline{z+4}\times\overline{u+3}-zu=3z+4u+12$, & summa Seriei exhibebitur per formulam $\frac{1}{z.\overline{z+2}\times u}$. Sit ipsorum z & u primus valor communis 1, & hinc elicietur Series $\frac{19}{1.3.5\times1.4}+\frac{37}{3.5.7\times4.7}+\frac{55}{5.7.9\times7.10}$ + &c. $=\frac{1}{3}$.

Scholium. In Seriebus jam expositis eadem ubique est differentia inter factores continuos ejusdem cujusvis termini, ac inter factores homologos termi-

terminorum continuorum. In sequentibus exempla quædam sunt Serierum, quarum summæ in terminis numero finitis exhiberi possunt, quamvis ea regula non observetur.

Prop. IV. *Prob.* Crescente z per differentias datas qn, invenire numeratorem integrum N, ut ad Integrale revocari possit fractio, cujus Denominator sit ex certo numero p terminorum z, $z+n$, $z+2n$, &c. Arithmetice proportionalium in invicem ductorum. Debet autem esse q numerus integer minor quam factorum numerus p.

Solutio. Erit $N = \overline{z+p-1\,n} \times \overline{z+p-2\,n} \times \&c. \times \overline{z+p-q\,n} - z \times \overline{z+n} \times \&c. \times \overline{z+q-1\,n}$, Integrale existente

$$\frac{1}{z \times \overline{z+n} \times \&c. \times \overline{z+p-q-1\,n}}.$$

Demonstratur ad modum propositionis præcedentis.

Sumptis ad libitum n, p, q, & primo valore z, hinc oriuntur infinitæ Series summabiles, cujusmodi sunt Series tres sequentes.

$$A = \frac{5}{1.2.3.4} + \frac{9}{3.4.5.6} + \frac{13}{5.6.7.8} + \frac{17}{7.8.9.10} \&c.$$

$$B = \frac{1}{1.2.3.4.5} + \frac{4}{4.5.6.7.8} + \frac{9}{7.8.9.10.11} + \frac{16}{10.11.12.13.14} + \&c.$$

$$C = \frac{1}{1.2.3.4.5} + \frac{14}{5.6.7.8.9} + \frac{55}{9.10.11.12.13} + \frac{140}{13.14.15.16.17} + \&c.$$

Has Series jampridem communicavi cum primariis quibusdam Geometris, à quibus minime contemni videntur. Sic ad me scribit peritissimus Geometra D. *Nicolaus Bernoulli* in epistola data 25 *Julii* 1716. " Vous me ferez un extreme plaisir, *Monsieur*, de me communiquer la " Solution de vostre probleme, *Etant donnée une suitte des Fractions dont* " *les Numerateurs soient des nombres figurés quelconque, & dont les Deno-* " *minateurs soient formés du produit d'un nombre egal de Facteurs qui* " *soient en Progression Arithmetique, trouver la somme*; & principale- " ment comment vous avez trouvé ces deux formules $\frac{p}{24 \times \overline{4p+1}}$, " $\frac{p \cdot \overline{p+1}}{12 \times \overline{3p+1} \times \overline{3p+2}}$. Hæ formulæ spectant ad Series C & B, designante p numerum terminorum, quorum summa requiritur. Sic etiam ad me scribit D. *Taylor* in epistola data 22 *Aug.* 1716. " Ut & qua ratione " incidisti in summationem Serierum a te exhibitarum, præsertim loquor " de Serie $\frac{1}{1.2.3.4.5} + \frac{4}{4.5.6.7.8} + \frac{9}{7.8.9.10.11} + \&c.$ quæ vide- " tur esse altioris indaginis.

Sed

Sed ut ad exempla jam redeamus. In Serie *A* sunt $p=4$, $q=2$, $n=1$, primo valore z existente 1. Est itaque $\overline{z+3} \times \overline{z+2} - z \times \overline{z+1} = 2 \times \overline{2z+3}$ formula, unde (rejecto dato numero 2) derivantur numeratores 5, 9, 13, 17, &c. Formula etiam summæ est $\frac{1}{z \times \overline{z+1}}$. Quare habita ratione numeri 2, quem ex numeratoribus rejecimus, summa totius Seriei, a termino in quo est z in infinitum continuatæ, exhibetur per formulam $\frac{1}{2z \times \overline{z+1}}$; adeoque summa Seriei integræ est $\frac{1}{2 \times 1 \times 2} = \frac{1}{4}$.

In Serie *B* sunt $n=1$, $p=5$, $q=3$, primo valore z existente 1. Est itaque $N = \overline{z+4} \times \overline{z+3} \times \overline{z+2} - z \times \overline{z+1} \times \overline{z+2} = 6 \times \overline{z+2}|^2$. Ipsius autem $\overline{z+2}$ valores continui sunt 3, 6, 9, &c. qui quoniam omnes sunt divisibiles per 3, ponendo $z+2=3x$, fit $N = 6 \times \overline{3x}|^2 = 6 \times 9x^2 = 54x^2$, ipsius x valoribus continuis existentibus 1, 2, 3, &c. Rejecto itaque numero dato 54, hinc prodeunt numeratores 1, 2^2, 3^2, &c. hoc est 1, 4, 9, &c. Formula etiam Integralis est $\frac{1}{z \times \overline{z+1}}$; quare habita ratione numeri 54 quem ex numeratoribus rejecimus, summa Seriei a termino in quo est z in infinitum continuatæ est $\frac{1}{54z \times \overline{z+1}}$. Unde summa Seriei integræ est $\frac{1}{108}$.

In Serie denique *C* sunt $n=1$, $p=5$, $q=4$, & primus valor $z=1$. Unde fit $N = \overline{z+4} \times \overline{z+3} \times \overline{z+2} \times \overline{z+1} - z \times \overline{z+1} \times \overline{z+2} \times \overline{z+3} = 4 \times \overline{z+1} \times \overline{z+2} \times \overline{z+3}$. Valores autem N per hanc formulam prodeuntes semper possunt dividi per $4 \times 2 \times 3 \times 4 = 96$. Ergo hoc divisore rejecto prodeunt numeratores 1, 14, 55, 140, &c. Et formula Summæ, habita ratione numeri 96, est $\frac{1}{96z}$. Adeoque Summa Seriei integræ est $\frac{1}{96}$.

Scholium 1. Per Propositiones has duas novissimas nullo negotio inveniri possunt Series quot libuerit summabiles. Et vicissim oblata Serie hujus speciei, si summari potest, ejus summa plerumque revocatur ad alterutram ex his Propositionibus. In examine tamen solertia est opus. Optime autem procedit si termini Seriei oblatæ revocentur ad formulam *Prop.* III. Sic *e. gr.* proposita Serie $\frac{7}{3.5.7.9.11} + \frac{11}{7.9.11.13.15} + \frac{15}{11.13.15.17.19} +$ &c. Denominatores sic scribi possunt $3.7.11 \times 5.9 . 7.11.15 \times 9.13.11.15.19 \times 13.17.$ &c.

Unde juxta *Prop.* III. fit $n=4$, $m=4$, $p=2$, $q=1$, primus valor $z=3$, primus valor $u=5$. Hinc formula Numeratoris invenitur $4 \times \overline{z+2u+8}$. Est autem $z+2u+8$ semper divisibile per 3; quare rejectis divisoribus datis 4 & 3, per hanc formulam prodeunt Numeratores 7, 11,

15, *&c.* iidem ac Numeratores in Serie propofita, quæ proinde fummabitur per illam propofitionem.

2. Cum Series illas A, B, C, communicaveram cum D. *Taylor*, refcripfit fe earum fummas inveniffe primam quidem A & tertiam C, eas revocando ad cafus fimplices Methodi Incrementorum, tertiam C, *e.g.* revocavit ad hanc formam $\frac{1}{24}\times\frac{1}{1.5}+\frac{1}{5.9}+\frac{1}{9.13}+\frac{1}{13.17}$ *&c.* ut habeatur fumma per præcepta tradita in *Scholio Prop.* 1. In Serie autem fecunda B, cum hoc non æque fucceffit, fequenti ufus eft Analyfi, quam, ipfius venia jam impetrata, ob ejus eximiam elegantiam huc transferre non piget. " Seriei iftius terminus [in Stylo ejus] exhibetur per formulam $\frac{\overline{z+2}\times\dot{z}}{27z\times\overline{z+1}\times z_{,}\times\overline{z_{,}+1}}$; pro $z+3$ in denominatore fcripto $z_{,}$, quoniam
" eft $\dot{z}=3$. Pone $\frac{B}{27C}$ æquale effe Integrali quæfito, hoc eft $\frac{B}{C}$ effe Inte-
" grale ipfius $\frac{\overline{z+2}\times\dot{z}}{z.\overline{z+1}\times z_{,}.\overline{z_{,}+1}}$, fepofito divifore dato 27. Ipfius au-
" tem $\frac{B}{C}$ incrementum eft $\frac{\dot{B}C-B\dot{C}}{CC_{,}}$. Debet ergo $\frac{\dot{B}C-B\dot{C}}{CC_{,}}$ i-
" dem effe ac $\frac{\overline{z+2}\times\dot{z}}{z.\overline{z+1}\times z_{,}.\overline{z_{,}+1}}$. Comparando denominatores inve-
" nitur $C=z\times\overline{z+1}$. Hinc itaque fumendo incrementa fit $\dot{C}=2z\dot{z}+\dot{z}^2$
" $+\dot{z}$ ($=2\dot{z}z+4\dot{z}$, quoniam eft $\dot{z}=3$,) His valoribus in locum C & $\dot{C}$
" fubftitutis prodit $\dot{B}C-B\dot{C}=\overline{zz+z}\,\dot{B}-2\dot{z}\times\overline{z+2}\,B$, quod debet
" effe idem ac $\overline{z+2}\times\dot{z}$. Sit $B=a+v$, exiftente a ipfius B parte in-
" variabili, & v parte variabili. Tum fumendo incrementa fit $\dot{B}=\dot{v}$.
" Unde ad invenienda a & v habetur æquatio $\overline{zz+z}\,\dot{v}-2\dot{z}\times\overline{z+2}\times$
" $\overline{a+v}=\overline{z+2}\times\dot{z}$, quæ fic fcribi poteft $\overline{zz+z}\,\dot{v}-2\dot{z}\times\overline{z+2}\,v=\dot{z}\times$
" $\overline{z+2}\times\overline{1+2a}$, vel etiam $C\dot{v}-\dot{C}v=\dot{z}\times\overline{z+2}\times\overline{1+2a}$. Pone
" $1+2a=0$ (unde fit $a=\frac{-1}{2}$,) & fit $C\dot{v}-\dot{C}v=0$; ubi fieri poteft
" $v=0$, (quoniam æquationis termini finguli afficiuntur vel ab v, vel ab
" $\dot{v}$.) Hinc ergo fit $B=a=\frac{-1}{2}$, adeoque $\frac{B}{C}=\frac{-1}{2z\times\overline{z+1}}$. Unde ha-
" bita ratione diviforis 27, Integrale quæfitum fit $\frac{-1}{54\times z\times\overline{z+1}}$. Sed
" & comparando æquationem $C\dot{v}-\dot{C}v=0$ cum formula generali
" $\frac{\dot{B}C-B\dot{C}}{CC_{,}}=0$, inde etiam concludere licet effe $\frac{v}{C}=$ quantitati datæ,

(quo-

“ (quoniam ipsius incrementum est 0.) Unde pro n sumpto quovis numero dato, fit $v = nC$, atque $B = -\frac{1}{2} + nC$. Quo pacto Integrale quæsitum fit $\frac{B}{C} = \frac{-\frac{1}{2}+nC}{C} = \frac{-1}{2C} + n$, quod ab Integrali prius invento differt quantitate data n. Hoc inde fit, quod, ut in quadratura Curvarum Area inventa augeri potest vel minui area data, sic in Methodo incrementorum Integrale inventum augeri potest vel minui quantitate data. Per Integrale autem primum, ubi deest n, exhibetur summa Seriei in infinitum continuatæ.

Prop. V. Crescente z per unitates, & existentibus a, b, c, *&c.* numeris datis integris, quorum nullæ inter se æquantur; invenire Integrale ipsius $\frac{1}{z\times\overline{z+a}\times\overline{z+b}\times\overline{z+c}\times \&c.}$

Solutio. Ducendo tam numeratorem quam denominatorem fractionis in terminos $z+1$, $z+2$, *&c.* $z+a+1$, $z+a+2$, *&c.* $z+b+1$, $z+b+2$, *&c.* $z+c+1$, $z+c+2$, *&c.* in denominatore deficientes, revocetur Denominator ad formulam $z\times\overline{z+1}\times\overline{z+2}\times$ *&c.* denominatoris in *Prop.* I. *Schol. n.* 3. Deinde revocetur Numerator ad formam $A+Bz+Cz\times\overline{z+1}+Dz\times\overline{z+1}\times\overline{z+2}+$ *&c.* Tum applicando terminos ad Denominatorem novum $z\times\overline{z+1}\times\overline{z+2}\times$ *&c.* revocetur fractio ad hanc formam $\frac{A}{z\times\overline{z+1}\times\&c.}+\frac{B}{\overline{z+1}\times\overline{z+2}\times\&c.}+\frac{C}{\overline{z+2}\times\overline{z+3}\times\&c.}+\frac{D}{\overline{z+3}\times\overline{z+4}\times\&c.}$ *&c.* Unde denique quæratur Integrale per *Schol. Prop.* I. *n.* 3.

Ratio Solutionis per se satis est manifesta.

Scholium 1. Hujus Solutionis tota difficultas latet in revocatione numeratoris ad formam requisitam, quod tamen quomodo sit faciendum uno exemplo patebit. Proponatur itaque factum $\overline{z+2}\times\overline{z+3}\times\overline{z+7}$, quod ad formam propositam sit revocandum. Terminos itaque evolvo gradatim ut sequitur. Factorem primum $z+2$ sic scribo $2+z$, cujus terminum primum 2 duco in $3+z$, unde fit $6+2z$: Terminum secundum z duco in $2+\overline{z+1}$ $(=\overline{z+3})$ unde fit $2z+z\times\overline{z+1}$. Dein facta in unam summam colligendo, fit $\overline{z+2}\times\overline{z+3}=\begin{matrix}6+2z\\+2z\end{matrix}+z\times\overline{z+1}=6+4z+z\times\overline{z+1}$. Superest ut hoc ducatur in $z+7$. Itaque terminum primum 6 duco in $7+z$ $(=z+7)$ unde fit $42+6z$; terminum secundum $4z$ duco in $6+\overline{z+1}$ $(=z+7)$ unde fit $24z+4z\times\overline{z+1}$; terminum tertium $z\times\overline{z+1}$, duco in $5+\overline{z+2}$ $(=z+7;)$ unde fit $5z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}$. Factis itaque in unum collectis ut prius, fit $\overline{z+2}\times\overline{z+3}\times\overline{z+4}=42+30z+9z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}$. Et ad eundem modum procedere licet in aliis casibus.

2. Sit autem exemplum Propositionis in fractione $\frac{1}{z\times\overline{z+2}\times\overline{z+5}}$.

Resti-

Restituendo factores $z+1$, $z+3$, $\overline{z+4}$ in Denominatore deficientes, fractio fit $\frac{\overline{z+1}\times\overline{z+3}\times\overline{z+4}}{z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}\times\overline{z+5}}$. Revocandus itaque est Numerator $\overline{z+1}\times\overline{z+3}\times\overline{z+4}$ ad formam requisitam. Itaque per methodum jam traditam fit primo $\overline{z+1}\times\overline{z+3}=1\times\overline{3+z}+z\times\overline{2+z+1}=3+\overline{z+2z}+z\times\overline{z+1}=3+3z+z\times\overline{z+1}$. Deinde $\overline{z+1}\times\overline{z+3}\times\overline{z+4}=3\times\overline{4+z}+3z\times\overline{3+z+1}+z\times\overline{z+1}\times\overline{2+z+2}=12+3z+9z+3z\times\overline{z+1}+2z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}=12+12z+5z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}$. Applicando hoc factum ad Denominatorem $z\times\overline{z+1}\times$ &c. $\times\overline{z+5}$ fractio tandem revocatur ad hanc formam $\frac{12}{z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}\times\overline{z+5}}+\frac{12}{\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}\times\overline{z+5}}+\frac{5}{\overline{z+2}\times\overline{z+3}\times\overline{z+4}\times\overline{z+5}}+\frac{1}{\overline{z+3}\times\overline{z+4}\times\overline{z+5}}$ Cujus denique Integrale est
$\frac{-12}{5z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}}+\frac{-12}{4.\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}}+\frac{-5}{3.\overline{z+2}\times\overline{z+3}\times\overline{z+4}}+\frac{-1}{2.\overline{z+3}\times\overline{z+4}}$.

3. Quando duo tantum sunt factores z & $z+a$, exhibebitur etiam Integrale per formulam $\frac{1}{2}-\frac{1-a}{2z\times\overline{z+1}}-\frac{\overline{1-a}\times\overline{2-a}}{3z\times\overline{z+1}\times\overline{z+2}}-\frac{\overline{1-a}\times\overline{2-a}\times\overline{3-a}}{4z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}}$ &c. Seriem nempe continuando donec abrumpatur per evanescentiam terminorum. Si Factores duo sint z & $z-a$ exhibebitur Integrale per formulam $\frac{-1}{z-1}-\frac{-1+a}{2.\overline{z-1}.\overline{z-2}}-\frac{\overline{-1+a}\times\overline{-2+a}}{3.\overline{z-1}.\overline{z-2}.\overline{z-3}}-$ &c. Potest idem Integrale exprimi utroque modo; prout fractionis oblatae factor vel minor vel major sumatur pro z.

4. Si primus valor z sit $a+1$, migrabit formula posterior in hanc $\frac{-1}{a}\times\frac{1}{1}\times\frac{1}{2}\times\frac{1}{3}+$ &c. usque $\frac{1}{a}$ inclusive, qua, cum signo contrario, exhibetur summa Seriei $\frac{1}{1\times\overline{1+a}}+\frac{1}{2\times\overline{2+a}}+\frac{1}{3\times\overline{3+a}}+$ &c. in infinitum continuatae. Sit *e. gr.* $a=1$, atque Series erit $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+$ &c. $=\frac{1}{1}\times\frac{1}{1}=1$. Si $a=2$, erit Series $\frac{1}{1\times3}+\frac{1}{2\times4}+$

$+ \frac{1}{3 \times 5} + \&c. = \frac{1}{2} \times \overline{\frac{1}{1} + \frac{1}{2}} = \frac{3}{4}$; Si $a = 3$, Series erit $\frac{1}{1 \times 4}$ $+ \frac{1}{2 \times 5} + \frac{1}{3 \times 6} + \frac{1}{4 \times 7}$ &c. $= \frac{1}{3} \times \overline{\frac{1}{1} + \frac{1}{2} + \frac{1}{3}} = \frac{11}{18}$.

5. Ex eadem Serie $\frac{1}{1 \times \overline{1+a}} + \frac{1}{2 \times \overline{2+a}} + \frac{1}{3 \times \overline{3+a}} + \&c.$ pro diverſo valore a oriuntur Series plures forma ſatis elegantes, quarum nonnullas Lectori ob oculos ſiſtere, credo, ingratum non erit.

Si pro a ſumantur ſucceſſive numeri pares, 2, 4, 6, 8, &c. Series erunt

$$\text{Si } a = 2)\ \frac{1}{1 \times \overline{1+2}} + \frac{1}{2 \times \overline{2+2}} + \frac{1}{3 \times \overline{3+2}} + \frac{1}{4 \times \overline{4+2}} + \&c.$$

$$4)\ \frac{1}{1 \times \overline{1+4}} + \frac{1}{2 \times \overline{2+4}} + \frac{1}{3 \times \overline{3+4}} + \frac{1}{4 \times \overline{4+4}} + \&c.$$

$$6)\ \frac{1}{1 \times \overline{1+6}} + \frac{1}{2 \times \overline{2+6}} + \frac{1}{3 \times \overline{3+6}} + \frac{1}{4 \times \overline{4+6}} + \&c.$$

$$8)\ \frac{1}{1 \times \overline{1+8}} + \frac{1}{2 \times \overline{2+8}} + \frac{1}{3 \times \overline{3+8}} + \frac{1}{4 \times \overline{4+8}} + \&c.$$

$$\text{Vel } \frac{1}{4-1} + \frac{1}{9-1} + \frac{1}{16-1} + \frac{1}{25-1} + \&c.$$

$$\frac{1}{9-4} + \frac{1}{16-4} + \frac{1}{25-4} + \frac{1}{36-4} + \&c.$$

$$\frac{1}{16-9} + \frac{1}{25-9} + \frac{1}{36-9} + \frac{1}{49-9} + \&c.$$

$$\frac{1}{25-16} + \frac{1}{36-16} + \frac{1}{49-16} + \frac{1}{64-16} + \&c.$$

$$\text{Vel } \frac{1}{4-1} + \frac{1}{9-1} + \frac{1}{16-1} + \frac{1}{25-1} + \&c.$$

$$\frac{1}{4+1} + \frac{1}{9+3} + \frac{1}{16+5} + \frac{1}{25+7} + \&c.$$

$$\frac{1}{4+3} + \frac{1}{9+7} + \frac{1}{16+11} + \frac{1}{25+15} + \&c.$$

$$\frac{1}{4+5} + \frac{1}{9+11} + \frac{1}{16+17} + \frac{1}{25+23} + \&c.$$

Si pro a ſumantur ſucceſſive numeri impares 1, 3, 5, 7, &c. Series erunt

$$a = 1)\ \frac{1}{1 \times \overline{1+1}} + \frac{1}{2 \times \overline{2+1}} + \frac{1}{3 \times \overline{3+1}} + \frac{1}{4 \times \overline{4+1}} + \&c.$$

$$3)\ \frac{1}{1 \times \overline{1+3}} + \frac{1}{2 \times \overline{2+3}} + \frac{1}{3 \times \overline{3+3}} + \frac{1}{4 \times \overline{4+3}} + \&c.$$

$$5)\ \frac{1}{1 \times \overline{1+5}} + \frac{1}{2 \times \overline{2+5}} + \frac{1}{3 \times \overline{3+5}} + \frac{1}{4 \times \overline{4+5}} + \&c.$$

$$7)\ \frac{1}{1 \times \overline{1+7}} + \frac{1}{2 \times \overline{2+7}} + \frac{1}{3 \times \overline{3+7}} + \frac{1}{4 \times \overline{4+7}} + \&c.$$

Vel

$$\text{Vel } \frac{1}{2} \times \frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \&c.$$

$$\frac{1}{2} \times \frac{1}{3-1} + \frac{1}{6-1} + \frac{1}{10-1} + \frac{1}{15-1} + \&c.$$

$$\frac{1}{2} \times \frac{1}{6-3} + \frac{1}{10-3} + \frac{1}{15-3} + \frac{1}{21-3} + \&c.$$

$$\frac{1}{2} \times \frac{1}{10-6} + \frac{1}{15-6} + \frac{1}{21-6} + \frac{1}{28-6} + \&c.$$

$$\text{Vel } \frac{1}{2} \times \frac{1}{1+0} + \frac{1}{3+0} + \frac{1}{6+0} + \frac{1}{10+0} + \&c.$$

$$\frac{1}{2} \times \frac{1}{1+1} + \frac{1}{3+2} + \frac{1}{6+3} + \frac{1}{10+4} + \&c.$$

$$\frac{1}{2} \times \frac{1}{1+2} + \frac{1}{3+4} + \frac{1}{6+6} + \frac{1}{10+8} + \&c.$$

$$\frac{1}{2} \times \frac{1}{1+3} + \frac{1}{3+6} + \frac{1}{6+9} + \frac{1}{10+12} + \&c.$$

6. Ante aliquot annos D. *Jac. Bernoulli* Geometra insignis invenit summam Seriei cujuslibet, cujus Numeratores constituunt Seriem æqualium, Denominatores vero constituunt, vel Seriem quadratorum dato aliquo quadrato Q minutorum, vel Seriem Triangulorum, dato aliquo Triangulo T minutorum. Hæc invenit ille observando quod hujusmodi Series oriantur ex ablatione Seriei Harmonice proportionalium truncatæ ab eadem Serie integra; nempe ita ut numerus terminorum deficientium in Serie truncata, sit, vel duplus lateris dati quadrati Q, vel duplus unitate auctus lateris dati Trianguli T. Idem etiam observavit frustra quæri summam Seriei reciprocæ Quadratorum. Hoc idem etiam verum est de reciprocis Cuborum, vel aliarum quarumlibet dignitatum numerorum in progressione Arithmetica. Ratio est, quod nulla intercedit differentia inter factores denominatorum, quod ad hujusmodi summationes semper requiri constat ex Methodo sumendi differentias in *Scholio Prop.* I. jam explicata. Nam si per formulam aliquam exhiberi posset summa quæsita, differentia istius formulæ exhiberet terminos Seriei propositæ: sed in tali differentia denominator semper afficitur per factores ab invicem diversos, quod quoniam in Seriebus prædictis non obtinet, summæ Serierum hujusmodi in terminis finitis haberi nequeunt. Ad eundem fere modum, argumento petito a *Prop.* III. & IV. demonstrari potest summas Serierum exhiberi non posse in terminis numero finitis, quarum Numeratores constituunt Seriem æqualium. Denominatores vero constant ex certo numero terminorum in progressione Arithmetica, maximo factore cujusvis termini minore existente quam factor minimus in termino proxime insequenti, cujusmodi est Series $\frac{1}{1 \cdot 2} + \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 6} + \&c.$

7. Jam liceret regulas nonnullas tradere, quas pro casibus quibusdam singularibus concinnavi; sed hæc nos longius abducerent. Sufficiat itaque quæ generaliora sunt explicasse, & simul monuisse, ad novæ hujusce Serierum infinitarum doctrinæ provectionem nihil magis facere, quam si

excogitentur formulæ generaliores ſummarum, ex quarum differentiis, per regulas ſupra traditas computatis, deinde conficiantur Canones quantitatum ſummabilium; ita fere ut jam factum eſt in Calculo Integrali, *h. e.* in Stylo *Newtoniano*, in Methodo Fluxionum.

8. Reſtituendo factores in Denominatore deficientes potuiſſet præſens Problema revocari ad *Propoſitionem* II. Sed & in terminis generalioribus proponi poteſt, nempe pro Numeratore ſumpta quavis Formula, cujus differentia aliqua datur. Sub ea tamen conditione ut dimenſiones Denominatoris ad minimum binario ſuperent Dimenſiones Numeratoris; alias enim ſumma Seriei in terminis numero finitis haberi nequit. Sit hujus rei exemplum in Serie $\frac{1}{1.3.5.7} + \frac{4}{2.4.6.8} + \frac{9}{3.5.7.9} + \frac{16}{4.6.8.10} +$ &c. ubi Numeratores ſunt numerorum naturalium quadrata. Applicando tum Numeratores tum Denominatores ad numeros naturales, Series revocatur ad formam ſimpliciorem $\frac{1}{3.5.7} + \frac{2}{4.6.8} + \frac{3}{5.7.9} + \frac{4}{6.8.10} +$ &c. Per p deſignatis numeris naturalibus 1, 2, 3, 4, &c. terminus Seriei deſignabitur per formulam $\frac{p}{\overline{p+2} \times \overline{p+4} \times \overline{p+6}}$; vel per formulam $\frac{z-2}{z \times \overline{z+2} \times \overline{z+4}}$, nempe pro $p+2$ ſcripto z. Quoniam progrediendo de termino in terminum augetur z per unitates, reſtituendi ſunt factores in denominatore deficientes $z+1$, $z+3$; & hoc pacto revocatur terminus Seriei ad formulam $\frac{\overline{z-2} \times \overline{z+1} \times \overline{z+3}}{z \times \overline{z+1} \times \overline{z+2} \times \overline{z+3} \times \overline{z+4}}$. Per methodum in hac Propoſitione jam explicatam revocatur numerator ad formam $-6-6z-z \times \overline{z+1} + z \times \overline{z+1} \times \overline{z+2}$. Unde habita ratione denominatoris Terminus revocatur ad formam $\frac{-6}{z \times \overline{z+1} \times \text{&c.} \times \overline{z+4}} + \frac{-6}{\overline{z+1} \times \overline{z+2} \times \overline{z+3} \times \overline{z+4}} + \frac{-1}{\overline{z+2} \times \overline{z+3} \times \overline{z+4}} + \frac{1}{\overline{z+3} \times \overline{z+4}}$. Adeoque ſumendo Integrale fit $\frac{6}{4z \times \overline{z+1} \times \overline{z+2} \times \overline{z+3}} + \frac{6}{3 \times \overline{z+1} \times \overline{z+2} \times \overline{z+3}} + \frac{1}{2 \times \overline{z+2} \times \overline{z+3}} + \frac{-1}{z+3}$; quo, ſub ſigno contrario, exhibetur ſumma Seriei in infinitum continuatæ, incipientis a termino $\frac{z-2}{z \times \overline{z+2} \times \overline{z+4}}$. Summa itaque Seriei integræ incipientis a termino $\frac{1}{3.5.7}$ eſt $\frac{31}{240}$.

Si per *Prop.* II. procedere eſſet animus, ex formula $\overline{z-2} \times \overline{z+1} \times \overline{z+3}$ collectis numeratoribus primis 24, 70, 144, 252, ſumendo eorum differentias haberentur $46=b$, $28=c$, $6=d$, $e=0=$ &c. exiſtente $M=24$; unde

unde per *Lem.* 2. prodiret formula $-6-6z-z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}$, qua designatur Terminus, eadem ac supra; atque pergendo per *Prop.* II. haberetur summa.

Prop. VI. *Prob.* Invenire summam quotlibet terminorum Seriei Fractionum, quarum Numeratores & Denominatores constituunt lineas duas quasvis transversas in Triangulo Arithmetico *Paschalii*; nempe cujus generatores sunt unitates.

Solutio. Per n designetur Ordo Seriei Numeratorum in Triangulo Arithmetico, & sit p differentia inter ordinem Numeratorum & Denominatorum, & per q designetur numerus terminorum quorum summa requiritur. Tum si Denominatores sint plurium dimensionum quam sunt Numeratores, Summa exhibebitur per formulam primam sequentem; si dimensiones Numeratorum plures sint quam dimensiones Denominatorum, Summa exhibebitur per formulam secundam.

Formula I. $\dfrac{n+p-1}{p-1}-\dfrac{n\,.\,\overline{n+1}\,.\,\overline{n+2}\,.\,\&c.\,\overline{n+p-1}}{\overline{p-1}\times\overline{n+q}\,.\,\overline{n+q+1}\,.\,\&c.\,\overline{n+q+p-2}}$

Formula II. $-\dfrac{n-p-1}{p+1}+\dfrac{\overline{q+n-1}\,.\,\overline{q+n-2}\,.\,\&c.\,\overline{q+n-p-1}}{\overline{p+1}\times\overline{n-1}\,.\,\overline{n-2}\,.\,\&c.\,\overline{n-p}}$

Ex. 1. Inveniendum sit aggregatum sex primorum terminorum Seriei $\frac{1}{1}+\frac{4}{7}+\frac{10}{28}+\frac{20}{84}+\frac{35}{210}+\frac{56}{462}+$ &c. ubi Numeratores constituunt lineam quartam, Denominatores constituunt lineam septimam in Triangulo Arithmetico. Sunt itaque $n=4$, $p=3$, $q=6$; & quoniam dimensiones Denominatorum superant dimensiones Numeratorum, dabitur summa per Formulam primam; nempe $\dfrac{4+3-1}{3-1}-\dfrac{4\,.\,5\,.\,6}{\overline{3-1}\times\overline{4+6}\times\overline{4+7}}$ sive $3-\frac{6}{11}=2\frac{5}{11}$.

Ex. 2. Quæratur summa sex primorum terminorum Seriei $\frac{1}{1}+\frac{7}{4}+\frac{28}{10}+\frac{84}{20}+\frac{210}{35}+\frac{462}{56}+$ &c. cujus termini sunt terminorum Seriei prioris reciproci. Sunt itaque $n=7$, $p=3$, $q=6$, adeoque per formulam secundam summa fit $-\frac{3}{4}+\frac{12\,.\,11\,.\,10\,.\,9}{4\times6\,.\,5\,.\,4}=24$.

Scholium 1. Formulas in hac propositione exhibitas ante biennium communicavi cum Viris celeberrimis *Moivreo* & *Bernoulliis*. Facile autem derivari possunt ex præceptis in *Prop.* I. traditis. Sit exemplum in Serie priori $\frac{1}{1}+\frac{4}{7}+\frac{10}{28}+$ &c. Per p designato loco Termini in Serie hac, exhibetur Terminus per formulam $\dfrac{4\,.\,5\,.\,6}{\overline{p+3}\,.\,\overline{p+4}\,.\,\overline{p+5}}$. Unde regrediendo ad Integrale, summa Seriei incipientis a termino illo exhibetur

tur per formulam $\frac{4.5.6}{2\times\overline{p+3}\times\overline{p+4}}$; adeoque pro p ſumpto 1, Series integra fit $\frac{4.5.6}{2.4.5}=3$, atque ſumma primorum ſex terminorum fit $3-\frac{4.5.6}{2.10.11}$, omnino ut per formulam jam exhibetur.

2. In formula prima ſumma Seriei in infinitum continuatæ eſt $\frac{n+p-1}{p-1}$, evaneſcente jam parte altera formulæ. Sed in caſu formulæ ſecundæ ſumma hæc eſt infinitum quid, cujus ſpecies, reſpectu numeri infiniti q, exhibetur per formulæ partem alteram, quæ in hoc caſu fit $\frac{q^{p+1}}{\overline{p+1}\times\overline{n-1}.\overline{n-2}.\&c.\overline{n-p}}$.

3. De hujuſmodi Seriebus in epiſtola data menſe *Maio* 1716, ſic ad me ſcripſit Vir. Ill. *D. Leibnitius*, quem magno Scientiarum damno nobis nuper ereptum lugemus. " Il me ſemble qu'autrefois j'ay auſſi ſommé " quelques Series ou ſuittes comme $\frac{1}{1}+\frac{2}{4}+\frac{3}{10}+\frac{4}{20}+\frac{5}{35}+\frac{6}{56}+$ " *&c.* Le terme de cette ſuitte exprimé Analytiquement eſt " $\frac{x}{x.\overline{x+1}.\overline{x+2}\times\frac{1}{1}.\frac{1}{2}.\frac{1}{3}}=\frac{1.2.3}{\overline{x+1}.\overline{x+2}}=\frac{6}{xx+3x+2}$. On demande " donc la ſomme d'une ſuitte donnée, dont un terme ſoit " $\frac{ll}{xx+3lx+2ll}$ ou x ſignifie les nombres naturales 1, 2, 3, 4, *&c.* & l " ſignifie l'Unité, ou la difference des x. Suppoſons que le terme de la " ſuitte ſommatrice demandée ſoit $\frac{fx}{mx+nl}=\frac{\odot}{D}$. Or Diff. $\frac{\odot}{D}=-\frac{\odot}{D}+$ " $\frac{\odot+d\odot}{D+dD}=\frac{D\,d\odot-\odot\,dD}{D.D+D\,dD}$: ſed $d\odot=fdx$, & $dD=mdx=ml$; " donc la Difference de $\frac{\odot}{D}$ eſt $=\frac{nfll}{\begin{matrix}mmxx+2mnlx+nnll\\+mmlx+mnll\end{matrix}}$. Maintenant il " faut faire $\frac{nfll}{\begin{matrix}mmxx+2mnlx+nnll\\+mmlx+mnll\end{matrix}}=\frac{mfll}{mmxx+3mmlx+2mmll}$ c'eſt " a dire, il faut identifier ces deux formules, ou la donnée eſt multipliée " per $\frac{nf}{mm}$: donc égalant les termes reſpectifs, puiſque les xx convien- " nent, on aura par les x, $2n+m=3m$, c'eſt a dire il y aura $m=n$, & " par les abſolus on aura $nn+mn=2mm$, ce qui donne encore $m=n$; " donc l'identification reüſſit, & nous pouvons faire $n=m=l=1$, & " $f=1$ (car f demeure arbitraire) & le terme de la ſuitte ſommatrice ſe- " ra $\frac{x}{x+1}$, car diff. $\frac{x}{x+1}$ donne $-\frac{x}{x+1}+\frac{x+1}{x+2}=\frac{1}{xx+3x+2}$, &

" par

" par conſequente $\frac{6x}{x+1}$ donne la ſomme des $\frac{x}{x\,.\,\overline{x+1}\,.\,\overline{x+2}\times\frac{1}{1}\,.\,\frac{1}{2}\,.\,\frac{1}{3}}$
" $3, 4, \frac{9}{2}, \frac{24}{5}, 5, \frac{36}{7}$, &c. *Series ſummatrix, cujus terminus* $\frac{6x}{x+1}$.
" $\frac{1}{1}+\frac{2}{4}+\frac{3}{10}+\frac{4}{20}+\frac{5}{35}+$ &c. *Series ſummanda, cujus terminus*
" $\frac{x}{x\,.\,\overline{x+1}\,.\,\overline{x+2}\times\frac{1}{1}\times\frac{1}{2}\,.\,\frac{1}{3}}$. Et pour s'en ſervir aux ſommations, les
" 5 termes, *par Ex.* de la ſuitte donnée ſeront $\frac{36}{7}-3=\frac{15}{7}$: Et gene-
" ralement la ſomme des termes juſqu'a quelque terme
" $\frac{x}{x\,.\,\overline{x+1}\,.\,\overline{x+2}\times\frac{1}{1}\,.\,\frac{1}{2}\,.\,\frac{1}{3}}$, excluſivement, ſera $\frac{6x}{x+1}-3$: Et pour la
" ſomme de la ſuitte entiere a l'infinie, x devient infini, & $\frac{6x}{x+1}$
" $=6$: donc la ſomme de toutte la ſuitte eſt $6-3=3$, comme vous
" l'avez trouvé.

" Cette methode eſt le calcul des differences appliqué aux Nombres; &
" il faut vous avouer qu'avant que de l'appliquer aux Figures, & même
" avant que d'avoir été Geometre, Je le prattiquai en quelque façon dans
" les nombres; ayant trouvé encore jeune garçon que les ſuites dont les
" Numerateurs fuſſent des Unites, & dont les Denominateurs fuſſent les
" Nombres figurés, comme Triangulaires Pyramidaux &c. etoient les
" differences 1eres, 2es, 3emes, &c. multipliées par les conſtantes de la ſuitte
" $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+$ &c. & par conſequent ſommables. Mais quand
" je devins un peu Geometre & Analyſte, Je vis qu'il y avoit moyen
" de venir a bout de telles ſommations par une Methode generale, au-
" tant qu'il etoit poſſible: & que le calcul des differences eſtoit encore
" plus commode dans la Geometrie que dans les Nombres, puis qu'il y a
" plus d'evanouiſſements, & que les differences repondent aux Tan-
" gentes, les ſommes aux Quadratures. Cette methode generalle de
" chercher la ſuitte ſommatrice de la ſuitte donnée, quand elle eſt
" poſſible, reüſſit toujours, quand le terme de la ſuitte donnée exprimé
" Analytiquement n'a point la quantité variable enveloppé dans une
" racine, ny entrant dans l'expoſant; & alors, on peut toujours deter-
" miner la ſuitte ſommatrice, ou prouver qu'il eſt impoſſible d'en trou-
" ver. Et la choſe reüſſit même bien ſouvent, lors même que la variable
" entre dans l'Expoſant. Mais comme il y a quelquefois des Quadra-
" tures particulieres de quelques portions d'une Figure, dont on ne ſçau-
" roit donner la Quadrature generalle ou la Figure quadratrice; de même
" on peut trouver quelquefois la ſomme de toute la ſuitte, ou d'un
" certaine partie, quoy qu'on ne puiſſe pas trouver la ſomme de chaque
" partie; & alors il faut avoir recours a des Methodes particulieres, dont

" on

" on n'eſt pas toujours le maiſtre, noſtre Analyſe n'eſtant pas encore por-
" tée a ſa perfection.

Prop. VII. *Prob.* Invenire ſummam Seriei cujus Numeratores conſtituunt lineam quamlibet erectam in Triangulo Arithmetico *Paſchalii*, Denominatores vero conſtituunt lineam quamlibet tranſverſam.

Solutio. Deſignetur ordo lineæ erectæ per p, ordo lineæ tranſverſæ per q, & ſit m aggregatum tot terminorum primorum in linea erecta ordinis $p+q-1$ quot ſunt unitates in $q-1$, atque ſumma quæſita erit $\overline{2^{p+q-2}-m} \times \frac{1.2.3.\&c.\overline{q-1}}{p.\overline{p+1}.\&c.\overline{p+q-2}}$.

Ex. 1. Proponatur Series $\frac{1}{1}+\frac{5}{4}+\frac{10}{10}+\frac{10}{20}+\frac{5}{35}+\frac{1}{56}$. Ubi Numeratores conſtituunt lineam ſextam erectam, Denominatores occupant lineam quartam tranſverſam. In hoc itaque caſu ſunt $p=6$, $q=4$, $p+q-1=9$, $q-1=3$, adeoque $m=1+8+28=37$ *i.e.* tribus terminis primis lineæ nonæ erectæ. Unde fit ſumma quæſita $\overline{2^8-37} \times \frac{1.2.3}{6.7.8} = \frac{219}{56}$.

Ex. 2. Conſtituant Numeratores lineam centeſimam erectam, & ſint Denominatores Numeri Trigonales, qui occupant lineam tertiam tranſverſam. Tum erunt $p=100$, $q=3$, $m=102$ atque adeo ſumma quæſita fit $\overline{2^{101}-102} \times \frac{1.2}{100.101}$.

Cor. Si $q=2$, formula fit $\frac{2^p-1}{p}$, qua exhibetur aggregatum primi termini, una cum ſemiſſe ſecundi, triente tertii, quadrante quarti, & ſic porro, lineæ cujuſvis erectæ ordinis p Trianguli Arithmetici *Paſchalii*. Sic *v. gr.* eſt $\frac{1}{1}+\frac{5}{2}+\frac{10}{3}+\frac{10}{4}+\frac{5}{5}+\frac{1}{6}=\frac{2^6-1}{6}=10\frac{1}{2}$.

Prop. VIII. *Prob.* Invenire ſummam ejuſdem Seriei, quando terminorum ſigna ſunt alternatim $+$ & $-$.

Solutio. Summa quæſita exhibetur per formulam ſimpliciſſimam $\frac{q-1}{p+q-2}$.

Ex. Invenienda ſit ſumma Seriei $\frac{1}{1}-\frac{6}{9}+\frac{15}{45}-\frac{20}{165}+\frac{15}{495}-\frac{6}{1287}+\frac{1}{3003}$, ubi Numeratores conſtituunt lineam ſeptimam erectam, Denominatores conſtituunt nonam tranſverſam. In formula itaque pro p & q ſcriptis 7 & 9, fit ſumma $\frac{8}{14}$.

Manente eadem Serie Numeratorum (nempe linea ſeptima erecta), ſi pro Serie Denominatorum ſumantur ſucceſſive lineæ tranſverſæ 2da, 3tia, 4ta, &c. Summæ erunt $\frac{1}{7}$, $\frac{2}{8}$, $\frac{3}{9}$, $\frac{4}{10}$, $\frac{5}{11}$, &c. quæ ſic poſſunt ſcribi,

bi, $\frac{1}{7}$, $\frac{7}{28}$, $\frac{28}{84}$, $\frac{84}{210}$, $\frac{210}{462}$, &c. ubi tam Numeratores, quam Denominatores excerpuntur ex linea tranſverſa ordinis ſeptimi. Idem eveniret ſi loco ſeptimæ, Numeratores conſtituiſſent aliam quamlibet lineam erectam ordinis p; Summæ quippe orirentur ex applicatione terminorum lineæ tranſverſæ ejuſdem ordinis p ad terminos proxime ſequentes in eadem linea.

Propoſitiones hæ duæ noviſſimæ potius elegantes ſunt quam utiles; quare Formularum noſtrarum demonſtrationem Lectoris ſolertia inveſtigandam relinquimus, ad Propoſitionem ultimam jam properantes, quæ tertiam continet Serierum ſpeciem, ob uſum multiplicem ſatis inſignem.

Lemma 5. Sit Series quævis $\frac{M}{h}$, $\frac{N}{h^2}$, $\frac{O}{h^3}$, $\frac{P}{h^4}$, &c. cujus terminorum Denominatores conſtituunt progreſſionem quamlibet Geometricam h, h^2, h^3, h^4, &c. Sint etiam Numeratorum primus A $(=M)$ prima differentiarum primarum B, prima ſecundarum C, prima tertiarum D, quartarum E, & ſic porro; & ſint $\frac{\alpha}{h}$, $\frac{\beta}{h^2}$, $\frac{\gamma}{h^3}$, $\frac{\delta}{h^4}$, &c. reſpective, aggregata, Unius, Duorum, Trium, Quatuor, vel plurium terminorum Seriei $\frac{M}{h}$, $\frac{N}{h^2}$, $\frac{O}{h^3}$, &c. atque ſint Numeratorum primus a $(=\alpha)$ prima differentiarum primarum b, prima ſecundarum c, prima tertiarum d, & ſic porro: & ſit $h-1=q$. Tum ipſorum a, b, c, d, &c. valores erunt,

$$\begin{aligned} a &= A = \alpha = M \\ b &= hA + B \\ c &= qhA + hB + C \\ d &= q^2hA + qhB + hC + D \end{aligned}$$

& ſic porro.

Demonſtratio. Satis conſtat eſſe $a=\alpha=A=M$.

Termini $\frac{M}{h}$, $\frac{N}{h^2}$, $\frac{O}{h^3}$, $\frac{P}{h^4}$, &c. Numeratoribus M, N, O, P, &c. expreſſis per $A, B, C, D,$ &c. transformantur in terminos $\frac{A}{h}$, $\frac{A+B}{h^2}$, $\frac{A+2B+C}{h^3}$, $\frac{A+3B+3C+D}{h^4}$ &c. Unde colligendo ſummas terminorum, inveniuntur Numeratores α, β, γ, δ, &c. nempe

$$\begin{aligned} \alpha &= A \\ \beta &= \overline{h+1}\,A + B \\ \gamma &= \overline{h^2+h+1}\,A + \overline{h+2}\,B + C \\ \delta &= \overline{h^3+h^2+h+1}\,A + \overline{h^2+2h+3}\,B + \overline{h+3}\,C + D, \text{ \&c.} \end{aligned}$$

Unde ſumendo differentias fiunt

$$\begin{aligned} b &= hA + B \\ c &= qhA + hB + C \end{aligned}$$

d =

$d = qqbA + qbB + bC + D$

& sic porro, ut in Propositione exhibentur.

Cor. 1. Si Numeratorum M, N, O, P, &c. differentia vel prima vel secunda, vel alia quædam detur, terminis omnibus post primos aliquot in Serie A, B, C, D, &c. evanescentibus, Differentiæ, b, c, d, &c. tandem incurrent in Progressionem Geometricam in ratione 1 ad q. Exempli gratia, si detur Numeratorum M, N, O, P, &c. differentia prima B, erunt c, d, &c. in ratione continua Geometrica 1 ad q; ut constat per ipsorum valores $qbA + bB$, $qqbA + qbB$, &c. existentibus $C = 0 = D =$ &c.

Cor. 2. Ordo autem primæ differentiarum B, C, D, &c. quæ hoc modo evanescunt, idem est ac ordo differentiæ vel b, vel c, &c. unde incipit Progressio illa Geometrica. Sic si $B = 0 = C =$ &c. erunt b, c, d, &c. in Progressione Geometrica; si $C = 0 = D =$ &c. erunt c, d, &c. in Progressione Geometrica. Et sic porro.

Lemma 6. Iisdem positis sit r terminus unde incipit Progressio Geometrica in Serie differentiarum b, c, d, &c. & per $p+1$ designetur ordo Termini in Serie $\frac{\alpha}{b}$, $\frac{\beta}{b^2}$, $\frac{\gamma}{b^3}$, $\frac{\delta}{b^4}$, &c. Tum Terminus ille designabitur per fractionem cujus Denominatore existente b^{p+1} Numerator est

$$\frac{a + bp + cp \times \frac{p-1}{2} + dp \times \frac{p-1}{2} \times \frac{p-2}{3} + \&c. + \frac{r}{q^n}}{\times\, b^p - 1 - qp - q^2 p \times \frac{p-1}{2} - q^3 p \times \frac{p-1}{2} \times \frac{p-2}{3} - \&c.}$$

nempe per n designato ordine differentiæ evanescentis in Serie B, C, D, &c. ut & Numero terminorum $a + bp$, &c. item terminorum $-1 - qp$, &c.

Demonstratio. Per *Lemma* 1. Termini istius Numerator exhibetur per formulam $a + bp + cp . \frac{p-1}{2} + dp \times \frac{p-1}{2} . \frac{p-2}{3} +$ &c. ($p+1$ subeunte vices x in Lemmate isto).

Ergo si sit, *ex. gr.* $n = 2$, per *Lemm.* 5. *Cor.* 2. erunt c, d, &c. in ratione continua 1 ad q. Numerator itaque in hoc casu est

$$a + bp + cp \times \frac{p-1}{2} + cqp \times \frac{p-1}{2} \times \frac{p-2}{3} + cq^2 p \times \frac{p-1}{2} \times \frac{p-2}{3} \times \frac{p-3}{4} + \&c.$$

Sed si termini $cp \times \frac{p-1}{2} + cqp \times \frac{p-1}{2} \times \frac{p-2}{3} +$ &c. ducantur in $\frac{q^2}{c}$, & productui addantur termini $1 + qp$, prodibit Series qua exprimitur binomii $1 + q$ dignitas $\overline{1+q}|^p = b^p$. Ergo productum illud æquale est $b^p - 1 - qp$; adeoque termini $cp \times \frac{p-1}{2} + cqp \times \frac{p-1}{2} \times \frac{p-2}{3} +$ &c. $= \frac{c}{q^2} \times \overline{b^p - 1 - qp}$. Quo pacto Numerator fit $a + bp + \frac{c}{q^2} \times \overline{b^p - 1 - qp}$, existentibus duobus terminis $a + bp$, ut & duobus

duobus $-1-qp$, juxta sensum Propositionis, quoniam $n=2$. Atque eadem est demonstratio in aliis casibus. De Denominatore vero per se satis constat.

Prop. IX. *Prob.* Invenire summam quotlibet terminorum Seriei cujusvis $\frac{M}{b}$, $\frac{N}{b^2}$, $\frac{O}{b^3}$, $\frac{P}{b^4}$, &c. cujus terminorum Denominatores constituunt progressionem quamlibet Geometricam b, b^2, b^3, b^4, &c. Numeratores autem sunt quantitates differentia aliqua constanti gaudentes.

Solutio. Sunto Numeratorum M, N, O, P, &c. primus A, prima differentiarum primarum B, prima secundarum C, prima tertiarum D, & sic porro; & sit ipsorum A, B, C, D, &c. numerus n, atque $b-1=q$, Tum fiat $a=A\,(=M)$ $b=bA+B$, $c=qbA+bB+C$, $d=q^2bA+qbB+bC+D$, &c. ut sint tot termini a, b, c, d, &c. quot sunt unitates in $n+1$. Terminorum istorum ultimus dicatur r, atque per $p+1$ designetur numerus terminorum $\frac{M}{b}$, $\frac{N}{b^2}$, $\frac{O}{b^3}$, $\frac{P}{b^4}$, &c. quorum summa requiritur; Dico summam illam exhiberi per fractionem, cujus Denominatore existente b^{p+1}, Numerator est

$$\overline{a+bp+cp\times\frac{p-1}{2}+dp\times\frac{p-1}{2}\times\frac{p-2}{3}+\&c.+\frac{r}{q^n}}$$
$$\times\, \overline{b^p-1-qp-q^2p\times\frac{p-1}{2}-q^3p\times\frac{p-1}{2}\times\frac{p-2}{3}-\&c.-q^{n-1}p\times}$$
$$\overline{\frac{p-1}{2}}\times\&c.$$

Demonstratio. Nam (per *Lem.* 6.) per hanc formulam repræsentatur terminus ordine $p+1$ Seriei $\frac{\alpha}{b}$, $\frac{\beta}{b^2}$, $\frac{\gamma}{b^3}$, $\frac{\delta}{b^4}$, &c. qui terminus (per constructionem *Lemmatis* 5.) æqualis est aggregato terminorum numero $p+1$ Seriei propositæ $\frac{M}{b}$, $\frac{N}{b^2}$, $\frac{O}{b^3}$, $\frac{P}{b^4}$. *Q. E. D.*

Ex. 1. Invenienda sit summa novem terminorum Seriei $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{8}$, $\frac{4}{16}$, &c. Sunt in hoc casu $b=2$, $q\,(=b-1)=1$, $p+1=9$, $p=8$, $A=1$, $B=1$, $C=0$, $=D=$&c. adeoque $n=2$, (quoniam sunt duo A, B,) Hinc fit $a\,(=A)=1$, $b\,(=bA+B=2\times1+1)=3$, $c\,(=qbA+bB+C=2\times1+2\times1+0)=4=r$, Adeoque per formulam fit summa quæsita

$$\frac{1+3\times8+\frac{4}{1^2}\times\overline{2^8-1-1\times8}}{2^9}=\frac{1013}{512}.$$

Ex. 2. Quæratur summa sex terminorum Seriei $1\times3+3\times3^2+6\times3^3+10\times3^4+15\times3^5+21\times3^6+$&c. In hoc casu sunt $b=\frac{1}{3}$, $q=$

$\frac{-2}{3}$, $p+1=6$, $p=5$, $A=1$, $B=2$, $C=1$, $D=0=E=$ &c. adeoque $n=3$, atque $a=1$, $b=\frac{1}{3}+2=\frac{7}{3}$, $c=\frac{-2}{9}+\frac{2}{3}+1=\frac{13}{9}$, $d=\frac{4}{27}-\frac{4}{9}+\frac{1}{3}=\frac{1}{27}=r$. Unde summa quæsita fit $=19956$, sive

$$\frac{1+\frac{7}{3}\times 5+\frac{13}{9}\times 5\times\frac{4}{2}+\frac{-1}{8}\times\frac{1}{3^5}-1+\frac{2}{3}\times 5-\frac{4}{9}\times 5\times\frac{4}{2}}{\overline{\frac{1}{3}}|^{6}}$$

Cor. 1. Ejusdem Seriei, a termino primo $\frac{M}{b}$ in infinitum continuatæ, summa exhibetur per formulam simplicissimam $\frac{A}{\overline{b-1}}+\frac{B}{\overline{b-1}|^2}+\frac{C}{\overline{b-1}|^3}+\frac{D}{\overline{b-1}|^4}$ &c.

Cor. 2. Si $b=2$, Seriei totius in infinitum continuatæ summa habetur sola additione terminorum A, B, C, D, &c. Et hæc summa eadem est ac summa lineæ erectæ respondentis termino primo A, in Triangulo Arithmetico, cujus lineam transversam occupant Numeratores M, N, O, P, &c. Quod facile constat ex contemplatione Trianguli. Si itaque fuerint M, N, O, &c. Numeri figurati cujusvis ordinis n, summa Seriei $\frac{M}{2}+\frac{N}{4}+\frac{O}{8}+\frac{P}{16}+$ &c. æqualis erit Numeri binarii dignitati $\overline{2}|^{n-1}$. Sic Series $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+$ &c. $=2^{1-1}=1$, ut vulgo notum; Series $\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+$ &c. $=2^{2-1}=2$; Series $\frac{1}{2}+\frac{3}{4}+\frac{6}{8}+\frac{10}{16}+$ &c. $=2^{3-1}=2^2=4$, & sic porro.

Scholium. Celeb. D. *Jac. Bernoulli*, in Tractatu suo de Seriebus infinitis, solvit illud Problema. "Invenire summam Seriei infinitæ Fractio-" "num quarum Denominatores crescunt in Progressione quacunque Geo-" "metrica, Numeratores vero progrediuntur vel juxta Numeros natura-" "les, 1, 2, 3, 4, &c. vel Trigonales 1, 3, 6, 10, &c. vel Pyramidales 1," "4, 10, 20, &c. aut juxta Quadratos 1, 4, 9, 16, &c. aut Cubos 1, 8, 27," "64, &c. eorumve multiplices." Ipsius solutionem consulat Lector. Aliam vero, & quidem multo generaliorem invenit D. *Nic. Bernoulli* illius Nepos, eamque (postquam ei hæc miseram, sed sine demonstratione) mecum communicare dignatus est, in epistola data 18° *Septembris* 1715, miris quidem inventis refertissima, qualibus me crebro dignatur vir Doctissimus. De hoc vero Problemate sic scribit. "Pour la somme d'un" "nombre determiné n de termes de la suitte de vostre Theoreme 7. [*Co-*

rollarium

" *rollarium* primum eſt hujus Propoſitionis] j'ay trouvé cette formule

" $\frac{1}{m^n} \times \frac{m-1}{m-1} a + \frac{A-n}{m-1} b + \frac{B-n \cdot \frac{n-1}{2}}{m-1} c + \frac{C-n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3}}{m-1} d$

" $+$ &c. ou les Lettres *A*, *B*, *C*, &c. marquent les Coefficients des ter-
" mes immediatement precedents. Et en mettant dans cette formule
" $p+1$ pour n, b^m pour m, & en multipliant tout encore par e^{n-1}, on
" a la ſolution de voſtre *Prob.* IX^me". Et me monuit Vir peritiſſimus hanc ſuam formulam generalem in noſtram particularem (*Cor.* 1. hujus propoſitionis) migrare quando $n = \infty$; quippe tum evaneſcunt $1, n, n \cdot \frac{n-1}{2}, n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3}$, &c. reſpectu ipſorum m^n, *A*, *B*, *C*, &c. adeo ut Series in eo caſu ſit $\frac{1}{m-1} a + \frac{A}{m-1} b + \frac{B}{m-1} c +$ &c. quæ omnino coincidit cum noſtra $\frac{a}{m-1} + \frac{b}{\overline{m-1}|^2} + \frac{c}{\overline{m-1}|^3} +$ &c.

2. *Definitiones.* 1. Quantitatis cujuſvis variabilis valorem præſentem deſigno litera ſimpliciter ſcripta, ut x; valores præcedentes lineolis eidem literæ ex parte ſuperiori poſitis, ſequentes lineolis ex parte inferiori ſcriptis. Ut vi hujus Definitionis ſint $\overset{\prime\prime}{x}$, $\overset{\prime}{x}$, x, $\underset{\prime}{x}$, $\underset{\prime\prime}{x}$, ejuſdem variabilis valores quinque continui, exiſtente x valore præſenti, $\overset{\prime}{x}$ proxime præterito, $\overset{\prime\prime}{x}$ ſecundo præterito; $\underset{\prime}{x}$ proxime, atque $\underset{\prime\prime}{x}$ ſecundo futuro. Et ſic de aliis. Ad eundem modum ſunt interpretandæ lineolæ quæ incrementis apponuntur. Sic ſunt $\overset{\prime\prime}{\underset{\cdot\cdot}{x}}$, $\overset{\prime}{\underset{\cdot\cdot}{x}}$, $\underset{\cdot\cdot}{x}$, $\underset{\cdot\cdot\,\prime}{x}$, $\underset{\cdot\cdot\,\prime\prime}{x}$, ipſius $\underset{\cdot\cdot}{x}$ valores quinque continui; ut ſit $\overset{\prime\prime}{\underset{\cdot\cdot}{x}}$ incrementum ſecundum ipſius $\overset{\prime\prime}{x}$, ſit $\overset{\prime}{\underset{\cdot\cdot}{x}}$ incrementum ſecundum ipſius $\overset{\prime}{x}$. Et ſic de aliis. Of the ſame, by Dr. Taylor, ib. p. 676.

Cor. Vi hujus Definitionis, $\overset{\prime}{x} + \overset{\prime}{\underset{\cdot}{x}} = x$, $x + \underset{\cdot}{x} = \underset{\prime}{x}$, $\underset{\cdot}{x} + \underset{\cdot\cdot}{x} = \underset{\prime\,\cdot}{x}$. Et ſic de aliis hujuſmodi.

Quando uſu venit ut variabilis quantitas, puta x, ſpectanda ſit tanquam Incrementum, ejus Integrale deſigno litera inter uncos [ ] incluſa. Iſtius etiam Integralis [x] Integrale (vel ipſius x integrale ſecundum,) deſigno numero binario uncorum priori ſuperimpoſito, ut $\overset{2}{[}\,x\,]$. Iſtius etiam Integralis Integrale (vel ipſius x Integrale tertium,) ad eundem modum deſigno numero ternario, ut $\overset{3}{[}\,x\,]$. Et ſic deinceps. Unde

vi hujus Definitionis constituunt $\overset{3}{[x]}$, $\overset{2}{[x]}$, $[x]$, x Seriem terminorum, quorum quilibet est ipsum immediate præcedentis incrementum primum, ut sit $\overset{2}{[x]} = \overset{3}{[\underset{\cdot}{x}]}$, $[x] = \overset{2}{[\underset{\cdot}{x}]}$, $x = [\underset{\cdot}{x}]$.

Lemma. Facti $x\,v$ ex Multiplicatione duorum variabilium x & v, incrementum est $\underset{\cdot}{x}\,v + \underset{\prime}{x}\,\underset{\cdot}{v}$.

Nam auctis variabilibus per propria incrementa, fit novum productum $\overline{x+\underset{\cdot}{x}} \times \overline{v+\underset{\cdot}{v}}$, sive $x\,v + \underset{\cdot}{x}\,v + \overline{x+\underset{\cdot}{x}} \times \underset{\cdot}{v}$, hoc est $x\,v + \underset{\cdot}{x}\,v + \underset{\prime}{x}\,\underset{\cdot}{v}$ (pro $x+\underset{\cdot}{x}$ scripto $\underset{\prime}{x}$ *per Def.* 1.) Unde dempto priori producto $x\,v$, restat Incrementum $\underset{\cdot}{x}\,v + \underset{\prime}{x}\,\underset{\cdot}{v}$.

Prop. I. *Theor.* Ejusdem Facti $x\,v$ Incrementum vel primum, vel secundum, vel tertium, vel aliud quodvis, cujus ordo designatur per symbolum n, exhibetur per formulam hanc generalem

$$\underset{n}{x}\,v + n\,\underset{\prime\,n-1}{x}\,\underset{\cdot}{v} + n \times \frac{n-1}{2}\,\underset{\prime\prime\,n-2}{x}\,\underset{\cdot\cdot}{v} + n \times \frac{n-1}{2} \times \frac{n-2}{3}\,\underset{\prime\prime\prime\,n-3}{x}\,\underset{\therefore}{v} + \&c.$$

In hac formula hæc sunt observanda, 1mo Terminorum numeri coefficientes 1, n, $n \times \frac{n-1}{2}$, $n \times \frac{n-1}{2} \times \frac{n-2}{3}$ &c. iidem sunt ac in binomii dignitate n. 2do Numeri n, $n-1$, $n-2$, $n-3$, &c. ipsis x infrascripti designant numeros punctorum quibus definiuntur Incrementa. 3^{tio} Lineolæ $\prime$, $\prime\prime$, $\prime\prime\prime$, &c. ipsis x infrascriptæ, interpretandæ sunt per Def. 1. 4to In quovis Termino numerus punctorum ipsis x & v simul infrascriptorum, est n. Sit *v. g.* $n = 4$: tum per formulam, ipsius $x\,v$ incrementum quartum prodit $\underset{::}{x}\,v + 4\,\underset{\prime\,\therefore}{x}\,\underset{\cdot}{v} + 6\,\underset{\prime\prime\,\cdot\cdot}{x}\,\underset{\cdot\cdot}{v} + 4\,\underset{\prime\prime\prime\,\cdot}{x}\,\underset{\therefore}{v} + \underset{\prime\prime\prime\prime}{x}\,\underset{::}{v}$.

Theorema hoc generale demonstrari potest per Inductionem, incrementis continuo sumptis juxta formam in Lemmate præcedenti traditam. Sed & collecta forma Seriei ex hujusmodi calculo, Theorema etiam demonstrari potest per Methodum Incrementorum, ad eum modum cujus specimen mox dabimus in demonstratione Propositionis tertiæ.

Prop. II. *Theor.* Ipsius $x\,v$ Integrale primum $[x\,v]$ exhibetur per Seriem $[x]\,v - \overset{2}{[\underset{\prime}{x}]}\,\underset{\cdot}{v} + \overset{3}{[\underset{\prime\prime}{x}]}\,\underset{\cdot\cdot}{v} - \overset{4}{[\underset{\prime\prime\prime}{x}]}\,\underset{\therefore}{v}$ &c.

Series autem ita terminatur, ut sit $[x\,v] = [x]\,v - \left[[\underset{\prime}{x}]\,\underset{\cdot}{v}\right] = [x]\,v - \overset{2}{[\underset{\prime}{x}]}\,\underset{\cdot}{v} + \left[\overset{2}{[\underset{\prime\prime}{x}]}\,\underset{\cdot\cdot}{v}\right] = \&c.$

Nam

Nam sumendo incrementa restituitur propositum $x v$.

Cor. 1. Datis duobus ex istis $[x]$, $[x v]$, $\left[[x_{\prime}]\, \underset{\cdot}{v}\right]$, datur tertium. Item datis tribus ex istis $[x]$, $\overset{2}{[x_{\prime}]}$, $[x v]$, $\left[\overset{2}{[x_{\prime\prime}]}\, \underset{\cdot\cdot}{v}\right]$, datur quartum. Et sic porro.

Cor. 2. Si $\underset{\cdot}{v}=0$, datur $[x v]$ ex dato $[x]$. Si $\underset{\cdot\cdot}{v}=0$ datur $[x v]$ ex datis duobus $[x]$, & $\overset{2}{[x]}$, Si $\underset{\because}{v}=0$, datur $[x v]$, ex datis tribus $[x]$, $\overset{2}{[x]}$, $\overset{3}{[x]}$. Et sic porro.

Ex. 1. Sit exemplum hujus formulæ in inventione Integralis ipsius $\frac{v}{z z_{\prime} z_{\prime\prime} z_{\prime\prime\prime}}$, dato nempe $\underset{\cdot}{z}$, atque existente $\underset{\because}{v}=0$, qui casus est specialis Propositionis secundæ Tractatus præcedentis D^ni *Monmort*. Facto itaque $x=\frac{1}{z z_{\prime} z_{\prime\prime} z_{\prime\prime\prime}}$, sunt $[x]=\frac{-1}{3\underset{\cdot}{z}\, z z_{\prime} z_{\prime\prime}}$, $\overset{2}{[x_{\prime}]}=\frac{1}{2\underset{\cdot}{z}\times 3\underset{\cdot}{z}\, z_{\prime} z_{\prime\prime}}$, atque $\overset{3}{[x_{\prime\prime}]}=\frac{-1}{1\underset{\cdot}{z}\times 2\underset{\cdot}{z}\times 3\underset{\cdot}{z}\, z_{\prime\prime}}$. Unde per formulam fit $[x v]$, hoc est $\frac{v}{z z_{\prime} z_{\prime\prime} z_{\prime\prime\prime}}=-$

$$\frac{v}{3\underset{\cdot}{z}\, z z_{\prime} z_{\prime\prime}}-\frac{\underset{\cdot}{v}}{2\underset{\cdot}{z}\times 3\underset{\cdot}{z}\, z_{\prime} z_{\prime\prime}}-\frac{\underset{\cdot\cdot}{v}}{1\underset{\cdot}{z}\times 2\underset{\cdot}{z}\times 3\underset{\cdot}{z}\, z_{\prime\prime}}.$$

Ex. 2. Sit aliud exemplum in inventione Integralis ipsius $n a^{z}$, ubi est $\underset{\cdot}{z}=1$, atque datur a. Tum pro x sumpto a^{z}, & pro v sumpto n, fit $x=a^{z\prime}$ hoc est $x_{\prime}=a x$, seu $x+\underset{\cdot}{x}=a x$, adeoque $\underset{\cdot}{x}=\overline{a-1}\, x$, atque $x=\frac{\underset{\cdot}{x}}{a-1}$ Regrediendo itaque ad Integralia fit $[x]=\frac{x}{a-1}$; item $\overset{2}{[x]}=\frac{[x]}{a-1}=\frac{x}{\overline{a-1}|^{2}}$, item $\overset{3}{[x]}=\frac{x}{\overline{a-1}|^{3}}$; & sic porro. Adeoque (quoniam $x_{\prime}=a x$,) sunt $[x]=\frac{x}{a-1}$, $[x_{\prime}]=\frac{a x}{\overline{a-1}|^{2}}$, $[x_{\prime\prime}]=\frac{a^{2} x}{\overline{a-1}|^{3}}$, &c. Unde per formulam prodit $[n a^{z}]=\frac{a^{z} n}{a-1}-\frac{a^{z+1}\underset{\cdot}{n}}{\overline{a-1}|^{2}}+\frac{a^{z+2}\underset{\cdot\cdot}{n}}{\overline{a-1}|^{3}}$, &c.

In hoc exemplo continetur Solutio Problematis, de quo agit D^nus de *Monmort* in Propositione nona. Coincidit autem formula cum ea quam exhibet ille in Corollario primo ejusdem Propositionis.

Scholium. Possunt etiam ex hac formula alii derivari valores Integralis quæsiti, pro vario modo quo interpretantur Incrementi propositi factores. Sic in exemplo secundo integrale ipsius $n a^{z}$ exhiberi potest per formulam

jam $a^x [n] - \overline{a-1}\, a^x [\overset{2}{\underset{,}{n}}] + \overline{a-1}|^2\, a^x [\overset{3}{\underset{,,}{n}}] -$ &c. pro x nempe sumpto n, & pro v sumpto a^x. Sed de his fortasse alia occasione fusius dicemus.

Prop. III. *Theor.* Ejusdem xv Integrale, vel primum, vel secundum, vel tertium, vel aliud quodvis cujus ordo designatur symbolo n, exhibetur per Seriem in hac forma generali prodeuntem $\overset{n}{[xv]} = \overset{n}{[x]}v - n \overset{n+1}{[\underset{,}{x}]}\dot{v} + n \times \frac{n+1}{2} \overset{n+2}{[\underset{,,}{x}]}\ddot{v} - n \times \frac{n+1}{2} \times \frac{n+2}{3} \overset{n+3}{[\underset{,,,}{x}]}\dddot{v} +$ &c.

Collecta forma Seriei ex Propositione præcedenti, Coefficientes $1, -n$, $n \times \frac{n+1}{2}, -n \times \frac{n+1}{2} \times \frac{n+2}{3}$, &c. sic inveniuntur per Methodum Incrementorum. Pone $\overset{n}{[xv]} = A \overset{n}{[x]} v + B \overset{n+1}{[\underset{,}{x}]} \dot{v} + C \overset{n+2}{[\underset{,,}{x}]} \ddot{v} + D \overset{n+3}{[\underset{,,,}{x}]} \dddot{v} +$ &c. Tum aucto n incremento suo $\underset{,}{n} = 1$, atque ipsis A, B, C, D, &c. incrementis suis contemporaneis $\dot{A}, \dot{B}, \dot{C}, \dot{D}$, &c. ut jam evadant $\underset{,}{n}$, $\underset{,}{A}, \underset{,}{B}, \underset{,}{C}, \underset{,}{D}$, &c. fiet novum Integrale (quod Integrale est ipsius $\overset{n}{[xv]}$,) $\overset{n+1}{[xv]} = \underset{,}{A} \overset{n+1}{[\underset{,}{x}]} v + \underset{,}{B} \overset{n+2}{[\underset{,}{x}]} \dot{v} + \underset{,}{C} \overset{n+3}{[\underset{,,}{x}]} \ddot{v} + \underset{,}{D} \overset{n+4}{[\underset{,,,}{x}]} \dddot{v} +$ &c. Hujus itaque Incrementum primum coincidere debet cum Integrali prius posito. Sumptis ergo incrementis, fit $\overset{n}{[xv]} = \underset{,}{A} [x] v \begin{matrix} + \underset{,}{A} \\ + \underset{,}{B} \end{matrix} \overset{n+1}{[\underset{,}{x}]} \dot{v} \begin{matrix} + \underset{,}{B} \\ + \underset{,}{C} \end{matrix} \overset{n+2}{[\underset{,,}{x}]} \ddot{v} \begin{matrix} + \underset{,}{C} \\ + \underset{,}{D} \end{matrix} \overset{n+3}{[\underset{,,,}{x}]} \dddot{v} +$ idem ac Integrale prius positum. Itaque terminos homologos inter se comparando fit 1^{mo} $A = \underset{,}{A}$. Unde est A datum quid. Sed ubi $n = 0$, est $A = 1$, ergo $A = 1$. 2^{do}. $\dot{B} = \underset{,}{B} + \underset{,}{A}$, hoc est $\dot{B} = B + \dot{B} + 1$, seu $\dot{B} = -1 = -\underset{,}{n}$. Ergo regrediendo ad Integralia, fit $B = -n + a$. Sed ubi $n = 0$, est $B = 0$. Ergo $a = 0$, atque $B = -n$. 3^{tio}. $\dot{C} = \underset{,}{C} + \underset{,}{B}$, hoc est $\dot{C} = \underset{,}{n}$. Regrediendo itaque ad integralia fit $C = \frac{n\underset{,}{n}}{2} + b$. Sed ubi $n = 0$, est $C = 0$. Ergo $b = 0$, atque $C = \frac{n\underset{,}{n}}{2}$, hoc est

$n \times$

$n \times \frac{n+1}{2}$. 4^{to}. Ad eundem modum invenitur $D = -n \times \frac{n+1}{2} \times \frac{n+2}{3}$. Et sic pergendo inveniuntur cæteri Coefficientes.

Scholium 1. In hac Propositione comparata cum Propositione prima, cernitur singularis quædam relatio Incrementa inter & Integralia. Ut enim in Arithmetica vulgari, Multiplicatio & Divisio sunt invicem ita contrariæ, ut si Multiplicatio designetur per Indicem affirmativum, Divisio designabitur per Indicem cum signo negativo; sic etiam in Methodo Incrementorum, si Incrementum designetur per Indicem affirmativum, Index negativus Integrale sistet. Sic in Propositione prima, si pro n sumatur Numerus binarius 2, per formulam exhibebitur ipsius $x\,v$ incrementum secundum, nempe $x\ddot{v} + 2\dot{x}_{\prime}\dot{v} + \ddot{x}_{\prime\prime}v$; Sed si pro n sumatur numerus negativus -2, ut jam quæratur ipsius $x\,v$ incrementum (ita loqui liceat) negative secundum, (quod idem est ac Integrale secundum) prodeunt coefficientes iidem ac si sumatur n affirmative in Propositione præsenti: atque interpretatis insuper ipsis $\underset{-2}{x}$, $\underset{\prime\,-3}{x}$, $\underset{\prime\prime\,-4}{x}$, &c. per $[x]^{2}$, $[\underset{\prime}{x}]^{3}$, $[\underset{\prime\prime}{x}]^{4}$, &c. Series fit omnino eadem ac per Propositionem præsentem prodit, ubi quæritur Integrale secundum.

2. Ex his autem formulis quasi sua sponte procedunt formulæ Propositionum undecimæ atque duodecimæ Libri de Methodo Incrementorum. Nam pro incrementis scribe Fluxiones, atque evanescentibus incrementis fiant jam omnes x, $x_{\prime}$, $x_{\prime\prime}$, $x_{\prime\prime\prime}$, &c. inter se æquales, atque migrabit statim hæc Propositio secunda in illam undecimam, atque præsens tertia in illam duodecimam. Quod quidem exemplum satis insigne est Methodi *Newtonianæ*, qua colligit ille rationes Fluxionum ex rationibus ultimis Incrementorum evanescentium, vel ex primis nascentium.

Præcedentium impressioni intentus dum in animo illa sæpius revolvo, subiit Artificium illud quo jam olim usus est D. *Jac. Bernoulli* in inventione quarundam Serierum, ope Progressionis Harmonicæ cujus meminit D. de *Monmort* in *Scholio 6. Prop.* V. præcedente, commode etiam applicari posse ad inventionem ipsius *Monmortii* Propositionum $2^{dæ}$, $3^{tiæ}$, 4^{te}, $5^{tæ}$, atque id genus aliarum aliquanto fortasse generaliorum. Hoc in sequentibus paucis ostendisse credebam Lectori non fore ingratum.

Theorema. Sit Progressio Arithmetica $p, p+n, p+2n$, &c. cujus termini singuli successive designentur per x, & sunto b, c, d, &c. quivis multiplices differentiæ datæ n terminorum Progressionis istius Arithmeticæ. Sint A, B, C, D, &c. Numeri quilibet dati, & constituantur fractiones quotvis $\frac{A}{x}$, $\frac{B}{x+b}$, $\frac{C}{x+c}$, $\frac{D}{x+d}$, &c. Pro x successive scriptis valoribus suis $p, p+n, p+2n$, &c. ex harum fractionum qualibet oritur Series Harmonice proportionalium. Sic *v.g.* ex fractione prima $\frac{A}{p}$, oritur

tur Series $\frac{A}{p}$, $\frac{A}{p+n}$, $\frac{A}{p+2n}$, &c. Dico quod aggregatum quotlibet hujusmodi Serierum in infinitum continuatarum in terminis numero finitis exhiberi potest, si modo fuerit numeratorum A, B, C, D, &c. aggregatum æquale nihilo. Duobus exemplis hoc fiet manifestum.

Ex. Sint duæ tantum fractiones $\frac{A}{x}$, atque $\frac{-A}{x+3n}$, existente $b=3n$. Scribantur Series harmonicæ ex his formulis ortæ, eo ordine, ut termini, in quibus sunt denominatores æquales, sibi invicem respondeant, & collectis summis terminorum homologorum, prodibit aggregatum Serierum in terminis numero finitis, ut in calculo apposito videre est.

$$\frac{A}{p}+\frac{A}{p+n}+\frac{A}{p+2n}+\frac{A}{p+3n}+\frac{A}{p+4n}+\&c. = \text{Seriei ortæ ex } \frac{A}{x}$$

$$+\frac{-A}{p+3n}+\frac{-A}{p+4n}+\&c. = \text{Seriei ex } \frac{-A}{x+3n}$$

$$\frac{A}{p}+\frac{A}{p+n}+\frac{A}{p+2n}+ \ 0 \ + \ 0 \ +\&c. = \text{Aggreg. Serierum.}$$

Ex. 2. Sint tres fractiones $\frac{A}{x}$, $\frac{B}{x+2n}$, $\frac{C}{x+3n}$, existentibus $b=2n$, $c=3n$, atque $A+B+C=0$. In hoc casu Calculus sic se habet.

$$\frac{A}{p}+\frac{A}{p+n}+\frac{A}{p+2n}+\frac{A}{p+3n}+\ldots+\&c. = \text{Seriei ortæ ex } \frac{A}{x}$$

$$+\frac{B}{p+2n}+\frac{B}{p+3n}+\ldots\ldots+\&c. = \text{Seriei ex } \frac{B}{x+2n}$$

$$+\frac{C}{p+3n}+\ldots\ldots+\&c. = \text{Seriei ex } \frac{C}{x+3n}$$

$$\frac{A}{p}+\frac{A}{p+n}+\frac{A+B}{p+2n}+\frac{A+B+C=0}{p+3n}+\&c. = \text{Aggregato Serierum.}$$

Ubi etiam prodit aggregatum Serierum in terminis numero finitis, nempe $\frac{A}{p}+\frac{A}{p+n}+\frac{A+B}{p+2n}$, ob Numeratorum A, B, C, aggregatum æquale nihilo. Et ad eundem modum demonstratur Theorema in aliis casibus quibusvis.

Cor. 1. Ex his principiis derivari possunt innumeræ Series in infinitum continuatæ, in terminis tamen numero finitis summabiles.

Cas. 1. Sint $\frac{A}{x}$ & $\frac{-A}{x+b}$ formulæ duarum Serierum harmonicarum, quarum aggregatum prodit in terminis numero finitis per superius demonstrata. Tum, formulis istis in unam summam collectis, fit $\frac{Ab}{x \times \overline{x+b}}$ formula Seriei summabilis. Sint *v. gr.* $A=\frac{1}{6}$, $p=1$, $n=2$, atque $b=3n=6$. Tum formulæ Serierum harmonicarum erunt $\frac{1}{6x}$, & $\frac{-1}{6\times\overline{x+6}}$, formula

mula Seriei compositæ summabilis erit $\frac{1}{x \times \overline{x+6}}$; Serie illa existente $\frac{1}{1 \times 7} + \frac{1}{3 \times 9} + \frac{1}{5 \times 11} + \frac{1}{7 \times 13} +$ &c. atque summa Seriei, per calculum in præmissis demonstratum, erit $\frac{1}{6 \times 1} + \frac{1}{6 \times 3} + \frac{1}{6 \times 5}$. Sint tres formulæ Serierum harmonicarum $\frac{A}{x}$, $\frac{B}{x+b}$, $\frac{C}{x+c}$ (existente $A + B + C = 0$, ut sit Serierum aggregatum finitum per præmissa.) Tum formulis in unam summam collectis fit

$$\frac{A \times \overline{x+b} \times \overline{x+c} + B \times x \times \overline{x+c} + C \times x \times \overline{x+b}}{x \times \overline{x+b} \times \overline{x+c}},$$

seu (terminis revocatis ad formam factorum x, $x \times \overline{x+b}$, $x \times \overline{x+b} \times \overline{x+c}$,)

$$\frac{Acb + \overline{Ac + \overline{c-b}\,B} \times x + \overline{A+B+C} \times x \times \overline{x+b}}{x \times \overline{x+b} \times \overline{x+c}},$$

hoc est (ob $A + B + C = 0$) $\frac{Acb + \overline{Ac + B \times \overline{c-b}} \times x}{x \times \overline{x+b} \times \overline{x+c}}$, formula Seriei summabilis. Si quatuor sint Fractiones $\frac{A}{x}$, $\frac{B}{x+b}$, $\frac{C}{x+c}$, $\frac{D}{x+d}$, (existente $A + B + C + D = 0$) ad eundem modum invenietur formula Seriei summabilis

$$\frac{Abcd + \overline{Acd + B \times \overline{c-b} \times \overline{d-b}}\,| \times x + \overline{Ad + B \times \overline{d-b} + C \times \overline{d-c}}\,| \times x \times \overline{x+b}}{x \times \overline{x+b} \times \overline{x+c} \times \overline{x+d}}.$$

Et sic pergere licet ad formulas adhuc magis compositas.

Cas. 2. Et si plures sint formulæ Serierum hujusmodi summabilium, quarum denominatorum factores excerpantur ex diversis progressionibus Arithmeticis, ex istarum formularum quotvis in unam summam additione, conficietur formula nova Seriei summabilis. Sint *e. gr.* formulæ duæ Serierum summabilium $\frac{1}{x \times \overline{x+3}}$ & $\frac{1}{z \times \overline{z+2}}$, excerptis x ex Progressione Arithmetica 1, 2, 3, 4, &c. z ex Progressione Arithmetica 1, 3, 5, &c. Tum ex his formulis in unam summam collectis fiet formula nova $\frac{z \times \overline{z+2} + x \times \overline{x+3}}{x \times \overline{x+3} \times z \times \overline{z+2}}$, vel (exposito z per x & numeros datos)

$$\frac{\overline{2x-1} \times \overline{2x+1} + x \times \overline{x+3}}{x \times \overline{x+3} \times \overline{2x-1} \times \overline{2x+1}}.$$

Cor. 2. Hinc omnis Series in infinitum continuata summabilis est, cujus termini designantur per Fractionem, cujus denominatoris factores excerpuntur ex data qualibet Progressione Arithmetica, numerator autem est multinomium, cujus dimensiones sunt ad minimum binario pauciores, quam sunt dimensiones Denominatoris. Nam omnis hujusmodi fractio resolvi potest in tot fractiones simplices, quot sunt dimensiones (hoc est, quot sunt factores) Denominatoris, quarum numeratorum aggregatum est nihil. Sit exempli gratia, formula oblata $\frac{\alpha + \beta x + \gamma x \times \overline{x+b}}{x \times \overline{x+b} \times \overline{x+c} \times \overline{x+d}}$. Pone

 hanc

hanc formulam æquari aggregato fractionum $\frac{A}{x} + \frac{B}{x+b} + \frac{C}{x+c} + \frac{D}{x+d}$. Tum fractionibus istis in unam summam collectis fiet $\overline{Abcd + \overline{Acd + B\,\overline{c-b} \times \overline{d-b}} \times x + \overline{Ad + B \times \overline{d-b} + C \times \overline{d-c}} \times x \times \overline{x+b}} + \overline{A+B+C+D} \times x \times \overline{x+b} \times \overline{x+c}$ applicatum ad $x \times \overline{x+b} \times \overline{x+c} \times \overline{x+d} = \frac{\alpha + \beta x + \gamma x \times \overline{x+b}}{x \times \overline{x+b} \times \overline{x+c} \times \overline{x+d}}$.

Unde per comparationem terminorum homologorum fit $Abcd = \alpha$, $Acd + B \times \overline{c-b} \times \overline{d-b} = \beta$, $Ad + B \times \overline{d-b} + C \times \overline{d-c} = \gamma$, $A + B + C + D = 0$, adeoque $A = \frac{\alpha}{bcd}$, $B = \frac{\beta - Acd}{\overline{c-b} \times \overline{d-b}}$, $C = \frac{\gamma - Ad - B \times \overline{d-b}}{d-c}$, $D = -A - B - C$. Quo pacto formula oblata resolvitur in fractiones simplices $\frac{\alpha}{bcdx} + \frac{\beta - Acd}{\overline{c-b} \times \overline{d-b} \times \overline{x+b}} + \frac{\gamma - Ad - B \times \overline{d-b}}{\overline{d-c} \times \overline{x+c}} + \frac{-A-B-C}{x+d}$, ex quibus ortarum Serierum aggregatum, hoc est, summa Seriei ortæ ex formula oblata $\frac{\alpha + \beta x + \gamma x \times \overline{x+b}}{x \times \overline{x+b} \times \overline{x+c} \times \overline{x+d}}$, per jam dicta prodit in terminis numero finitis. Quod vero dimensiones numeratoris in formula oblata debeant esse binario ad minimum pauciores, quam sunt dimensiones Denominatoris, hinc constat, quod in reductione fractionum $\frac{A}{x}$, $\frac{B}{x+b}$, $\frac{C}{x+c}$, $\frac{D}{x+d}$, quilibet numerator A, B, C, D, ducitur in omnes denominatores excepto uno, nempe suo; unde prodeunt Numeratoris Dimensiones unitate pauciores quam sunt dimensiones Denominatoris. Sed per æquationem $A + B + C + D = 0$ perit altissima dimensio in numeratore; Unde supersunt Numeratoris Dimensiones ad minimum binario pauciores quam sunt dimensiones Denominatoris. Ad hoc vero Corollarium revocari possunt D. de *Monmort* Propositiones 2[da] & 5[ta].

Cor. 3. Item oblata formula juxta *Cas.* 2. *Cor.* 1. adhuc magis composita, ex iisdem principiis perspici potest an sit Series summabilis. Sint progressiones duæ Arithmeticæ 1, 3, 5, &c. 2, 4, 6, &c. quarum termini homologi designentur per x & z, & sit formula Seriei oblata $\frac{\alpha + \beta x + \gamma x^2}{x \times \overline{x+2} \times z \times \overline{z+2}}$, vel (pro z scripto $x+1$, & factoribus Denominatoris in ordinem coactis) $\frac{\alpha + \beta x + \gamma x^2}{x \times \overline{x+1} \times \overline{x+2} \times \overline{x+3}}$. Pone formulam hanc æquari aggregato formularum $\frac{P}{x \times \overline{x+2}}$, $\frac{Q}{\overline{x+1} \times \overline{x+3}}$, Serierum per supe-

superius dicta summabilium, ut (formulis his novissimis in unam summam collectis) sit $\frac{P \times \overline{x+1} \times \overline{x+3} + Q \times x \times \overline{x+2}}{x \times \overline{x+1} \times \overline{x+2} \times \overline{x+3}}$ seu $\frac{3P + \overline{4P+2Q}\,x + \overline{P+Q}\,x^2}{x \times \overline{x+1} \times \overline{x+2} \times \overline{x+3}} = \frac{\alpha + \beta x + \gamma x^2}{x \times \overline{x+1} \times \overline{x+2} \times \overline{x+3}}$. Hinc comparando terminos homologos oriuntur æquationes $3P = \alpha$, $4P + 2Q = \beta$, $P + Q = \gamma$. Unde eliminatis P & Q per debitas operationes Analyticas, prodit æquatio $2\alpha - 3\beta + \gamma = 0$, qua definitur relatio quæ inter coefficientes α, β, γ intercedere debet, ut Series orta ex formula oblata $\frac{\alpha + \beta x + \gamma x^2}{x \times \overline{x+1} \times \overline{x+2} \times \overline{x+3}}$ sit summabilis. Ad eundem modum si formulæ oblatæ Denominatoris factores excerpantur ex tribus Progressionibus Arithmeticis, invenientur duæ æquationes quibus definiantur relationes coefficientium Numeratoris, ut sit Series summabilis. Si quatuor sint Progressiones Arithmeticæ, Coefficientium relatio definietur per tres æquationes. Et sic porro. Et in hujusmodi formulis ut sint Series summabiles, hæc insuper observanda sunt, Primo ut Numeratorum dimensiones sint ad minimum binario pauciores quam sunt dimensiones Denominatorum. Deinde ut ex singulis Progressionibus Arithmeticis excerpantur ad minimum duo factores Denominatoris. Denique, quod si sint duo vel plures factores Denominatoris inter se æquales, ponendum sit tot etiam Progressiones Arithmeticas, ex quibus excerpuntur, esse inter se æquales. Præmissis attentius perpensis, hæc obvia erunt. Ad hoc vero Corollarium facile revocantur D. de *Monmort* Propositiones 3^{tia} & 4^{ta}.

XXII. Arithmeticæ pars præcipua consistit in invenienda in numeris quantitate quacunque determinata; cum vero quantitatum & numerorum natura non patiatur ut omnes quantitates exhibeantur in numeris accurate, necesse habemus ad Approximationes confugere. Hoc est ubi quantitatum valores mathematice accurati nequeunt obtineri, quærenda sunt ii qui ab accuratis distant minus data quavis differentia.

Sir Isaac Newton's *Differential Method illustrated, by Mr.* James Sterling. *n.* 362 *p.* 1050.

Quicquid hac de re a Veteribus ad nos pervenit, vel est particulare, ut Methodus eorum reducendi Æquationes Quadraticas; vel saltem usibus generalibus male destinatum, ut Methodus Exhaustionum. *Vieta* quidem primus erat qui aliquid generale in hac arte assequutus est: quippe invenit methodum reducendi Æquationes Rationales, quæ solæ tunc in usu erant. In hac acquievere omnes Geometræ ex ejus temporibus usque ad ea *Newtoni*. Hic ex Interpolationibus primo pervenit ad Series: quas postea ad reductionem Æquationum omnium omnino generum universaliter applicuit. Hæc autem methodus procedit per quantitatum nascentium & evanescentium rationes primas & ultimas, seu si ita loqui liceat, per quantitatum coincidentium differentias infinite parvas. Sed & ulterius promovit *Newtonus* hanc methodum; docuitque qua ratione approximandum sit ad quantitates quæ determinantur per regularem seriem termino-

 rum,

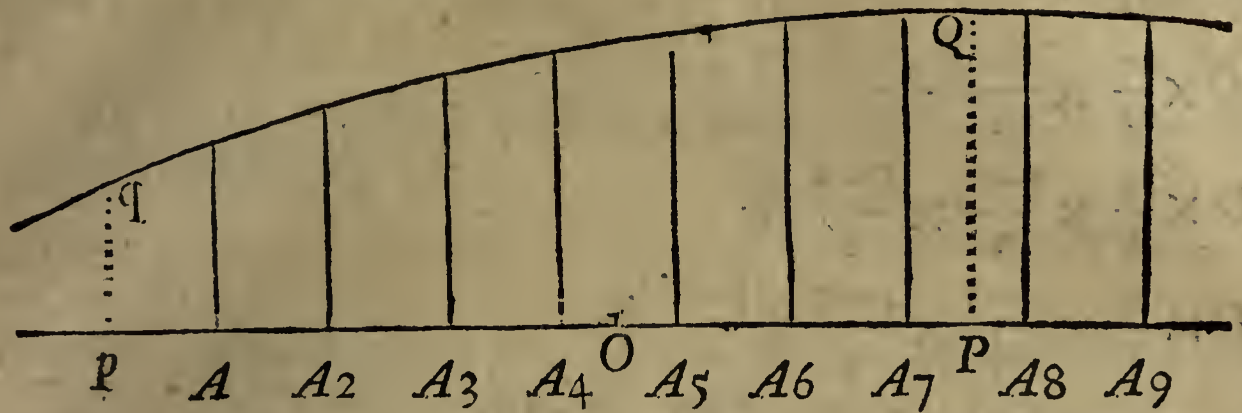
rum, non per Æquationem ut vulgo fit. Atque sic posuit fundamenta calculi hujus Differentialis, qui procedit per quantitatum differentias cujuscunque magnitudinis; ideoque est methodo Serierum universalior. Per hasce artes *Newtonianas* universa doctrina Approximationum reducitur ad solutionem Problematis, *Invenire Lineam Geometricam quæ per data quotcunque puncta transibit.* Ex hujus inquam solutione inveniuntur radices Æquationum quarumcunque, & etiam quantitates quarum relationes ad alias datas per nullas Æquationes hactenus notas possunt exprimi. Existimo igitur *Newtonum* perduxisse methodum Approximandi ad summum perfectionis fastigium; dum ex unico simplicissimo principio totam hanc doctrinam longe lateque patentem deducit.

Author noster, in Epistola ad *Oldenburgum, Octob.* 24. 1676. data, mentionem fecit de methodo expedita ducendi Lineam Parabolicam per data quotcunque puncta; qua dixit se usum fuisse ubi Series simplices non sunt satis tractabiles. Et hanc methodum primo publicavit in Lemmate quinto Libri tertii *Principiorum.* Atque in Lectionibus publicis, circa idem tempus quo dicta Epistola scripta est, *Cantabrigiæ* habitis, exposuit modum generalem determinandi Curvas cujuscunque generis quæ transibunt per totidem data puncta quot earum natura patitur. Hæ Lectiones sub titulo *Arithmeticæ Universalis* anno 1707. publicatæ sunt, ubi habetur methodus exemplis illustrata in sectionibus Conicis. Anno vero 1711. tandem prodiit, inter alios ejusdem Authoris tractatus, ipsa Methodus Differentialis plenius quam ante exposita, cum fundamento ejus demonstrato.

Archimedes in methodo Exhaustionum, *Cavallerius* in methodo Indivisibilium, & *Wallisius* noster in Arithmetica infinitorum, posuerunt fundamenta doctrinæ de determinanda quantitate quæsita per locum quem obtinet inter terminos in data Serie: at qua ratione approximandum esset ad valores quantitatum sic determinatarum, horum nemo docuit; Hoc primus & solus perfecit *Newtonus*: atque exinde haud parum ampliata est universa Analysis. Nam sicut ante hoc inventum, ea Problemata Arithmetica sola pro solutis habebantur, ubi relatio quantitatis quæsitæ ad alias datas definiebatur Æquatione, jam pro solutis habenda sunt non minus ea, in quibus quantitas quæsita locum datum sortitur inter terminos datæ Seriei; siquidem numeri desiderati non minus accurate obtinentur per Methodum Differentialem, quam per extractionem Radicum: hisce vero habitis, parum interest quomodo ad eos deventum est. Et experientia multiplex docuit, quod plurima Problemata ad Æquationes ægre deducuntur, dum ad methodum Differentialem facillime. Qualis est ex multis aliis toties decantata Circuli Quadratura; quam tam perfectam, mea opinione, *Wallisius* in Arithmetica Infinitorum exhibuit quam *Archimedes* illam Parabolæ.

Pro-

Propositio. *Invenire Lineam Parabolicam quæ transibit per extrema Ordinatorum quotcunque æquidistantium.*



B *B*2 *B*3 *B*4 *B*5 *B*6 *B*7 *B*8

C *C*2 *C*3 *C*4 *C*5 *C*6 *C*7

D *D*2 *D*3 *D*4 *D*5 *D*6

E *E*2 *E*3 *E*4 *E*5

F *F*2 *F*3 *F*4

G *G*2 *G*3

H *H*2

I

Casus Primus. Designent *A*, *A*2, *A*3, *A*4, *A*5, *A*6, *A*7, *A*8, *A*9, &c. Ordinatas æquidistantes insistentes Abscissæ in dato angulo. Collige earum differentias *B*, *B*2, *B*3, *B*4, *B*5, *B*6, *B*7, *B*8, &c. harumque differentias *C*, *C*2, *C*3, *C*4, *C*5, *C*6, *C*7, &c. harumque differentias *D*, *D*2, *D*3, *D*4, *D*5, *D*6, &c. harumque differentias *E*, *E*2, *E*3, *E*4, *E*5, &c. harumque *F*, *F*2, *F*3, *F*4, &c. Et sic porro. Differentiæ autem colligi debent auferendo priores semper de posterioribus. Hoc est ponendo $B = A2 - A$, $B2 = A3 - A2$, $B3 = A4 - A3$, $B4 = A5 - A4$, $B5 = A6 - A5$, &c. Tum $C = B2 - B$, $C2 = B3 - B2$, $C3 = B4 - B3$, $C4 = B5 - B4$, &c. deinde $D = C2 - C$, $D2 = C3 - C2$, $D3 = C4 - C3$, &c. Et similiter sunt omnes differentiæ sequentes colligendæ. Vel sint α, β, γ, δ, ε, ζ, η, &c. æquales *A*, *A*2, *A*3, *A*4, *A*5, *A*6, *A*7, &c. Eritque $A = \alpha$, $B = \beta - \alpha$, $C = \gamma - 2\beta + \alpha$, $D = \delta - 3\gamma + 3\beta - \alpha$, $E = \varepsilon - 4\delta + 6\gamma - 4\beta + \alpha$, $F = \zeta - 5\varepsilon + 10\delta - 10\gamma + 5\beta - \alpha$, $G = \eta - 6\zeta + 15\varepsilon - 20\delta + 15\gamma - 6\beta + \alpha$, &c. In hisce valoribus numerales Coefficientes ipsorum $\alpha, \beta, \gamma, \delta, \varepsilon$, &c. generantur ut in dignitatibus integris Binomii $\overline{1-z}|^{0}$, $\overline{1-z}|^{1}$, $\overline{1-z}|^{2}$, $\overline{1-z}|^{3}$, $\overline{1-z}|^{4}$, &c. Scribendo numeros 1, 2, 3, 4, 5, &c. in Serie $1 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} \times \frac{n-4}{5} \times$ &c. successive pro n. Sit jam *PQ* quælibet Ordinata reliquis intermedia, & *AP* ejus distantia ab Ordinata prima *A* appelletur z, tum erit

PQ

$$PQ = A +$$
$$B \times \frac{z}{1} +$$
$$C \times \frac{z}{1} \times \frac{z-1}{2} +$$
$$D \times \frac{z}{1} \times \frac{z-1}{2} \times \frac{z-2}{3} +$$
$$E \times \frac{z}{1} \times \frac{z-1}{2} \times \frac{z-2}{3} \times \frac{z-3}{4} +$$
$$F \times \frac{z}{1} \times \frac{z-1}{2} \times \frac{z-2}{3} \times \frac{z-3}{4} \times \frac{z-4}{5} +$$
$$G \times \frac{z}{1} \times \frac{z-1}{2} \times \frac{z-2}{3} \times \frac{z-3}{4} \times \frac{z-4}{5} \times \frac{z-5}{6} + \&c.$$

Adeoque ſignum ipſius z mutandum eſt, quando PQ cadit ad alteras partes Ordinatæ primæ, ut pq.

Caſus Secundus. Sit jam $A5$ Ordinata in medio omnium; pone $A = B4 + B5$, $B = D3 + D4$, $C = F2 + F3$, $D = H + H2$, *&c.* & $a = C4$, $b = E3$, $c = G2$, $d = I$, *&c.* id eſt, ſi ſint $A6 = \alpha$, $A7 = \beta$, $A8 = \gamma$, $A9 = \delta$, *&c.* $A4 = \varkappa$, $A3 = \lambda$, $A2 = \mu$, $A = \nu$, *&c.* Pone $A = \alpha - \varkappa$, $B = \beta - 2\alpha + 2\varkappa - \lambda$, $C = \gamma - 4\beta + 5\alpha - 5\varkappa + 4\lambda - \mu$, $D = \delta - 6\gamma + 14\beta - 14\alpha + 14\varkappa - 14\lambda + 6\mu - \nu$, *&c.* $a = \alpha - 2\,A5 + \varkappa$, $b = \beta - 4\alpha + 6\,A5 - 4\varkappa + \lambda$, $c = \gamma - 6\beta + 15\alpha - 20\,A5 + 15\varkappa - 6\lambda + \mu$, $d = \delta - 8\gamma + 28\beta - 56\alpha + 70\,A5 - 56\varkappa + 28\lambda - 8\mu + \nu$, *&c.* Et dicatur $A5\,P$, z, tum erit

$$PQ = A5 + \frac{Az + azz}{1.2} +$$
$$\frac{2Bz + bzz}{1.2} \times \frac{zz-1}{3.4} +$$
$$\frac{3Cz + czz}{1.2} \times \frac{zz-1}{3.4} \times \frac{zz-4}{5.6} +$$
$$\frac{4Dz + dzz}{1.2} \times \frac{zz-1}{3.4} \times \frac{zz-4}{5.6} \times \frac{zz-9}{7.8} +$$
$$\frac{5Ez + ezz}{1.2} \times \frac{zz-1}{3.4} \times \frac{zz-4}{5.6} \times \frac{zz-9}{7.8} \times \frac{zz-16}{9.10} + \&c.$$

Caſus Tertius. Sint jam $A4$, $A5$, Ordinatæ duæ in medio omnium. Pone $A = \frac{A4 + A5}{2}$, $B = \frac{C3 + C4}{2}$, $C = \frac{E2 + E3}{2}$, $D = \frac{G + G2}{2}$, &c. $a = B4$, $b = D3$, $c = F2$, $d = H$, &c. Vel ſint $A5 = \alpha$, $A6 = \beta$, $A7 = \gamma$, $A8 = \delta$, &c. $A4 = \varkappa$, $A3 = \lambda$, $A2 = \mu$, $A = \nu$, &c.

Dein-

Deinde erunt $2A = \alpha + \varkappa$, $2B = \beta - \alpha - \varkappa + \lambda$, $2C = \gamma - 3\beta + 2\alpha + 2\varkappa - 3\lambda + \mu$, $2D = \delta - 5\gamma + 9\beta - 5\alpha - 5\varkappa + 9\lambda - 5\mu + \nu$, &c. Et $a = \alpha - \varkappa$, $b = \beta - 3\alpha + 3\varkappa - \lambda$, $c = \gamma - 5\beta + 10\alpha - 10\varkappa + 5\lambda - \mu$, $d = \delta - 7\gamma + 21\beta - 35\alpha + 35\varkappa - 21\lambda + 7\mu - \nu$, &c. Et sit O punctum medium inter $A4$, $A5$, atque appelletur OP, z; eritque Ordinata

$$PQ = \frac{A + az}{4^0} +$$
$$\frac{3B + bz}{4^1} \times \frac{4zz - 1}{2 \cdot 3} +$$
$$\frac{5C + cz}{4^2} \times \frac{4zz - 1}{2 \cdot 3} \times \frac{4zz - 9}{4 \cdot 5} +$$
$$\frac{7D + dz}{4^3} \times \frac{4zz - 1}{2 \cdot 3} \times \frac{4zz - 9}{4 \cdot 5} \times \frac{4zz - 25}{6 \cdot 7} +$$
$$\frac{9E + ez}{4^4} \times \frac{4zz - 1}{2 \cdot 3} \times \frac{4zz - 9}{4 \cdot 5} \times \frac{4zz - 25}{6 \cdot 7} \times \frac{4zz - 49}{8 \cdot 9} + \&c.$$

In hisce duobus etiam casibus z est negativa, quando Ordinata PQ cadit ad alteras partes initii Abscissæ. Et in omnibus tribus casibus distantia communis Ordinatarum ponitur unitas.

Omnes tres casus demonstrantur facillime per calculum. In casu primo pro PQ scribo successive α, β, γ, δ, ε, &c. & pro z interea 0, 1, 2, 3, 4, *&c.* quæ sunt longitudines Abscissæ ordine sequentes; & provenient æquationes,

$\alpha = A$, $\beta = A + B$, $\gamma = A + 2B + C$, $\delta = A + 3B + 3C + D$, $\varepsilon = A + 4B + 6C + 4D + E$, &c.

$\beta - \alpha = B$, $\gamma - \beta = B + C$, $\delta - \gamma = B + 2C + D$, $\varepsilon - \delta = B + 3C + 3D + E$, &c.

$\gamma - 2\beta + \alpha = C$, $\delta - 2\gamma + \beta = C + D$, $\varepsilon - 2\delta + \gamma = C + 2D + E$, &c.

$\delta - 3\gamma + 3\beta - \alpha = D$, $\varepsilon - 3\delta + 3\gamma - \beta = D + E$, &c.

$\varepsilon - 4\delta + 6\gamma - 4\beta + \alpha = E$, &c.

Hæ Æquationes, capiendo earum differentias, nullo labore resolvuntur, uti videre est. Et dant eosdem ipsorum A, B, C, D, *&c.* valores, qui antea positi sunt in solutione. Et ad eundem modum demonstrantur casus duo reliqui.

Harum trium serierum unaquæque converget ad valorem Ordinatæ PQ, ubi Ordinatarum datarum differentiæ sunt justæ magnitudinis. At ubi non convergunt, aliæ artes adhibendæ sunt. Sed impræsentiarum de hujus Propositionis usu pauca adjiciamus.

Designent α, β, γ, δ, ε, ζ, η, θ, $\varkappa$, λ, &c. terminos quoscunque æquidistantes, quorum differentiæ sunt perexiguæ; & relationes, quas inter se obtinent, definiuntur quamproxime per Æquationes sequentes, quæ oriuntur

tur capiendo differentias & differentias differentiarum continuo, & ponendo eas æquales nihilo.

$$\alpha - \beta = 0$$
$$\alpha - 2\beta + \gamma = 0$$
$$\alpha - 3\beta + 3\gamma - \delta = 0$$
$$\alpha - 4\beta + 6\gamma - 4\delta + \varepsilon = 0$$
$$\alpha - 5\beta + 10\gamma - 10\delta + 5\varepsilon - \zeta = 0$$
$$\alpha - 6\beta + 15\gamma - 20\delta + 15\varepsilon - 6\zeta + \eta = 0$$
$$\alpha - 7\beta + 21\gamma - 35\delta + 35\varepsilon - 21\zeta + 7\eta - \theta = 0$$
$$\alpha - 8\beta + 28\gamma - 56\delta + 70\varepsilon - 56\zeta + 28\eta - 8\theta + \kappa = 0$$
$$\alpha - 9\beta + 36\gamma - 84\delta + 126\varepsilon - 126\zeta + 84\eta - 36\theta + 9\kappa - \lambda = 0.$$

&c.

Hæc Tabula in uſum reſervanda eſt, ut conſulatur quoties opus ſit. Quod autem hæ Æquationes vel obtinent accurate, vel ad verum approximant, ubi differentiæ terminorum ſunt parvæ, patet ex demonſtratione caſus primi Propoſitionis.

Aſſumatur quælibet Series $\frac{1}{101}$, $\frac{1}{102}$, $\frac{1}{103}$, $\frac{1}{104}$, $\frac{1}{105}$, $\frac{1}{106}$, &c. Et quæratur terminus qui ſtat proximus ante $\frac{1}{101}$: patet quod ille eſt $\frac{1}{100}$; videamus ergo qualem hæc methodus exhibebit eundem. Repræſentet α terminum quæſitum, eritque

$$\begin{array}{l} \frac{1}{101} = \beta = 0099,0099,0099,0, \\ \frac{1}{102} = \gamma = 0098,0392,1568,7, \\ \frac{1}{103} = \delta = 0097,0873,7864,1, \\ \frac{1}{104} = \varepsilon = 0096,1538,4615,4, \\ \frac{1}{105} = \zeta = 0095,2380,9523,8, \\ \frac{1}{106} = \eta = 0094,3396,2264,2, \end{array} \quad \text{Æquatio} \left\{ \begin{array}{l} \text{1ma} \\ \text{2da} \\ \text{3tia} \\ \text{4ta} \\ \text{5ta} \\ \text{6ta} \end{array} \right\} \text{dat } \alpha \left\{ \begin{array}{l} 0099,0099,0099,0, \\ 0099,9805,8629,3, \\ 0099,9994,3455,0, \\ 0099,9999,7824,8, \\ 0099,9999,9895,8, \\ 0099,9999,9993,1, \end{array} \right.$$

Patet ergo quod hæc methodus continue approximat. Si terminorum differentiæ fuiſſent minores, valores acceſſiſſent citius ad verum, & contra tardius quando differentiæ ſunt majores. Hinc ſi in Tabulis numericis deſit terminus, poteſt is per hanc methodum inſeri.

Hoc modo etiam prodeunt ipſiſſimæ Series Specioſæ, quæ per alias methodos prodire ſolent. Proponatur $\overline{1 + zz}|^{-1}$ Ordinata Curvæ quadrandæ: Ea eſt prima in ſerie regulari $\overline{1 + zz}|^{-1}$, $\overline{1 + zz}|^{0}$, $\overline{1 + zz}|^{1}$, $\overline{1 + zz}|^{2}$, $\overline{1 + zz}|^{3}$, &c. Ordinatarum, quæ omnes præter primam dant ſuas areas z, $z + \frac{1}{3}z^3$, $z + \frac{2}{3}z^3 + \frac{1}{5}z^5$, $z + \frac{3}{3}z^3 + \frac{3}{5}z^5 + \frac{1}{7}z^7$, *&c.* conſtituentes novam ſeriem cujus primus terminus erit Area quæſita: quæ ideo invenietur ponendo pro ea α, & pro reliquis in ſuo Ordine β, γ, δ, ε, &c. Prima Æquatio dat $\alpha = z$, ſecunda $\alpha = z - \frac{1}{3}z^3$, tertia $\alpha = z - \frac{1}{3}z^3 + \frac{1}{5}z^5$, quarta $\alpha = z - \frac{1}{3}z^3 + \frac{1}{5}z^5 - \frac{1}{7}z^7$, &c. Eſt ergo univerſim area quæſita $z - \frac{1}{3}z^3 + \frac{1}{5}z^5 - \frac{1}{7}z^7 + \frac{1}{9}z^9 - \frac{1}{11}z^{11}$, &c. Eſtque hæc Series arcus ad Tangentem z, in circulo radium habente unitati æqualem. Eam invenit *Jacobus Gregorius* noſter, & cum *Collinio* communicavit initio anni 1671. a quo, mediante *Oldenburgo* ad *Leibnitium* delata eſt.

Sit

Sit jam &c. $e, d, c, b, a, P, \alpha, \beta, \gamma, \delta, \epsilon$, &c. Series utrinque excurrens in infinitum, ubi dantur omnes termini præter P in medio omnium. Sit $A = \alpha + a$, $B = \beta + b$, $C = \gamma + c$, $D = \delta + d$, $E = \epsilon + e$, &c. atque erit

$$P = \frac{A}{2} + \frac{A-B}{6} + \frac{5A-8B+3C}{60} + \frac{7A-14B+9C-2D}{140} + \frac{42A-96B+81C-32D+5E}{1260} + \frac{66A-165B+165C-88D+25E-3F}{2772} + \frac{429A-1144B+1287C-832D+325E-72F+7G}{24024} + \text{\&c.}$$

Investigatur hæc Series ex Æquationibus, excerpendo alternas in quibus numerus terminorum est impar. Nam earum differentiæ relinquent terminos in hac Serie; quæ itaque ad libitum produci potest.

Sit $\overline{1+z}|^{-1}$ Ordinata Hyperbolæ, & quæratur Area ejus quæ jacet supra Abscissam z, quando ea evadit unitas. Hæc Ordinata est media in Serie Ordinatarum, &c. $\overline{1+z}|^{-5}$, $\overline{1+z}|^{-4}$, $\overline{1+z}|^{-3}$, $\overline{1+z}|^{-2}$, $\overline{1+z}|^{-1}$, $\overline{1+z}|^{0}$, $\overline{1+z}|^{1}$, $\overline{1+z}|^{2}$, $\overline{1+z}|^{3}$ &c. æquidistantium, hinc inde excurrente in infinitum. Adeoque Areæ ab hisce Ordinatis genitæ constituent seriem consimilem, cujus medius terminus erit Area quæsita; quæ proinde obtinebitur per Seriem modo expositam. Quando z est unitas, ut in casu præsente, areæ curvarum evadunt, &c. $\frac{15}{64}, \frac{7}{24}, \frac{3}{8}, \frac{1}{2}$, & $1, \frac{3}{2}, \frac{7}{3}, \frac{15}{4}$, &c. Hinc est $A = 1 + \frac{1}{2} = \frac{3}{2}$, $B = \frac{3}{2} + \frac{3}{8} = \frac{15}{8}$, $C = \frac{7}{3} + \frac{7}{24} = \frac{21}{8}$, $D = \frac{15}{4} + \frac{15}{64} = \frac{255}{64}$, &c. Hisce in Serie substitutis, prodit P, id est, area Hyperbolæ, $\frac{3}{4} - \frac{3}{48} + \frac{3}{480} - \frac{3}{4480} +$ &c. id est, $\frac{3}{4} - \frac{A}{4 \cdot 3} - \frac{2B}{4 \cdot 5} - \frac{3C}{4 \cdot 7} - \frac{4D}{4 \cdot 9} - \frac{5E}{4 \cdot 11} -$ &c. Ubi jam A, B, C, D, &c. more *Newtoniano*, designant terminos in suo ordine ab initio. Calculum appono.

TERMINI.

Affirmativi	*Negativi.*
7500,0000,0000,0000,0	0625,0000,0000,0000,0
62,5000,0000,0000,0	6,6964,2857,1428,5
7440,4761,9047,6	845,5086,5800,8
97,5586,9130,8	11,3818,4731,9
1,3390,4086,1	1585,7062,8
188,7745,5	22,5708,7
2,7085,0	3260,2
393,4	47,5
5,7	7
+7563,2539,3930,7494,1	−0631,7821,3370,8041,1

Summam negativam ſubducens ab affirmativa, habeo pro Area, id eſt, pro Logarithmo Hyperbolico Binarii, numerum 6931,4718,0559,9453.

Pro conſtructione Tabularum quarumvis numericarum percommoda eſt Series quæ ſequitur. Deſignent &c. $e, d, c, b, a, \alpha, \beta, \gamma, \delta, \epsilon$, &c. terminos alternos in Serie utrinque ſerpente in infinitum; Pone $A = \alpha + a$, $B = \beta + b$, $C = \gamma + c$, $D = \delta + d$, $E = \epsilon + e$, &c. Et terminus inter α & a erit

$$\frac{A}{2} +$$

$$\frac{1}{1} \times \frac{A - B}{2^4} +$$

$$\frac{1 \cdot 3}{1 \cdot 2} \times \frac{2A - 3B + C}{2^7} +$$

$$\frac{1 \cdot 3 \cdot 5}{1 \cdot 2 \cdot 3} \times \frac{5A - 9B + 5C - D}{2^{10}} +$$

$$\frac{1 \cdot 3 \cdot 5 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4} \times \frac{14A - 28B + 20C - 7D + E}{2^{13}} +$$

$$\frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \times \frac{42A - 90B + 75C - 35D + 9E - F}{2^{16}} +$$

$$\frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} \times \frac{132A - 297B + 275C - 154D + 44E - 11F + G}{2^{19}} + \text{\&c.}$$

Hæc Series ſequitur ex caſu tertio Propoſitionis, ponendo $z = 0$. Coefficientes numerales literarum ſic producuntur, exempli gratia, in quarto termino coefficiens literæ penultimæ C eſt 5; pone $5 + 1 = n$, & numeri qui proveniunt ex multiplicatione terminorum $1 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} \times \frac{n-4}{5} \times$ &c. erunt 1, 6, 15, 20, &c. Horum differentiæ 5, 9, 5, ſunt numeri quæſiti. Atque adeo Series ad libitum produci poteſt.

Datis

Datis Logarithmis numerorum 46, 48, 50, 52, 54, 56, 58 & 60; invenire Logarithmum numeri 53 qui consistit in medio omnium. Pone $l\,52 + l\,54 = A = 3{,}4483{,}9710{,}34$, $l\,50 + l\,56 = B = 3{,}4471{,}5803{,}13$, $l\,48 + l\,58 = C = 3{,}4446{,}6923{,}08$, $l\,46 + l\,60 = D = 3{,}4409{,}0908{,}19$. Hisce valoribus in Serie scriptis, primi quatuor termini dabunt 1,7242, 2586,96 pro Logarithmo numeri 53. Et eadem ratione invenire licet quemvis alium intermedium.

In Constructione ergo Tabularum sufficit primo quærere aliquos terminos in debitis distantiis, nam reliqui possunt hoc modo interseri. Etenim continuo sunt intercalandi termini primo inventi, usque dum perventum fuerit ad ultimos qui desiderantur. Hoc modo habebitur tota Tabula ex datis paucis terminis sub initio pro fundamento operationis. Sed non convenit ut termini quos primo quærimus, sint omnes per totam Tabulam æquidistantes; nam si omittimus alternos ubi eorum differentia est maxima, possumus alibi per saltum omittere duos, tres, viginti aut forte plures terminos. Numerus autem terminorum inter duos datos consistentium, qui omittuntur, debet semper esse aliquis sequentium 1, 3, 7, 15, 31, 63, &c. dummodo volumus inserere eos per hanc Seriem; hoc vero neutiquam incommodabit opus.

Possunt autem pro Praxi termini in unam summam colligi, ut factum vides in hac Tabella. Prima expressio est primus terminus; secunda est summa primi & secundi; tertia est summa primi, secundi & tertii: & sic porro.

2	$\frac{A}{2}$
4	$\frac{9A - B}{16}$
6	$\frac{150A - 25B + 3C}{256}$
8	$\frac{1225A - 245B + 49C - 5D}{2048}$
10	$\frac{39690A - 8820B + 2268C - 305D + 35E}{65536}$

Sic datis aliquibus terminis alternis, intermedii confestim dabuntur per hasce expressiones, nulla ratione habita naturæ Tabulæ particularis. Nam hæ regulæ sunt eædem in omnibus. Areæ curvarum sunt proxime æquales areis Parabolicæ figuræ quæ transit per extrema Ordinatarum suarum. Sed quoniam laboriosum nimis esset semper recurrere ad Parabolam, computavi Tabulam sequentem, qua Areæ directe exhibentur ex datis Ordinatis.

$$\begin{array}{r|l}
1 & \dfrac{A}{1}R \\
3 & \dfrac{A+4B}{6}R \\
5 & \dfrac{7A+32B+12C}{90}R \\
7 & \dfrac{41A+216B+27C+272D}{840}R \\
9 & \dfrac{989A+5888B-928C+10496D-4540E}{28350}R \\
11 & \dfrac{16067A+106300B-48525C+272400D-260550E+427368F}{598752}R.
\end{array}$$

Hic numerus Ordinatarum eſt impar, A eſt ſumma primæ & ultimæ, B ſecundæ & penultimæ, C tertiæ & antepenultimæ; & ſic porro, uſque dum deventum ſit ad eam in medio omnium, quæ per ultimam literam in quaque expreſſione repræſentatur. R eſt baſis ſeu pars Abſciſſæ inter primam & ultimam Ordinatam interceptæ. Expreſſiones ſunt Areæ contentæ inter Curvam, Baſin & Ordinatas hinc inde extremas. Tabulam pro pare numero Ordinatarum non appoſui, quoniam Area cæteris paribus ex impare earum numero accuratius definitur.

Quæratur area quæ generatur ab Ordinata $\overline{1+zz}|^{-1}$ & jacet ſupra Abſciſſam z quando ea evadit unitas. In $\overline{1+zz}|^{-1}$, pro z ſcribe $\frac{0}{10}, \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}, \frac{10}{10}$; & prodibunt undecim Ordinatæ $1, \frac{100}{101}, \frac{25}{26}, \frac{100}{109}, \frac{25}{29}, \frac{4}{5}, \frac{25}{34}, \frac{100}{149}, \frac{25}{41}, \frac{100}{181}, \frac{1}{2}$. Hinc eſt $A = 1 + \frac{1}{2} = \frac{3}{2}$, $B = \frac{100}{101} + \frac{100}{181} = \frac{28200}{18281}$, $C = \frac{25}{26} + \frac{25}{41} = \frac{1675}{1066}$, $D = \frac{100}{109} + \frac{100}{149} = \frac{25800}{16241}$, $E = \frac{25}{29} + \frac{25}{34} = \frac{1575}{986}$, $F = \frac{4}{5}$. Hiſce valoribus ſubſtitutis in ultima expreſſione, & unitate pro R, invenies aream eſſe 785398187. Juſtus eſt hic numerus in ſeptima figura, in octava verum ſuperans Binario.

Si undecim Ordinatæ non dent aream ſatis exactam, erige plures; & concipe aream diviſam eſſe in plures partes, quarum quamque ſeorſum quærens habebis pro lubitu juſtam.

Valor ipſius $\overline{1+Q}|^n$ exprimi poteſt per quamcunque trium ſerierum ſequentium.

$$\begin{aligned}
\overline{1+Q}|^n = 1 &+ \\
&Q \times \frac{n}{1} + \\
&Q^2 \times \frac{n}{1} \times \frac{n-1}{2} + \\
&Q^3 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} + \\
&Q^4 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} + \\
&Q^5 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} \times \frac{n-4}{5} + \&c.
\end{aligned}$$

Vel

Vel $\overline{1+Q}|^n = 1 +$

$$R\times\frac{n}{1} +$$
$$R^2\times\frac{n}{1}\times\frac{n+1}{2} +$$
$$R^3\times\frac{n}{1}\times\frac{n+1}{2}\times\frac{n+2}{3} +$$
$$R^4\times\frac{n}{1}\times\frac{n+1}{2}\times\frac{n+2}{3}\times\frac{n+3}{4} +$$
$$R^5\times\frac{n}{1}\times\frac{n+1}{2}\times\frac{n+2}{3}\times\frac{n+3}{4}\times\frac{n+4}{5} + \text{\&c.}$$ posito scilicet $R = \frac{1+Q}{Q}$. Vel

$\overline{1+Q}|^n = 1 +$

$$\frac{2+\overline{n+1}\times Q}{\overline{1+Q}|^1}\times Q\times\frac{n}{1.2} +$$
$$\frac{4+\overline{n+2}\times Q}{\overline{1+Q}|^2}\times Q^3\times\frac{n}{1.2}\times\frac{nn-1}{3.4} +$$
$$\frac{6+\overline{n+3}\times Q}{\overline{1+Q}|^3}\times Q^5\times\frac{n}{1.2}\times\frac{nn-1}{3.4}\times\frac{nn-4}{5.6} +$$
$$\frac{8+\overline{n+4}\times Q}{\overline{1+Q}|^4}\times Q^7\times\frac{n}{1.2}\times\frac{nn-1}{3.4}\times\frac{nn-4}{5.6}\times\frac{nn-9}{7.8} +$$
$$\frac{10+\overline{n+5}\times Q}{\overline{1+Q}|^5}\times Q^9\times\frac{n}{1.2}\times\frac{nn-1}{3.4}\times\frac{nn-4}{5.6}\times\frac{nn-9}{7.8}\times\frac{nn-16}{9.10} + \text{\&c.}$$

Primæ duæ Series demonſtrantur per Caſum primum Propoſitionis. Nam ſi $\overline{1+Q}|^0$, $\overline{1+Q}|^1$, $\overline{1+Q}|^2$, $\overline{1+Q}|^3$, $\overline{1+Q}|^4$, *&c.* deſignent Ordinatas totidem æquidiſtantes in Parabolica figura, erit $\overline{1+Q}|^n$ ejuſdem Ordinata, cujus diſtantia a $\overline{1+Q}|^0$ eſt n. Et ſic prodit Series prima. At ſi in alia Parabola $\overline{1+Q}|^0$, $\overline{1+Q}|^{-1}$, $\overline{1+Q}|^{-2}$, $\overline{1+Q}|^{-3}$, $\overline{1+Q}|^{-4}$, *&c.* ſint æquidiſtantes Ordinatæ, erit $\overline{1+Q}|^n$ Ordinata in eadem, cujus diſtantia a $\overline{1+Q}|^0$ eſt $-n$; ſic proveniet Series ſecunda. Sit jam in tertia Parabola *&c.* $\overline{1+Q}|^{-4}$, $\overline{1+Q}|^{-3}$, $\overline{1+Q}|^{-2}$, $\overline{1+Q}|^{-1}$, $\overline{1+Q}|^0$, $\overline{1+Q}|^1$, $\overline{1+Q}|^2$, $\overline{1+Q}|^3$, $\overline{1+Q}|^4$, *&c.* Series Ordinatarum æquidiſtantium hinc inde progrediens in infinitum, eritque in eadem $\overline{1+Q}|^n$ Ordinata, diſtantia n a termino medio $\overline{1+Q}|^0$ remota. Et ſic provenit Series tertia per Caſum Secundum Propoſitionis. Prima abrumpit quando eſt n integer & affirmativus, ſecunda quando eſt n integer & negativus, & tertia in caſu utroque abrumpit. Per harum quamque radices numerales commode evolvuntur in Series. Tertia reliquis multo

multo citius convergit: ejus terminus ſecundus adhiberi poteſt pro correctione, ubi fit extractio per repetitionem calculi.

Halleius in ſua methodo conſtruendi Logarithmos, ex prima harum ſerierum demonſtrat Seriem *Mercatoris* pro Quadratura Hyperbolæ. Sit ejus Ordinata $\overline{1+z}|^{-1}$, vel $\overline{1+z}|^{n-1}$, exiſtente n numero infinite parvo; unde per methodos Quadrandi, area quæ jacet ſupra Abſciſſam z, id eſt, Logarithmus numeri $1+z$, erit $\frac{\overline{1+z}|^{n}-1}{n}$. Eſt vero per primam Seriem $\overline{1+z}|^{n} = 1 + \frac{n}{1}z + \frac{n}{1} \times \frac{n-1}{2} z^2 + \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} z^3 + \&c.$ adeoque in caſu præſente, ubi eſt n infinite parvus, eſt $\overline{1+z}|^{n} = 1 + \frac{n}{1} z - \frac{n}{2} z^2 + \frac{n}{3} z^3 - \frac{n}{4} z^4$ &c. quo ſubſtituto in valore areæ, ea prodit $z - \frac{1}{2}z^2 + \frac{1}{3}z^3 - \frac{1}{4}z^4 + \frac{1}{5}z^5 -$ &c. quæ eſt Series *Mercatoris.*

Similiter per Seriem ſecundam prodit hæc regula; Sit datus numerus $1+z$, pone $R = \frac{z}{1+z}$, eritque ejus Logarithmus $R + \frac{1}{2}R^2 + \frac{1}{3}R^3 + \frac{1}{4}R^4 + \frac{1}{5}R^5 +$ &c.

Per Seriem tertiam provenit ſequens regula. Sit quilibet numerus R, pone $z = \frac{\overline{R-1}|^2}{2R}$, eritque ejus Logarithmus $\frac{RR-1}{2R} - \frac{1}{3}Az - \frac{2}{5}Bz - \frac{3}{7}Cz - \frac{4}{9}Dz - \frac{5}{11}Ez -$ &c. Ubi $A, B, C, D, E,$ &c. more *Newtoniano* deſignant terminos Seriei ſicut ab initio. Hæc Series, ut ea ex qua deducitur, reliquis duabus multis vicibus celerius approximat: eſtque eadem generalius expreſſa quam, ex fundamento haud abſimili, pro inventione Logarithmi Binarii prius dedimus.

Methodus inveniendi valores Serierum Arithmeticarum utcunque tarde convergentium.

In aliquibus Seriebus ſumma terminorum haberi nequit niſi ad pauciſſima figurarum loca, dummodo præter ſimplicem eorum additionem aliæ artes non adhibeantur. Proponatur jam Series quælibet cujus termini omnes iiſdem ſignis afficiuntur, & quorum proximi continue tendunt eſſe inter ſe æquales; quales ſunt ſequentes $\frac{1}{1\cdot 2} + \frac{1}{3\cdot 4} + \frac{1}{5\cdot 6} + \frac{1}{7\cdot 8} +$ &c. $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} +$ &c. Collige ſummam aliquot terminorum ſub initio, ii proxime addendi ſint $\alpha, \beta, \gamma, \delta, \epsilon, \zeta,$ &c. In numeris proximis ſit $r = \frac{\alpha\gamma - \beta\beta}{\alpha\beta - 2\alpha\gamma + \beta\gamma}$, & quantitatum $\alpha \times \frac{\alpha + r\beta}{\alpha - \beta}$, $\alpha + \beta \times \frac{\beta + r\gamma}{\beta - \gamma}$, $\alpha + \beta + \gamma \times \frac{\gamma + r\delta}{\gamma - \delta}$, $\alpha + \beta + \gamma + \delta \times \frac{\delta + \epsilon}{\delta - \epsilon}$, $\alpha + \beta + \gamma + \delta + \epsilon \times \frac{\epsilon + r\zeta}{\epsilon - \zeta}$, &c. differentiæ ſint $a, b, c, d, e,$ &c. Deinde in numeris proximis ſit $s =$

a c

$\frac{ac-bb}{ab-2ac+bc}$, & ipſorum $a \times \frac{a+sb}{a-b}$, $a+b \times \frac{b+sc}{b-c}$, $a+b+c \times \frac{c+sd}{c-d}$, $a+b+c+d \times \frac{d+se}{d-e}$, &c. differentiæ ſint A, B, C, D, &c. & ſit $t = \frac{AC-BB}{AB-2AC+BC}$: atque ſic procede quoad libuerit. Tum erit $\alpha+\beta+\gamma+\delta+\epsilon+$ &c. $= \alpha \times \frac{\alpha+r\beta}{\alpha-\beta} + a \times \frac{a+sb}{a-b} + A \times \frac{A+tB}{A-B} +$ &c. atque ultra duos primos terminos hujus novæ Seriei raro opus erit progredi.

Ut ſi deſideretur valor Seriei $\frac{1}{1 \cdot 2} + \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 6} + \frac{1}{7 \cdot 8} +$ &c. collige primos 21 terminos, quorum ſummam reperio fore 6813,8410,1885. Termini proxime addendi ſunt $\alpha =$,0005,2854,1226, $\beta =$,0004,8309,1787, $\gamma =$,0004,4326,2411, $\delta =$,0004,0816,3265, &c. Hinc fit $r = 1$ proxime, & $\alpha \times \frac{\alpha+r\beta}{\alpha-\beta} =$,0117,6449,6282, $a = -$,0000,0017,5096, $b = -$,0000,0014,7410, $c = -$,0000,0012,4986, &c. Unde $s = \frac{1}{3}$ prope, & $a \times \frac{a+sb}{a-b} = -$,0000,0141,8111, quem propter ſignum negativum ſubduco ab $\alpha \times \frac{\alpha+r\beta}{\alpha-\beta}$, & remanet 0117,6307,8171: hic additus ſummæ primo inventæ 6813,8410,1885, dat pro ſumma totius Seriei numerum 6931,4718,0056, qui juſtus eſt in nona decimali; at ante duas haſce correctiones ſumma erat juſta in prima figura ſola. Si animus ſit propius ſcopum attingere, pergendum erit ad approximationes ſequentes. Si termini Seriei diverſa habeant ſigna, conjungendi ſunt, ut omnes eadem tandem habeant, ut in Serie $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} -$ &c. conjunctis terminis ea evadit $\frac{2}{1 \cdot 3} + \frac{2}{5 \cdot 7} + \frac{2}{9 \cdot 11} + \frac{2}{13 \cdot 15} +$ &c. Sed hic notandum eſt quod differentiæ, a, b, c, d, e, &c. ut & A, B, C, D, &c. colligi debent ſubducendo quantitates antecedentes de ſubſequentibus. Et in omnibus hujuſmodi Seriebus ſi p, q, r, repræſentent tres terminos ordine ſequentes, p primum, q ſecundum, r tertium, & rectangulum $\frac{p+r}{2} \times q$ non ſit majus pr, valor Seriei erit infinite magnus: at magnitudinis ſemper finitæ ubi accidit contrarium. Poteſt hæc regula nonnunquam fallere, ubi termini p, q, r parum diſtant ab initio Seriei, at ſi conſiſtant inter eos ab initio aliquantum remotos, evadet regula certiſſima.

Ad alia Serierum genera debent aliæ regulæ adhiberi. Sit Series regularium Polygonorum Circulo Inſcriptorum, exiſtente Radio unitate.

$H=$

$H = 2,0000,0000,0000,000$	4
$G = 2,8284,2712,4746,190$	8
$F = 3,0614,6745,8920,718$	16
$E = 3,1214,4515,2258,051$	32
$D = 3,1365,4849,0545,938$	64
$C = 3,1403,3115,6954,752$	128
$B = 3,1412,7725,0932,772$	256
$A = 3,1415,1380,1144,299$	512

Dicatur jam ultimum Polygonum A, penultimum B, antepenultimum C, & reliqua in suo ordine retrorsum D, E, F, &c. atque area Circuli quæsita erit $A + \frac{A-B}{3} + \frac{4A-5B+C}{3.15} + \frac{64A-84B+21C-D}{3.15.63} + \frac{4096A-5440B+1428C-85D+E}{3.15.63.255} +$ &c. Ubi si pro $A, B, C, D, E,$ &c. scribantur proprii valores, primi quatuor termini dabunt 3,1415,9265, 3589,790 pro area Circuli. Hæc autem Series est generalis, ex natura Circuli neutiquam dependens: applicabilis est quotiescunque numerorum approximantium differentiæ priores sunt posteriorum quasi quadruplæ. Factores in Denominatoribus sunt dignitates integræ numeri 4 unitatibus minutæ: quibus datis, coefficientes literarum in diversis terminis formantur ex multiplicatione continua numerorum $1, \frac{n}{3}, \frac{n-3}{15}, \frac{n-15}{63}, \frac{n-63}{255}$, &c. Ubi pro n substituendus est ultimus Factorum in Denominatore.

Ultima quantitatum $x-1$, $2\sqrt[2]{x}-2$, $4\sqrt[4]{x}-4$, $8\sqrt[8]{x}-8$, $\sqrt[16]{x}-16$, &c. æqualis est Logarithmo numeri x. Pro x scribe 2, & per repetitam extractionem radicis quadratæ exibunt numeri,

$M =$	1,0000,0000,0000,0000.
$L =$	,8284,2712,4746,1901.
$I =$	7568,2864,0010,8843.
$H =$	7240,6186,1322,0613.
$G =$	7083,8051,8838,6214.
$F =$	7007,0875,6931,7337.
$E =$	6969,1430,7308,8294.
$D =$	6950,2734,2438,7611.
$C =$	6940,8641,2851,8363.
$B =$	6936,1658,4759,4014.
$A =$	6933,8182,9699,9493.

Dicatur ultimus numerorum A, penultimus B, & sic retro, atque Logarithmus quæsitus erit $A + \frac{A-B}{1} + \frac{2A-3B+C}{1.3} + \frac{8A-14B+7C-D}{1.3.7}$

+

$+\frac{64A-120B+70C-15D+E}{1.3.7.15}+$ &c. Primi quinque termini dant 6931,4718,0559,9457 pro Logarithmo Hyperbolico Binarii. Et quomodo hæc Series procedit in infinitum facile colligitur ex eo quod de priore diximus: eſtque etiam univerſalis, proprietates Hyperbolæ minime reſpiciens.

Extenditur quoque Methodus hæcce Differentialis ad Reſolutionem Æquationum & alia quamplurima quorum hic non fit mentio.

XXVI. § 1. Æquationis Cubicæ Univerſalis $\left.\right\}$ $x^3 = \begin{array}{l} 3px^2 + 3qx + 2r, \\ \quad\; - 3p^2 + p^3 \\ \qquad\qquad\; - 3pq \end{array}$

A univerſal Solution of Cubick and Biquadratick Æquations, viz. Analytical, Geometrical, and Mechanical. by Mr. J. Colſon, *n.* 309. *p.* 2353.

Radices Tres ſunt,

$$x = p + \sqrt[3]{r + \sqrt{r^2 - q^3}} + \sqrt[3]{r - \sqrt{r^2 - q^3}}$$

$$x = p - \frac{1 - \sqrt{-3}}{2} \times \sqrt[3]{r + \sqrt{r^2 - q^3}} - \frac{1 + \sqrt{-3}}{2} \times \sqrt[3]{r - \sqrt{r^2 - q^3}}$$

$$x = p - \frac{1 + \sqrt{-3}}{2} \times \sqrt[3]{r + \sqrt{r^2 - q^3}} - \frac{1 - \sqrt{-3}}{2} \times \sqrt[3]{r - \sqrt{r^2 - q^3}}$$

Vel ut Calculus Arithmeticus facilior ac paratior evadat, ſi poſueris Binomii irrationalis $r + \sqrt{r^2 - q^3}$ Radicem Cubicam eſſe $m + \sqrt{n}$, erunt ejuſdem Æquationis Radices tres $x = p + 2m$, & $x = p - m \pm \sqrt{-3n}$.

Igitur data Æquatione quavis Cubica, inter ejus hujuſque Æquationis Univerſalis terminos ſingulos inſtituenda eſt comparatio, quo pacto facillime invenientur ipſæ p, q, r; & hiſce cognitis, innoteſcent Æquationis datæ Radices omnes. Hujus vero Solutionis Exempla ſint ſequentia in Numeris.

1. Æquationis Cubicæ $x^3 = 2x^2 + 3x + 4$ ſit Radix x indaganda. Erit primo juxta præſcriptum $3p = 2$, ſive $p = \frac{2}{3}$. Secundo $3q - (3p^2)$ $\frac{4}{3} = 3$, ſive $q = \frac{13}{9}$. Tertio $2r\ (+\overline{p^2 - 3q} \times p) - \frac{70}{27} = 4$, ſive $r = \frac{89}{27}$, & $r^2 - q^3 = \frac{212}{27}$. Et propterea $x = \frac{2}{3} + \sqrt[3]{\frac{89}{27} + \sqrt{\frac{212}{27}}} + \sqrt[3]{\frac{89}{27} - \sqrt{\frac{212}{27}}}$. Reliquæ duæ Radices ſunt impoſſibiles.

2. In Æquatione $x^3 = 12x^2 - 41x + 42$, erit primo $3p = 12$, ſive $p = 4$. Secundo $3q - (3p^2)\ 48 = -41$, ſive $q = \frac{7}{3}$. Tertio $2r + (\overline{p^2 - 3q} \times p)\ 36 = 42$, ſive $r = 3$; Et inde $r^2 - q^3 = -\frac{100}{27}$. At Binomii ſurdi $3 + \sqrt{-\frac{100}{27}}$ $(= r + \sqrt{r^2 - q^3})$ Radix Cubica, per Metho-

 dos

dos ex Arithmetica petendas extracta, est $-1+\sqrt{-\frac{4}{3}}$, $(=m+\sqrt{n},)$ & proinde Radix $x=(p+2m=4-2=)\ 2$, vel etiam $x=(p-m+\sqrt{-3n}=4+1\pm(\sqrt{4})\ 2=)\ 7$ vel 3. Vel rursus, ejusdem Binomii $3+\sqrt{-\frac{100}{27}}$. Radix alia Cubica (tres enim agnoscit) est $\frac{3}{2}+\sqrt{-\frac{1}{12}}$ $(=m+\sqrt{n},)$ & proinde Radix $x=(p+2m=4+3=)\ 7$, & etiam $x=(p-m\pm\sqrt{-3n}=4-\frac{3}{2}\pm(\sqrt{\frac{1}{4}})\ \frac{1}{2}=3$ vel 2. Vel denuo, ejusdem Binomii $3+\sqrt{-\frac{100}{27}}$ Radix Cubica tertia est $-\frac{1}{2}-\sqrt{-\frac{25}{12}}$ $(=m+\sqrt{n},)$ & proinde Radix $x=(p+2m=4-1=)\ 3$, atque etiam $x=(p-m\pm\sqrt{-3n}=4+\frac{1}{2}\pm(\sqrt{\frac{25}{4}})\ \frac{5}{2}=)\ 7$ vel 2.

3. In Æquatione $x^3=-15x^2-84x+100$, erit $p=-5$, $q=-3$, $r=135$; & Binomii $135+\sqrt{18252}$ Radix Cubica est $3+\sqrt{12}$. Igitur Radix $x=-5+6=1$, & $x=-5-3\pm\sqrt{-36}=-8\pm\sqrt{-36}$, impossibiles.

4. In Æquatione $x^3=34x^2-310x+1012$, erit $p=\frac{34}{3}$, $q=\frac{226}{9}$, $r=\frac{5536}{27}$; & Binomii $\frac{5536}{27}+\sqrt{\frac{707560}{27}}$ Radix Cubica est $\frac{16}{3}+\sqrt{\frac{10}{3}}$. Igitur Radix $x=\frac{34}{3}+\frac{32}{3}=22$, & $x=\frac{34}{3}-\frac{16}{3}\pm\sqrt{-10}=6\pm\sqrt{-10}$, impossibiles.

5. In Æquatione $x^3=28x^2+61x-4048$, erit $p=\frac{28}{3}$, $q=\frac{967}{9}$, $r=-\frac{25010}{27}$; & Binomii $-\frac{25010}{27}+\sqrt{-382347}$ Radix Cubica est $\frac{41}{6}+\sqrt{-\frac{243}{4}}$. Igitur $x=\frac{28}{3}+\frac{41}{3}=23$, & $x=\frac{28}{3}-\frac{41}{6}\pm(\sqrt{\frac{729}{4}})\ \frac{27}{2}=16$ vel -11.

6. In Æquatione $x^3=-x^2+166x-660$, erit $p=-\frac{1}{3}$, $q=\frac{499}{9}$, $r=-\frac{9658}{27}$; & Binomii $-\frac{9658}{27}+\sqrt{-\frac{1147205}{27}}$ Radix Cubica est $-\frac{22}{3}+\sqrt{-\frac{5}{3}}$. Igitur $x=-\frac{1}{3}-\frac{44}{3}=-15$, & $x=-\frac{1}{3}+\frac{22}{3}\pm\sqrt{5}=7\pm\sqrt{5}$, irrationales.

7. In

7. In Æquatione $x^3 = 63x^2 + 99673x + 9951705$, erit $p = 21$, $q = \frac{100996}{3}$, $r = 6031680$; & Binomii $6031680 + \sqrt{-\frac{47887175043136}{27}}$ Radix Cubica est $183 + \sqrt{-\frac{529}{3}}$. Igitur $x = 21 + 366 = 387$, & $x = 21 - 183 \pm (\sqrt{529})\ 23 = -139$ vel 185.

Nec secus in cæteris procedendum: Investigatur autem Theorema ad modum sequentem. Pono Æquationis cujusdam Cubicæ Radicem $z = a + b$, & cubice multiplicando proveniet $z^3 = (a^3 + 3a^2b + 3ab^2 + b^3 =)\ a^3 + 3ab \times \overline{a+b} + b^3$. Jam loco ipsius $a + b$ valorem ejus z substituendo, fiet $z^3 = 3abz + a^3 + b^3$, quæ est Æquatio Cubica ex Radice $z = a + b$ constructa, cui terminus secundus deest. Ut hæc vero ad formam magis commodam magisque concinnam revocentur, sumo Æquationem $z^3 = 3qz + 2r$, quæ posthac ipsius $z^3 = 3abz + a^3 + b^3$, vices gerat. Igitur transmutatione hujus in illam, fiet primo $3q = 3ab$, sive $q^3 = a^3b^3$; & secundo $2r = a^3 + b^3$, sive $2ra^3 = (a^6 + a^3b^3 =)\ a^6 + q^3$. Et soluta hac æquatione quadratica, erit $a^3 = r + \sqrt{r^2 - q^3}$, & hinc $b^3 = (2r - a^3 =)\ r - \sqrt{r^2 - q^3}$: Atque igitur tandem $a = \sqrt[3]{r + \sqrt{r^2 - q^3}}$ & $b = \sqrt[3]{r - \sqrt{r^2 - q^3}}$. Et propterea in Æquatione Cubica $z^3 = 3qz + 2r$ erit Radix $z = (a + b =)\ \sqrt[3]{r^2 + \sqrt{r - q^3}} + \sqrt[3]{r^2 - \sqrt{r - q^3}}$.

At vero hæc Radix revera triplex est, pro triplici valore quem induere potest, & $\sqrt[3]{r + \sqrt{r^2 - q^3}}$ & $\sqrt[3]{r - \sqrt{r^2 - q^3}}$. Cujusvis enim quantitatis Radix Cubica triplex erit, & ipsius Unitatis Radix Cubica vel est 1, vel $-\frac{1}{2} + \frac{1}{2}\sqrt{-3}$, vel $-\frac{1}{2} - \frac{1}{2}\sqrt{-3}$. Atque id adeo, propterea quod harum alicujus Cubus sit Unitas. Igitur si $1 \times \sqrt[3]{r + \sqrt{r^2 - q^3}}$ aut $\sqrt[3]{r + \sqrt{r^2 - q^3}}$ $(= \sqrt[3]{1 \times r + \sqrt{r^2 - q^3}} = \sqrt[3]{1} \times \sqrt[3]{r + \sqrt{r^2 - q^3}})$ Radicem aliquam [quam supra nominavimus $m + \sqrt{n}$, aut $1 \times \overline{m + \sqrt{n}}$,] Cubi $r + \sqrt{r^2 - q^3}$ designet; ipsæ $\frac{-1 + \sqrt{-3}}{2} \times \sqrt[3]{r + \sqrt{r^2 - q^3}}$ & $\frac{-1 - \sqrt{-3}}{2} \times \sqrt[3]{r + \sqrt{r^2 - q^3}}$ [i. e. $\frac{-1 + \sqrt{-3}}{2} \times \overline{m + \sqrt{n}}$ & $\frac{-1 - \sqrt{-3}}{2} \times \overline{m + \sqrt{n}}$] alias duas ejusdem Cubi Radices designabunt. Similiter & $\sqrt[3]{r - \sqrt{r^2 - q^3}}$, $\frac{-1 + \sqrt{-3}}{2} \times \sqrt[3]{r - \sqrt{r^2 - q^3}}$ & $\frac{-1 - \sqrt{-3}}{2} \times \sqrt[3]{r - \sqrt{r^2 - q^3}}$, [i. e. $\overline{m - \sqrt{n}}$, $\frac{-1 + \sqrt{-3}}{2} \times \overline{m - \sqrt{n}}$, $\frac{-1 - \sqrt{-3}}{2} \times \overline{m - \sqrt{n}}$,] tres Cubicæ Radices erunt Apotomes $r - \sqrt{r^2 - q^3}$. Atque has Radices debite connectendo, fiet $z = \sqrt[3]{r + \sqrt{r^2 - q^3}} +$

$\sqrt[3]{r - \sqrt{r^2 - q^3}}$, [i. e. $z = \overline{m + \sqrt{n}} + \overline{m - \sqrt{n}} = 2m$,] $z = \frac{-1 + \sqrt{-3}}{2} \times \sqrt[3]{r + \sqrt{r^2 - q^3}} + \frac{-1 - \sqrt{-3}}{2} \times \sqrt[3]{r - \sqrt{r^2 - q^3}}$, [i. e. $z = \frac{-1 + \sqrt{-3}}{2} \times \overline{m + \sqrt{n}} + \frac{-1 - \sqrt{-3}}{2} \times \overline{m - \sqrt{n}} = -m + \sqrt{-3n}$,] & $z = \frac{-1 - \sqrt{-3}}{2} \times \sqrt[3]{r + \sqrt{r^2 - q^3}} + \frac{-1 + \sqrt{-3}}{2} \times \sqrt[3]{r - \sqrt{r^2 - q^3}}$ [i. e. $z = \frac{-1 - \sqrt{-3}}{2} \times \overline{m + \sqrt{n}} + \frac{-1 + \sqrt{-3}}{2} \times \overline{m - \sqrt{n}} = -m - \sqrt{-3n}$,] quæ tres erunt Radices Æquationis Cubicæ $z^3 = 3qz + 2r$. Debite autem connectuntur Radices istæ ad modum præcedentem, quippe quæ sic connexæ, & more vulgari in se invicem continue ductæ, Æquationem $z^3 = 3qz + 2r$ restituunt. Denique fac $z = x - p$, & Æquatio fiet $x^3 - 3px^2 + 3p^2x - p^3 = 3qx - 3pq + 2r$, quæ universalis est, & cujus Radices evadunt ut supra fuerunt exhibitæ.

Hic obiter notatu dignum est, quod Æquationis Cubicæ cujuscunque Radices omnes sint possibiles & reales, quoties Binomii membrum irrationale $\sqrt{r^2 - q^3}$ impossibilitatem in se complectitur; hoc est, quoties q est quantitas affirmativa, & simul cubus ejus major est quadrato ex latere r. At si membrum istud $\sqrt{r^2 - q^3}$ sit possibile, hoc est si q sit quantitas negativa, aut etiam si affirmativæ cubus sit minor quadrato ex latere r, tunc unicam tantum agnoscit Æquatio Radicem possibilem & realem, reliquæque duæ erunt impossibiles.

In hoc Theoremate si fiat $p = 0$, hoc est, si desit Æquationis terminus secundus, tunc deventum erit ad casum Regularum quæ dicuntur *Cardani*, cujus solutio continetur in præcedentibus.

§ 2. Æquationis Biquadraticæ Universalis

$$x^4 = 4px^3 \begin{array}{l} + 2qx^2 \\ - 4p^2 \end{array} \begin{array}{l} + 8rx \\ - 4pq \end{array} \begin{array}{l} + 4s, \\ - q^2 \end{array}$$

Radices quatuor sunt $x = p - a \pm \sqrt{p^2 + q - a^2 - \frac{2r}{a}}$, & $x = p + a \pm \sqrt{p^2 + q - a^2 + \frac{2r}{a}}$, Ubi a^2 est Radix Æquationis Cubicæ

$$a^6 = \begin{array}{l} p^2 a^4 \\ + q \end{array} - 2pra^2 \begin{array}{l} + r^2 \\ - s \end{array}$$

Jam data Æquatione quavis Biquadratica, inter ejus hujusque Æquationis Universalis terminos singulos instituenda est comparatio, quo pacto citissime invenientur ipsæ p, q, r, s; & hisce cognitis, non latebit valor ipsius a, ex Theoremate superiori inveniendus, & tum demum innotescent Æquationis datæ Radices omnes.

Huic Solutioni illustrandæ Exemplum unum aut alterum sufficiat.

1. Æquationis Biquadraticæ $x^4 = 8x^3 + 83x^2 - 162x - 936$ sint Radices extrahendæ. Erit primo juxta præscriptum $4p = 8$, sive $p = 2$. Secundo $2q - (4p^2)\ 16 = 83$, sive $q = \frac{99}{2}$. Tertio $8r - (4pq)\ 396 =$

-162, five $r = \frac{117}{4}$. Quarto $4s - (q^2)\frac{9801}{4} = -936$, five $s = \frac{6057}{16}$. Hinc $p^2 + q = \frac{107}{2}$, $2pr + s = \frac{7929}{16}$, $r^2 = \frac{13689}{19}$, & proinde $a^6 = \frac{107}{2}a^4 - \frac{7929}{16}a^2 + \frac{13689}{16}$. Jam ut Æquatio hæc aliquatenus Cubica in Radices ejus resolvatur, ad Theorema præcedens recurrendum est, in quo erit $p = \frac{107}{2}$, $q = \frac{22009}{144}$, $r = \frac{2903923}{1728}$, & $r^2 - q^3 = -\frac{11940075}{16}$. Atqui Binomii $\frac{2903923}{1728} + \sqrt{-\frac{11940075}{16}}$ Radix Cubica est $-\frac{53}{12} + \sqrt{-\frac{400}{3}}$ & propterea $a^2 = \frac{107}{6} - \frac{53}{6} = 9$, & etiam $a^2 = \frac{107}{6} + \frac{53}{12} \pm (\sqrt{400})\, 20 = \frac{169}{4}$ vel $\frac{9}{4}$: Vel quod perinde est, Æquationis præmissæ revera Cubo-Cubicæ sex Radices sunt $a = \pm 3$, $a = \pm \frac{13}{2}$, & $a = \pm \frac{3}{2}$, quarum quævis indiscriminatim proposito nostro faciet satis. Puta si in præsenti casu fiat $a = 3$, erit juxta Theorema $x = (p - a \pm \sqrt{p^2 + q - a^2 - \frac{2r}{a}} = 2 - 3 \pm \sqrt{4 + \frac{99}{2} - 9 - \frac{39}{2}} = -1 \pm (\sqrt{25})\, 5 =)\, 4$ vel -6, & $x = (p + a \pm \sqrt{p^2 + q - a^2 + \frac{2r}{a}} = 2 + 3 \pm \sqrt{4 + \frac{99}{2} - 9 + \frac{39}{2}} = 5 \pm (\sqrt{64})\, 8 =)\, 13$ vel -3, quæ sunt Æquationis datæ Radices quatuor.

2. In Æquatione $x^4 = 20x^3 + 252x^2 - 6592x + 21312$, erit $p = 5$, $q = 176$, $r = -384$, & $s = 13072$. Hinc $p^2 + q = 201$, $2pr + s = 9232$, & $r^2 = 147456$; & inde $a^6 = 201a^4 - 9232a^2 + 147456$. Jam in Theoremate pro Cubicis erit $p = 67$, $q = \frac{4235}{3}$, & $r = 65219$; eritque Binomii $65219 + \sqrt{\frac{38889307072}{27}}$ Radix Cubica $\frac{77}{2} + \sqrt{\frac{847}{12}}$. Igitur $a^2 = 67 + 77 = 144$, five $a = 12$; & proinde $x = 5 - 12 \pm \sqrt{25 + 176 - 144 + 64} = -7 + (\sqrt{121})\, 11 = 4$ vel -18, & $x = 5 + 12 \pm \sqrt{25 + 176 - 144 - 64} = 17 \pm \sqrt{-7}$, impossibiles.

Hujus autem Theorematis Inventio est hujusmodi. Ex duarum Æquationum Quadraticarum $z^2 + 2az - b = 0$, & $z^2 - 2az - c = 0$, in se invicem multiplicatione, Æquationem conficio Biquadraticam $z^4 = \overline{4a^2 + b + c} \times z^2 + \overline{2ac - 2ab} \times z - bc$, cui terminus secundus deest, quamque huic Æquationi $z^4 = ez^2 + fz + g$ statuo æquipollere. Unde primo $4a^2 + b + c = e$, five $b = e - 4a^2 - c$. Secundo $2ac - 2ab = f$, hoc

hoc est, $2ac - 2ae + 8a^3 + 2ac = f$, sive $c = \frac{f}{4a} + \frac{e}{2} - 2a^2$, & inde $b = (e - 4a^2 - c =) - \frac{f}{4a} + \frac{e}{2} - 2a^2$. Tertio $-bc = g$, sive $-\frac{f^2}{16a^2} + \frac{e^2}{4} - 2ea^2 + 4a^4 = -g$, hoc est, $a^6 = \frac{1}{2}ea^4 - \frac{1}{4}ga^2 - \frac{1}{16}ea^2 + \frac{f^2}{64}$, quæ Æquatio quasi Cubica est, ex Radice a^2 & notis vel assumptis e, f, g conflata. Ea vero Radix per Theorema superius exhiberi potest, & eodem Calculo innotescent ipsæ b & c. At Æquationum $z^2 + 2az - b = 0$ & $z^2 - 2az - c = 0$ Radices sunt $z = -a \pm \sqrt{a^2 + b}$ & $z = a \pm \sqrt{a^2 + c}$, sive $z = -a \pm \sqrt{\frac{1}{2}e - a^2 - 4a}$, & $z = a \pm \sqrt{\frac{1}{2}e - a^2 + \frac{f}{4a}}$, quæ proinde erunt Radices Equationis $z^4 = ez^2 + fz + g$; cognita videlicet a vel a^2 ex Equatione $a^6 = \frac{1}{2}ea^4 - \frac{1}{4}ga^2 - \frac{1}{16}ea^2 + \frac{f^2}{64}$. Jam ut Equatio ista evadat universalis, & omnibus suis terminis instructa, fac $z = x - p$, eritque $x^4 - 4px^3 + 6p^2x^2 - 4p^3x + p^4 = ex^2 - 2pex + p^2e + fx - fp + g$, item & $x = p - a \pm \sqrt{\frac{1}{2}e - a^2 - \frac{f}{4a}}$ & $x = p + a \pm \sqrt{\frac{1}{2}e - a^2 + \frac{f}{4a}}$. Tandem concinnitatis & compendii gratia, fac $e = 2q + 2p^2$ & $f = 8r$; tum $x^4 - 4px^3 + 4p^2x^2 = 2qx^2 - 4pqx + 2p^2q + p^4 + 8rx - 8pr + g$, $x = p - a \pm \sqrt{p^2 + q - a^2 - \frac{2r}{a}}$, $x = p + a \pm \sqrt{p^2 + q - a^2 + \frac{2r}{a}}$, & $a^6 = \overline{p^2 + q} \times a^4 - \overline{\frac{1}{4}g + \frac{1}{4}p^4 + \frac{1}{2}p^2q + \frac{1}{4}q^2} \times a^2 + r^2$. Denique fac $g = 4s - q^2 + 8pr - p^4 - 2p^2q$, & fiunt Equationes præcedentes

$$\begin{array}{llllll} x^4 = & 4px^3 & + 2qx^2 & + 8rx & + 4s & \text{\&}\ a^6 = p^2a^4 - 2pra^2 + r^2. \\ & & - 4p^2 & - 4pq & - q^2 & \qquad\quad + q \qquad - s \end{array}$$

Scilicet omnia evadunt ut supra sunt posita.

§ 3. Hactenus de Æquationum Cubicarum & Biquadraticarum Resolutione Analytica. Quoniam autem earundem *Effectio Geometrica* per Parabolam vulgo tradi solet, & nonnullis in pretio est, ipsam συνθετικῶς, & quidem universalius, non pigebit hic exhibere.

Data Æquatione quavis vel Cubica vel Biquadratica, instituenda est comparatio inter terminos ejus, terminosque respondentes hujus Æquationis

$$\begin{array}{lllll} x^4 = & \frac{2p}{q}x^3 & + \frac{4pr}{q}x^2 & + \frac{2p2}{q}x & + p^2, \\ & - 4r & - 4r^2 & - \frac{2ps}{q} & - q^2 \\ & & + 2s & + 4rs & - s^2 \\ & & - 1 & - 2q & + t^2 \end{array}$$

quo pacto facile satis eruen-

eruentur ipsæ p, q, r, s, t; earum interim una aliqua utcunque pro lubitu assumpta. Tum in Parabola quavis data AVB, cujus Vertex principalis V, Axis VS, & Axi perpendicularis VT; capiatur $VS = p$ versus interiora Parabolæ, & in angulo SVT inscribatur $ST = q$, quæ producta Parabolam secet in punctis binis N & O. Bisecetur ON in M, & per M agatur MA Axi parallela & Parabolæ occurrens in A. Ipsi ON parallela ducatur AL, ut sit AL Latus rectum Parabolæ ad Diametrum AM, sitque hæc eadem Unitas. In AL (utrinque si opus est producta) capiatur $AG = r$, & a puncto G ducatur GR Axi parallela, quæ Parabolam secet in B, a quo capiatur $BR = s$. A novissime invento puncto R ducatur RE ipsi VT parallela & æqualis, quæ sinistram versus jaceat respectu ipsius R si q sit quantitas affirmativa, at versus dextram si q sit negativa. Atque idem de ipsis AG & BR intelligatur, quæ ad contrarias itidem partes duci debent, si modo valores ipsarum r & s prodeant negativi. Denique Centro E & Radio $EC = t$ describatur Circulus $CK\kappa c$, qui Parabolam in totidem secabit punctis, quot sunt Æquationis datæ Radices reales. Etenim a punctis istis C, K, &c. ducantur CP, $\kappa\pi$, &c. ipsi ST parallelæ, & ad rectam GR (si opus est productam) terminatæ, eritque harum quævis x, seu Æquationis datæ Radix quæsita; eæ scilicet ad dextram jacentes erunt Radices affirmativæ, quæ vero ad sinistram sunt positæ erunt Radices negativæ. Punctum contactus, si quod fuerit, hic sumitur pro intersectionis punctis duobus ad invicem vicinissimis. Plate 3. Fig. 1.

Inter Æquationes Cubicas & Biquadraticas ita constructas hoc tantum intercedit discriminis, quod in prioribus, ob terminum ultimum in præcedente Æquatione deficientem, semper sit $p^2 - q^2 - s^2 + t^2 = 0$, sive $t = \sqrt{s^2 + q^2 - p^2}$. Igitur Centro E & Radio EB $(= \sqrt{BRq + (ERq)} STq - VSq)$ descripto Circulo $C\kappa\kappa c$, Radicum una CP in priori constructione in nihilum abit.

Hæc autem demonstrantur ad modum sequentem. Manentibus jam constructis, & producta CP, si opus est, donec secat AM in H, erit CH Ordinata Parabolæ ad Diametrum AH, & proinde $CHq = AL \times AH = AH$, ob $AL = 1$. At $CH = CP + AG$, & $AH = GB + BP$, & propterea $CPq + 2AG \times CP + AGq = GB + BP$; sed ob naturam Parabolæ erit $AGq = GB$, unde $CPq + 2AG \times CP = BP$. Jam a puncto C ad ipsam BP demittatur normalis CD, quæ occurrat etiam ipsi EI, ad BP actæ parallelæ, in puncto I. Propter similia Triangula CDP & TVS, erit $DP = \dfrac{VS \times CP}{ST}$ & $CD = \dfrac{VT \times CP}{ST}$, & proinde $CPq + 2AG \times CP = BP = DP + BD = \dfrac{VS \times CP}{ST} + BR - IE$, sive $CPq + 2AG \times CP - \dfrac{VS}{ST} CP - BR = -IE$. Ast $IEq = CEq - CIq = CEq - CDq - VTq - 2CD \times VT = CEq - \dfrac{VTq \times CPq}{STq} - VTq - \dfrac{2VTq \times CP}{ST}$

$=$

$= (\text{ob } VTq = STq - SVq)\ CEq - CPq + \frac{SVq}{STq}CPq - STq + SVq - 2ST \times CP + \frac{2SVq}{ST}CP$, quæ igitur æqualis erit Quadrato ex Latere $CPq + 2AG \times CP - \frac{VS}{ST}CP - BR$. Atque hæc Æquatio, ad terminos p, q, r, s, t, revocata, ipsissima fit Æquatio proposita.

Hinc liquet, quod eadem quævis Æquatio Biquadratica innumeras per Parabolam constructiones sortiri possit, pro indefinito valore quantitatis istius, quam ad arbitrium assumi posse jam diximus. Sed casus est simplicissimus faciendo $VS = p = 0$, & migrat constructio, si rem ipsam spectes, in vulgarem istam, in qua Radicum repræsentatrices rectæ CP, *&c.* sunt ad Axem perpendiculares. Æquatio autem fit

$$x^4 = \begin{array}{l} -4rx^3 \\ \end{array} \begin{array}{l} -4r^2x^2 \\ +2s \\ -1 \end{array} \begin{array}{l} +4rsx \\ -2q \\ \end{array} \begin{array}{l} -q^2, \\ -s^2 \\ +t^2 \end{array}$$

quæ facile construitur ut supra.

§ 4. Sed ne Parabolæ descriptio Organica difficilis nimium videatur, in promptu est Artificium quoddam Mechanicum, ope Fili penduli pondere instructi peractum, cujus auxilio quam exactissime & facillime Æquatio novissima construi potest, & proinde Æquationum quarumcunque Cubicarum & Biquadraticarum Radices inveniri; idque sine ullo linearum ductu nisi Rectarum & Circuli. Constructio autem, quam appellare libet *Mechanicam*, est ad hunc modum.

Plate 3. Fig. 2. Contra Parietem erectum, vel planum aliud quodvis Horizonti perpendiculare, ad punctum aliquod F suspendatur filum tenuissimum & flexile FP; pondere quovis P ad extremitatem P appenso. In hoc filo notetur punctum aliquod N, a puncto suspensionis F satis remotum; vel filo parvulus, si id mavis, innectatur Nodus N. Et sumpta utcunque NP pro Unitate, ad punctum medium A ducatur (in plano prædicto) recta $A\mathcal{Q}$ Horizonti parallela, & utrinque quantum satis producta. Hisce generaliter paratis, pro particulari jam applicatione fac $A\mathcal{Q} = r$; ipsis q, r, s, t, ut sæpius inculcatum, vel Arithmetice vel Geometrice, pro datæ cujusvis Æquationis exigentia, in Æquatione novissima prius determinatis. Tunc Acu vel Stylo tenuissimo, aut etiam cuspide Circini admodum gracili, flectatur filum a loco suo ad punctum quoddam B, ita ut punctum N cadat in novissime invento puncto $\mathcal{Q}$. In $B\mathcal{Q}$ ab isto B capiatur $BR = s$, & in R ad ipsam BR perpendicularis erigatur $ER = q$. Verum enimvero istæ $A\mathcal{Q}$, BR, RE ad contrarias partes ab earum initiis cadere debent, si forte valores ipsarum r, s, q prodeant negativi. Denique in puncto invento E figatur Circini crus unum, &, ad distantiam $EZ = t$ extentum, agatur crus alterum in orbem, secumque circumducat filum FZP. Hac fili circulatione pondus P nunc ascendet nunc descendet motu reciproco, ut & Nodus N nunc supra rectam $A\mathcal{Q}$ extabit, nunc vero infra eandem deprimetur. Quoties autem reperietur Nodus ille N in ipsa $A\mathcal{Q}$, puta in punctis D, d, Δ, δ, abscindet is rectas $D\mathcal{Q}$, $d\mathcal{Q}$, $\Delta\mathcal{Q}$, $\delta\mathcal{Q}$, quæ erunt Æqua-

Æquationis datæ Radices omnes reales; hæ nempe ad dextram erunt Radices affirmativæ, illæ vero ad finistram Radices negativæ. Demonstratio est manifesta ex præcedentibus, habita tantum ratione Parabolæ per puncta B, C, c, κ, K transeuntis. Nam posito F foco Parabolæ, (cujus distantia a Vertice est $\frac{1}{4} ON$,) notum est quod lineæ omnes ut $FB + BQ$, $FC + CD$, &c. eandem ubique conficiant summam.

Atque ex principiis hic positis proclive erit Instrumentum haud inconcinnum & quantumvis accuratum fabricari, cujus beneficio hujusmodi Æquationum quarumcunque Radices nullo fere negotio inveniri possint, & præ oculis exhiberi.

Rules for correcting the common computus of Interest. by Mr. Tho. Watkins. *n.* 340. *p.* 111. I. *Of Compound Interest.*

XXVII. The Supposition whereon the Method of computing by Compound Interest is founded; *viz.* That all Interest Money, Rents, &c. are or may be constantly receiv'd, and put out again at Interest, the Moment they become due, without any Charge, or Trouble, being impracticable; therefore all Computations by this Method (except of Fee-Simples or other Perpetuities) must needs be erroneous. Thus for Instance, the Amount of a Sum of Money, or Annuity, for want of Deductions out of the Profits, for the unavoidable Trouble, Charge, and Delay in the Management, will be too great: and for the same reason, the present value of a Sum of Money payable in any time to come, will be too little; also the present value of an Annuity (being only the Amount of the difference between the Annuity, and Interest of the said present value) will be too much. But in long terms of Years, as that difference becomes less, so does the Error, as the term is greater; and in Fee-Simples it vanishes; the contrary to which happens in Amounts of Sums of Money, and Annuities.

All which is propos'd to be rectify'd, only by a just reduction of the Rate, and Annuity; (which is done by deducting so much *per Cent.* thereout, as the whole Trouble and Charge of Management is suppos'd to amount to, and reducing the Remainder, by a Discount equivalent to the suppos'd loss of time) and then by working with the Rate so reduc'd for Sums of Money, and with the Rate and Annuity reduc'd in the like proportion for Annuities, according to the common Method of Compound Interest; as follows. Put r for the rate of Interest of 1 *l.* c for the Charge and Trouble of the Management of 1 *l.* Then is $r - cr =$ the Rate after deducting the said Charge, $=$ (putting d for $1 - c$) dr. And for the Discount put t for the time lost, that is for such part of the Period of time in which the Payments are made (whether Yearly, $\frac{1}{2}$ Yearly, Quarterly, or otherwise) as is suppos'd to be spent in receiving and putting them out again at Interest. Then, dtr being $=$ the Interest of 1 *l.* for that time; say, as $1 + dtr : 1 :: dr : \frac{dr}{1 \times dtr} =$ (putting e for $1 + dtr$) $\frac{dr}{e}$, which is equal to the reduc'd Rate, near enough for practice, for which put r. But if the utmost accuracy be requir'd, the Discount itself

muſt be made with regard to the like loſs of time, which is done by a Series of Diſcount rais'd thus; $e\,(=1+tdr) : tdr :: dr : \frac{td^2r^2}{e} :: \frac{td^2r^2}{e} : \frac{t^2d^3r^3}{e^2}$ &c. Whence $dr - \frac{td^2r^2}{e} + \frac{t^2d^3r^3}{e^2} - \frac{t^3d^4r^4}{e^3}$ &c. $=$ (putting q for $\frac{tdr}{e}$) $\overline{1-q+q^2-q^3\text{ \&c.}} \times dr. = \frac{dr}{e}\,(=\overline{1-q}\times dr) + \overline{q^2-q^3\text{ \&c.}} \times dr$, is $=\mathfrak{r}$, $=$ the true Rate reduced. Put $s=1+\mathfrak{r}$, $n=$ the time, $p=$ the preſent Sum or Value, $m=$ the Amount. Then will $\overline{1+\frac{dr}{e}+\overline{q^2-q^3\text{ \&c.}}\times dr}\big|^n \times p = \overline{1+\mathfrak{r}}\big|^n \times p = ps^n$, be exactly $=m$: But $\overline{1+\frac{dr}{e}}\big|^n \times p = m$ is ſufficient for practice. And for the Amounts and preſent values of Annuities.

Put $A=$ Annuity *per annum.*
$a=$ Annuity ½ yearly, quarterly, &c.
$R=$ Yearly rate of Intereſt of 1 *l.*
$r=$ Rate ½ yearly, quarterly, &c.
$\mathfrak{r}=$ Reduc'd rate yearly, ½ yearly, quarterly, &c.
$n=$ Number of Years, ½ years, quarters, &c.

Then will $A\frac{\mathfrak{r}}{R} = a\frac{\mathfrak{r}}{r}$ be $=$ reduc'd Annuity taken yearly, ½ yearly, quarterly, or otherwiſe; and by Compound Intereſt 'twill be $\frac{\overline{1+\mathfrak{r}}|^n-1}{\mathfrak{r}} \times a\frac{\mathfrak{r}}{r} = \frac{\overline{1+\mathfrak{r}}|^n-1}{r}a = \frac{\overline{1+\mathfrak{r}}|^n-1}{R}A = \frac{s^n-1}{R}A = m$, and $\frac{s^n-1}{Rs^n}A\,(=\frac{m}{s^n}) = p$. Whence the Theorems for ſolving all the other Caſes are eaſily deduc'd. And if the Rate be requir'd, when 'tis for a Sum of Money, the Solution is obvious: when for the Amount or Value of an Annuity, ſince $\overline{1+r}|^n = \frac{mr+a}{a} = \frac{a}{a-pr}$ are the Equations whence Theorems for the Rate are uſually deriv'd, which by this Correction become $\overline{1+\mathfrak{r}}^n = \frac{mr+a}{a} = \frac{a}{a-pr}$. That the ſame $\mathfrak{r}$ may be had on both ſides the Equation, put $u\mathfrak{r}$ for r, and 'twill be $\overline{1+\mathfrak{r}}|^n = \frac{mu\mathfrak{r}+a}{a} = \frac{a}{a-pu\mathfrak{r}}$; then, by the Rate aſſum'd as near the truth as may be, find the value of $u\,(=\frac{1}{d}+tr=\frac{1}{d-td\mathfrak{r}})$ and in any Theorem for the Rate, putting mu for m, and pu for p, the reſult will be the Rate reduced nearly: and by repeated Operations correcting $\mathfrak{r}$ and thereby u, the true $\mathfrak{r}$, and thence R the whole Rate may be found.

The

The only difficulty that remains, is the right aſſuming the Quantities *c* and *t*, the impoſſibility of doing which with perfect Exactneſs, I ſuppoſe to be the reaſon why neither this, nor any Method of Correction to the like purpoſe, has yet been taken notice of by the Writers on this Subject; and what may therefore be very likely to be objected to this. But the ſame Objection I take to be of equal force againſt the Eſtimates of any other Uncertainties whatſoever, as Eſtates for Lives, Inſurances, *&c.*

Firſt then, for the Quantity *c*, which is put for the Trouble and Charge of Management, *viz.* of collecting and placing out the Money on good Security, together with all Contingencies attending the ſame, as travelling Charges, Expences, Attorney's Bills, *&c.* of all which, the principal Article is the Charge of Collection, or Receiver's Fees, which is commonly a fix'd Rate, cuſtomarily allow'd in the Place, or upon the Eſtate itſelf out of which the Purchaſe is made, if it be a Rent; and for Intereſt Money, or any other Annuity, the like Eſtimate is to be made, whether the Proprietor acts for himſelf, or by another. Then for the Charge of placing out the Money at Intereſt when receiv'd; though this be for the moſt part defray'd by the Borrower, yet becauſe it highly concerns the Lender to ſee it be ſecurely done, there are uſual Allowances made to Agents and Scriveners, to encourage their Care and Fidelity therein; beſides the Time, Expence and Trouble of the Proprietor himſelf, in finiſhing Contracts, inſpecting Securities, *&c.* and whatever is ſav'd in this Article, we muſt ſuppoſe to be fully made up by an Equivalent Degree of Riſque in the Security.

In the next place, for the loſs of time; if the uſual times of Payment of the particular Rent or Annuity to be purchaſed, with a moderate degree of diligence in the Manager, and the uſual indulgence practis'd by Men of Buſineſs in this Caſe towards one another be obſerv'd, a reaſonable Eſtimate may be alſo made of the loſs of time; in which 'tis to be noted, that Intereſt Money being uſually paid in ſmall Sums, when any Sum of Money to be made up of ſeveral ſuch Payments, is intended to be put out at Intereſt, the whole muſt lie dead till the laſt Payment be made; alſo that the Principal lies dead ſometimes as well as the Intereſt; and that on the other hand, to ſave time, Borrowers may be found out, and treated with during the time of Collection; but this Advantage is in a great Meaſure loſt by the difficulty of fixing the time or *Quantum* of a Loan, till the whole be paid in. Note alſo, that if the Charge of Collection, or loſs of time, on the Rent, or Annuity, of any particular Eſtate or Place, be found to differ from that of the Intereſt of the Purchaſe Money, and ſo much exactneſs be required, as that the Computation be made with regard to ſuch difference: It muſt be done, either by aſſuming a proper Medium for both, or more accurately thus: For the reduc'd Rate of the $\left\{\begin{matrix}\text{Annuity}\\ \text{Intereſt-Money}\end{matrix}\right\}$ put $\left\{\begin{matrix} z\\ \mathfrak{r}\end{matrix}\right\}$ Then $r : z :: a : a\frac{z}{\mathfrak{r}}$, and

$$\mathfrak{r} : \overline{1+\mathfrak{r}}|^{n} - 1 :: a\frac{z}{r} : \frac{\overline{1+\mathfrak{r}}|^{n}-1}{r} a \times \frac{z}{\mathfrak{r}} = m = \overline{1+\mathfrak{r}}|^{n} \times p.$$

To

To give a Specimen of this Method in Numbers, firſt the Quantities c and t are to be aſſum'd, which are not here to be accommodated to any particular Place or Eſtate, but to be taken in general: And firſt for c the Charge of Management; the uſual Rates of Collectors Fees in theſe Kingdoms, as I am inform'd, are 6*d.* 12*d.* and 18*d.* *per* Pound, but the moſt uſual 12*d.* which is 5 *per Cent.* However to be within compaſs, I ſhall take 4 *per Cent.* for the Medium, including what further trouble and charge may attend the Receipt of the Money, beſides Receivers Fees; and 2 *per Cent.* for all the other Charges before mention'd, in placing it out at Intereſt; both which make 6 *per Cent.* ſo that c is $= 0{,}06$, and $d\ (= 1 - c) = 0{,}94$. Next for t the loſs of time; ſince few Annuities are paid yearly, and a Diſcount being given for the loſs of time, we are to loſe no more than is diſcounted for; therefore I chooſe Half-yearly Payments for the Examples, being the moſt uſual, which with little Alteration may ſerve for quarterly; and conſidering the before-mention'd Circumſtances relating to the Time, I look upon two Months the leaſt, and ſeven or eight Months the moſt, that can well be ſuppos'd to be ſpent, one time with another, in receiving and putting out the Money, upon a moderate Management; between which the Medium is about four Months and half, which being $\frac{3}{4}$ of $\frac{1}{2}$ a Year, gives $\frac{3}{4} = t$: and if $\frac{dr}{e}$ be $= \mathfrak{r}$, the yearly Rates of 4, 5, 6, 8 and 10 *per Cent.* will produce for half-yearly Rates reduc'd of 1 *l.* 0,018539, 0,023093, 0,027616, 0,036569 and 0,0454 each $= \mathfrak{r}$. But if $\mathfrak{r}$ be $= \frac{dr}{e} + \overline{q^2 - q^3 \,\&c.} \times dr$, 'twill be, 0,01854219, 0,0230999155, 0,027627775, 0,036596289 and 0,04545235, each $= \mathfrak{r}$, (ſo that each Rate loſes by this Eſtimate about $\frac{1}{12}$ Part.) Whence the following Amounts, and preſent values of 1 *l.* *per Annum* computed Half-yearly, are produc'd, and compared with thoſe of the uſual Method computed yearly, to agree with the common Tables.

Years

Years.	Amounts of 1 l. per An. at 5 per Cent. by Comp. Int.	Cor. Int. cor.	Differences.	Amounts of 1 l. per An. at 6 per Cent. by Comp. Int.	Co. Int. cor.	Differences.
5	5,52563	5,13104	,39459	5,63709	5,22138	,41571
10	12,57789	11,57846	,99943	13,18079	12,07854	1,10225
20	33,06595	29,85996	3,20599	36,78559	32,91054	3,87505
30	66,43885	58,72546	7,71339	79,05819	68,83976	10,21843
40	120,79977	104,30079	16,49898	154,76197	130,80733	23,95464
60	353,58372	289,88164	63,70208	533,12818	422,01429	111,11389
80	971,22882	752,53436	218,69446	1746, 5999	1288, 2484	458,3515
100	2610,02516	1905,92671	704,09845	5638, 3680	3864, 9753	1773,3927
	The ſame at 8 *per Cent.*			The ſame at 10 *per Cent.*		
5	5,86660	5,40633	,46027	6,10510	5,59705	,50805
10	14 48656	13,15092	1,33564	15,93743	14,32680	1,61063
20	45,76196	40,13759	5,62437	57,27500	49,17930	8,09570
30	113,28321	95,51623	17,76698	164,49402	133,96428	30,52974
40	259,05652	209,15737	49,89915	442.59257	340,21900	102,37357
60	1253,2133	920,90040	332, 3129	3034,81648	2062,57167	972,24481
80	5886,9354	3918,0578	1968, 8776	20474,0027	12255,3348	8218, 6679
100	27484,5157	16539,0989	10945,4168	137796,127	72575,3926	65220,7344

Years.	Pref. Values of 1 l. per An. at 5 per Cent. by Comp. Int.	Co. Int. cor.	Differences.	Pref. Values of 1 l. per An. at 6 per Cent. by Comp. Int.	Co. Int. cor.	Differences.
5	4,32948	4,08343	,24605	4,21236	3,97582	,23654
10	7,72174	7,33314	,38860	7,36009	7,00322	,35687
15	10,37966	9,91935	,46031	9,71225	9.30843	,40382
20	12,46221	11,97753	,48468	11,46992	11,06373	,40619
30	15,37245	14.91902	,45343	13,76483	13,41805	,34678
40	17,15909	16,78200	,37709	15,04630	14,78309	,26321
50	18,25593	17,96190	,29403	15,76186	15,57456	,18730
70	19,34268	19,18247	,16021	16,38454	16,29953	,08501
100	19,84791	19,79231	,05560	16,61755	16,59510	,02245
F.S.	20.00000	20,00000	,00000	16,66666	16.66666	,00000

Note, That this Correction is alſo applicable to the Valuations of Eſtates for Lives, in which the firſt Step being to find an Equivalent in Years of Continuance, brings them to the Caſe of Eſtates for Years.

2. The abſurdity of the Suppoſition, on which the uſual Method of computing preſent Values by Simple-Intereſt, is founded, *viz.* That the Rent or Annuity is conſtantly received, and put out again at Intereſt, as it becomes due; but that the Intereſt of the Purchaſe Money lies dead during the whole Term, is ſo apparent, and the Errors ariſing from it ſo groſs, that the Writers who have laid down this Method, have at the ſame time caution'd againſt the Uſe of it for any more than 6 or 7 Years, the Error for that time being not conſiderable. *Of Simple Intereſt.*

The ſame Suppoſition does alſo occaſion the miſcomputation of Amounts, or rather the miſapplication of them to their proper Caſes.

Where-

Wherefore, ſince the Simple-Intereſt of Money is of equal Value *pro rata*, and of the ſame regard with Rents or Annuities, being each the Original Profits iſſuing alike from a principal Stock, Eſtate or Value, and equally improveable; This general Rule may ſerve for a juſt Correction of this Method, *viz.* That ſuppoſing in any Caſe an Intereſt ought to be, or not to be allowed to either of thoſe Profits, the ſame be done in the like Caſe to the other. Thus, in the Caſe of Debts, or Amounts of Sums of Money, Rents, or Annuities for the time paſt, its uſual in practiſe to allow no Intereſt to either: For though the Law, to curb the exorbitant Avarice of Uſurers, and for other Reaſons, does more expreſly diſallow Intereſt upon Intereſt for a Debt; our Courts both of Law and Equity, as I am inform'd, will be as far from allowing the Charge of Intereſt againſt a Tenant for Rent in Arrear; except on a *nomine pœnæ* (which is now become almoſt obſolete) ſo that in this Caſe (putting a and r for the Annuity and Rate yearly, half-yearly, or otherwiſe) as $prn+p$ is the Amount of a Sum of Money, ſo is an the Amount of an Annuity or Rent in Arrear, and not $\overline{\frac{n-1}{2}r+1}\times an$, as Arithmeticians commonly make it. But in the Computation of preſent Values, or Amounts for the time to come, the ſame being made on the Expectation of a conſtant regular Income of the Profits, without any extraordinary Interruption, an Intereſt ought to allow'd to both, eſpecially in preſent Values, which are found by ſetting the Amounts of both againſt each other: ſo that in theſe Caſes, putting $x=n-1$, if $\overline{\frac{1}{2}xr+1}\times an$, be made the Amount of a Rent or Annuity; then $\overline{\frac{1}{2}xr+1}\times prn+p$, will be the proper Amount of a Sum of Money, and not $prn+p$: and conſequently $\frac{\overline{\frac{1}{2}xr+1}\times an}{\overline{\frac{1}{2}xr+1}\times rn+1}=p$ will be the preſent Value of a Rent or Annuity, the ſubſequent Intereſt being remitted on both ſides in lieu of the loſs of Time and Charge of Management; which ſuch as are apt to depreciate long Futurities, may think the propereſt Method of approaching the true Value; but I rather look upon the former Method of Compound Intereſt corrected as more exact, as well as more general, the Intereſt remitted in this being in ſhort Terms leſs, and in long Terms more than an Equivalent for the Trouble, Charge and Delay in the Management. But it is however the moſt exact of any of the Methods, that have yet been deduc'd from Simple-Intereſt. The reduc'd Rate may alſo in ſome Caſes be perly made uſe of for Amounts, but not for preſent Values, except for ſhort Terms; and then, ſince $r : a :: \overline{\frac{1}{2}x\mathfrak{r}+1}\times \mathfrak{r}n : \overline{\frac{1}{2}x\mathfrak{r}+1}\times an\frac{\mathfrak{r}}{r}$, 'twill be $\overline{\frac{1}{2}x\mathfrak{r}+1}\times an\frac{\mathfrak{r}}{r}=m=\overline{\frac{1}{2}x\mathfrak{r}+1}\times p\mathfrak{r}n+p$, and $\frac{\overline{\frac{1}{2}x\mathfrak{r}+1}\times an}{\overline{\frac{1}{2}x\mathfrak{r}+1}\times \mathfrak{r}n+1}\times\frac{\mathfrak{r}}{r}=p$.

Examples

Examples of this Method compar'd with those of the former will stand as follows; in which all is computed Half-yearly, except the last Column of Compound Interest.

Amounts of 1 *l.* at 5 *per Cent.* computed 6 several Ways.

Years.	1	2	3	4	5	6
	Simple Int.	*Id. for Bonds*	*Sim. Int. cor.*	*Id. by the red rat.*	*Co. Int. cor.*	*Comp. Int.*
	$prn+p=m$	Id. till $prn=p$	$\frac{1}{2}xr+1\times prn+p=m$	$\frac{1}{2}xr+1\times prn+p=m$	$\overline{r+\frac{1}{2}l}^{\,n}\times p=m$	$\overline{r+Rl}^{\,N}\times p=m$
5	1,25	1,25	1,27813	1,25501	1,25655	1,27628
10	1,5	1,5	1,61875	1,56338	1,57892	1,62889
20	2,	2,	2,48750	2,34021	2,49300	2,65330
30	2,5	2,	3,60625	3,33048	3,93625	4,32194
40	3,	2,	4,97500	4,53419	6,21504	7,04000
60	4,	2,	8,46250	7,58194	15,49408	18,67919
80	5,	2,	12,95000	11,48345	38,62672	49,56144
100	6,	2,	18,43750	16,23874	96,29634	131,50126
			Amounts of 1 l. *at* 6 per Cent. *by the same Theorems.*			
5	1,3	1,3	1,3405	1,31063	1,31328	1,33822
10	1,6	1,6	1,7710	1,69758	1,72471	1,79084
20	2,2	2,	2,9020	2,70048	2,97463	3,20713
30	2,8	2,	4,3930	4,00870	5,13039	5,74349
40	3,4	2,	6,2440	5,62223	8,84844	10,28572
60	4,6	2,	11,0260	9,76525	26,32086	32,98769
80	5,8	2,	17,2480	15,12954	78,29488	105,79599
100	7,	2,	24,9100	21,71510	232,89852	339,30208
			Amounts of 1 l. *at* 10 per Cent. *by the same Theorems.*			
5	1,5	1,5	1,6125	1,54749	1,55970	1,61051
10	2,	2,	2,4750	2,30157	2,43268	2,59374
20	3,	2,	4,9500	4,42951	5,91793	6,7275[illegible]
30	4,	2,	8,4250	7,38381	14,39643	17,44940
40	5,	2,	12,9000	11,16449	35,02190	45,25925
60	7,	2,	24,8500	21,20493	207,25717	304,48165
80	9,	2,	40 8000	34,55084	1225,5335	2048,4003
100	11,	2,	60,7500	51,20222	7258,5398	13780,6127

Years.	*Pres. Values of* 1 l. per An. *at* 5 per Cent. *by*			*Pres. Values of* 1 l. per An. *at* 6 per Cent. *by*		
	Sim. In. cor.	*Co. Int. cor.*	*Comp. Int.*	*Sim. In. Cor.*	*Co. Int. Cor.*	*Comp. Int.*
5	4,35208	4,08343	4,32948	4,23349	3,97582	4,21236
10	7,64478	7,33314	7,72174	7,25579	7,00322	7,36009
15	10,10821	9,91935	10,37966	9,39341	9,30843	9,71225
20	11,95980	11,97753	12,46221	10,92350	11,06373	11,46992
30	14,45407	14,91902	15,37245	12,87275	13,41805	13,76483
40	15,97990	16,78200	17 15909	13,99744	14,78309	15,04630
50	16,96682	17,96190	18,25[illegible]93	14,69544	15,57456	15,76186
70	18,10986	19,18247	19,34268	15,47252	16,29953	16,38454
100	18,91525	19,79231	19,84791	15,99759	16,59510	16,61755
F.S.	20,00000	20,00000	20,00000	16,66666	16,66666	16,66666

The

The Theorems to the preceding Columns of Amounts (of which the fourth and fifth are infinitely variable in the Refult, by affuming *c* and *t* in the reduc'd Rates at Pleafure) may ferve to anfwer all the fimple Cafes of Amounts that occur in Bufinefs: To inftance in fome.

1. The firft Column contains the Amounts of fuch Debts, or Sums of Money as carry a fimple Intereft till the Principal be paid.

2. The fecond Column anfwers the common Cafe of Debts due by Bond, that by Law are allow'd not to exceed the Penalty, which is generally double the principal Debt.

3. The third Column anfwers the Cafe of a Security or joynt Obligor, that has duly and conftantly paid the Intereft, and at laft the principal Sum of a Debt, from which he has a Counter-Bond from the principal Debtor to fave him harmlefs, againft whom he may make his Charge from this Column.

4. In cafe the Parties fhall agree that the Debt fhall lie for any time certain, or uncertain; and for the much greater eafe, advantage and fatisfaction of both of them, no Intereft to be call'd for, till the Principal it felf is paid; but to carry Intereft as it becomes due, the Lender allowing for the time and charge he muft have been at, in receiving and putting out his Intereft, the fourth Column will fit this Cafe, or elfe the fifth as a greater or lefs degree of Lenity is agreed upon in favour of the Borrower.

5. The fifth Column is alfo proper in the following Cafe, *viz.* if it be demanded, what Eftate in reverfion, after a certain number of Years, any Sum in Hand will Purchafe; the firft Step being to find the Amount of that Sum to the time the Reverfion commences, its had in this Column.

6. The laft Column gives the Amount of a Sum of Money, according to the common Method of Compound Intereft, but being computed with that extraordinary rigor as has been faid, (except fome fmall allowance for the lofs of time, by being done Yearly) 'tis hardly fuitable to any Cafe.

Of Intereft Accounts.

3. The Inequality of the ufual Method of ftating Intereft Accounts, as practis'd in our Courts of Equity, will beft appear by an Example; for which I fhall take the common general Cafe, of an Intereft Account to be ftated on a Mortgage, *viz.* fuppofe 1000 *l.* to be let out at 6 *per Cent.* on a Mortgage of 120 *l. per annum*, payable Half-Yearly, and the Mortgagee after five Years end, to have Poffeffion till the Arrear of Intereft, accruing Intereft and Principal be difcharged: *Quære*, How long that will be? fuppofing alfo, for the fake of brevity in the Account, the Payments to be equally and punctually made as they become due. By the Chancery Method, the Rent is firft apply'd to difcharge the Arrear of Intereft; and then the remainder of every Half-Years Rent, after deducting the fame Half-Years Intereft, is apply'd towards the Difcharge of the Principal, and thereby the Principal and Intereft continually leffens, till the whole be paid off. Now by this means the Mortgagee, after the Arrear is difcharged, pays Compound Intereft, with the utmoft rigour, for fo much *per annum* of the Rent, as exceeds the Intereft of the

whole

whole Principal Money, and receives but Simple Intereſt for his Debt; which, however ſtrange it may ſeem, is eaſily prov'd, by applying the proper Theorems of Simple and Compound Intereſt to this Caſe, in which the Annuity, Principal Money, Rate and Arrear of Intereſt are given, and the time requir'd; the reſult being the ſame with that of the Chancery Method, except a very ſmall difference only when any part of the time is expreſs'd by a Fraction: *viz.* putting L for Logarithm, $\alpha = a - pr = 30$, $s = 1 + r = 1{,}03$, $t =$ time of contracting the Arrear $= 10$ Half-years, $n =$ any number of Half-years ſpent in diſcharging the whole or any part, $N =$ number of Years required; the Equation for the Arrear will be $prt + prn = \alpha n$; and for the Principal and accruing Intereſt $prn + p = prn + \frac{s^n - 1}{r} \times \alpha$. Whence $\frac{prt}{2\alpha} + \frac{La - L\alpha}{2Ls} = N = 16{,}7249$ Years $=$ the time demanded; *i. e.* $\frac{prt}{2\alpha} = 5$ Years $=$ the time of diſcharging the Arrear and $\frac{La - L\alpha}{2Ls} = 11{,}7249$ Years, $=$ the time in which the Principal and accruing Intereſt is diſcharged; during which its evident, the Mortgagee pays full Compound Intereſt for 60 *l. per annum* of the Rent. For the Correction of which inequality, in the firſt Place, to the end that neither Branch may exceed, or be depriv'd of its due Profits; This general Rule is propos'd as neceſſary to be always obſerv'd, *viz.* That Amounts of the Produce on each ſide be ſtated ſeparately, and ſet againſt each other in the Account, in order to a Ballance. And in the common Caſes of Mortgages, Government, and Stock-Securities, *&c.* where the Debt is paid off by a Rent, Annuity, Penſion, Dividend, or other Payments iſſuing in the ſame manner, and with the like trouble, charge, *&c.* as Intereſt Money does; I preſume this Rule will alſo be eaſily admitted, *viz.* That the ſame equitable Advantage be allowed on both ſides; for which the Method of Simple Intereſt, as corrected under the foregoing Head, ſeems truly adapted; whereby the Original Profits on each ſide are ſuppos'd to be deem'd, either as Intereſt, or elſe as Principal Money; and ſince the Amounts both of an Annuity and Sum of Money, for the time paſt, as there ſtated, on the firſt of theſe Suppoſitions (t being there $= 0$) are likewiſe vouch'd by our Laws, and the practice of our Courts, to be good when ſeparately us'd; I think its very evident, that the Account ought to be ſtated by ſetting thoſe Amounts againſt each other thus, $prt + prn + p = \alpha n$, (whence $\frac{1 + tr}{2\alpha} p = N$, $= 21{,}6666$ Years) and that this Method is moſt proper for general Uſe, in the Caſes mention'd: Unleſs it ſhould be thought fit, in conſideration of the various Ways found out for the ready improvement of Money, to allow a further Advantage on both ſides, by charging the Original Profits as Principal Money, and giving a Simple Intereſt thereto, which ſtill falls ſhort of the

Advantage allow'd to Rents by the Chancery Method. And this is to be done two ways, *viz.* either by applying an Amount of Rent to pay off the Arrear firſt, and afterwards another Amount of Rent to diſcharge the Principal, and accruing Intereſt; or elſe by letting the Profits with all Arrears and other Charges run on at Simple Intereſt on each ſide, till the end of the Term: *viz.* putting $x = n - 1$, $y = t - 1$, $\alpha = a - pr$, $f = \overline{2 - r} \times \alpha$, $g = f - 2ptr^2$, $\mu = \overline{\tfrac{1}{2} yr + 1} \times prt =$ Arrear of Intereſt; by the firſt of theſe 'twill be, for the Arrear, $\mu + ptr^2 n = \overline{\tfrac{1}{2} xr + 1} \times \alpha n$, and for the Principal and accruing Intereſt, $\overline{\tfrac{1}{2} xr + 1} \times \alpha n = p$; Whence $\dfrac{\sqrt{8\mu\alpha r + g^2} - \sqrt{8p\alpha r + f^2} - f + g}{4\alpha r} = N = 18{,}7653$ Years. By the other 'twill be, for the whole, $\mu + ptr^2 n + p = \overline{\tfrac{1}{2} xr + 1} \times \alpha n$. Whence $\dfrac{\sqrt{\overline{\mu + p} \times 8\alpha r + g^2} - g}{4\alpha r} = N = 18{,}1648$ Years. The Lender will have to alledge for the firſt of theſe two Ways, that as the Rent is not hindred by any other parallel Charge from making the utmoſt produce it can, ſo for that reaſon ought his Principal Money to have the Advantage of the Arrears being firſt diſcharged, which alſo agrees with the Chancery Method in this particular.

Laſtly, another way of ſtating this Account, may be taken from that Notion of Simple-Intereſt, whereby the Annuity only is charged as Principal Money: and then 'twill be, for the Arrear, $prt + prn = \alpha n$, and for the Principal and accruing Intereſt, $\overline{\tfrac{1}{2} xr + 1} \times \alpha n = p$; Whence $\dfrac{\sqrt{8p\alpha r + f^2} - g}{4\alpha r} \left(= \dfrac{prt}{2\alpha} + \dfrac{\sqrt{8p\alpha r + f^2} - f}{4\alpha r}\right) = N = 17{,}3072$ Years; which appears to be the ſame with the Chancery Method, only that the Compound Intereſt in that, is turned into Simple in this; and as it ſtill retains part of the ſame inequality, to the Advantage of the Borrower, it ſeems only fit to be obſerved in ſuch Caſes wherein the Borrower may be thought to merit favour, as when the Debt is paid out of the Profits of Trade, ariſing by extraordinary Riſque or Induſtry.

The following Specimen ſhews at ſight, how the Reſults of the ſeveral Methods differ, as the Rent, Arrear, or Rate of Intereſt, is greater or leſs, and conſequently of how much more or leſs concern it is to the Parties, as well as to the due Adminiſtration of Juſtice, to have regard thereto.

The time requir'd in	Years (computed Half-yearly)			
To diſcharge a Mort. of 1000 *l.* by a Rent of	120*l. per annum.*		90 *l. per annum.*	
At the Rate of Int. *per annum* of	5 *per* C	6 *per* C	5 *per* C.	6 *per* C.
No Arrear of Intereſt.				
By the Chancery Method	10,9141	11,7249	16,4205	18,5835
By the ſame turning the Comp. Int. into Sim.				
BySimp.Int.cor. } Princ. and { contin^d at Int.	11,2579	12,3072	17,5335	20,7400
theOriginalPro- } theArrear { firſt diſcharg'd				
fits charged as } Intereſt	14,2857	16,6666	25,0000	33,3333
5 Years Arrear of Intereſt.				
By the Chancery Method	14,4855	16,7249	22,6705	28,5835
By the ſame turning the Comp. Int. into Sim.	14,8293	17,3072	23,7835	30,7400
BySimp.Int.cor. } Princ. and { continued at Int.	15,3191	18,1648	24,7148	32,7064
theOriginalPro- } the Arrear { firſt diſcharg'd	15,5897	18,7653	25,4899	34,8052
fits charged as } Intereſt	17,8571	21,6666	31,2500	43,3333
10 Years Arrear of Intereſt.				
By the Chancery Method	18,0569	21,7249	28,9205	38,5835
By the ſame turning the Comp. Int. into Sim.	18,4007	22,3072	30,0335	40,7400
BySimp.Int.cor. } Princ. and { continued at Int	20,3161	25,5998	33,6320	48,0874
theOriginalPro- } theArrear { firſt diſcharg'd	21,2895	27,5587	36,1932	53,8107
fits charged as } Intereſt	21,4286	26,6666	37,5000	53,3333
15 Years Arrear of Int.				
By the Chancery Method	21,6283	26,7249	35,1705	48,5835
By the ſame turning the Comp. Int. into Sim.	21,9721	27,3072	36,2835	50,7400
BySimp.Int.cor. } Princ.and { continued at Int	26,1353	34 2913	43,9764	65,8386
theOriginalPro- } theArrear { firſt diſcharg'd	28,0209	37,7141	48,5159	74,5664
fits charged as } Intereſt	25.0000	31,6666	43,7500	63,3333

I muſt alſo obſerve for the ſake of ſuch as are unacquainted with ſpecious Arithmetick, that though for brevities ſake, the foregoing Theorems and Examples are laid down, and wrought in Algebraic Terms: Yet the ſame Accounts may be ſtated after the Chancery manner it ſelf, according to the ſeveral Principles before deliver'd, and with the ſame Reſults, with this only caution, that (inſtead of a continual deduction of the Rent or Annuity out of the Principal and Intereſt of the Debt, which occaſions the Error before mention'd) the preceding General Rule of ſtating ſeparate Amounts be obſerv'd, which may be done by continually adding the profits together on each ſide, in the ſame manner, as if the Parties were to make a ſeparate Charge againſt each other, which is the rather to be noted, as being the only Courſe that can be taken, in caſe the Sums or Times of payment ſhould differ, but the reſpective Reſults will notwithſtanding be analogous to the above Examples.

XXVIII. Ad Logarithmorum inventum perficiendum, hoc tantum inveniendum ſupereſſe videtur; ut ſcil. omnes Series Logarithmicas inveniendi Methodum habeamus generalem; talis autem eſt hæc quæ ſequitur, facilis quidem illa & genuina, utpote ex ipſa Logarithmorum Natura deducta. *Logarithmotechnia Generalis. by Mr.* Craig. *n.* 328. *p.* 191.

Per literam *l* numero cuilibet præfixam denotetur (ut vulgo ſolet) iſtius Numeri Logarithmus. Jam quoniam Numeri cujuſvis propoſiti Logarithmus

rithmus duobus modis investigari potest, ideo Logarithmotechniæ hujus duas partes constituemus: in priori Logarithmum immediate ex ipso numero deducimus; in posteriori vero Numerorum aliquot antecedentium Logarithmi adhibentur, ut ex iis propositi Numeri Logarithmus inveniatur.

Pars Prior. Sit $a+1$ numerus quilibet propositus, & x ejus Logarithmus inveniendus. Jam ex hypothesi $x = l.\overline{a+1}$, quæ æquatio vocetur Canon generalis. (1.) Fiat æquatio inter terminos ex a & y utcunque compositos, & cum aliis quibusvis numeris quovis modo per Additionem, Substractionem Multiplicationem, Divisionem aut Radicum extractionem combinatos. (2.) Ope æquationis sic ad libitum assumptæ exterminetur a ex Canone generali, & habebitur æquatio exprimens relationem inter indeterminatos x, y. (3.) Hujus æquationis (per regulam *Bernoullianam*) inveniatur Differentialis, & hujus Integralis (per methodos notissimas) per Seriem Infinitam expressa dabit Logarithmi quæsiti x valorem cognitum.

Exemplum 1. Assumatur $a = y$, unde per Canonem generalem $x = l.\overline{1+y}$, cujus differentialis est $\dot{x} = \frac{\dot{y}}{1+y}$, & hujus integralis per Seriem infinitam expressa dat

$$x = y - \tfrac{1}{2}y^2 + \tfrac{1}{3}y^3 - \tfrac{1}{4}y^4 + \tfrac{1}{5}y^5 - \tfrac{1}{6}y^6 + \tfrac{1}{7}y^7 \text{ \&c.}$$

Exemplum 2. Assumatur $y = \frac{a}{a+2}$, unde $a+1 = \frac{1+y}{1-y}$, ideoque per Canonem generalem $x = l.\frac{1+y}{1-y}$, cujus Differentialis est $\dot{x} = \frac{2\dot{y}}{1-yy}$; & hujus Integralis in Seriem resoluta dat.

$$x = 2 \times y + \tfrac{1}{3}y^3 + \tfrac{1}{5}y^5 + \tfrac{1}{7}y^7 + \tfrac{1}{9}y^9 \text{ \&c.}$$

Ubi numerus 2 Seriei præfixus multiplicari supponitur in singulos Seriei terminos. Nec plura addere exempla opus hic erit, cum ex his pateat Methodus inveniendi innumeras Series Logarithmicas, quæ, absque ullo ad aliorum numerorum Logarithmos respectu, exhibent numeri propositi Logarithmum. *Q.E.I.*

Lemma. 1. Sit z Logarithmus cujusvis fractionis $\frac{b}{a+1}$, x Logarithmus denominatoris $a+1$; erit $lb - z = x$: Vel si sit z Logarithmus fractionis $\frac{a+1}{b}$, erit $lb + z = x$.

Lemma 2. Sit e exponens cujusvis potestatis numeri b; erit $l.\,b^e = e \times l.\,b$; ideoque datis Logarithmo numeri b^e & exponente e, datur ipsius b Logarithmus: Et ex Natura Logarithmorum constat utrumque Lemma.

Pars Posterior. Sit (ut prius) $a+1$ Numerus cujus Logarithmus x est inveniendus; sitque b Numerus productus ex Multiplicatione Numerorum, quorum maximus est minor quam $a+1$; & z Logarithmus fractionis $\frac{b}{a+1}$, id est $z = l.\frac{b}{a+1}$, quæ æquatio vocetur Canon generalis. Tum

(1.)

(1.) pro b sumatur quantitas ex a & numeris quibusvis determinatis utcunque composita, & hic valor numeri b sic ad libitum sumptus substituatur in fractione $\frac{b}{a+1}$, unde illa per a & numeros datos exprimetur. (2.) Fiat quælibet æquatio inter y & a cum numeris ad libitum sumendis; & ope hujus exterminetur a ex Canone generali, unde habetur æquatio exprimens relationem inter indeterminatos z, y. (3.) Hujus æquationis inveniatur (per Regulam *Bernoullianam*) Differentialis, hujusque Integralis (juxta Methodos notissimas) per Seriem infinitam expressa dabit fractionis $\frac{b}{a+1}$ Logarithmum z; & ex invento z habebitur (per *Lem.* 1.) numeri propositi $a+1$ Logarithmus $x = l.\, b - z$. Nam ex hypothesi, b^e producitur ex Multiplicatione Numerorum quorum maximus est minor quam $a+1$; & ex hypothesi dantur Logarithmi omnium numerorum proposito $a+1$ minorum, ergo & Logarithmus Numeri ex omnibus producti seu b^e, & proinde (per *Lem.* 2.) ipsius b Logarithmus datur.

Exemplum 1. Sumatur si placet $b=a$, unde $z=l.\frac{a}{a+1}$: Dein (per art. 2.) fiat ad libitum $y=2a+1$, per hanc exterminetur a, & erit $z=l.\frac{y-1}{y+1}$, cujus Differentialis est $\dot{z}=\frac{2\dot{y}}{yy-1}$; cujus Integralis per Seriem expressa dat $z=-2\times\frac{1}{y}+\frac{1}{3y^3}+\frac{1}{5y^5}+\frac{1}{7y^7}$ &c. Unde per *Lemma* 1. $x=lb+2\times\frac{1}{y}+\frac{1}{3y^3}+\frac{1}{5y^5}+\frac{1}{7y^7}+\frac{1}{9y^9}$ &c.

Exemplum 2. Fiat $b=\sqrt{aa+2a}$, unde $z=l.\frac{\sqrt{aa+2a}}{a+1}$, sumatur etiam ad libitum $y=2a+2a$, unde $z=l.\frac{1}{y}\sqrt{yy-4}$, cujus Differentialis est $\dot{z}=4\dot{y}\times\overline{y^3-4y}|^{-1}$, & hujus Integralis est $z=-2\times\frac{1}{y^2}+\frac{2^2}{2y^4}+\frac{2^4}{3y^6}+\frac{2^6}{4y^8}$ &c. Unde *Lemma* 1.

$$x=l.\,b+2\times\frac{1}{y^2}+\frac{2^2}{2y^4}+\frac{2^4}{3y^6}+\frac{2^6}{4y^8}+\frac{2^8}{5y^{10}} \text{ &c.}$$

Exemplum 3. Fiat $b=\sqrt{aa+2a}$, ut in præcedenti, sed jam assumatur $y^2=2aa+4aa+1$; Si per has duas æquationes exterminentur b & a ex Canone generali, erit $z=l\frac{\sqrt{yy-1}}{\sqrt{yy+1}}$, cujus Differentialis est $\dot{z}=2y\dot{y}\times\overline{y^4-1}|^{-1}$; & hujus Integralis per Seriem expressa est $z=-\frac{1}{y^2}-\frac{1}{3y^6}-\frac{1}{5y^{10}}-\frac{1}{7y^{14}}$ &c. Unde per *Lem.* 1.

$$x=l.\,b+\frac{1}{y^2}+\frac{1}{3y^6}+\frac{1}{5y^{10}}+\frac{1}{7y^{14}}+\frac{1}{9y^{18}} \text{ &c.}$$

Notan-

Notandum vero eſt quod numerus 2 Seriebus *Exemp.* 1 & 2. præfixus multiplicari ſupponitur in ſingulos ſerierum terminos. Similesque Series deduci poſſunt eodem modo ex $z = l\,\frac{a+1}{b}$, atqui tum $x = l,\ b + z$, ut conſtat ex *Lem.* 1. *Part.* 2. Ex his igitur ſatis ſuperque conſtat Logarithmotechniam jam expoſitam eſſe facillimam & maxime genuinam, necnon adeo generalem, ut duobus modis innumeræ Series inveniri poſſunt, Numeri cujuſvis propoſiti Logarithmum exhibentes: Nam innumeras (ad libitum) aſſumere licet æquationes relationem inter y & a exprimentes, quarum unaquæque exhibet novam Seriem Logarithmicam. Summa tamen adhibenda eſt Cura, ut tales aſſumantur, quæ efficient ut Serierum termini quam celerrime convergant, *i. e.* ut Logarithmus quam minimo Calculi labore inveniatur. Ad hoc præſtandum, perquam apta eſt Series in exemplo poſtremo exhibita, & quæ eadem eſt cum illa quam exhibuit Celeberrimus *Halleius* * in eleganti ſua Logarithmos conſtruendi Methodo.

* *Abr.* Vol. I. p. 108.

Logometria, ſive de menſura Rationis. by Mr. Cotes. *n.* 338. *p.* 5.

XXIX. Agitur in hoc Tractatu de *Menſuris Rationum.* Hæ Menſuræ ſunt quantitates cujuſcunque generis, quarum magnitudines magnitudinibus rationum ſunt analogæ. In dato itaque Syſtemate, rationis ejuſdem eadem eſt menſura, duplicatæ dupla, triplicatæ tripla, ſubduplicatæ ſubdupla, ſeſquiplicatæ ſeſquialtera: denique quocunque modo per compoſitionem vel reſolutionem auctæ vel diminutæ rationis, ſimiliter aucta eſt vel diminuta menſura. Æqualitatis ratio nullam habet magnitudinem, quia nullam addita vel detracta mutationem inducit; rationes quæ dicuntur majoris & minoris inæqualitatis contrarias habent magnitudinum ſuarum affectiones, quoniam in compoſitione & reſolutione contraria ſemper efficiunt: itaque ſi menſura rationis quam habet terminus major ad minorem poſitiva cenſeatur, menſura rationis quam habet terminus minor ad majorem erit negativa, menſura vero rationis inter æquales terminos nullius erit magnitudinis. Porro diverſa menſurarum oriuntur *Syſtemata*, prout modis diverſis exponitur analogia illa determinata & immutabilis quæ eſt inter magnitudines rationum. Inde vero patet, exhiberi poſſe numero infinita Syſtemata, minuendo vel augendo Syſtematis cujuſvis dati menſuras omnes in eadem data quacunque proportione, aut etiam pro menſuris adhibendo quantitates diverſi generis. In tanta autem varietate confuſionem aliquam oboriri neceſſe eſt, ni probe conſtiterit ad quodnam Syſtema referendæ ſint menſuræ ſingulæ de quibus contingat ſermonem inſtitui. Huic malo remedium optime parari poteſt ſi menſura datæ alicujus rationis, quæ commodiſſima videbitur, pro *Modulo* habeatur ad quem conſtanter in omni Syſtemate menſuræ reliquarum rationum exigantur. Id enim ſi fiat, ſtatim ex dato illo Modulo determinabitur Syſtema totum: nam ex menſuris conſtabit quæ Modulo erunt homogeneæ, quæque eo majores habebunt magnitudines vel minores quo major ille fuerit vel minor, ut ita menſurandarum rationum invariata magnitudinum ſervetur analogia inter ipſas menſuras. Patebit igitur in ſequentibus rationem quandam dari, dupli inter & tripli rationes intermediam, ad rationem

vero

vero tripli aliquanto propius accedentem, quæ proposito nostro non immerito aptissima judicetur, siquidem ipsa rei natura hujus usum suadere ac non incertis indiciis efflagitare quodammodo videatur. Hanc ego, ex officio ejus desumpto nomine, *Modularem Rationem* appellabo; quo autem pacto ipsa sit accuratius definienda, ostendetur inferius, nunc enim de Logarithmis pauca sunt addenda.

Logarithmi sunt rationum mensuræ Numerales: solent autem in Canone sic disponi, ut singulis numeris naturali ordine crescentibus, & in serie continua positis adscribatur Logarithmus, non quidem ipsius numeri uti vulgo dicitur, sed rationis quam habet numerus ad Unitatem. Exinde vero rationis per quoscunque terminos designatæ facilis est inventio Logarithmi. Nam cum ratio antecedentis ad consequentem sit excessus rationis antecedentis ad Unitatem supra rationem consequentis ad Unitatem: Logarithmus ejus similiter erit excessus Logarithmi rationis quam habet antecedens ad Unitatem supra Logarithmum rationis quam consequens habet ad Unitatem; hoc est, ut vulgari sermone utamur, excessus Logarithmi antecedentis supra Logarithmum consequentis; neutiquam enim displicet loquendi modus jam a multis annis receptus, si recte intelligatur. Exinde porro peregregium enascitur compendium ad operationes Arithmeticas. Datis enim duobus quibuscunque numeris in se multiplicandis, si quæratur numerus ex multiplicatione productus; quoniam rationes numerorum datorum ad Unitatem, conficiunt simul additæ rationem producti ad Unitatem, & rationum componendarum mensuræ simul additæ conficiunt rationis compositæ mensuram: Logarithmus producti æquabitur Logarithmis numerorum datorum simul sumptis. Ad eundem modum si quæratur numerus ex divisione ortus; quoniam ratio divisoris ad Unitatem e ratione dividendi ad Unitatem detracta relinquit rationem quoti ad Unitatem: habebitur quoti Logarithmus subducendo Logarithmum divisoris e Logarithmo dividendi. Et eodem argumento, si quæratur dati cujusvis numeri quælibet potestas; quoniam ratio dati numeri ad Unitatem per Indicem potestatis multiplicata rationem efficit quam habet numeri potestas ad Unitatem, & mensura prioris rationis multiplicata per eundem Indicem efficit pariter mensuram rationis posterioris: Logarithmus potestatis æquabitur Logarithmo numeri dati per Indicem potestatis multiplicato. Et similiter Logarithmus cujuslibet radicis numeri dati æquabitur Logarithmo numeri dati per Indicem radicis diviso. Igitur ope Canonis peragetur inventio potestatum & radicum per multiplicationem & divisionem, multiplicatio autem & divisio per additionem & subductionem. Cæterum de hisce vulgo notis Logarithmorum usibus non est mei instituti fusius differere: missis ergo ambagibus, ad alia nunc me confero, & rem ipsam protinus aggredior.

Propositio I. *Invenire Mensuram Rationis cujuscunque propositæ.*

Proponatur Ratio inter AC & AB, cujus Mensuram oportet invenire. Terminorum differentia BC divisa concipiatur in particulas innumeras quam minimas PQ, atque ratio inter AC & AB in totidem rationes

quam

quam minimas inter AQ & AP: & si detur magnitudo rationis inter AQ & AP, dividendo dabitur ratio quam habet PQ ad AP; atque adeo data illa magnitudo rationis inter AQ & AP, per datam quantitatem $\frac{PQ}{AP}$ exponi potest. Manente AP, augeri vel minui intelligatur particula PQ in proportione quavis; & in eadem proportione augebitur vel mi-
Plate 3. *Fig.* 3. nuetur magnitudo rationis inter AQ & AP: capiatur particula dupla vel tripla, subdupla vel subtripla, & evadet ratio duplicata vel triplicata, subduplicata vel subtriplicata; etiamnum igitur exponetur per quantitatem $\frac{PQ}{AP}$. Sed &, assumpta determinata quavis quantitate M, exponi potest per $M \times \frac{PQ}{AP}$: erit ergo quantitas $M \times \frac{PQ}{AP}$ mensura rationis inter AQ & AP. Hæc vero mensura diversam habebit magnitudinem, & ad Systema diversum accommodabitur, pro diversa magnitudine quantitatis assumptæ M, quæ adeo vocetur Systematis *Modulus*. Jam quemadmodum summa rationum omnium inter AQ & AP æqualis est propositæ rationi, quam utique habet AC ad AB: ita summa mensurarum omnium $M \times \frac{PQ}{AP}$ (per Methodos satis notas invenienda) æqualis erit ejusdem propositæ rationis mensuræ quæsitæ. *Q. E. I.*

Corol. 1. Terminis AP, AQ ita ad æqualitatem accedentibus, ut quam minima sit eorundem differentia PQ: erit $M \times \frac{PQ}{AP}$ vel $M \times \frac{PQ}{AQ}$ æqualis mensuræ rationis inter AQ & AP ad Modulum M.

Corol. 2. Unde Modulus ille M est ad mensuram rationis inter terminos AQ & AP, ut terminorum alteruter AP vel AQ ad terminorum differentiam PQ.

Corol. 3. Data ratione inter AC & AB, datur summa omnium $\frac{PQ}{AP}$, & summa omnium $M \times \frac{PQ}{AP}$ est ut M. Itaque mensura datæ cujuscunque rationis est ut Modulus Systematis ex quo desumitur.

Corol. 4. Modulus ergo, in omni mensurarum Systemate, semper æqualis fit mensuræ rationis cujusdam determinatæ atque immutabilis: Quam proinde *Rationem Modularem* vocabo.

Scholium 1. Problematis solutio per Exemplum illustrabitur. Sit z quantitas determinata quævis & permanens, sit vero x quantitas indeterminata fluxuque perpetuo variabilis, ejusque fluxio sit $\dot{x}$; & quæratur mensura rationis inter $z + x$ & $z - x$. Statuatur hæc ratio æqualis rationi inter y & 1, exponatur autem numerus y per AP, fluxio ejus $\dot{y}$ per PQ, 1 per AB: & ex Corollario primo colligetur fluxionem quæsitæ mensuræ rationis inter y & 1 esse $M \times \frac{\dot{y}}{y}$. Reponatur jam pro y valor

ejus

ejus $\frac{z+x}{z-x}$, itemque pro y valoris fluxio $\frac{2z\dot{x}}{\overline{z-x}|^2}$: & fluxio menfuræ evadet $2M \times \frac{z\dot{x}}{zz-xx}$ vel $2M \times \frac{\dot{x}}{z-\frac{xx}{z}}$ five $2M$ in $\frac{\dot{x}}{z}+\frac{\dot{x}x^2}{z^3}+\frac{\dot{x}x^4}{z^5}+$ &c. Atque adeo menfura illa fiet $2M$ in $\frac{x}{z}+\frac{x^3}{3z^3}+\frac{x^5}{5z^5}+$ &c. Unde patet Corollarium fequens.

Corol. 5. Si duarum quantitatum fumma fit z & differentia fit x; & fumatur $2M\frac{x}{z}=A$, $A\frac{xx}{zz}=B$, $B\frac{xx}{zz}=C$, $C\frac{xx}{zz}=D$, &c. Menfura rationis quam habet quantitas major ad quantitatem minorem, erit $A+\frac{1}{3}B+\frac{1}{5}C+\frac{1}{7}D+$ &c.

Scholium 2. Non abfimili computo menfura rationis inter $1+v$ & 1 erit M in $v-\frac{1}{2}v^2+\frac{1}{3}v^3-\frac{1}{4}v^4+\frac{1}{5}v^5-$ &c. Unde fi menfura illa vocetur m, erit $\frac{m}{M}=v-\frac{1}{2}vv+\frac{1}{3}v^3-\frac{1}{4}v^4+\frac{1}{5}v^5$, &c. ac proinde $\frac{mm}{MM}=vv-v^3+\frac{11}{12}v^4-\frac{5}{6}v^5$, &c. fimiliterque $\frac{m^3}{M^3}=v^3-\frac{3}{2}v^4+\frac{7}{4}v^5$, &c. quinetiam $\frac{m^4}{M^4}=v^4-2v^5$, &c. ac denique $\frac{m^5}{M^5}=v^5$, &c. Ut igitur viciffim, ex data menfura m, inveniatur ratio quam metitur; addendo æqualia æqualibus habebitur $\frac{m}{M}+\frac{mm}{2MM}=v*-\frac{1}{6}v^3+\frac{5}{24}v^4-\frac{13}{60}v^5$, &c. atque iterum $\frac{m}{M}+\frac{mm}{2MM}+\frac{m^3}{6M^3}=v**-\frac{1}{24}v^4+\frac{3}{40}v^5$, &c. rurfumque $\frac{m}{M}+\frac{mm}{2MM}+\frac{m^3}{6M^3}+\frac{m^4}{24M^4}=v***-\frac{1}{120}v^5$, &c. atque tandem $\frac{m}{M}+\frac{mm}{2MM}+\frac{m^3}{6M^3}+\frac{m^4}{24M^4}+\frac{m^5}{120M^5}=v****$, &c. id eft, $\frac{m}{M}+\frac{mm}{2MM}+\frac{m^3}{6M^3}+\frac{m^4}{24M^4}+\frac{m^5}{120M^5}+$ &c. $=v$. Itaque ratio quæfita inter $1+v$ & 1, eft ea quam habet $1+\frac{m}{M}+\frac{mm}{2MM}+\frac{m^3}{6M^3}+\frac{m^4}{24M^4}+\frac{m^5}{120M^5}+$ &c. ad 1. Ponatur $m=M$, five $\frac{m}{M}=1$; & exinde ratio Modularis erit ea quam habet $1+\frac{1}{1}+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\frac{1}{120}+$ &c. ad 1.

Eodem modo, fi detur ratio inter 1 & $1-v$, menfura hujus rationis erit M in $v+\frac{1}{2}v^2+\frac{1}{3}v^3+\frac{1}{4}v^4+\frac{1}{5}v^5$, &c. Et viciffim fi detur rationis menfura m, ratio erit ea quam habet 1 ad $1-\frac{m}{M}+\frac{mm}{2MM}-\frac{m^3}{6M^3}+\frac{m^4}{24M^4}-\frac{m^5}{120M^5}+$ &c. Ponatur $m=M$, five $\frac{m}{M}=1$; & exinde

Ratio Modularis erit ea quam habet 1 ad $1 - \frac{1}{1} + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} + \&c.$ Ex hisce vero patet Corollarium sequens.

Corol. 6. Exposito termino R, si sumatur $\frac{1}{1} R = A$, $\frac{1}{2} A = B$, $\frac{1}{3} B = C$, $\frac{1}{4} C = D$, $\frac{1}{5} D = E$, *&c.* in infinitum; & capiatur $S = R + A + B + C + D + E + \&c$: Ratio Modularis erit ea quæ est inter terminum minorem expositum R & majorem inventum S. Vel exposito termino S, si sumatur $\frac{1}{1} S = A$, $\frac{1}{2} A = B$, $\frac{1}{3} B = C$, $\frac{1}{4} C = D$, $\frac{1}{5} D = E$, *&c.* in infinitum; & capiatur $R = S - A + B - C + D - E + \&c$: Ratio Modularis erit ea quæ est inter terminum majorem expositum S & minorem inventum R. Porro eadem ratio est inter 2,718281828459 *&c.* & 1, vel inter 1 & 0,367879441171 *&c.*

Scholium 3. Si forte termini minores desiderentur, qui eandem proxime Rationem Modularem ita exhibeant, ut nulli ipsis non majores propius: instituenda erit operatio ad modum sequentem. Dividatur terminus major 2,71828 &c. per minorem 1, vel etiam major 1 per minorem 0,367879 &c. & rursus minor per numerum qui reliquus est, & hic rursus per ultimum residuum, atque ita porro pergatur: & prodibunt quotientes 2, 1, 2,

Rationes vera Majores.		*Rationes vera Minores.*	
1	0 × 2	0	1
2	1	2	0
3	1 × 2	2	1 × 1
8	3	6	2
11	4 × 1	8	3 × 1
76	28	11	4
87	32 × 1	19	7 × 4
106	39	87	32
193	71 × 6	106	39 × 1
1264	465	1158	426
1457	536 × 1	1264	465 × 1
21768	8008	1457	536
23225	8544 × 1	2721	1001 × 8
25946	9545	23225	8544
49171	18089 × 10	25946	9545 × 1
&c.	&c.	&c.	&c.

1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1, 16, 1, 1, &c. His inventis, perficiendæ sunt binæ rationum columnæ, quarum altera terminos continet rationem habentes vera majorem, altera terminos quorum ratio est vera minor; ineundo computationem a rationibus 1 ad 0, 0 ad 1, quæ remotissimæ sunt a vera; inde autem exorsam deducendo ad rationes reliquas, quæ continuæ ad veram propius accedunt. Multiplicentur itaque termini 1 & 0 per quotientem primum 2, & scribantur facti 2 & 0 infra termi-

terminos o & 1; & addendo prodibit ratio 2 + o ad o + 1, sive 2 ad 1. Hujus termini multiplicentur per quotientem secundum 1, factique 2 & 1 addantur terminis 1 & o; & habebitur ratio 2 + 1 ad 1 + o, sive 3 ad 1. Hujus termini multiplicentur per quotientem tertium 2, factique 6 & 2 addantur terminis præcedentibus 2 & 1; & habebitur ratio 8 ad 3. Hujus termini multiplicentur per quotientem quartum 1, factique 8 & 3 addantur terminis præcedentibus 3 & 1; & habebitur ratio 11 ad 4. Hujus termini multiplicentur per quotientem quintum 1, factique 11 & 4 addantur præcedentibus 8 & 3; & habebitur ratio 19 ad 7. Hujus termini rursus multiplicentur per quotientem sextum 4, factique 76 & 28 addantur præcedentibus 11 & 4, ad inveniendam rationem 87 ad 32; & sic porro pergendum quousque libuerit, transitu alternis facto in alteram columnam. Hisce peractis, habebuntur rationes vera majores 3 ad 1, 11 ad 4, 87 ad 32, 193 ad 71, 1457 ad 536, 23225 ad 8544, 49171 ad 18089, &c. Vera autem minores erunt 2 ad 1, 8 ad 3, 19 ad 7, 106 ad 39, 1264 ad 465, 2721 ad 1001, 25946 ad 9545, &c. Atque hæ quidem sunt præcipuæ & primariæ rationes, quibus ad rationem propositam continue appropinquatur.

Quod si exquiratur integra series rationum omnium vera majorum quæ ita dari possint, ut nulla minoribus terminis designata ratio vera major ad veram propius accedat; & similiter series integra rationum omnium vera minorem quæ ita dari possint, ut nulla minoribus terminis designata ratio vera minor ad veram propius accedat, inter primarias illas modo inventas inserendæ sunt aliæ secundariæ rationes. Hæ vero locum habent ubi quotiens unitatem superat. Inveniuntur autem mutata multiplicatione, quæ supra per quotientem facta est, in continuam additionem terminorum tot vicibus quot sunt unitates in quotiente. Sic quia quotiens primus erat 2, termini 1 & o bis addendi sunt terminis o & 1; & summæ dabunt rationes 1 ad 1, 2 ad 1. Hi ultimi termini 2 & 1, quia quotiens secundus erat 1, semel addendi sunt terminis 1 & o; & summæ dabunt rationem 3 ad 1. Hi termini 3 & 1, quia quotiens tertius erat 2, bis addendi sunt terminis 2 & 1; & summæ dabunt rationes 5 ad 2, 8 ad 3. Hi ultimi termini 8 & 3, quia quotiens quartus erat 1, semel addendi sunt terminis 3 & 1; & summæ dabunt rationem 11 ad 4. Hi termini 11 & 4, quia quotiens quintus erat 1, semel addendi sunt terminis 8 & 3; & summæ dabunt rationem 19 ad 7. Hi denique termini 19 & 7, quia quotiens sextus erat 4, quater addendi sunt terminis 11 & 4; & summæ dabunt rationes 30 ad 11, 49 ad 18, 68 ad 25, 87 ad 32. Et sic porro procedere licebit quousque commodum videbitur. Ista tandem operatione peracta, series integra rationum omnium vera majorum, erit 1 ad o, 3 ad 1, 11 ad 4, 30 ad 11, 49 ad 18, 68 ad 25, 87 ad 32, *&c.* similiterque series integra rationum omnium vera minorum, erit o ad 1, 1 ad 1, 2 ad 1, 5 ad 2, 8 ad 3, 19 ad 7, *&c.*

Rationes Vera Majores.		*Rationes Vera Minores.*	
1	0 × 2	0	1
2	1	1	0
3	1 × 2	1	1
8	3	1	0
11	4 × 1	2	1 × 1
19	7	3	1
30	11	5	2
19	7	3	1
49	18	8	3 × 1
19	7	11	4
68	25	19	7 × 4
19	7	87	32
87	32 × 1	106	39 × 1.
&c.	&c.	&c.	&c.

Harum approximationum utilitas ad alia multa ſeſe diffundit: quapropter earum inventionem aliquanto prolixius expoſitam dedi, per Methodum quæ mihi ſimpliciſſima & facillima videtur. Idem argumentum paulo aliter pertractarunt Viri celeberrimi *Walliſius* & *Hugenius*.

Propoſitio II. *Logarithmorum Canonem* Briggianum *conſtruere.*

Numerorum Compoſitorum Logarithmi derivantur ex Logarithmis Primorum componentium, per additionem ſolam; horum autem inveſtigatio pluribus modis inſtitui poteſt: Exemplum unicum appono.

Per Corollarium quintum Propoſitionis ſuperioris, ſcribendo 1 pro M, inveniantur Logarithmi rationum inter 126 & 125, 225 & 224, 2401 & 2400, 4375 & 4374; qui vocentur reſpective p, q, r, s: & Logarithmus denarii ſeu rationis decupli erit $239p + 90q - 63r + 103s$, ſive 2,302585092994 *&c.* Itaque cum Logarithmus *Briggianus* denarii ſit 1; fiat (per *Corol.* 3. *Prop.* 1.) ut denarii Logarithmus modo inventus 2,302585092994 *&c.* ad Modulum ſuum 1, ita denarii Logarithmus *Briggianus* 1, ad Modulum *Briggianum*, qui adeo erit 0,434294481903 *&c.* Ponatur ergo deinceps iſte valor pro M, & erunt $M \times \overline{202p + 76q - 53r + 87s}$, $M \times \overline{167p + 63q - 44r + 72s}$, $M \times \overline{114p + 43q - 30r + 49s}$ Logarithmi *Briggiani* numerorum 7, 5, 3. Logarithmus numeri 2 habetur, ſubducendo Logarithmum numeri 5 a Logarithmo numeri 10. Atqua ita dantur & Modulus *Briggianus* & Logarithmi Primorum omnium qui ſunt minores denario.

Logarithmi numerorum ſequentium Primorum 11, 13, 17, 19, 23, *&c.* ita computari poſſunt. Quæratur tum factus a numeris Primo propoſito utrinque proxime adjacentibus, tum Primi ipſius quadratum, quod ſemper unitate factum illud ſuperabit. Logarithmo rationis quadrati ad factum

ctum per *Corol.* 5. *Prop.* 1. inveniendo) addatur ipſius facti Logarithmus, qui ſemper componetur ex datis Logarithmis Primorum qui propoſito Primo ſunt minores: & ſemiſumma erit Logarithmus Primi quæſitus.

Corol. Canonis *Briggiani* Modulus eſt 0,434294481903 *&c:* Hujus vero Reciprocus eſt 2,302585092994 *&c.*

Scholium. Ad hunc itaque modum perfici poſſet Logarithmorum Tabula ampliſſima, qualis edita eſt a *Briggio* vel *Vlacco.* Inventioni autem Numerorum & Logarithmorum ſibi invicem congruentium, qui intermedii ſunt & ultra Tabulæ limites excurrunt, abunde ſufficiet terminus primus Seriei quæ in Corollario quinto Propoſitionis præcedentis exhibetur.

Si dato Numero intermedio quæratur ejus Logarithmus; pone a & e pro Numero intermedio propoſito atque huic proximo tabulari, ita ut a deſignet majorem, e minorem; ſit eorum ſumma z, differentia x; pone λ pro Logarithmo rationis quam habet a ad e, hoc eſt, pro exceſſu Logarithmi Numeri a ſupra Logarithmum Numeri e: & erit $\lambda = 2M\frac{x}{z}$ quamproxime.

Si quæratur Numerus qui congruit Logarithmo intermedio; quoniam eſt $\lambda = \frac{2Mx}{z} = \frac{2Mx}{2a - x}$ vel $\frac{2Mx}{2e + x}$; erit $x = \frac{\lambda}{M + \frac{1}{2}\lambda}a$ vel $\frac{\lambda}{M - \frac{1}{2}\lambda}e$ quamproxime.

Propoſitio III. *Syſtematis cujuſvis Logometrici conſtructionem exponere per Canonem Logarithmorum.*

Caſ. 1. Si detur, e Syſtemate propoſito, menſura rationis alicujus determinatæ: rationis cujuſvis oblatæ menſura, erit ad menſuram illam datam determinatæ rationis, ut oblatæ rationis Logarithmus, ad Logarithmum rationis ejuſdem determinatæ.

Caſ. 2. Si non detur, e Syſtemate propoſito, menſura rationis alicujus determinatæ: inveniendus erit Modulus propoſiti Syſtematis, per Corollarium ſecundum Propoſitionis primæ. Et menſura cujuſvis oblatæ rationis, erit ad Modulum inventum, ut oblatæ rationis Logarithmus, ad Canonis Modulum.

Caſus hujus ultimi habentur Exempla in ſequentibus.

Propoſitio IV. *Spatium quodvis Hyperbolicum quadrare per Canonem Logarithmorum.*

Sit Hyperbola quævis $ERSF$ centro A, Aſymptotis ABC, AD, deſcripta; & quæratur area $BEFC$ quam claudunt rectæ BE, CF, ad Aſymptoton AD parallelæ. Compleatur parallelogrammum $ABED$, & ad hunc Modulum inveniatur (per Propoſitionem tertiam) menſura rationis inter AC & AB vel inter BE & CF: Dico menſuram inventam æqualem fore magnitudini areæ quæſitæ $BEFC$. Nam diviſa concipiatur hujus areæ baſis BC in particulas innumeras quam minimas PQ, ea lege, ut ubique detur ratio illa quæ eſt inter AQ & AP, & ducantur Aſymptoto AD parallelæ PR, QS. Quoniam itaque eſt AQ ut AP; erit diviſim *Fig.* 4.

PQ

PQ ut AP, hoc est, ut PR reciproce. Unde data est area $PRSQ$, quæ proinde potest haberi pro mensura rationis datæ quæ est inter AQ & AP. Hujus autem mensuræ Modulus erit parallelogrammum $ABED$, per *Corol.* 2. *Prop.* 1. Nam si compleatur æquale parallelogrammum $APRT$; statim intelligetur, ita illud se habere ad aream $PRSQ$, ut se habet AP ad PQ. Similes ergo summas arearum atque rationum utrinque colligendo; area tota $BEFC$ erit mensura rationis totius quæ est inter AC & AB, vel inter BE & CF, ad eundem Modulum $ABED$.

Aliter. Sit rursus Hyperbola quævis AP, centro C atque Asymptoto CB descripta; & quæratur area Sectoris cujuslibet CAP, semidiametris CA, CP curvæque AP interjecti. Producta semidiametro utravis CAQ ultra verticem A, ducatur illius conjugata CR; & ad ipsas CQ, CR ordi-
Fig. 5. natim applicentur a puncto P rectæ PQ, PR, quæ Asymptoto CB occurrant in Z & X, deinde agatur AB quæ Hyperbolam tangat in A, Asymptoton secet in B, rectamque CP in D; & Triangulo ABC existente Modulo, area quæsita sectoris CAP erit mensura rationis inter $QZ + QP$ & AB, sive rationis inter AB & $QZ - QP$, sive subduplicatæ rationis inter $QZ + QP$ & $QZ - QP$, sive subduplicatæ rationis inter $AB + AD$ & $AB - AD$; vel erit mensura rationis inter $RP + RX$ & CA, vel rationis inter CA & $RP - RX$, vel subduplicatæ rationis inter $RP + RX$ & $RP - RX$. Nam si ducantur rectæ AE, PF quæ secent Asymptoton CB in E & F, alterique Asymptoto parallelæ sint: æquales erunt hæ omnes rationes rationi quam habet AE ad PF, vel CF ad CE; erit & sector CAP areæ $EAPF$ æqualis, similiterque triangulum ABC duplicato triangulo AEC, sive parallelogrammo Asymptotis & Hyperbolæ inscripto æquabitur. Quare patet propositum ex supra demonstratis.

Data vero per modum priorem area $BEFC$, vel per modum posteriorem area CAP; dabitur alia quævis area Hyperbolica ad arcum EF, vel ad arcum AP terminata: quippe quæ semper est areæ modo inventæ & areæ alicujus rectilineæ vel summa vel differentia. *Q. E. I.*

Scholium. Hinc facilem habent solutionem Problemata omnia, quæcunque pendent ab Hyperbolæ quadratura. Exemplum satis luculentum præbebit descensus gravium in Mediis, quorum resistentia est in duplicata ratione velocitatis corporis moti. Sit V velocitas maxima quam corpus
Fig. 6. in hujusmodi Medio, infinite descendendo, potest acquirere; T dimidium temporis quo corpus idem in eodem Medio, vi sola ponderis sui relativi, absque resistentia cadendo velocitatem illam acquiret; S spatium hocce casu descriptum; R pondus relativum corporis in Medio resistente: & quæratur spatium s quod corpus descendens, tempore quovis t, describet in Medio resistente; & resistentia r quam patitur in fine illius temporis; & velocitas v ex isto descensu acquisita.

Centro D, vertice A describatur Hyperbola æquilatera AT, cujus una Asymptotorum est DC & ad verticem tangens AC semiaxi AD æqualis. Capiatur area DAT ad dimidium trianguli DAC ut t ad T, secetque DT tangentem AC in P: & erit v ad V ut AP ad AC. Sit AK ipsis AC, AP tertia proportionalis: & erit r ad R ut AK ad AC. Ad tangentem

gentem AC erigantur normales CZ, KN, AB; centroque C & Asymptotis CA, CZ describatur Hyperbola quævis BN: & erit s ad S ut area $ABNK$ ad rectangulum CKN. Patent hæc omnia per Propositiones octavam & nonam Libri secundi Philosophiæ *Newtonianæ*.

Est itaque t ad T ut area Hyperbolica DAT ad dimidium trianguli DAC, hoc est, ut dimidiata mensura rationis inter $AC + AP$ & $AC - AP$ ad illius mensuræ dimidiatum Modulum. Ergo si recta quævis EF producatur ad f, ita ut t sit mensura rationis inter Ef & EF ad Modulum T, & bisecetur Ff in G: erit GF ad GE ut AP ad AC, hoc est, ut v ad V. Sumantur GE, GF, GH continue proportionales: & erit GH ad GE ut AK ad AC, hoc est, ut r ad R. Erit insuper EG ad EH ut CA ad CK; unde cum sit s ad S ut area $ABNK$ ad rectangulum CKN, hoc est, ut mensura rationis inter CA & CK vel inter EG & EH ad mensuræ Modulum: erit s mensura rationis inter EG & EH ad Modulum S, atque inde dabitur. Fig. 7.

Ex hisce porro facillime se prodit, per unicam quamvis Hyperbolam, constructio non inconcinna; quam & adscribere visum est ob dignitatem Problematis. In recta quavis GE sumatur utcunque punctum F inter E & G, & ab altera parte capiatur Gf ipsi GF æqualis, & sint GE, GF, GH continue proportionales. Deinde per puncta E, F, H, G, f ducantur sibi invicem parallelæ rectæ ER, FL, HM, GQ, fl, quas secet Hyperbola quævis $LMQl$ centro E, Asymptotis ER, EG descripta, & compleatur parallelogrammum $EGQR$. Jam si sit t ad T ut area Hyperbolica $LFfl$ ad parallelogrammum EQ: erit s ad S ut area $MHGQ$ ad EQ; v ad V ut GF ad GE; r ad R ut GH ad GE.

Libet & casum alterum adjicere ubi corpus ascendit; ne forte analogia illa, quæ inter utrumque servari debet, in allata constructione quodammodo perire videatur. Ergo eadem atque prius denotantibus V, R, T, S, ponantur v & r pro velocitate & resistentia sub ascensus initio, s pro spatio quod corpus ascendendo describere possit antequam tota velocitas amittatur, t pro tempore hujus ascensus. Ad EG erigatur perpendicularis GO ipsi EG æqualis, & sumendo puncta F, f, ad easdem distantias hinc inde a puncto G, jungantur OF, Of, quibus occurrat in T & t circuli arcus TGt centro O descriptus, & sint Gh, Gf, GE continue proportionales, & ducatur ipsi ER parallela hm Hyperbolæ occurrens in m. Deinde si t sit mensura anguli FOf ad Modulum T, hoc est, si t sit ad T ut arcus TGt ad radium OG: erit s mensura rationis inter Eh & EG ad Modulum S, vel erit s ad S ut area Hyperbolica $mhGQ$ ad EQ; & v erit ad V ut Gf ad GE; atque r ad R ut Gh ad GE. Fig. 8.

Propositio V. *Logisticam describere per Canonem Logarithmorum.*

Si ad Logisticæ $BQDG$ Asymptoton $APCF$ ordinatim applicentur binæ quævis rectæ AB, FG intercludentes Asymptoti portionem quamvis AF: erit illa portio mensura rationis quam ad invicem habent ordinatæ; hæc utique est natura Curvæ notissima. Integrum ergo & perfectum Systema Logometricum per hanc Lineam exhibetur; id quod

etiam

etiam de Hyperbola dici potest per Propositionem præcedentem, de Spirali Æquiangula per subsequentem; nam omitto complures alias Figuras, quæ & ipsæ dudum sunt in Geometriam receptæ. Itaque si detur Asymptoti positio & simul duo puncta per quæ Curva transire debet, dabuntur puncta reliqua per casum priorem Propositionis tertiæ. Quod si data positione Asymptoti, detur insuper Systematis Modulus atque unicum punctum per quod ducenda erit Curva; invenientur puncta reliqua per Casum posteriorem Propositionis ejusdem. Iste vero Modulus quo pacto definiendus sit, & qualem habeat magnitudinem, jam oportet exponere.

Ducatur recta BC quæ Curvam tangat in B & Asymptoton secet in C. Dico primo, magnitudinem subtangentis AC eandem permanere ubicunque sumatur punctum B. Intelligatur enim Ordinata PQ vicinissima Ordinatæ ARB, recta vero QR parallela Asymptoto AC, ac detur Ordinatarum intervallum illud quam minimum AP. Ob datam igitur lineolam AP, dabitur ratio quam habet AB ad PQ, & divisim ratio quam habet AB ad RB, atque adeo (propter similia triangula BAC, BRQ) ratio quam habet AC ad RQ sive AP, atque inde magnitudo ipsius AC.

Fig. 9.

Dico secundo, determinatam hanc & immutabilem subtangentem AC, esse Modulum ad quem exigendæ sunt mensuræ illæ interceptæ AF. Patet hoc per Corollarium secundum Propositionis primæ: nam dum termini AB & PQ ad æqualitatem proxime accedunt, erit AC ad AP, quæ metitur rationem inter AB & PQ, ut terminus AB ad terminorum differentiam BR. Unde data subtangente, facilis est descriptio Curvæ & solutio Problematum omnium quæ exhinc pendent.

Si Curva jam descripta habeatur, subtangentis magnitudo sic determinabitur. Producatur Ordinata quævis CD ad E, ita ut CE ad CD rationem habeat Modularem, per Corollarium sextum Propositionis primæ definitam; & recta EB quæ a puncto E parallela ducitur Asymptoto, quæque Curvæ occurrit in puncto B, æqualis erit subtangenti quæsitæ.

Corol. 1. Area $ABIH$, quæ inter Curvam BDI & Asymptoton ejus ACH infinite versus HI extenditur, & ad alteram partem ab Ordinata AB terminatur, æqualis est parallelogrammo $ABEC$ ab Ordinata eadem AB & subtangente AC comprehenso. Componuntur enim area & parallelogrammum ex elementis quæ sunt ut $AP \times AB$ & $AC \times RB$, quæque adeo æquantur propter analogiam inter AP & RB, AC & AB.

Corol. 2. Atque hinc, ob datam subtangentis magnitudinem, area illa indefinita erit ut Ordinata ad quam terminatur.

Scholium. Hujus Propositionis usus per Exemplum declarabitur. Proponatur ad quamlibet altitudinem a superficie telluris, invenire densitatem Atmosphæræ. Sit AB telluris superficies, & abinde sursum producatur perpendicularis AH, atque ad hujus puncta singula ductæ concipiantur Ordinatæ FG, quæ sint ut Aeris densitates in locis F; & Ordinatarum termini omnes G in Linea Logistica $BDGI$ siti erunt. Patet hoc per Corollarium secundum hujus Propositionis. Nam area indefinita $FGIH$ est ut quantitas seu pondus Atmosphæræ supra locum F, & pondus illud

est

eſt vis quæ comprimit Aerem in hoc loco, iſthæc vero vis (uti docet Experientia multiplex) eſt ut Aeris compreſſi denſitas FG.

Itaque ſi quotlibet altitudines ſumantur in Arithmetica progreſſione: denſitates Aeris in his altitudinibus erunt in progreſſione Geometrica; & differentia binarum quarumvis altitudinum, erit menſura rationis quæ eſt inter denſitates Aeris in iſtis altitudinibus.

Ceſſante vi gravitatis, ita jam per vim aliquam extraneam intelligatur Aeris facta compreſſio, ut eandem habeat ubique denſitatem quam ad terræ ſuperficiem; & quantitas ejus, quæ modo erat expoſita per aream indefinitam $HABI$, nunc per æquale rectangulum $ABEC$ exhibebitur. Atmoſphæræ hujus homogeneæ altitudo AC, eſt ad altitudinem Hydrargyri in tubo *Torricellii*, ut gravitas Hydrargyri ad gravitatem Aeris; atque inde datur. Huic autem datæ altitudini æquatur (per *Corol.* 1.) ſubtangens Curvæ $BDGI$, atque adeo Modulus Syſtematis menſurarum omnium AF. Eſt ergo Logarithmus rationis inter denſitates Aeris in binis quibuſvis altitudinibus, ad Modulum Canonis, ut altitudinum earundem differentia, ad Atmoſphæræ prædictæ homogeneæ altitudinem illam datam AC.

Hæc ita ſe habent ex Hypotheſi, quod vis gravitatis eadem ſit ad omnes altitudines. Cæterum ex Philoſophia *Newtoniana* conſtat eam diminui, in receſſu a centro telluris, in duplicata ratione diſtantiæ: concluſio itaque paulo aliter ſe habebit. Sit S centrum telluris, & AB ſuperficies ejuſdem; ſumatur ipſis SF, SA tertia proportionalis Sf, erigatur ordinata fg quæ ſit ut Aeris denſitas in F: & Curva Bgn quam punctum g perpetuo tangit, erit eadem atque prius Logiſtica, ſed inverſo ſitu. Augeatur enim altitudo AF particula quam minima FM, capiatur Sm ad SA ut SA ad SM, ducatur Ordinata mn quæ ſit ut Aeris denſitas in M; & erit Sm ad Sf ut SF ad SM, & diviſim fm ad FM ut Sf ad SM, ſive ut Sf ad SF, hoc eſt, ut SAq ad SFq. Unde fm eſt ut SFq inverſe & FM directe, id eſt, ut gravitatio & moles Aeris inter F & M conjunctim; adeoque $fm \times fg$ ſive area $fgnm$ eſt ut gravitatio, moles & denſitas ejuſdem Aeris conjunctim, hoc eſt, ut preſſio illius in Aerem inferiorem: & ſumma ſimilium omnium arearum infra fg eſt ut ſumma preſſionum omnium ſupra F, id eſt, ut Aeris in F denſitas fg: & ſummarum differentia $fgnm$ ut denſitatum differentia $fg - mn$. Detur lineola fm; & erit fg ut area $fgnm$, adeoque ut $fg - mn$, atque inde (componendo) ut mn. Ergo data lineola fm erit menſura datæ illius rationis quæ eſt inter fg & mn: atque hinc patet Curvam Bgn eſſe Logiſticam. Sed & eandem eſſe cum ſupra deſcripta Logiſtica, facile abinde colligitur, quod ordinatæ baſi AB viciniſſimæ & ad æqualia intervalla quam minima diſpoſitæ, reſpective ſint æquales in utraque Curva; ac proinde eadem curvatura, eadem inclinatio tangentis ad punctum B, eademque ſubtangentis magnitudo. Fig. 10.

Ergo ſi diſtantiæ SF a centro telluris capiantur in Muſica progreſſione; harum reciprocæ, nempe diſtantiæ Sf, erunt in progreſſione Arithmetica; & Aeris denſitates fg erunt in progreſſione Geometrica.

Ad inveniendam itaque densitatem in loco quovis F, minuenda est altitudo AF in ratione distantiæ SF ad telluris semidiametrum SA: & Logarithmus rationis inter densitates Aeris in A & F, erit ad Modulum Canonis, ut altitudo illa diminuta Af, ad Atmosphæræ homogeneæ altitudinem AC.

Quæ supra demonstrata sunt, accurate obtinebunt, si modò Atmosphæra ex Aere pariter Elastico tota constet: rationes igitur allatas paululum conturbabunt admisti vapores atque exhalationes, quibus etiam accedet Caloris Frigorisque diversa temperies ad altitudines diversas.

Propositio VI. *Logarithmorum Canonem ad Spiralem Æquiangulam accommodare.*

Fig. 11. Æquiangula Spiralis appellatur Linea illa curva ADE, quæ polo P descripta, in eodem dato angulo secat exeuntes a polo radios PA, PD, PE, &c.

Si centro P & intervallo quovis PA describatur circulus ABC, qui radiis PA, PD, PE occurrat in A, B, C: Dico interceptum arcum BC mensuram fore rationis quam habet PD ad PE, & interceptum arcum AB mensuram rationis quam habet PA ad PD. Dividatur enim arcus AB in particulas quam minimas & æquales QR, & jungantur PQ, PR secantes Spiralem ad S & T in angulis datis PST, PTS: & ob datam particulam QR, dabitur angulus QPR, atque adeo species Figuræ SPT, & ratio laterum PS, PT. Data ergo particula QR mensura erit rationis datæ quam habet PS ad PT; & summa particularum, nempe arcus AB, mensura erit summæ similis rationum, hoc est, rationis quam habet PA ad PD. Et eodem argumento, erit arcus BC mensura rationis quam habet PD ad PE.

Ducatur AF Spiralem tangens ad Circuli & Spiralis intersectionem A, huic vero in F occurrat recta PC quæ ad radium PA normalis erigitur: & subtangens PF erit mensurarum Modulus, per *Corol.* 2. *Prop.* 1. Nam si in recta PS sumatur PV ipsi PT æqualis, & jungantur puncta V, T; similia erunt triangula PAF, VST. Unde PF est ad VT ut PA ad VS, sed & VT est ad QR ut PT ad PA: ergo ex æquo perturbate, PF est ad QR quæ metitur rationem inter PS ad PT, ut terminus PT ad terminorum differentiam VS.

Scholium. Spiralem æquiangulam, ad Meridianæ Nauticæ divisionem demonstrandam, feliciter adhibuit Geometra clarissimus *Edmundus Halleius*. Sit acp pars octava Sphæræ terrestris, p Polus, ac quadrans Æquatoris, ap quadrans Meridiani; & quæratur magnitudo rectæ, quæ propositum quemlibet hujus arcum designet in Planisphærio. Per Æquatoris & Meridiani intersectionem a, ducta intelligatur linea Helicoeides ade quæ secet omnes Meridianos ad angulum semirectum, huic occurrat in d parallelus Æquatori circulus gd, per idem punctum d agatur Meridianus pdb; & longitudo intercepti arcus Æquatoris ab, erit magnitudo Nautica quæsita arcus ag. Resolvatur enim arcus ag in particulas innumeras quam minimas gk, ducatur parallelus kmn, secans Meridianum pdb in n;

Lineam

Lineam ade in m; & arcus Meridianus pmh abscindet Æquatoris particulam bh, quæ erit ad mn, sive huic (ob angulum semirectum mdn) æqualem dn vel gk, ut peripheria Æquatoris ad peripheriam paralleli kmn. Est ergo particula bh magnitudo Nautica particulæ gk, & summa particularum omnium bh, nempe longitudo arcus ab, magnitudo Nautica summæ particularum omnium gk, id est, arcus ag. Manente jam Æquatore abc vel ABC, concipiatur Sphærica superficies in plano ejus Stereographice depingi; & Polo p occupante centrum P, projicientur Meridiani pga, pdb, pec in totidem rectas PA, PDB, PEC a centro P exeuntes, ita ut distantia abinde puncti cujusvis D vel A, tangens sit arcus dimidiati pd vel pa quem distantia illa repræsentat. Linea vero Helicoeides ade convertet se in spiralem æquiangulam ADE, polo P descriptam, & secantem radios suos omnes ad angulum semirectum. Hoc siquidem postulat nota Lex hujusce Projectionis, ut anguli omnes eandem in Plano ac in Sphærica superficie magnitudinem servent. Arcus itaque propositi ag magnitudo Nautica ab vel AB, est ad subtangentem PF vel huic jam æqualem Sphæræ radium PC, ut Logarithmus rationis inter PA & PD, hoc est, inter tangentes dimidiatorum arcuum pa & pd, vel pa & pg, ad Modulum Canonis. Fig. 12.

Hinc quoniam longitudo Radii est ad longitudinem arcus minuti unius primi, ut 3437,746770784939 &c. ad 1, & reciprocus Moduli Canonis est 2,302585092994 &c. atque hi numeri in se multiplicati efficiunt 7915,704467897819 &c: si magnitudo illa Nautica AB in minutis primis exhibenda sit, uti mos exigit; subducta tangente artificiali dimidiati arcus pg a tangente artificiali dimidiati arcus pa, multiplicetur residuum per numerum 7915,704467897819 &c. & factus dabit partes Meridionales desideratas. Perinde vero se habebit conclusio, sive in Æquatore, sive extra hunc alibi ad utramvis partem locetur punctum a.

Scholium Generale. In eum potissimum finem præcedentia conscripsi, ut allatis aliquot Exemplis ostenderem, qua commodissima ratione Logarithmorum usus in Geometriam recipi, & ad resolutionem Problematum difficiliorum adhiberi possit. Visum est hoc loco nonnullas adjicere porro constructiones, eodem consilio effectas, quæ mihi ista tractanti subinde sese obviam non invitæ dederunt: ut ita, ex uberiore specimine, de præstantia Methodi hujus Logometricæ judicium feratur.

Parabolæ *Apollonianæ* AP sit A vertex, F focus, AQ axis, PQ ordinatim applicata ad axem. Ducatur AL quæ bifariam secet PQ in L, & productæ adjiciatur LM quæ sit mensura rationis inter $LA + AQ$ & QL ad Modulum AF: & recta AM æqualis erit arcui Parabolico AP. Fig. 13.

Spiralis *Archimedea* PQ similem habet extensionem in rectam. Sit Q polus ejus, QP radius a polo ductus ad Curvæ quodlibet punctum P, & ad eum radium normalis QA. Ducatur LA parallela tangenti Spiralem in P, quæ radium PQ bifariam secet in L; & ponendo AF ad QL ut QL ad QA, ipsi AL adjiciatur LM quæ sit mensura rationis inter LA Fig. 14.

 $+AQ$

$+ AQ$ & QL ad Modulum AF: & recta AM æquabitur Spiralis arcui PQ.

Spiralis Reciprocæ AeE sit A polus, AB radius primus & infinitus, CD asymptotos radio primo parallela ad distantiam AC; & invenienda proponatur hujusce Curvæ longitudo. Inter Spiralem illam vulgarem *Archimedis* atque hanc, quam Reciprocam appello, isthæc intercedit differentia, quod cum illius radii sint ut anguli quos faciunt cum radio suo primo, hujus radii e contrario sunt reciproce ut iidem anguli: eandem utique proportionem habet radius AE ad radium Ae quam habet angulus eAB ad angulum EAB. Unde facile colligitur, si ad puncta E & e ducantur tangentes EF, ef, & ad radios AE, ae erigantur normales AF, Af, fore normales istas sibi invicem & Asymptoti intervallo AC æquales. Invenietur autem longitudo cujusvis arcus Ee, ponendo LM mensuram rationis inter AE & $EF - AF$ ad Modulum AF, & similiter lm mensuram rationis inter Ae & $ef - Af$ ad æqualem Modulum Af. Nam si tangentium differentiæ $EF - ef$ adjiciatur mensurarum differentia $lm - LM$, aggregatum æquabitur arcui Ee.

Fig. 15.

Linea illa Logistica, cujus aliquas exposuimus affectiones in Propositione quinta, non absimilem habet longitudinis suæ determinationem; quam & hoc loco apponam in eorum gratiam qui hujusmodi contemplationibus delectantur. Oblata sit igitur Logistica $EMem$, cujus Asymptotos $AFaf$: & quæratur longitudo cujusvis arcus Ee. Demissis in Asymptoton perpendiculis ELA, ela, & ductis tangentibus EF, ef, capiatur AL æqualis excessui quo tangens EF superat subtangentem AF, & similiter al æqualis excessui quo tangens ef superat subtangentem af: & actis LM, lm Asymptoto parallelis, si tangentium differentiæ $Ef - ef$ adjiciatur parallelarum differentia $lm - LM$, aggregatum æquabitur arcui Ee.

Fig. 16.

Accedo ad Cissoidem *Diocleam*. Sit A vertex ejus, AB diameter Circuli genitoris, BC Asymptotos, PQ perpendicularis in diametrum demissa, Cissoidi in P & diametro in Q occurrens. Agatur AC quæ secet Asymptoton in C ac faciat angulum BAC qui sit recti pars tertia, sumptaque inter BQ & BA media proportionali BD jungatur CD; denique per medium perpendiculum PQ ducatur AE recta, quæ occurrat Asymptoto in E: & Cissoidis arcus AP æquabitur duplicato excessui rectæ AE supra diametrum AB, & simul triplicatæ mensuræ rationis inter $BA + AC$ & $BD + DC$ ad Modulum BC.

Fig. 17.

Si Cissoidis area APQ convertatur circum axem AQ; generabitur solidum cujus dimensio pendet a Logometria, & sic construitur. Sint AQ, AB, AR, AS, AT continue proportionales; deinde ad Modulum TS capiatur QX mensura rationis inter AB & BQ, & retro ponatur XZ æqualis ipsi SR una cum dimidio ipsius RB ac triente simul ipsius BQ: & solidum Cissoidale axem habens AQ basisque semidiametrum PQ, æquabitur Cylindro cujus eadem est basis & cujus altitudo est QZ.

Fig. 18. Adjungam solidum ex Conchoide *Nicomedis* genitum. Sint AE, ae Curvæ conjugatæ, polo P, regula CD, intervallo CA vel Ca, axe $PaCA$ ad

ad regulam normali, verticibusque A & a descriptæ. Per polum P ducatur ad libitum recta $PeDE$, regulæ occurrens in D, Lineæ vero in E & e: & ex natura Conchoidis, erunt segmenta DE, De intervallo CA vel Ca æqualia. Eodem intervallo centroque P describatur circuli arcus RS secans axem PC in R & rectam PD in S: & semisumma solidorum Conchoidalium quæ generantur ex conversione Figurarum $AEDC$, $aeDC$ circum axem AaP, erit ad sectorem Sphæræ genitum ex circuli sectore PRS circum axem eundem converso, ut $3PC \times PD + PRq$ ad PRq. Eorundem vero semidifferentia Cylindro æquatur, cujus basis est circulus diametro Aa descriptus, & cujus altitudo est mensura duplicata rationis inter PD & PC ad Modulum PC.

Area vero Figuræ totius $AEea$ æquatur rectangulo cujus basis est Aa, & cujus altitudo CM est mensura rationis inter $PD + DC$ & PC ad Modulum PC. Quod si desideretur quadratura partium $AEDC$, $aeDC$; ductis ad axem normalibus AF, af, in regula CD sumenda est CN quæ sit anguli CPD mensura ad eundem Modulum PC: & acta per punctum M recta FMf quæ parallela sit rectæ jungenti puncta P, N, quæque occurrat normalibus in F & f; erit area $AEDC$ æqualis Trapezio $AFMC$, & area $aeDC$ æqualis Trapezio $afMC$.

Hyperbolæ quadraturam in superioribus expositam dedi, eo modo, qui mihi visus est ad propositum quam maxime accommodatus. Libet aliam constructionem hoc loco apponere, & simul adjicere gravitatis centrum. Oblata sit portio interior ADB, interclusa curvæ ADB & rectæ cuivis AB ad diametrum PQ parallelæ. A Figuræ centro C producatur diameter CDE, quæ basin AB bifariam secet in E; deinde si in diametro producta sumantur CR ad CD, & CD ad CS, ut basis AB ad diametrum PQ, & ad Modulum CS fiat CN mensura rationis quam habet CD ad ER: triangulum rectilineum ANB æquabitur areæ curvilineæ ADB. Fig. 19.

Hujus autem areæ centrum gravitatis Z invenietur, capiendo CZ ad CR ut $2CR$ ad $3EN$.

Sit nunc oblata portio exterior $APQB$, interclusa curvis oppositis AP, BQ, diametro PQ, & rectæ cuivis AB ad diametrum illam parallelæ. Esto CD conjugatæ semidiametri longitudo extra portionem oblatam $APQB$ posita, quæ producta in contrariam partem centri C bifariam secet basim AB in E. Deinde in diametro producta si sumantur CR ad CD, & CD ad CS, & CS ad CT, ut basis AB ad diametrum PQ, ponantur vero CR & CT ad eandem centri partem cum basi AB; & ad Modulum CS, in contrariam centri partem, sumatur CN mensura rationis quam habet CD ad ER: triangulum rectilineum ANB æquabitur areæ curvilineæ $APQB$. Fig. 20.

Hujus autem areæ centrum gravitatis Z invenietur, capiendo CZ ad CR ut $2TR$ ad $3EN$.

Pergo ad superficies ab Hyperbola circum axes suos convoluta genitas. Sit AN Hyperbola descripta vertice A, centro C, Asymptoto CB, foco F, semiaxe principali AC, semiaxe conjugato AB normali ad AC; & ad axis AC punctum quodvis X sit XN ordinatim applicata, quæ Hyperbolæ Fig. 21.

bolæ occurrat ad N. In axe CA capiatur CE ad CA ut CA ad CF; & ad eundem axem erecta perpendiculari EZ, quæ Asymptoto occurrat in G, angulo CEZ inscribatur æqualis ipsi CX recta CZ, quæ porro producta secet ordinatim applicatam XN ad O. Tum sumatur KL quæ sit æqualis excessui quo XO superat AB, atque LM quæ sit mensura rationis inter $CZ + ZE$ & $CG + GE$ ad Modulum CE: & superficies genita ex arcus AN conversione circum axem AX, erit ad Circulum semidiametro AB descriptum, ut excessus KM quo KL superat LM, ad semidiametrum illam AB.

Sit rursus BN Hyperbola descripta vertice B, centro C, foco F, semiaxe principali CB, semiaxe conjugato CA normali ad CB; & ad axis
Fig. 22. AC punctum quodvis X sit XN ordinatim applicata, quæ Hyperbolæ occurrat ad N. In axe CB capiatur CE ad CA ut CA ad CF, & jungatur EX. Tum sumatur KL quæ sit ad XC ut XE ad CE, & LM quæ rationis inter $EX + XC$ & CE mensura sit ad Modulum CE: & superficies genita ex arcus BN conversione circum axem CX, erit ad Circulum semidiametro CB descriptum, ut linearum KL & LM aggregatum KM, ad semidiametrum illam CB.

His addere licebit ab Ellipsi genitas superficies. Sit ANB Ellipsis descripta centro C, verticibus A & B, foco F, semiaxe principali CB, semiaxe conjugato CA; & ad axis CA punctum quodvis X sit XN ordina-
Fig. 23. tim applicata, quæ Ellipsi occurrat ad N. In axe CB capiatur CE ad CA ut CA ad CF, & jungatur EX. Tum sumatur KL quæ sit ad XC ut XE ad CE, & LM quæ rationis inter $EX + XC$ & CE mensura sit ad Modulum CE: & superficies genita ex arcus BN conversione circum axem CX, erit ad Circulum semidiametro CB descriptum, ut linearum KL & LM aggregatum KM, ad semidiametrum illam CB. Ut hæc ultima constructio locum habeat, oportet semiaxem CA circa quem conversio facta est, minorem esse altero semiaxe CB; aliter enim Moduli CE quantitas $\frac{CAq}{\sqrt{CBq - CAq}}$ evadet impossibilis, & constructio illa Logometrica (quod in hujusmodi casibus fieri solet) convertet se in Trigonometricam, qualis illa est quæ jam sequitur.

Sit ANB Ellipsis descripta centro C, verticibus A & B, foco F, semiaxe principali CA, semiaxe conjugato CB; & ad axis CA punctum quodvis X sit XN ordinatim applicata, quæ Ellipsi occurrat ad N. Angulo CXN inscribatur recta CE, quæ sit ad CA ut CA ad CF. Tum sumatur
Fig. 24. KL quæ sit ad XC ut XE ad CE, & LM quæ anguli XEC mensura sit ad Modulum CE, hoc est, quæ sit æqualis arcui cujus sinus est XC ad radium CE: & superficies genita ex arcus BN conversione circum axem CX, erit ad Circulum semidiametro CB descriptum, ut linearum KL & LM aggregatum KM, ad semidiametrum illam CB. Posset hujus etiam superficiei dimensio per Logometriam designari, sed modo inexplicabili. Nam si quadrantis circuli quilibet arcus, radio CE descriptus, sinum habeat CX sinumque complementi ad quadrantem XE: sumendo radium

CE

CE pro Modulo, arcus erit rationis inter $EX + XC\sqrt{-1}$ & CE menſura ducta in $\sqrt{-1}$. Verum iſthæc aliis, quibus operæ pretium videbitur, diligentius excutienda relinquo. Ceterum ex præcedentibus intelligi poteſt, quanta ſit cognatio inter angulorum atque rationum menſuras, quamque levi mutatione in ſe invicem facillime convertantur pro variis ejuſdem Problematis caſibus. De Cubicarum æquationum radicibus dudum ab Analyſtis obſervatum eſt; vel eas exprimi poſſe per *Cardani* regulas, atque adeo per duarum mediarum proportionalium inventionem; vel per diviſionem arcus circularis in tres æquales partes, ſi forte fuerint inexplicabiles per memoratas regulas. * Hoc animadvertit *Carteſius*, ſed & ante *Carteſium* idem obſervabit *Franciſcus Vieta* ſub finem Supplementi Geometriæ. Exhinc autem aperte colligitur, qualis ſit ordo Naturæ tranſeuntis ad Anguli triſectionem a triſectione Rationis.

Mirabilem illam Harmoniam ulterius declarare lubet, Exemplo deſumpto ab eadem Figura circum axes ſuos convoluta. Sit igitur $APBQ$ Ellipſis, axis ejus major AB, minor PQ, centrum C, focus F. Hæc circum axem utrumvis convoluta Solidum generet, cujus particulæ conſtantes ex materia homogenea, vires attractivas habeant in duplicata diſtantiarum ratione decreſcentes: & quæratur vis qua Solidum illud attrahit corpuſculum quodvis, in ejus ſuperficie locatum ad axis illius terminum. Jungantur puncta P, F, ac ſumatur CD quæ ſit menſura rationis inter $PF + FC$ & CP ad Modulum CA, pariterque ſumatur CE quæ ſit anguli CPF menſura ad Modulum CP; ſitque FD exceſſus menſuræ CD ſupra CF, atque FE exceſſus ipſius CF ſupra menſuram CE: & Solidi convolutione circum axem majorem AB geniti vis in corpuſculum ad A locatum, erit ad Sphæræ homogeneæ & eodem axe deſcriptæ vim in idem corpuſculum, ut $3\,FD \times CPq$ ad $CF\,cub$; Solidi autem converſione circum axem minorem PQ geniti vis in corpuſculum ad P locatum, erit ad Sphæræ homogeneæ & eodem axe deſcriptæ vim in idem corpuſculum, ut $3\,FE \times CAq$ ad $CF\,cub$. Unde cum vis Sphæræ prioris in corpuſculum Fig. 26.

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* Sublato etenim termino ſecundo, tres habentur Æquationum caſus. Hi vero reſolvuntur ope trianguli rectanguli ABC, rectum habentis angulum ad A, in quo inſuper triangulo ſemper data ſunt duo latera.

Caſ. 1. Nam ſi ſit $x^3 + 3aax = \pm 2aab$. ponantur $AB = a$, $AC = b$; & ſumantur M & N binæ mediæ proportionales inter $BC + AC$ & $BC - AC$: & erit $M + N$ radix unica poſſibilis affirmativa, ſi habeatur $+ 2aab$; vel $N - M$ radix unica poſſibilis negativa, ſi habeatur $- 2aab$. Fig. 25.

Caſ. 2. Si ſit $x^3 - 3aax = \pm 2aab$; exiſtente a minore quam b: ponantur $AB = a$, $BC = b$; & ſumantur M & N binæ mediæ proportionales inter $BC + AC$ & $BC - AC$: & erit $M + N$ radix unica poſſibilis affirmativa, ſi habeatur $+ 2aab$; vel $- M - N$ radix unica poſſibilis negativa, ſi habeatur $- 2aab$.

Caſ. 3. Denique ſi ſit $x^3 - 3aax = \pm 2aab$, exiſtente a majore quam b: ponantur $AB = b$, $BC = a$; & ſumatur M ſinus trientis angulorum ſummæ $A + B$, atque N ſinus trientis angulorum differentiæ $A - B$, exiſtente radio $2\,BC$: & erunt $-M$, $-N$, & $M + N$ tres radices poſſibiles, ſi habeatur $+ 2aab$; vel M, N, & $-M - N$, tres radices poſſibiles, ſi habeatur $- 2aab$.

Atque ita Problemata omnia Solida ſolutionem facilem recipiunt, vel per Canonem Logarithmicum, vel per Canonem Trigonometricum.

ad-

ad A, sit ad vim Sphæræ posterioris in corpusculum ad P, ut CA ad CP: erit vis Solidi prioris in corpusculum ad A, ad vim Solidi posterioris in corpusculum ad P, ut $FD \times CP$ ad $FE \times CA$.

Hinc quoniam Solidum posterius medium est proportionale inter Solidum prius & Sphæram priorem: vis Solidi posterioris in corpusculum ad A, erit media proportionalis quamproxime inter vires Solidi prioris & Sphæræ prioris in idem corpusculum ad A, si modo axes Ellipseos sint prope æquales. Itaque in hoc casu, ponendo CG mediam proportionalem inter CF & $3FD$, & capiendo CH ad $3FE$ ut CA ad CF; posterioris Solidi vires ad A & P, vel ad B & Q, erunt ad invicem quamproxime ut CG ad CH. Id quod non inutile præbet compendium ad inventionem Figuræ Telluris, qualem eam subtiliter instituit celeberrimus *Newtonus*, summus ille Philosophiæ sanioris Instaurator.

Consideratio virium centripetarum aliud porro mihi suggerit Exemplum, in quo satis ampla se prodit mutationum varietas. Proponatur Trajectoriarum species enumerare, in quibus corpora moveri possunt, quæ a viribus centripetis in ratione distantiarum triplicata decrescentibus agitantur, quæque de loco dato, cum data velocitate, secundum datam rectam egrediuntur.

Cas. 1. Sit S centrum virium, exeatque corpus de loco P secundum rectam PQ vel QP, ea cum velocitate quam acquirere posset ab iisdem viribus, libere cadendo versus centrum S de loco C, & casu suo describendo altitudinem CP. In datam rectam QPT demittantur perpendicula SQ, CT, centroque S & intervallo $\sqrt{SQq + QTq}$ describatur circulus RTA, rectæ SPC occurrens in R: deinde ad Modulum $\sqrt{SCq - SRq}$ sit arcus RA mensura rationis inter $SR + \sqrt{SRq - SPq}$ & SP, jaceant autem arcus ille RA & punctum Q ad diversas partes rectæ SR; & punctum A erit Apsis summa Trajectoriæ. Exhinc vero Trajectoria dabitur, sumendo SM æqualem ipsi $\sqrt{SCq - SRq}$, deinde in recta SA capiendo longitudinem quamvis SD quæ sit minor quam SA, ad eandem erigendo perpendiculum DE secans circulum in E, & jungendo SE. Nam si ad utras-
Fig. 27.
que partes puncti A ponatur arcus circularis AR, cujus longitudo sit mensura rationis inter $SE + ED$ & SD ad Modulum SM, & in semidiametris SR capiantur distantiæ SP æquales ipsi SD: erunt puncta P ad Trajectoriam describendam. Tempus autem quo radius SP, a centro ad corpus motum ductus, percurret aream quamvis SAP, erit ut recta DE: nam area percursa æquatur ipsi DE in Modulum dimidiatum $\frac{1}{2} SM$ ductæ. Velocitas vero corporis in loco quovis P, erit ad velocitatem qua in Circulo, ad eandem distantiam SP, cum iisdem viribus revolvi posset, ut $\sqrt{SCq - SPq}$ ad SC. Ex ipsa constructione patet, hanc Spiralem primam infinitis gyris circa centrum virium contorqueri, quin & seipsam infinitis vicibus decussare, & siti erunt Nodi omnes ad Apsidis lineam AS.

Cas. 2. Recedat punctum C ad infinitam distantiam a centro S; & corporis de loco P secundum rectam PM vel MP exeuntis ea sit velocitas, quam

quam acquirere poſſet cadendo libere ad eundem locum P ab infinita diſtantia. Ad rectam SP ducatur normalis SM, quæ ſecet PM in M; deinde centro S & intervallo SP deſcribatur circulus, & in ejus circumferentia capiatur arcus PX, cujus longitudo ſit menſura rationis inter diſtantiam quamvis SD, & diſtantiam datam SP ad Modulum SM, jaceant autem arcus ille PX & punctum M ad diverſas partes rectæ SP ſi SD fuerit major quam SP, aliter ad eaſdem, inque ſemidiametro SX ponatur SZ æqualis ipſi SD; & punctum Z erit ad Trajectoriam deſcribendam. Tempus autem quo radius SZ, a centro ad corpus motum ductus, percurret aream quamvis SPZ, erit ut differentia quadratorum ex SZ & SP: Nam area percurſa eſt ad illam differentiam, in data ratione Moduli dimidiati $\frac{1}{2}SM$ ad SP. Velocitas vero corporis in loco quovis P, æqualis erit velocitati qua in Circulo, ad eandem diſtantiam SP, cum iiſdem viribus revolvi poſſet. Ex conſtructione patet hanc ſecundam Spiralem eſſe Æquiangulam illam Propoſitionis ſextæ; ea vero migrabit in Circulum ubi angulus SPM ſit rectus. Fig. 28.

Caſ. 3. Ut velocitas ſit adhuc major, abeat jam punctum C ad diſtantiam pluſquam infinitam a centro S, vel (quod perinde eſt) accedat a parte contraria eidem centro, ad finitam diſtantiam; & corporis de loco P ſecundum rectam PQ vel QP exeuntis, ea ſit velocitas, quam acquirere poſſet aſcendendo libere de loco C ad infinitam diſtantiam, & deinde ab infinita diſtantia ex altera centri parte deſcendendo ad locum P, viribus centripetis inter aſcendendum in æquales vires centrifugas converſis. In datam rectam PQT demittantur perpendicula SQ, CT; & erit TQ vel major, vel æqualis, vel minor quam SQ. Si TQ fuerit major quam SQ; centro S & intervallo $\sqrt{TQq - SQq}$ deſcribatur circulus RBE rectæ SP occurrens in R, deinde ad Modulum $\sqrt{SCq - SRq}$ ſit arcus RB menſura rationis inter $SR \pm \sqrt{SRq + SPq}$ & SP, jaceant autem arcus ille RB & punctum Q ad partes diverſas rectæ SP. Exhinc Trajectoria dabitur, ſumendo SM æqualem ipſi $\sqrt{SCq - SRq}$, in recta SB capiendo longitudinem quamvis SD, ad eandem erigendo perpendiculum SE circulum ſecans in E, & jungendo DE. Nam ſi retro ponatur a puncto B circularis arcus BR, cujus longitudo menſura ſit rationis inter $SE + ED$ & SD ad Modulum SM, & in ſemidiametro SR capiatur diſtantia SP æqualis ipſi SD: erit punctum P ad Trajectoriam deſcribendam. Tempus autem quo radius SP, a centro ad corpus motum ductus, percurret aream quamvis hujus Trajectoriæ, erit ut incrementum vel decrementum rectæ DE per tempus illud factum: nam area percurſa æquatur huic incremento vel decremento in Modulum dimidiatum $\frac{1}{2}SM$ ducto. Velocitas vero corporis in loco quovis P, erit ad velocitatem qua in Circulo, ad eandem diſtantiam SP, cum iiſdem viribus revolvi poſſet, ut $\sqrt{SCq + SPq}$ ad SC. Ex conſtructione patet, hanc Spiralem tertiam infinitis gyris centrum cingere infra punctum datum P; at ſupra idem punctum vel non undique cinget, ſi arcus RB minor fuerit quam circumferentia tota $RBER$; vel toties cinget, quoties arcus ille circumferentiam excedit. Fig. 29.

Cas. 4. Reliquis manentibus, sint jam $T\mathcal{Q}$ & $S\mathcal{Q}$ æquales. Centro S & intervallo SP describatur circulus PXB, & sit arcus PB æqualis ipsi SC, jaceant autem arcus PB & punctum $\mathcal{Q}$ ad partes diversas rectæ SP. Exhinc Trajectoria dabitur, sumendo in recta SB longitudinem quamvis SD, centroque S & intervallo SD describendo circuli arcum DZ æqualem ipsi SC. Nam si ordine circulari contrario ponantur arcus PB a puncto
Fig. 30. P & arcus DZ a puncto D: erit punctum Z ad Trajectoriam describendam. Tempus autem quo radius SZ, a centro ad corpus motum ductus, percurret aream quamvis SPZ, erit ut differentia radiorum SZ & SP: nam area percursa æquatur huic differentiæ ductæ in semissem distantiæ SC. Velocitas vero corporis in loco quovis P, erit ad velocitatem qua in Circulo, ad eandem distantiam SP, cum iisdem viribus revolvi posset, ut $\sqrt{SCq + SPq}$ ad SC. Ex constructione patet, hanc Spiralem quartam esse Reciprocam illam, cujus longitudinem supra dimensam dedimus.

Cas. 5. Reliquis adhuc manentibus, sit jam $T\mathcal{Q}$ minor quam $S\mathcal{Q}$. Centro S & intervallo $\sqrt{S\mathcal{Q}q - T\mathcal{Q}q}$ describatur circulus RAE rectæ SP occurrens in R; deinde sit arcus RA, ad ejusdem circuli arcum cujus secans est SP, ut $\sqrt{SCq + SRq}$ ad SR; ponatur autem arcus ille RA ad easdem partes rectæ SP cum puncto $\mathcal{Q}$: & A erit Apsis ima Trajectoriæ. Exhinc vero Trajectoria dabitur, sumendo SM æqualem ipsi $\sqrt{SCq + SRq}$, in recta SA capiendo longitudinem quamvis SD quæ sit major quam SA, ducendo DE quæ circulum tangat in E, & jungendo SE. Nam si ad utrasque partes puncti A ponatur arcus circularis AR, cujus longitudo mensura sit anguli DSE ad Modulum SM, & in semidiametris SR capiantur distantiæ SP æquales ipsis SD: erunt puncta P ad Trajectoriam describendam. Tempus autem quo radius SP, a centro ad corpus motum ductus, percurret aream quamvis SAP, erit ut recta DE: nam area percursa æquatur ipsi DE in Modulum dimidiatum $\frac{1}{2} SM$ ductæ.
Fig. 31. Velocitas vero corporis in loco quovis P, erit ad velocitatem qua in Circulo, ad eandem distantiam SP, cum iisdem viribus revolvi posset, ut $\sqrt{SCq + SPq}$ ad SC. Ex constructione patet, hanc quintam Spiralem vel nullum habere Nodum, vel unicum, vel plures, pro varia proportione rectæ SM ad diametrum circuli EAR: toties enim Trajectoria sese decussabit, quoties illa recta diametrum excedit, & Nodi omnes siti erunt ad Apsidis lineam AS.

Sunt itaque Trajectoriarum quinque Species. Harum primam atque ultimam descripsit olim *Newtonus* per Hyperbolæ & Ellipseos quadraturam. Geometris integrum erit, ex adductis hactenus Exemplis de Methodo nostra judicare; quam quidem, si proba fuerit, ulterius excolere pergent & excolendo latius promovebunt. Patet utique campus amplissimus in quo vires suas experiri poterunt, præsertim si Logometriæ Trigonometriam insuper adjungant, quibus miram quandam affinitatem in se invicem euntibus intercedere notabam. Hisce quidem Principiis haud facile crediderim generaliora dari posse; cum tota Mathesis vix quicquam in universo suo ambitu complectatur, præter angulorum & rationum Theoriam.

Ne-

Neque ſane commodiora ſperabit, qui animadverterit Effectionis facilitatem per ampliſſimas illas, omnibuſque ſuis numeris abſolutas, tum Logarithmorum tum Sinuum & Tangentium Tabulas, quas anteceſſorum noſtrorum laudatiſſimæ ſolertiæ debemus acceptas. Ut vero tanti beneficii uberior nobis exſurgat fructus, id nunc exponendum reſtat, quibus artibus ad iſtiuſmodi concluſiones rectiſſime perveniatur. In hunc finem Theoremata quædam, tum Logometrica tum Trigonometrica adjeciſſem, quæ parata ad uſum aſſervo; ni conſultius viſum eſſet, quum abſque nimiis ambagibus ea tradi non poſſent, intacta potius præterire atque aliis denuo inveſtiganda relinquere. Cæterum iſthoc apparatu non ſemper eſt opus; nam in Methodo Fluxionum ſæpe evenit ut ipſæ Fluentes, omiſſis hujuſmodi ſubſidiis, ad Logometriam ſatis commode revocentur: id quod uno atque altero Exemplo oſtendam.

Egimus in præcedentibus de rectilineo Gravium deſcenſu, per Medii reſiſtentiam continuam retardato, ex Hypotheſi quod illa reſiſtentia eſſet in duplicata ratione velocitatis. Ex eadem Hypotheſi reſiſtentiam corporis penduli, in Cycloide oſcillantis, jam ſit propoſitum invenire. Cycloidis itaque in rectam explicatæ ſit AC dimidium, C punctum infimum, B punctum a quo cadere incipit corpus pendulum, BC, CD arcus deſcenſu ejus & ſubſequente aſcenſu deſcripti. Hiſce poſitis, exquirenda eſt ratio quam habet reſiſtentia corporis in loco quovis E, ad pondus ejus relativum in Medio reſiſtente. Exponatur pondus illud per AC; & vis ab eodem oriunda, qua pendulum acceleratur ad E, exponetur per CE: quæ ſi dicatur x, & momentum ejus $+\dot{x}$; momentum arcus jam deſcripti BE erit $-\dot{x}$. Exponatur vis reſiſtentiæ per z; & vis qua pendulum vere accelera- *Fig.* 32.
tur, erit ut exceſſus vis prioris ſupra reſiſtentiam, hoc eſt, ut $x-z$. Itaque cum reſiſtentia ſit ut quadratum velocitatis, reſiſtentiæ momentum $\dot{z}$ erit ut velocitas & velocitatis momentum, hoc eſt, ut $-\dot{x}$ & $x-z$, ſive ut $z\dot{x}-x\dot{x}$. Nam ſi tempus in particulas æquales dividatur, erit velocitas ut arcus deſcripti momentum $-\dot{x}$, & velocitatis momentum ut vis acceleratrix $x-z$ quæ momentum illud generat. Quoniam ergo $\dot{z}$ eſt ut $z\dot{x}-x\dot{x}$, ſi capiatur quantitas invariabilis a, quæ ſit idoneæ magnitudinis: erit $a\dot{z}=z\dot{x}-x\dot{x}$.

Ad hanc æquationem conſtruendam, aſſumatur quantitas v quæ ſit variabilis, & fingatur æquatio $z=p+qx+rv$, in qua notæ p, q, r deſignent alias novas quantitates invariabiles; & erit $\dot{z}=q\dot{x}+r\dot{v}$. Hiſce porro valoribus ipſarum z & $\dot{z}$ ſubſtitutis in æquatione prima $a\dot{z}=z\dot{x}-x\dot{x}$, habebitur $\overline{aq-p}\,\dot{x}+ar\dot{v}=\overline{q-1}\,x\dot{x}+rv\dot{x}$. Ut hæc æquatio ſimplicior evadat, ponatur $q-1=0$, & $aq-p=0$; ſive $q=1$, & $p=a$: & fiet $a\frac{\dot{v}}{v}=\dot{x}$, ac præterea $z=a+x+rv$. Jacentibus punctis D & S ad eandem partem puncti C, intelligatur CS æqualis ipſi a: & erit $z=SE+rv$, atque $CS\frac{\dot{v}}{v}=\dot{x}$. Sit c valor quantitatis v, dum incidit punctum E in punctum C: & quantitas x, ſive CE, æquabitur menſuræ rationis quam

quam habet v ad c pro Modulo CS, per Propositionem primam: quam æqualitatem sic designare soleo, $CE = CS \left| \frac{v}{c} \right.$. Tota ergo Problematis difficultas jam revocatur ad binas illas æquationes $CE = CS \left| \frac{v}{c} \right.$, atque $z = SE + rv$: hæ vero deduci non poterunt in usum priusquam determinatæ fuerint quantitates r & CS. Ad hoc efficiendum, duæ restant conditiones nondum adimpletæ; oportet enim resistentiam esse nullam, atque adeo quantitatem z sive $SE + rv$ evanescere, ubi punctum E in puncta B & D inciderit.

Sint ergo b & d valores ipsius v, dum incidit punctum E in puncta B & D respective: & in his casibus habebuntur $SB + rb = 0$, $SD + rd = 0$. Unde $r = -\frac{SB}{b}$, $r = -\frac{SD}{d}$, atque $z = SE + rv = SE - \frac{v}{b} SB = SE - \frac{v}{d} SD$. Porro erit $\frac{SB}{SD} = \frac{b}{d}$; atque adeo $CS \left| \frac{SB}{SD} \right. = (CS \left| \frac{b}{d} \right. = CS \left| \frac{b}{c} \right. - CS \left| \frac{d}{c} \right. = CB + CD =) BD$: unde dabitur punctum S.

Componetur itaque Problema hunc in modum. Producatur BD versus D ad S, eo usque donec BD fuerit mensura rationis inter SB & SD ad Modulum CS. Deinde ad arbitrium posita quantitate c, ita capiantur quantitates b & v, ut eodem Modulo CS, fiat CB mensura rationis quam habet b ad c, fiat quoque CE mensura rationis quam habet v ad c: & erit vis resistentiæ in loco E, ad pondus relativum corporis penduli, ut $SE - \frac{v}{b} SB$ ad CA.

Hujus Problematis solutio utilitatem habet in Physica non contemnendam: quapropter constructionem ejusdem Linearem, ex eadem Analysi deductam, subjungere visum est. Invento uti supra puncto S; ad rectam SA erigantur perpendicula DH, Cc, EK, BF, AN, rectæ SN utcunque per S ductæ occurrentia in H, c, K, F, N. Per punctum c ducatur recta da parallela rectæ DA, quæ iisdem perpendiculis occurrat in d, c, e, b, a; & ad Asymptoton SA ducatur Logistica $HGIF$, quæ transeat per puncta H & F, secetque perpendicula Cc, EK in G & I, ac parallelam da in m: namque his positis, erit pondus relativum corporis penduli, ad vim
Fig. 33. illam qua pendulum acceleratur ad punctum E in Medio non resistente, ut aN ad eK; erit autem ad vim resistentiæ in loco E, ut aN ad KI; atque adeo ad vim qua pendulum acceleratur ad punctum E in Medio resistente, ut aN ad eI. Porro, si per punctum m ducatur ad rectam SMA perpendicularis LmM, quæ secet SN in L: erit M locus ubi resistentia sit maxima: atque adeo resistentia illa maxima, erit ad pondus relativum penduli, ut Lm ad Na, hoc est, ut CM ad CA.

Ceterum si ita ducatur recta SN, ut abscindat rectam DH quæ sit dupla ipsius SD, centroque C & intervallo CB describatur Circulus BOP, qui occurrat perpendiculo KE in O: erit penduli in Medio resistente oscillantis

cillantis velocitas in loco E, ad velocitatem penduli ejusdem ad eundem locum E delati per idem pondus relativum in Medio non resistente, ut media proportionalis inter CS & KI, ad EO.

Adhæc si jungatur CO, & in perpendiculo KE sumatur ER, quæ sit ad CB ut CB ad mediam proportionalem inter $4CS$, & KI; continuoque ductu rectæ ER in basin BE generetur area $BQRE$: erit tempus quo Cycloidis arcus BE describitur in Medio resistente, ad tempus quo idem arcus describeretur in Medio non resistente, ut area illa $BQRE$, ad Circuli sectorem BOC. Pergo nunc ad alia.

Densitatem Aeris invenimus ad quamvis altitudinem, ubi vis Gravitatis vel erat uniformis, vel decrescebat in recessu a centro telluris in duplicata ratione distantiæ: libet eandem exquirere denuo, ubi gravitatio vel augetur vel diminuitur in ratione datæ cujusvis dignitatis distantiæ. Sit S centrum telluris, A punctum in ejus superficie vel alibi utcunque situm, $SAF2$ recta a centro ad summitatem Atmosphæræ producta: & quærenda sit ratio densitatis in loco A, ad densitatem in loco quovis F, ex Hypothesi quod vis gravitatis in F sit ut distantiæ SF dignitas quæcunque SF^n, cujus index est n. Pro SF scribatur x, ac designent d & v densitates Aeris ad A & F; & cum densitas sit ubique ut pressura totius Aeris incumbentis, erit densitatis momentum ut momentum pressuræ, Fig. 34.

hoc est, $\dot{v}$ ut $v\dot{x}x^n$, atque adeo $\frac{\dot{v}}{v}$ ut $\dot{x}x^n$. Sit AC altitudo Atmosphæræ, cujus uniformis densitas eadem esset ac densitas loci A; vel sit AC ad altitudinem Hydrargyri barometrici in loco A, ut densitas Hydrargyri ad densitatem Aeris in eodem loco A: & si punctum F accedere intelligatur ad punctum A; erit altitudo Hydrargyri barometrici in loco A, ad altitudinem Hydrargyri barometrici in loco F, ut AC ad FC. Aeris ergo in loco A densitas d, est ad Aeris in loco F densitatem v, ut AC ad FC: unde consequitur ut sit $d-v$ sive $\dot{v}$ ad d sive v, ut AF sive $\dot{x}$, ad AC. Erit itaque, in hoc casu, $AC\frac{\dot{v}}{v} = \dot{x} = \frac{\dot{x}x^n}{SA^n}$. Quoniam ergo, ubicunque sumeretur punctum F, erat $\frac{\dot{v}}{v}$ ut $\dot{x}x^n$: erit porro $AC\frac{\dot{v}}{v} = \frac{\dot{x}x^n}{SA^n}$, ubicunque sumatur punctum F.

Jam si gravitatio sit reciproce ut distantia a centro, sive ut $\frac{1}{x}$ vel x^{-1}; erit $n = -1$, atque inde $AC\frac{\dot{v}}{v} = SA\frac{\dot{x}}{x}$; unde si Fluentes statuantur æquales, mensura rationis inter densitates d & v ad Modulum AC, æquabitur mensuræ rationis inter distantias SF & SA ad Modulum SA.

Si gravitationis sit alia quævis Lex: quoniam est $AC\frac{\dot{v}}{v} = \frac{\dot{x}x^n}{SA^n}$; si Fluentes statuantur æquales, erit $\frac{1}{n+1}$ in $\frac{SF^{n+1}}{SA^n} - SA$ mensura rationis inter densitates d & v ad Modulum AC. Itaque si sumantur in progressi-

one Geometrica termini crescentes SA, SF, $SF1$, $SF2$, &c. decrescentes SF, SA, $Sf2$, $Sf3$, &c. mensura rationis inter densitates Aeris in A & F ad Modulum AC, erit $\frac{1}{2}Af3$, si gravitatio sit reciproce in triplicata ratione distantiæ; erit $Af2$, si gravitatio sit reciproce in duplicata ratione distantiæ; erit AF, si gravitatio uniformis statuatur; erit $\frac{1}{2}AF1$, si gravitatio sit ut distantia; erit $\frac{1}{3}AF2$, si gravitatio sit in duplicata ratione distantiæ. Et sic proceditur in infinitum.

Denique ut plenius constet, Syntheticas etiam demonstrationes ex elementis præmissis levi negotio concinnari posse; sufficiet unicum insuper addidisse Exemplum, tædet utique plura jam proferre. Repetatur itaque divisio illa Nautica Meridianæ quam supra attigimus, & videamus etiam absque ope Curvæ cujuspiam Logometricæ, annon simplicior aliquanto sit futura demonstratio ad modum sequentem. Sit $PXCQ$ Telluris axis, CO semidiameter Æquatoris, $PAOBQ$ Meridianus; & invenienda sit in planisphærio Nautico magnitudo cujusvis arcus AB. Ad arcus illius terminos A & B ducantur ab alterutro Polorum P vel Q rectæ QA, QB, semidiametro CO occurrentes in D & E: Dico magnitudinem Nauticam arcus AB æqualem esse mensuræ rationis inter EC & DC ad Modulum OC. Nam divisus intelligatur arcus AB in particulas quam minimas RS, & jungantur QR, QS quæ secent CO in T & V; & demisso in axem perpendiculo SX quod rectæ QR occurrat in Z, erit lineola SZ æqualis particulæ RS. Itaque magnitudo Nautica nascentis arcus RS, erit ad Sphæræ semidiametrum OC, ut arcus ille RS sive lineola SZ ad SX, hoc est, ut VT ad VC. Unde per *Corol.* 2. *Prop.* 1.) magnitudo illa Nautica æquatur mensuræ rationis inter VC & TC ad Modulum OC: & similes utrobique summas colligendo, magnitudo Nautica totius arcus AB æquabitur mensuræ totius rationis inter EC & DC ad eundem Modulum OC.

Fig. 35.

XXX. *A newMethod and Table for Logarithms. Com. by Mr.* Long. *n.* 339. *p.* 52.

Log.	*Nat. Num.*	*Log.*	*Nat. Num.*
0,9	7.943282347	0,09	1.230268771
0,8	6.309573445	0,08	1.202264435
0,7	5.011872336	0,07	1.174897555
0,6	3.981071706	0,06	1.148153621
0,5	3.162277660	0,05	1.122018454
0,4	2.511886432	0,04	1.096478196
0,3	1.995262315	0,03	1.071519305
0,2	1.584893193	0,02	1.047128548
0,1	1.258925412	0,01	1.023292992

Log.

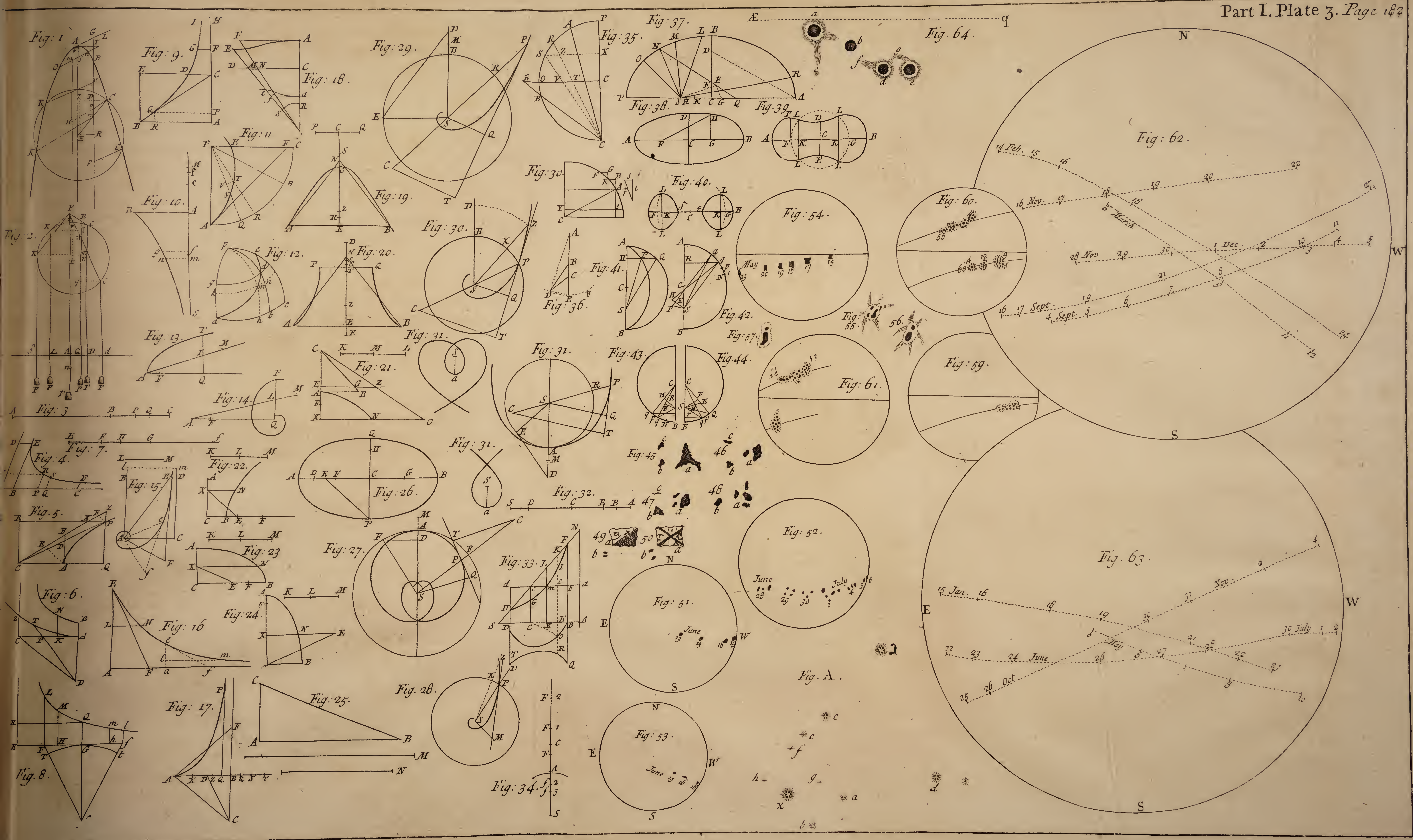


Fig: 1
Fig: 2.
Fig: 3
Fig: 4.
Fig: 5.
Fig: 6.
Fig: 7.
Fig. 8.
Fig: 9.
Fig: 10.
Fig: 11.
Fig: 12.
Fig: 13.
Fig: 14.
Fig: 15.
Fig: 16
Fig: 17.
Fig: 18.
Fig: 19.
Fig: 20.
Fig: 21.
Fig: 22.
Fig: 23
Fig: 24.
Fig: 25.
Fig: 26.
Fig: 27.
Fig. 28.
Fig: 29.
Fig: 30.
Fig: 31.
Fig: 32.
Fig: 33.
Fig: 34.
Fig: 35.
Fig: 36.
Fig: 37.
Fig: 38.
Fig: 39.
Fig: 40.
Fig: 41.
Fig. 42.
Fig: 43.
Fig: 44.
Fig: 45
Fig: 51.
Fig: 52.
Fig: 53.
Fig: 54.
Fig: 55
Fig: 57.
Fig: 59.
Fig: 60.
Fig: 61.
Fig: 62.
Fig. 63.
Fig. 64.
Fig. A.
June
July
May
Nov.
Dec.
Sept.
Feb.
March
Jan.
Oct.
N
S
E
W

Log.	*Nat Num.*	*Log.*	*Nat Num.*
0,009	1.020939484	0,000009	1.000020724
0,008	1.018591388	0,000008	1.000018421
0,007	1.016248694	0,000007	1.000016118
0,006	1.013911386	0,000006	1.000013816
0,005	1.011579454	0,000005	1.000011513
0,004	1.009252886	0,000004	1.000009210
0,003	1.006931669	0,000003	1.000006908
0,002	1.004615794	0,000002	1.000004605
0,001	1.002305238	0,000001	1.000002302
0,0009	1.002074475	0,0000009	1.000002072
0,0008	1.001843766	0,0000008	1.000001842
0,0007	1.001613109	0,0000007	1.000001611
0,0006	1.001382506	0,0000006	1.000001381
0,0005	1.001151956	0,0000005	1.000001151
0,0004	1.000921459	0,0000004	1.000000921
0,0003	1.000691015	0,0000003	1.000000690
0,0002	1.000460623	0,0000002	1.000000460
0,0001	1.000230285	0,0000001	1.000000230
0,00009	1.000207254	0,00000009	1.000000207
0,00008	1.000184224	0,00000008	1.000000184
0,00007	1.000161194	0,00000007	1.000000161
0,00006	1.000138165	0,00000006	1.000000138
0,00005	1.000115136	0,00000005	1.000000115
0,00004	1.000092106	0,00000004	1.000000092
0,00003	1.000069080	0,00000003	1.000000069
0,00002	1.000046053	0,00000002	1.000000046
0,00001	1.000023026	0,00000001	1.000000023

This Table I ſometimes make uſe of for finding the Logarithm of any Number propos'd, and *vice verſa.* Suppoſe I had occaſion to find the Logarithm of 2000. I look in the firſt Claſs of my Table (the whole Table conſiſts of 8 Claſſes) for the next leſs to 2, which is .995262315, and againſt it is 3, which conſequently is the firſt Figure of the Logarithm ſought. Again, dividing the Number propos'd 2, by 1.995262315 the Number found in the Table, the Quotient is 1.002374467; which being look'd for in the ſecond Claſs of the Table, and finding neither its equal, nor a leſſer, I add 0 to the part of the Logarithm before found, and look for the ſaid Quotient 1.002374467 in the third Claſs, where the next leſs is 1,002305238, and againſt it is 1, to be added to the part of the Logarithm already found; and dividing the Quotient 1.002374467, by 1.002305238, laſt found in the Table, the Quotient is 1,000069070; which being ſought in the fourth Claſs gives 0, but being ſought in the fifth Claſs gives 2, to be added to the part of the Logarithm already found; and dividing the laſt Quotient by the Number laſt found in the Table,

viz.

viz. 1.000046053, the Quotient is 1.000023015, which being ſought in the ſixth Claſs, gives 9 to the part of the Logarithm already found: And dividing the laſt Quotient by the new Diviſor, *viz.* 1.000002072, the Quotient is 1.000000219, which being greater than 1.000000115, ſhews that the Logarithm already found, *viz.* 3,3010299 is leſs than the Truth by more than half an Unit; wherefore adding 1, you have *Briggs*'s Logarithm of 2000, *viz.* 3,3010300.

If any Logarithm be given, ſuppoſe 3,3010300, throw away the Characteriſtic, then over againſt theſe Figures 3 ... 0 .. 1 ... 0 .. 3 .. 0 .. 0, you have in their reſpective Claſſes 1,995262315 ..... 0 ..... 1,002305238 ...... 0 ..... 1,000069080 .... 0 ... 0 which multiplied continually into one another, the Product is 2.000000019966, which by reaſon the Characteriſtic is 3, becomes 2.000,000019966, *&c.* that is, 2000, the natural Number deſired. I ſhall not mention the Method by which this Table is fram'd, becauſe you will eaſily ſee that from the uſe of it.

It is obvious to the intelligent Reader, that theſe Claſſes of Numbers are no other than ſo many Scales of mean Proportionals: In the firſt Claſs, between 1 and 10; ſo that the laſt Number thereof, *viz.* 1,258925412 is the tenth Root of 10, and the reſt in order aſcending are the Powers thereof. So in the ſecond Claſs, the laſt Number 1,023292992 is the hundredth Root of 10, and the reſt in the ſame manner are Powers thereof. So 1,002305238 in the third Claſs, is the tenth Root of the laſt of the ſecond, and the reſt its Powers, *&c.* Or, which is all one, each Number in the preceeding Claſs, is the tenth Power of the correſponding Number in the next following Claſs: Whence 'tis plain, that to conſtruct theſe Tables requires no more than one Extraction of the fifth or furſolid Root for each Claſs, the reſt of the Work being done by the common Rules of Arithmetick; and for extracting the fifth Root, you will find more than one very compendious Rule in *Num.* * 210 of theſe *Tranſactions*, if any one ſhall deſire to examin the *computus* of theſe Tables.

* *Abr.* Vol. I. *p.* 81.

The Doctrine of Combinations and Alternations improv'd, and compleated by Maj. Thornycroft. *n.* 299. *p.* 1951.

XXXI. In order to underſtand what follows, it muſt be obſerved, 1ſt, That as in the notation of Powers, $aaaabbbcc$ is deſign'd by $a^4 b^3 c^2$, and univerſally p times the poſition of a, q times the poſition of b, r times the poſition of c, by $a^p b^q c^r$, ſo in things expos'd likewiſe: (unleſs where 'tis propos'd they ſhould be all different) which Indices, as they have here no relation to Powers, but expreſs only the Occurrences of thoſe things to which they reſpectively belong; I therefore call Indices of Occurrences.

2dly, That as often as I ſhall hereafter mention the Combinations or Alternations of the $p' \; q' \; r'$ or s', (which conſider'd by themſelves are capable of no variation) I mean of thoſe things whoſe Indices they are.

3dly, That m is generally put for the whole number of things expos'd, whether all different or not, *i. e.* equal to the ſum of their Indices; and n, for ſuch a number of them, as each Combination and Alternation muſt conſiſt of; (unleſs preſuppoſed equal) which explains what is hereafter meant

meant by the Combinations and Alternations of m things taken n and n; or of m things taken m and m; and the like Expreſſion, by whatever Symbols the number of things out of which the Combinations and Alternations are to be made, or of which they are to conſiſt, may be deſign'd.

Lemma 1ſt. If in a right Line, at any diſtances, be plac'd any number of things, $a\,b\,c\,d$, &c. the number of the Intervals ab, bc, cd, &c. terminated each by two adjacent things, is one leſs than the number of things.

For, whereas every Interval is terminated by two adjacent things, if to any number of things, be added one thing more, one Interval only is thereby added. *Q. E. D.*

Lemma 2. The number of the Alternations of m things $a\,b\,c\,d$, &c. different each from other, taken m and m, is m times the number of the Alternations of $m-1$ things $a\,b\,c$, taken $m-1$ and $m-1$.

For, (by *Lem.* 1ſt) the laſt Letter d, beſides the poſition it hath, may have $m-2$ poſitions, *viz.* in the Intervals which are between $m-1$ things $a\,b\,c$; but it may alſo have one more, for it may be put firſt of all, it may therefore have m poſitions; and thoſe in all the different Orders, whereof $m-1$ things are capable; which being all the poſſible poſitions of d, in all the varieties of $a\,b\,c$, is all the variety whereof the whole number of things expoſed $a\,b\,c\,d$, &c. is capable. *Q. E. D.*

Lemma 3d. The number of the Alternations of m things $a\,b\,c\,d$, &c. different each from other, taken m and m, is equal to $m \times \overline{m-1} \times m\,\overline{m-2} \times \overline{m-3} \times \overline{m-4}$, &c. continu'd to m places.

For let mO, expreſs the number of the Alternations of m things different each from other; $\overline{m-1}\,O$, of $m-1$ things and the like.

'Tis evident that if $m=1$, it will be $mO=m$; for there can be but one order of one thing.

And if m be greater than unity, then will it be (by *Lem.* 2.) $mO = m \times \overline{m-1}\,O = m \times \overline{m-1} \times \overline{m-2}\,O = m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3}\,O =$, &c. till we have an Equation conſiſting of m places; *i. e.* $= m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3} \times$ &c. continu'd to m places. *Q. E. D.*

Lemma 4th. If $m\omega$ expreſs the number of the Alternations of m things $a^p\,b^p\,c^p\,d^q\,e^r\,f^r$ &c. taken m and m, and α the number of p's, β the number of q's, γ the number of r's, it will be $m\omega =$

$$\frac{m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3} \times \overline{m-4} \times \overline{m-5} \times \text{\&c. continu'd to } m \text{ places.}}{\overline{p \times \overline{p-1} \times \overline{p-2} \times \text{\&c.}}\,\alpha \times \overline{q \times \overline{q-1} \times \text{\&c.}}\,\beta \times \overline{r \times \overline{r-1} \times \text{\&c.}}\,\gamma}$$

each Series continued to p, q, r, &c. places reſpectively.

For the number of the Alternations of any number of things, however divided into parts, is produc'd by a continual Multiplication of the Alternations of thoſe things amongſt themſelves reſpectively, which compoſe each part, into the number of their Alternations one amongſt the other; *i. e.* in the preſent Caſe (the ſeveral occurrences being ſuppoſed to compoſe the ſeveral Parts, and conſequently the number of the Alternations of the things compoſing each part equal to unity) $m\omega =$ to

 the

the number of the Alternations of the things compofing the parts one amongft the other; but the number of their Alternations one amongft the other, is the fame in this Cafe, as if the things expos'd, being all different, were divided into the fame parts; for the things which compofe each part in both Cafes, are different from the reft of the things expos'd; *i. e.* by *Lem.* 3*d.*

$$m\,\omega = \frac{m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3} \times \overline{m-4} \times \overline{m-5} \times \&c. \text{ continu'd to } m \text{ places}}{\overline{p \times \overline{p-1} \times \overline{p-2} \times \&c}|^{\alpha} \times \overline{q \times \overline{q-1} \times \&c.}|^{\beta} \times \overline{r \times \overline{r-1} \times \&c.}|^{\gamma}}$$ each Series continu'd to p, q, r places refpectively. *Q. E. D.*

Lemma 5th. The number of the Combinations of m things $a\,b\,c\,d$, &c. different each from other, taken n and n, is equal to $\frac{m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3} \times \&c.}{n \times \overline{n-1} \times \overline{n-2} \times \overline{n-3} \times \&c.}$ each Series continued to n places.

For if the things expos'd be divided in two parts, *viz.* in the *Ratio* of n and $m-n$, 'tis evident that their different Combinations taken n and n, are produc'd by the Alternations of the things compofing the Parts one amongft the other: And therefore the number of thofe = to the number of thefe = to the number of the Alternations of m things taken m and m, the Indices of whofe occurrences are n and $m-n$ = $\frac{m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3} \times \&c. \text{ continued to } m \text{ places.}}{n \times \overline{n-1} \times \&c. \times \overline{m-n} \times \overline{m-n-1} \times \&c.}$ each Series continued to n and $m-n$ places refpectively (by *Lem.* 4th) *i. e.* becaufe $n + \overline{m-n} = m$ = $\frac{m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3}\ \&c.}{n \times \overline{n-1} \times \overline{n-2} \times \overline{n-3}\ \&c.}$ each Series continued to n places. *Q. E. D.*

But the number of the Alternations in every Combination is $= n \times \overline{n-1} \times \overline{n-2} \times \overline{n-3} \times$, &c. continued to n places, by *Lem.* 3d therefore

Lemma 6th. The number of the Alternations of m things $a\,b\,c\,d$, &c. different each from other, taken n and n, is $= m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3} \times$, &c. continued to n places. *Q. E. D.*

Scholium. Since in the things expos'd the fame things may occur more than once, and alfo n be lefs than m, the Indices of the occurrences which are in fome of the Combinations of m things taken n and n, may differ from thofe which are in others; but thofe Combinations, the Indices of whofe occurrences are the fame, are faid to be in the fame Form: Therefore, whereas n is equal to the Sum of the Indices, which are in each Combination taken n and n, if n be exprefs'd by all the different Combinations of fuch Indices only (being integer Numbers) whereof no one may exceed the higheft Index of the things expos'd, and being more than one in a Combination, are each of them, which are in the fame Combination, comprehended in a diftinct Index thereof; thefe Expreffions of n will neceffarily be the feveral Forms of the Combinations taken n and n, whereof m things are capable: Whence is deriv'd a General Theorem for finding the Combinations and Alternations of m things taken n and n univerfally: *i. e.* Whether m confift of things all different or not, and whether n be equal to, or lefs than m.

Theo-

Theorem. If n be exprefs'd, according to all the different Forms of Combination which the things expos'd are capable of,

$$\text{and}\left\{\begin{array}{l} p = \text{the higheft Index} \\ q = \text{the next higheft} \\ r = \text{the next higheft} \\ f = \text{the next higheft} \\ \&c. \end{array}\right. \left\{\begin{array}{l} \alpha = \text{the number of } p^s \\ \beta = \text{the number of } q^s \\ \gamma = \text{the number of } r^s \\ \delta = \text{the number of } f^s \end{array}\right\} \text{In every form of Combination.}$$

$$\left\{\begin{array}{l} A = \text{the number of all Indices not lefs than } p \\ B = \text{the number of all Indices not lefs than } q \\ C = \text{the number of all Indices not lefs than } r \\ D = \text{the number of all Indices not lefs than } f \\ \&c. \end{array}\right\} \text{Which are in the things Expos'd}$$

and $b = \alpha + \beta$, $c = b + \gamma$, $d = c + \delta$, &c.

I fay the number of the Combinations of m things taken n and n, in any one Form of Combination, fhall be $\frac{A \times A-1 \times A-2}{\alpha \times \alpha-1 \times \alpha-2}$ &c. $\times$ $\frac{B-\alpha \times B-\alpha-1}{\beta \times \beta-1} \times$ &c. $\times \frac{C-b \times C-b-1}{\gamma \times \gamma-1} \times$ &c. $\times \frac{D-c \times D-c-1}{\delta \times \delta-1}$ &c. continued to fo many Terms as there are different Indices in the form of Combination, and each Term to $\alpha, \beta, \gamma, \delta,$ &c. places refpectively, and this number multiply'd into

$\frac{\times\, n-1 \times n-2 \times n-3 \times n-4 \times n-5 \times n-6 \text{ \&c. continu'd to } n \text{ places}}{p \times p-1 \times p-2 \times \text{\&c.}|^{\alpha} \times q \times q-1 \times \text{\&c}|^{\beta} \times 1 \times r-1 \times \text{\&c}|^{\gamma} \times \text{\&c.}}$ each Series continued to p, q, r, *&c.* places refpectively, fhall be the number of their Alternations.

But the Sum of all the Combinations and Alternations which are in every Form of n, fhall be the whole number of Combinations and Alternations of m things taken n and n.

Demonftration. Firft, Then 'tis evident, that thofe Combinations, which are in different Forms, differ from each other.

Again, 'Tis evident that the Combinations of m things, as $a^p\, b^p\, c^p\, d^p\, e^q\, f^q\, g^q\, h^r\, i^r$, *&c.* (the Indices fimply confider'd) taken n and n, in a Form wherein are p^s q^s and r^s, fhall be equal to the number of the Combinations of the p^s, which are in the things expos'd, taken α and α, multiply'd into the number of the Combinations of the q^s taken β and β multiply'd into the number of the Combinations of the r^s taken γ and γ.

But becaufe p and all leffer Indices are comprehended in every Index which is greater than themfelves; therefore is $A =$ to the number of p^s which are in the things expos'd, and for the fame reafon, would $B =$ the number of the q^s, and C the number of r^s: But the number of the p^s, which are in every Form of Combination, is $= \alpha$; therefore is $B - \alpha =$ to the number of q^s; alfo becaufe the number of p^s and q^s together, which are in every Form of Combination, wherein there are q^s, is $= \alpha + \beta = b$; therefore is $C - b =$ to the number of r^s, and fo on, how many foever were the different Indices in any form of Combination.

 But

But (by *Lem.* 5th) the number of the Combinations of the p', which are in the things expos'd, whose number is A, taken α and α, is $= \frac{A \times \overline{A-1} \times \overline{A-2}}{\alpha \times \overline{\alpha-1} \times \overline{\alpha-2}}$ &c. continu'd to α places, and the number of the Combinations of the q', whose number is $B-\alpha$, taken β and β, is $= \frac{\overline{B-\alpha} \times \overline{B-\alpha-1}}{\beta \times \overline{\beta-1}} \frac{\times \overline{B-\alpha-2}}{\times \overline{\beta-2}}$ &c. continued to β places, and the number of the Combinations of the r', whose number is $C-b$, taken γ and γ, is $= \frac{\overline{C-b} \times \overline{C-b-1}}{\gamma \times \overline{\gamma-1}}$ &c. continued to γ places. *Q. E. D.*

But every Combination, in one and the same Form, affords the same number of Alternations: Therefore the number of Alternations, in any one Form, is so many times the number of Combinations, as is the number of Alternations in any one of these Combinations.

But (by *Lem.* 4th) the number of Alternations in any of those Combinations shall be

$$\frac{n \times \overline{n-1} \times \overline{n-2} \times \overline{n-3} \times \overline{n-4} \times \overline{n-5} \times \overline{n-6} \times \&c. \text{ continued to } n \text{ places}}{\overline{p \times \overline{p-1} \times \overline{p-2} \times \&c.}|^{\alpha} \times \overline{q \times \overline{q-2} \times \&c.}|^{\beta}\ \overline{r \times \overline{r-1} \times \&c.}|^{\gamma}}$$ × each Series continued to $p\ q\ r$ &c. places respectively. *Q. E. D.*

Now to make an application of this general Rule to those particular Cases which have already been consider'd by others, and which are contain'd in our 3*d*, 4*th*, 5*th*, and 6*th* *Lemma's*, and by us more generally demonstrated; I say

If $n=m$, there can be but one form of Combination, and but one Combination in that form; and therefore the number of Alternations $=$

$$\frac{m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3} \times \overline{m-4} \times \&c. \text{ continu'd to } m \text{ places}}{\overline{p \times \overline{p-1} \times \&c.}|^{\alpha} \times \overline{q \times \overline{q-1} \times \&c.}|^{\beta} \times \overline{r \times \&c.}|^{\gamma} \times \&c.}$$ each Series to $p\ q\ r$, &c. places respectively, *i. e.* (if $p=1$) $= m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3} \times \overline{m-4} \times$ &c. continu'd to m places, which are the Cases of the 4th and 3d *Lemma's*.

But if the things expos'd are all different, and n be less than m, which is the case of the 5th and 6th *Lemma*'s, then also can there be but one form of Combination, and it will be $A=m$ and $\alpha=n$, and the whole number of Combinations $= \frac{A \times \overline{A-1} \times \overline{A-2} \times \&c.}{\alpha \times \overline{\alpha-1} \times \overline{\alpha-2} \times \&c.}$ *i. e.* $=$ $\frac{m \times \overline{m-1} \times \overline{m-2} \times \&c.}{n \times \overline{n-1} \times \overline{n-2} \times \&c.}$ each Series continued to n places, and therefore the number of Alternations $= m \times \overline{m-1} \times \overline{m-2} \times$ &c. continu'd to n places.

To illustrate this *Theorem*, which, as delivered in general, may seem somewhat too Abstracted to be commonly understood, I shall subjoyn one short Example.

Example.

Example. Let the things expos'd be $aaabbbcc$, or according to our way of notation $a^3\, b^3\, c^2$; 'Tis requir'd to find the number of their Combinations and Alternations taken 4 and 4.

Then (becaufe in the things expos'd, there is no one thing occurs more than thrice, nor more than three things different from each other) will all the forms of Combination, which the things expos'd are capable of, be thefe, *viz.* $\left\{\begin{array}{l} 3\,.\,1 \\ 2\,.\,2 \\ 2\,.\,1\,.\,1 \end{array}\right\}$ Then

In the firft form will $p=3$, $q=1$, $\alpha=1$, $\beta=1$, $A=2$, $B=3$
In the fecond form will $p=2$, ——, $\alpha=2$, ——, $A=3$, ——
In the third form will $p=2$, $q=1$, $\alpha=1$, $\beta=2$, $A=3$, $B=3$

The number of Combinations in the firft Form $=\frac{A}{\alpha}\times\frac{B-\alpha}{\beta}=\frac{2}{1}\times\frac{2}{1}=4$

The number of Combinations in the 2d Form $=\frac{A\times \overline{A-1}}{\alpha\times\overline{\alpha-1}}=\frac{3\times 2}{2\times 1}=3$

The number of Combinations in the third Form $=\frac{A}{\alpha}\times\frac{\overline{B-\alpha}\times\overline{B-\alpha-1}}{\beta\times\overline{\beta-1}}=\frac{2\times 1}{2\times 1}=3$

And the whole number of Combinations $= 10$

Alfo the number of Alternations.

In the firft Form $=4\times\frac{n\times \overline{n-1}\times\overline{n-2}\times\overline{n-3}}{\overline{p\times \overline{p-1}\times\overline{p-2}}|^{\alpha}\times q^{\beta}}=4\times\frac{4\times 3\times 2\times 1}{\overline{3\times 2\times 1}|^{1}}=4\times 4=16$

In the 2d Form $=3\times\frac{n\times\overline{n-1}\times\overline{n-2}\times\overline{n-3}}{\overline{p\times \overline{p-1}}|^{\alpha}}=4\times\frac{4\times 3\times 2\times 1}{\overline{2\times 1}|^{2}}=3\times 6=18$

In the 3d Form $=3\times\frac{n\times\overline{n-1}\times\overline{n-2}\times\overline{n-3}}{\overline{p\times\overline{p-1}}|^{\alpha}\times q^{\beta}}=3\times\frac{4\times 3\times 2\times 1}{\overline{2\times 1}|^{1}\times 1}=3\times 12=36$

And the whole number of Alternations $= 70$

Many are the Properties of this *Theorem*, in common with others, as, to find the *Unciæ* of a Multinomial rais'd to any integer Power. To raife an infinite Series to an integer Power, though of an interrupted Order, without introducing any thing immaterial, or which muft afterwards be expung'd; and many others. But then fo many Terms of the Series muft be taken in at firft as fhall ferve to the purpofes of the intended Approximation, otherwife as often as it fhall fall fhort of that, the Operation muft be begun *de novo*.

Many likewife are the Properties peculiar to this *Theorem*, and great variety of Problems might be fram'd; and I fcruple not to fay, many may occur in Practice, which are folvable by this, and no other Method whatever.

Hence

Hence may be found the number of Words whereof the 24 Letters are capable, from one Letter in each Word, to any number of Letters given.

Hence may be found the number of all Numbers, to any given number of Places, which may be produc'd from any number of Figures given.

Hence alſo the Compaſs of a Muſical Inſtrument being given, the Time and number of the Bars, whereof each Tune ſhall conſiſt, the number of Tunes may be found which that Inſtrument is capable of.

To give an inſtance of the prodigious variety that there is in Muſick, I have calculated the number of Tunes in Common Time, conſiſting of eight Bars each, which may be play'd on an Inſtrument of one Note Compaſs only, and it is this, *viz*, 27584 270157. 013570. 368586. 999728. 299176. whereas the Changes on 24 Bells is but 620448. 401733. 239439. 360000, which is but $\frac{1}{444588.\,604583}$ of the number of Tunes, and yet Dr. *Wallis* in his *Algebra* demonſtrates, could not be diſpatch'd in 31557. 600000. 000000 years.

If then the Inſtrument were of as many Notes Compaſs as any Inſtrument now in uſe, how prodigiouſly muſt the number of Tunes be encreas'd; the Calculation of which (tho' much more intricate and operoſe) would be equally attainable by our *Theorem.*

Of the Laws of Chance. by Mr. De Moivre. *n.* 329. *p.*215. XXXII. 1. Si p ſit numerus caſuum quibus eventus aliquis contingere poſſit, & q numerus caſuum quibus poſſit non-contingere; tam contingentia quam non-contingentia eventus ſuum habent probabilitatis gradum: Quod ſi caſus omnes quibus eventus contingere vel non-contingere poteſt, ſint æque faciles; probabilitas contingentiæ erit ad probabilitatem non-contingentiæ ut p ad q.

Si *A* & *B*, colluſores duo ita de eventibus certent, ut ſi caſus p contingant, *A* vicerit; ſin caſus q contingant, *B* vicerit; atque ſit a ſumma depoſita, ſors ſeu expectatio ipſius *A* erit $\frac{pa}{p+q}$, ſors vero ſeu expectatio ipſius *B* erit $\frac{qa}{p+q}$, adeoque ſi *A* vel *B* expectationes ſuas vendant, æquum eſt ut pro illis recipiant $\frac{pa}{p+q}$ & $\frac{qa}{p+q}$ reſpective.

Si præmium aliquod a proponatur victori concedendum, ita ut ſi caſus p contigerint, præmium concedatur ipſi *A*, ſin vero caſus q contigerint, præmium ipſi *B* concedatur, atque *A* & *B* hoc pactum ineant, ut ante eventum, præmium dividatur pro ratione ſortium, *A* debebit ſumere partem $\frac{pa}{p+q}$, *B* vero partem $\frac{qa}{p+q}$.

Si eventus duo nullo modo ex ſe invicem pendeant, ita ut p ſit numerus caſuum quibus eventus primus contingere poſſit, & q numerus caſuum quibus poſſit non-contingere; & ſit r numerus caſuum quibus eventus ſecundus

cundus contingere poſſit, & s numerus caſuum quibus poſſit non-contingere: Multiplicetur $p + q$ per $r + s$, & Productum Multiplicationis, *viz.* $pr + qr + ps + qs$ erit numerus caſuum omnium quibus contingentia & non-contingentia eventuum inter ſe variari poſſunt.

Ergo ſi *A* & *B* inter ſe ita de his eventibus certent, ut *A* contendat fore ut uterque contingat, ratio ſortium erit ut pr ad $qr + ps + qs$.

Sed ſi *A* contendat fore ut alteruter contingat, ratio ſortium erit ut $pr + qr + ps$ ad qs.

Si vero *A* contendat fore ut eventus primus contingat, ſecundus autem non contingat, ratio ſortium erit ut ps ad $pr + qr + qs$.

Et eodem argumentandi modo, ſi tres vel plures ſint eventus de quibus *A* & *B* certent, ratio ſortium invenietur Multiplicatione ſola.

Si eventus omnes habeant datum numerum caſuum quibus contingere poſſint, & datum itidem numerum caſuum quibus poſſint non-contingere, & ſit a numerus caſuum quibus eventus aliquis poſſit contingere, & b numerus caſuum quibus poſſit non-contingere, & ſit n numerus eventuum omnium; elevetur $a + b$ ad poteſtatem n.

Et ſi *A* cum *B* certet ea conditione ut ſi eventus unus vel plures contigerint, ipſe *A* vicerit; ſin nullus, tum *B* vicerit; ratio ſortium erit ut $\overline{a + b}|^{n} - b^{n}$ ad b^{n}; etenim terminus unicus, in quo a non reperitur eſt b^{n}.

Si *A* cum *B* certet ea conditione, ut ſi eventus duo vel plures contigerint, *A* vicerit; ſin nullus vel unus, tum *B* vicerit; ratio ſortium erit ut $\overline{a + b}|^{n} - b^{n} - nab^{n-1}$, ad $b^{n} + nab^{n-1}$: Etenim termini duo in quibus aa non reperitur, ſunt b^{n} & nab^{n-1}; & ſic deinceps de cæteris.

Prob. I. A & B *una teſſera ludunt, ea conditione, ut ſi* A *bis vel pluries, octo jactibus teſſeræ monada jecerit, ipſe* A *vincat; ſin ſemel tantum, vel non omnino,* B *vincat; quænam erit ratio ſortium?*

Solutio. Quoniam eſt caſus unicus quo monas contingere poteſt, & quinque caſus quibus poteſt non-contingere, fiat $a = 1$, & $b = 5$. Rurſus quoniam ſunt octo jactus teſſeræ, fiat $n = 8$, & erit $\overline{a + b}|^{n} - b^{n} - nab^{n-1}$ ad $b^{n} + nab^{n-1}$ ut 663991 ad 1015625. hoc eſt, ut 2 ad 3 circiter.

Prob. II. A & B *ſingulis globis ea conditione certant, ut qui globum propius ad metam miſerit, unum ludum vincat; jam poſt ludos aliquot peractos, ipſi* A *deſunt ludi* 4 *quo minus victor abeat, ipſi vero* B, 6; *at ea eſt ipſius* A *in mittendis globis dexteritas, ut ſors illius foret ad ſortem ipſius* B *ut* 3 *ad* 2, *ſi de unico ludo contenderent; quænam eſt ratio ſortium in caſu propoſito?*

Solutio. Quoniam ipſi *A* deſunt 4 ludi quominus victor abeat, ipſi vero *B* 6, ſequitur fore ut certamen futuris concludatur ludis ad plurimum 9 videlicet ſumma deficientium ludorum minus unitate, ergo elevetur $a + b$ ad poteſtatem nonam, hæc erit; $a^{9}\,a + 9a^{8}b + 36a^{7}bb + 84a^{6}b^{3} + 126a^{5}b^{4} + 126a^{4}b^{5} + 84a^{3}b^{6} + 36aab^{7} + 9ab^{8} + b^{9}$. Et ſumantur pro *A* termini omnes in quibus a habet 4 vel plures dimenſiones, & pro *B* termini omnes in quibus *B* habet 6 vel plures dimenſiones, ergo

ratio

ratio fortium erit ut $a^9 + 9a^8b + 36a^7bb + 84a^6b^3 + 126a^5b^4 + 126a^4b^5$ ad $84a^3b^6 + 36aab^7 + 9ab^8 + b^9$. Exponatur a per 3, & b per 2, & habebitur ratio fortium in numeris, videlicet 1759077 ad 194048.

Et generaliter, pofito quod p & q fint numeri deficientium ludorum refpective; elevetur $a+b$ ad poteftatem $p+q-1$, & fumantur pro A & B refpective tot termini quot ipfis defunt ludi reciproce, hoc eft, pro A fumantur tot termini quot funt unitates in q, pro B vero tot termini quot funt unitates in p.

Prob. III. Si A & B *fingulis globis ludant, & ea fit ipfius* A *in mittendis globis dexteritas, ut poffit ipfi* B *duos ludos ex tribus largiri; quæritur quænam foret ratio fortium fi de de ludo uno contenderent.*

Solutio. Sint fortes quæfitæ ut z ad 1, & elevetur $z+1$ ad Cubum; hic erit, $z^3 + 3zz + 3z + 1$. Jam cum A poffit duos ludos ex tribus ipfi B largiri, A in fe id fufcipere poterit, ut tres ludos continuos vincat, adeoque fortes hoc in cafu erunt ut z^3 ad $3zz + 3z + 1$. Ergo $z^3 = 3zz + 3z + 1$. Sive $2z^3 = z^3 + 3zz + 3z + 1$. Ergo $z\sqrt[3]{2} = z + 1$, adeoque $z = \frac{1}{\sqrt[3]{2} - 1}$: Igitur fortes quæfitæ erunt $\frac{1}{\sqrt[3]{2} - 1}$ & 1 refpective.

Et generaliter, fi ea fit ipfius A dexteritas, ut poffit æquali forte in fe fufcipere ut n vices continuas vincat, A poterit deponere $\frac{1}{\sqrt[n]{2} - 1}$ contra 1, fore ut prima vice vincat.

Prob. IV. *Si* A *poffit æqua forte unum ex tribus ludis ipfi* B *largiri, quæritur ratio fortium ipforum* A & B *cum de ludo unico contendunt, hoc eft requiritur ratio dexteritatum.*

Solutio. Sit ratio dexteritatum ut z ad 1. Si autem A unum ludum ex tribus ipfi B largiatur, ergo fufcipit A fe ter victurum, priufquam B bis vicerit; elevetur itaque $z+1$ ad poteftatem quartam, videlicet, $z^4 + 4z^3 + 6zz + 4z + 1$, ergo ratio fortium erit ut $z^4 + 4z^3$ ad $6zz + 4z + 1$; Ergo cum æqua forte contendant, fiat $z^4 + 4z^3 = 6zz + 4z + 1$; Qua æquatione foluta, obtinebitur $z = 1.6$ prope. Ergo ratio dexteritatum erit circiter ut 8 ad 5.

Prob. V. *Invenire quotenis tentaminibus futurum fit probabile ut eventus aliquis contingat, pofito quod fint cafus* a *quibus primo tentamine contingere poffit, & cafus* b *quibus poffit non-contingere, ita ut fi* A & B *de eventu contendant, poffint* A & B *æqua forte eventum affirmare & negare.*

Solutio. Sit x numerus tentaminum quibus eventus aliquis poffit æquali expectatione contingere vel non-contingere, ergo per jam demonftrata erit $\overline{a+b}|^x - b^x = b^x$, five $\overline{a+b}|^x = 2b^x$, ergo $x = \frac{Log. 2.}{Log. \overline{a+b} - Log. b}$.

Infuper refumatur æquatio $\overline{a+b}|^x = 2b^x$, & fit $a : b :: 1 : q$, & æquatio migrat in iftam, $\overline{1 + \frac{1}{q}}|^x = 2$. Elevetur $1 + \frac{1}{q}$ ad poteftatem x, ope

Theo-

Theorematis *Newtoniani*, & fiet $1 + \frac{x}{q} + \frac{x}{1} \times \frac{x-1}{2qq} + \frac{x}{1} \times \frac{x-1}{2} \times \frac{x-2}{3q^3}$ &c. $= 2$. In hac æquatione si sit $q = 1$, erit $x = 1$; si q sit infinita, erit x infinita. Sit x infinita, ergo æquatio superior fiet, $1 + \frac{x}{q} + \frac{xx}{2qq} + \frac{x^3}{6q^3}$ &c. $= 2$. Iterum sit $\frac{x}{q} = z$, & erit $1 + z + \frac{1}{2}zz + \frac{1}{6}z^3$ &c. $= 2$. Sed $1 + z + \frac{1}{2}zz + \frac{1}{6}z^3$ &c. est numerus cujus Logarithmus Hyperbolicus est z, ergo $z = Log.\ 2$. Sed Logarithmus Hyperbolicus ipsius 2 est 7 proxime, ergo $z = 7$ proxime.

Igitur ubi q est 1, erit $x = 1q$; & ubi q est infinita, erit $x = 7q$ proxime.

Jam ergo definivimus limites arctissimos intra quos ratio x ad q consistet, etenim ratio illa orditur ab æqualitate, & cum ad infinitum est provecta, desinit tandem in ratione 7 ad 10 proxime.

Exemp. I. *Inveniendum sit quotenis jactibus* A *suscipere in se possit, ut duas monadas duabus tesseris jaciat.*

Solutio. Quoniam *A* habet casum unicum quo duas monadas jacere possit, & 35 quibus illas non jaciat, erit $q = 35$; Multiplicetur igitur 35 per .7, & productum 24.5 indicabit numerum jactuum quæsitum fore inter 24 & 25.

Exemp. II. *Inveniendum sit quotenis jactibus* A *suscipere in se possit, ut tres monadas tribus tesseris jaciat.*

Solutio. Quoniam *A* habet casum unicum quo monadas tres tribus tesseris jacere possit, & casus 215 quibus illas non jaciat; Multiplicetur 215 per .7, & productum 150.5 indicabit numerum jactuum quæsitum fore inter 150 & 151.

Lemma. *Invenire numerum casuum quibus datus punctorum numerus dato tesserarum numero jaci possit.*

Solutio. Sit $p + 1$ datus punctorum numerus, n numerus tesserarum, f numerus facierum in tessera: fiat $p - f = q$, $q - f = r$, $r - f = s$, $s - f = t$, &c. Numerus casuum quæsitus erit,

$$+\frac{p}{1} \times \frac{p-1}{2} \times \frac{p-2}{3} \text{ \&c.}$$

$$-\frac{q}{1} \times \frac{q-1}{2} \times \frac{q-2}{3} \text{ \&c.} \times \frac{n}{1}.$$

$$+\frac{r}{1} \times \frac{r-1}{2} \times \frac{r-2}{3} \text{ \&c.} \times \frac{n}{1} \times \frac{n-1}{2}.$$

$$-\frac{s}{1} \times \frac{s-1}{2} \times \frac{s-2}{3} \text{ \&c.} \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3}.$$

Quam

Quam ſeriem continuari oportebit, donec aliqui factorum fiant vel æquales nihilo, vel negativi.

N.B. Tot factores ſingulorum productorum, $\frac{q}{1} \times \frac{p-1}{2} \times \frac{p-2}{3}$ &c. $\frac{r}{1} \times \frac{r-1}{2} \times \frac{r-2}{3}$ &c. $\frac{s}{1} \times \frac{s-1}{2} \times \frac{s-2}{3}$ &c. ſumendi ſunt, quot ſunt unitates in $n-1$.

Praxis. Requiratur *v. g.* numerus caſuum, quibus 16 puncta 4 teſſeris jaci poſſint.

$$+\frac{15}{1} \times \frac{14}{2} \times \frac{13}{3} = +455$$

$$-\frac{9}{1} \times \frac{8}{2} \times \frac{7}{3} \times \frac{4}{1} = -336$$

$$+\frac{3}{1} \times \frac{2}{2} \times \frac{1}{3} \times \frac{4}{1} \times \frac{3}{2} = +\ 6$$

Jam $455 - 336 + 6 = 125$. Ergo 125 eſt numerus caſuum quæſitus.

Requiratur numerus caſuum quibus 15 puncta 6 teſſeris jaci poſſint.

$$+\frac{14}{1} \times \frac{13}{2} \times \frac{12}{3} \times \frac{11}{4} \times \frac{10}{5} = +2002$$

$$-\frac{8}{1} \times \frac{7}{2} \times \frac{6}{3} \times \frac{5}{4} \times \frac{4}{5} \times \frac{6}{1} = -\ 336$$

Jam $2002 - 336 = 1666$ numerus caſuum quæſitus.

Requiratur numerus caſuum quibus 27 puncta 6 teſſeris jaci poſſint.

$$+\frac{26}{1} \times \frac{25}{2} \times \frac{24}{3} \times \frac{23}{4} \times \frac{22}{5} = +65780$$

$$-\frac{20}{1} \times \frac{19}{2} \times \frac{18}{3} \times \frac{17}{4} \times \frac{16}{5} \times \frac{6}{1} = -93024$$

$$+\frac{14}{1} \times \frac{13}{2} \times \frac{12}{3} \times \frac{11}{4} \times \frac{10}{5} \times \frac{6}{1} \times \frac{5}{2} = +30030$$

$$-\frac{8}{1} \times \frac{7}{2} \times \frac{6}{3} \times \frac{5}{4} \times \frac{4}{5} \times \frac{6}{1} \times \frac{5}{2} \times \frac{4}{3} = -\ 1120$$

Jam $65780 - 93024 + 30030 - 1120 = 1666$ numerus caſuum quæſitus.

Corollarium. Puncta omnia æqualiter ab extremis diſtantia habent eundem numerum caſuum quibus producantur, adeoque ſi numerus punctorum datus vicinior ſit majori extremo quam minori, ſubtrahatur numerus iſte ex ſumma extremorum, & inveniatur numerus caſuum quibus reſiduus numerus producatur, & fiet operatio brevior.

Exem. III. *Invenire quotenis jactibus A ſuſcipere in ſe poſſit ut 15 puncta 6 teſſeris jaciat.*

Solu-

Solutio. Quoniam *A* habet caſus 1666 quibus jacere poſſit 15 puncta, & 44990 quibus illa non jaciat, dividatur 44990 per 1666, & quotus 27 erit $= q$. Ergo multiplicetur 27 per 7, & productum multiplicationis, 18,9 indicabit numerum jactuum quæſitum eſſe 19 fere.

Prob. IV. *Invenire quotenis tentaminibus futurum ſit probabile, ut eventus aliquis bis contingat, poſito quod ſint caſus* a *quibus primo tentamine contingere poſſit, & caſus* b *quibus poſſit non-contingere; ita ut ſi* A & B *de eventu contendant, poſſint* A & B *æqua ſorte eventum affirmare & negare.*

Solutio. Sit x numerus tentaminum, ergo per jam demonſtrata patebit fore $\overline{a+b}|^x = 2b^x + 2axb^{x-1}$. Sive faciendo $a:b::1:q$, $\overline{1+\frac{1}{q}}|^x = 2 + \frac{2x}{q}$. 1°. Sit $q = 1$, & erit $x = 3$. 2°. Sit q infinita, & erit x infinita: Pone x infinitam, & $\frac{x}{q} = z$, & erit $1 + z + \frac{1}{2}z^2 + \frac{1}{6}z^3$, &c. $= 2 + 2z$, adeoque $z = Log.\ 2. + Log.\ \overline{1+z}$; jam ſi $Log.\ 2.$ vocetur y, æquatio iſta in hanc Fluxionalem transformabitur $\dot{z} - \frac{\dot{z}}{1+z} = \dot{y}$. Si autem valor ipſius z, inveſtigetur per Poteſtates ipſius y, invenietur $z = 1.678$ proxime, ergo x ſemper conſiſtet intra limites $3q$ & $1.678q$; ſed x citiſſime converget ad $1.678q$, adeoque ſi q ad 1 habuerit rationem non adeo parvam, poterit aſſumi $x = 1.678q$. Si vero ſit aliqua ſuſpicio ne x ſit juſto minor, ſubſtituatur ipſius valor in æquatione $\overline{1+\frac{1}{q}}|^x = 2 + \frac{2x}{q}$, & notetur error, ſi quis ſit notatu dignus, tunc augeatur x aliquantulum, & ſubſtituatur valor ſic auctus pro x in prædicta æquatione, & notetur novus error, & ope duorum errorum, valor ipſius x poterit ſatis accurate corrigi.

Exemp. I. *Inveniendum ſit quotenis vicibus,* A *in ſe ſuſcipere poſſit, ut tres monadas tribus teſſeris bis jaciat.*

Solutio. Quoniam *A* caſum habet unicum quo tres monadas jaciat, & 215 quibus illas non jaciat, erit $q = 215$: Ergo multiplicetur 215 per 1.678, & productum multiplicationis 360.7 indicabit numerum jactuum quæſitum, fore inter 360 & 361.

Exemp. II. *Inveniendum ſit quotenis vicibus,* A *in ſe ſuſcipere poſſit ut 15 puncta, 6 teſſeris bis jaciat.*

Solutio. Quoniam *A* habet caſus 1666 quibus jacere poſſit 15 puncta, & 44990 quibus illa non jaciat; dividatur 44990 per 1666, & quotus 27 erit $= q$: Ergo multiplicetur 27 per 1.678, & productum multiplicationis 45.3, indicabit numerum jactuum quæſitum, fore inter 45 & 46.

Prob. VII. *Invenire quotenis temtaminibus futurum ſit probabile, ut eventus aliquis, ter, quater, quinquies, &c. contingat, poſito quod ſint caſus* a

quibus primo tentamine contingere possit, & casus b *quibus possit non contingere.*

Solutio. Sit x numerus tentaminum quæsitus; & ex jam demonstratis si de triplici eventu contendatur, facto $a : b :: 1 : q$, erit $\overline{1 + \frac{1}{q}}\Big|^{x} = 2 \times \overline{1 + \frac{x}{q} + \frac{x}{1} \times \frac{x-1}{2qq}}$. Si de quadruplici, $\overline{1 + \frac{1}{q}}\Big|^{x} = 2 \times \overline{1 + \frac{x}{q} + \frac{x}{1} \times \frac{x-1}{2qq} + \frac{x}{1} \times \frac{x-1}{2} \times \frac{x-2}{3q^3}}$. Et continuatio istarum æquationum est manifesta. Jam in priori æquatione, si sit $q = 1$, erit $x = 5q$; si vero q sit infinita, vel ad unitatem habuerit rationem satis magnam, æquatio prædicta, ponendo $\frac{x}{q} = z$, migrabit in istam $z = Log.\ 2 + Log.\ \overline{1 + z + \frac{1}{2}z^2}$, vel in istam Fluxionalem posito $Log.\ 2 = y$, $\frac{\frac{1}{2}z^2 \dot{z}}{1 + z + \frac{1}{2}zz} = \dot{y}$; ubi reperietur $z = 2.675$ proxime; ergo x semper consistet intra $5q$ & $2.675q$.

In æquatione posteriori, si q sit $= 1$, erit $x = 7q$; si vero x sit infinita, vel ad unitatem habuerit rationem satis magnam, erit $z = Log.\ 2 + Log.\ \overline{1 + z + \frac{1}{2}z^2 + \frac{1}{6}z^3}$, vel $\frac{\frac{1}{6}z^3 \dot{z}}{1 + z + \frac{1}{2}zz + \frac{1}{6}z^3} = \dot{y}$, ubi reperietur $z = 3.67199q$ proxime; & par est ratio omnium sequentium, & limites semper approximant ad rationem numeri binarii ad unitatem.

Tabella Limitum. Si de eventu simplici contendatur, numerus tentaminum erit intra $1q$ & $0.693q$.

Si de duplici, intra	$3q$ & $1.678q$
Si de triplici, intra	$5q$ & $2.675q$
Si de quadruplici, intra	$7q$ & $3.67199q$
Si de quintuplici, intra	$9q$ & $4.67q$
Si de sextuplici, intra	$11q$ & $5.668q$

Si de pluribus, quorum numerus sit n, contendatur; modo n & q ad unitatem habuerint rationem satis magnam, conjectura de numero tentaminum non multum a vero aberrans facile fiet, ponendo numerum tentaminum $= \frac{2n-1}{2}q$. Etenim x cito converget ad limitem minorem.

Prob. VIII. *Tres collusores* A, B, C, *singulis globis certant, ea conditione ut qui primus datum ludorum numerum vicerit depositum lucretur; jam post ludos aliquot peractos, desunt ipsi* A, 1; *ipsi* B, 2; *ipsi* C, 3 *ludi; rationes vero dexteritatum sunt ut* a, b, c *respective, quæritur ratio expectationum.*

Solutio. Elevetur $a + b + c$ ad potestatem quartam (etenim 4 ad plurimum ludis certamen necessario concludetur) hæc erit $a^4 + 4a^3b + 6aabb + 4ab^3 + b^4 + 4a^3c + 12aabc + 6aacc + 12abcc + 6bbcc + 4ac^3 + c^4$.

Termini

Termini $a^4 + 4a^3b + 12aabc + 4a^3c + 12abcc$, ubi a ad dimenſionem æque altam ac eſt numerus ludorum ipſi *A* deſideratus, vel altiorem aſcendit; & ubi b & c ad pauciores dimenſiones, quam ſunt numeri ludorum ipſis *B* & *C* deſiderati, aſcendunt; componunt partem expectationis ipſius *A*. Eodem modo termini $b^4 + 4b^3c + 6bbcc$ componunt partem expectationis ipſius *B*. Et termini $4bc^3 + c^4$ componunt partem expectationis ipſius *C*: Reliqui omnes termini ſunt communes, & ita dividi debent, ut partes illæ omnes quæ favent uni colluſorum illi ipſi tribuantur.

Jam cum ipſi *A* deſit 1 ludus, ipſi *B* 2, ipſi *C* 3, partes illæ omnes in quibus a dimenſionem 1^{am} vel altiorem aſſecutus fuerit, priuſquam b 2^{am} & c 3^{m} aſſecuti fuerint, ipſi *A* favent; & eadem eſt ratio partium quæ ipſis *B* & *C* favent, adeoque ſi terminus $6aabb$ in partes ſuas $aabb$, $abab$, $abba$, $baab$, $baba$, $bbaa$, ſit diviſus, partes 5 priores ipſi *A* ſunt tribuendæ, pars unica poſterior ipſi *B*; ergo jam $5aabb$ addi debebit expectationi ipſius *A*, & $1aabb$ expectationi ipſius *B*. Si terminus $4ab^3$ in partes ſuas $abbb$, $babb$, $bbab$, $bbba$, ſit diviſus, pars prima & ſecunda favent ipſi *A*, pars tertia & quarta favent ipſi *B*, adeoque $2ab^3$ utrique eſt tribuenda. Si terminus $12abbc$ in partes ſuas ſit diviſus, partes 8 ipſi *A*, partes vero 4 ipſi *B* ſunt tribuendæ; ſi terminus $4ac^3$ in partes ſuas ſit diviſus, partes 3 ipſi *A* ſunt tribuendæ, pars vero unica ipſi *C*, adeoque expectationes totales jam erunt

1^a. $a^4 + 4a^3b + 5aabb + 2ab^3 + 12aabc + 4a^3c + 6aacc + 8abbc + 3ac^3$.
2^a. $b^4 + 4b^3c + 6bbcc + aabb + 2ab^3 + 4abbc$.
3^a. $4bc^3 + ac^3 + c^4$.

Sit n numerus deficientium ludorum, p numerus colluſorum, rationes expectationum ut a, b, c, d, &c. elevetur $a + b + c + d$, &c. ad poteſtatem $n + 1 - p$, & eodem modo procedatur.

Prob. IX. A & B *aſſumentes uterque* 12 *nummos, ludunt tribus teſſeris, hac conditione, ut ſi* 11 *puncta jaciantur,* A *tradat unum nummum ipſi* B; *at ſi* 14 *puncta jaciantur,* B *tradat unum nummum ipſi* A, *& ut ille ludum victurus ſit qui primus nummos habuerit omnes: Quæritur ratio ſortis ipſius* A *ad ſortem ipſius* B.

Solutio. Sit p numerus nummorum quos uterque ſingulatim aſſumit, ſint a & b numerus caſuum quibus *A* & *B* reſpective nummum unum obtinere poſſunt, & ratio ſortium erit ut a^p ad b^p; hoc in caſu eſt $p = 12$, $a = 27$, $b = 15$; ſive cum ſit $27 : 15 :: 9 : 5$, fiat $a = 9$, $b = 5$, adeoque ratio expectationum erit ut 9^{12} ad 5^{12}, ſive ut 244140625 ad 282429536481 qualem *Hugenius* fore aſſeruit.

Solutio Generalior. Sit p numerus nummorum ipſius *A*, q vero numerus nummorum ipſius *B*; & *A* in ſe ſuſcipiat ut prius nummos q, quam *B* nummos p lucretur, erunt ſortes ut $a^q \times \overline{a^p - b^p}$, ab $b^p \times \overline{a^q - b^q}$. Fingatur enim *A* nummos habere *E*, *F*, *G*, *H*, &c. quorum numerus p; & *B* nummos habere *I*, *K*, *L*, &c. quorum numerus q; fingatur inſuper, valorem cujuſlibet

cujuſlibet nummi eſſe ad valorem ſequentis ut a ad b, ita ut $E, F, G, H, I, K, L,$ ſint in progreſſione Geometrica; his ita poſitis, poterunt A & B qualibet vice deponere nummos quorum valor ſit proportionalis numero caſuum quibus alter alterum vincere poſſit; etenim prima vice poterit A deponere H, B vero I; at H ad I ex hypotheſi eſt ut a ad b; ergo jam A & B æquali conditione certant; ſi vicerit A, poterit ille deponere I, B vero K; ſed I ad K ex Hypotheſi eſt ut a ad b; ſin B vicerit, poterit A deponere G, B vero H, quorum ipſorum G & H ratio eſt ut a ad b, & ſic deinceps. Ergo quamdiu A & B certant, ſemper certant æquali conditione: Igitur eorum expectationes ſunt inter ſe ut ſumma terminorum $E, F, G, H,$ &c. quorum numerus eſt p; ad ſummam terminorum $I, K, L,$ quorum numerus eſt q; hoc eſt, ut $a^q \times \overline{a^p - b^p}$ ad $b^p \times \overline{a^q - b^q}$, quod facile conſtabit, ſi ſummentur progreſſiones iſtæ Geometricæ: Jam poſito, quemlibet nummum eſſe ad ſequentem ut a ad b, non exinde mutantur probabilitates vincendi, ergo poſito, valorem nummorum eſſe æqualem, probabilitates vincendi, ſeu ſortes ipſorum A & B etiamnum erunt in illa ipſa ratione quam determinavimus.

Maxime cavendum eſt ne Problemata propter ſpeciem aliquam affinitatis inter ſe confundantur. Problema ſequens videtur affine ſuperiori.

Prob. X. *C aſſumptis 24 calculis, tres teſſeras jaciat; jam quoties 27 puncta jecerit, tradat calculum unum ipſi* A, *quoties vero 14 puncta jecerit, tradat calculum unum ipſi* B, *at* A *&* B *hoc pacto certent, ut qui prior calculos 12 habuerit, depoſitum obtineat; quæritur ratio expectationum.*

Problema iſtud a ſuperiore in hoc differt, quod 23 ad plurimum teſſerarum jactibus, ludus neceſſario finietur; cum ludus ex lege ſuperioris problematis, poſſet in æternum continuari, propter reciprocationem lucri & jacturæ ſe invicem perpetuo deſtruentium.

Solutio. Elevetur $a + b$ ad poteſtatem 23$^{\text{tam}}$, & termini 12 priores erunt ad 12 poſteriores, ut expectatio ipſius A ad expectationem ipſius B.

Prob. XI. *Tres colluſores* A, B, C, *aſſumentes duodecim calculos, quorum 4 albi, & 8 nigri ſint, ludant hac conditione, ut qui primus ipſorum, velatis oculis, album calculum elegerit, vincat; & ut prima electio ſit penes* A, *ſecunda penes* B, *tertia penes* C; *& tum ſequens rurſus penes* A, *& ſic deinceps ordine: Quæritur quænam futura ſit ratio ſortium ipſorum* A, B, C.

Solutio. Sit n numerus calculorum omnium, a numerus alborum, b numerus nigrorum, 1 ſumma depoſita, ſeu præmium victori concedendum.

1°. A habet caſus a quibus album, & caſus b quibus nigrum eligat adeoque ejus expectatio ex prima electione oriunda eſt $\frac{a}{a+b}$ ſive $\frac{a}{n}$. Igitur ſi $\frac{a}{n}$ ex 1 ſubtrahatur, valor reſiduarum expectationum erit $1 - \frac{a}{n} = \frac{n-a}{n} = \frac{b}{n}$.

2°. B habet caſus a quibus album, & caſus $b - 1$ quibus nigrum eligat; ſed prima electio eſt penes A, & incertum eſt utrum ille victurus ſit nec-

ne,

ne, adeoque præmium reſpectu ipſius *B* non eſt 1, ſed tantummodo $\frac{b}{n}$, igitur illius expectatio ex ſecunda electione oriunda eſt $\frac{a}{a+\overline{b-1}} \times \frac{b}{n} = \frac{ab}{n \times \overline{n-1}}$. Subtrahatur $\frac{ab}{n \times \overline{n-1}}$ ex $\frac{b}{n}$, & valor reſiduarum expectationum erit $\frac{nb-b-ab}{n \times \overline{n-1}} = \frac{b \times \overline{b-1}}{n \times \overline{n-1}}$.

3°. *C* habet caſus *a* quibus album, & caſus $b-2$, quibus nigrum eligat, adeoque ejus expectatio ex tertia electione eſt $\frac{a \times b \times \overline{b-1}}{n \times \overline{n-1} \times \overline{n-2}}$.

4°. Eodem modo *A* habet caſus *a* quibus album, & caſus $b-3$ quibus nigrum eligat, adeoque ejus expectatio ex quarta electione erit $\frac{a \times b \times \overline{b-1} \times \overline{b-2}}{n \times \overline{n-1} \times \overline{n-2} \times \overline{n-3}}$. Et ſic deinceps de cæteris.

Scribatur ergo ſeries $\frac{a}{n} + \frac{b}{n-1}P + \frac{b-1}{n-2}Q + \frac{b-2}{n-3}R + \frac{b-3}{n-4}S$ &c. ubi *P*, *Q*, *R*, *S*, &c. denotant terminos præcedentes cum ſuis ſignis; & ſumantur tot termini hujus ſeriei quot ſunt unitates in $b+1$ (etenim non plures erunt electiones quam ſunt unitates in $b+1$) Et ſumma tertiorum omnium, intermiſſis binis, terminorum, incipiendo ab $\frac{a}{n}$, erit tota expectatio ipſius *A*, ſumma tertiorum itidem omnium incipiendo a $\frac{b}{n-1}P$, erit tota expectatio ipſius *B*; ſumma tertiorum omnium incipiendo a $\frac{b-1}{n-2}Q$, erit tota expectatio ipſius *C*.

Si plures ſint colluſores, *A*, *B*, *C*, *D*, &c. ſive calculum unum, ſive plures, ſive eundem calculorum numerum, ſive diverſum unaquaque vice elegerint, illorum expectationes, ope præcedentis ſeriei, facili negotio itidem determinabuntur.

Sed ut ad caſum in Problemate propoſitum revertamur, fiat $a=4$, $b=8$, $n=12$, & ſeries generalis jam in iſtam migrabit, $\frac{4}{12} + \frac{8}{11}P + \frac{7}{10}Q + \frac{6}{9}R + \frac{5}{8}S + \frac{4}{7}T + \frac{3}{6}V + \frac{2}{5}X + \frac{1}{4}Y$.

Sive in alteram iſtam (multiplicando terminos omnes per numerum iſtum qui tollendis fractionibus magis idoneus judicabitur, nempe hoc in caſu per 495)

$$165 + 120 + 84 + 56 + 35 + 20 + 10 + 4 + 1.$$

adeoque tribuantur ipſi *A*, $165 + 56 + 10 = 231$; ipſi *B*, $120 + 35 + 4 = 159$; ipſi *C*, $84 + 20 + 1 = 105$. Adeoque expectationes erunt ut 231, 159, 105; ſive ut 77, 53, 35.

Corollarium. Si numerus caſuum quibus *A*, *B*, *C*, vel colluſores quotcunque vincere poſſunt, tandem aliquando exhauriatur, expreſſiones ſortium erunt finitæ.

Prob.

Prob. XII. *Si collusores tres,* A, B, C, *vicibus suis Dodecaedron* 4 *albis faciebus,* & 8 *nigris, jaciant, ea conditione ut qui primus faciem albam jecerit, vincat; quæritur ratio expectationum.*

Solutio. Ratiocinia circa hanc Propositionem eadem sunt atque illa quibus uti sumus in præcedenti, sed cum jactus Dodecaedri nihil detrahant de numero facierum, pro $b-1$, $b-2$, $b-3$, $b-4$, &c. $n-1$, $n-2$, $n-3$, $n-4$, &c substituantur b & n respective, & series præcedentis Problematis evadet $\frac{a}{n}+\frac{ab}{nn}+\frac{abb}{n^3}+\frac{ab^3}{n^4}+\frac{ab^4}{n^5}+\frac{ab^5}{n^6}$ &c quæ series in infinitum est continuanda. Et sumendo tertios quosque terminos, expectationes erunt

$$\frac{a}{n}+\frac{ab^3}{n^4}+\frac{ab^6}{n^7}\ \&c.$$

$$\frac{ab}{nn}+\frac{ab^4}{n^5}+\frac{ab^7}{n^8}\ \&c.$$

$$\frac{abb}{n^3}+\frac{ab^5}{n^6}+\frac{ab^8}{n^9}\ \&c.$$

Sed termini ex quibus expectationes singulæ componuntur sunt in progressione geometrica, & ratio cujuslibet termini ad sequentem eadem est in singulis seriebus, nempe ut n^3 ad b^3; ergo summæ serierum sunt ut primi serierum termini, nempe ut $\frac{a}{n}$, $\frac{ab}{nn}$, $\frac{abb}{n^3}$, sive ut nn, bn, bb. Hoc est, in casu istius Problematis, ut 9, 6, 4.

Corollarium. Si plures sint collusores, *A*, *B*, *C*, *D*, &c. iisdem conditionibus ac supra certantes, sumantur tot termini in ratione n ad b, quot sunt collusores, & termini illi denotabunt expectationes collusorum respective.

Prob. XIII. A & B *ludant binis tesseris, hac conditione, ut* A *vincat si punctum senarium jecerit;* B, *si septenarium.* A *primo jactum unum instituat, deinde* B *duos jactus simul; tum rursus* A *duos jactus, atque sic deinceps, donec hic vel ille victor evadat: Quæritur ratio sortis ipsius* A, *ad sortem ipsius* B.

Solutio. Ponatur a numerus casuum quibus *A* vincere possit, & b numerus casuum quibus *B* vincere possit, n numerus variationum in tesseris datis; sit insuper $n-a=d$, & $n-b=e$; sit etiam 1 præmium victori concedendum.

1°. A habet casus a quibus vincere possit, & casus $n-a$ quibus non vincat, adeoque illius expectatio ex primo jactu oriunda est $\frac{a}{n}$; igitur si $\frac{a}{n}$ ex 1 subtrahatur, valor residuarum expectationum erit $1-\frac{a}{n}=\frac{n-a}{n}=\frac{d}{n}$.

2°.

2°. Si B ad jactum suum perveniat, ejus expectatio ex jactu ipsius oriunda, erit $\frac{b}{n}$; sed quoniam incertum est utrum ille ad jactum suum sit perventurus nec ne, expectatio $\frac{b}{n}$ minuenda est in ratione d ad n; Etenim præmium illius respectu, non 1, sed tantummodo $\frac{d}{n}$ censendum est, adeoque expectatio ipsius B priusquam A jactum suum instituat, erit $\frac{bd}{nn}$; subtrahatur $\frac{bd}{nn}$ ex $\frac{d}{n}$, & valor residuarum expectationum erit $\frac{d}{n} - \frac{bd}{nn} = \frac{nd - bd}{nn} = \frac{ed}{nn}$.

3°. Eodem argumentandi modo, expectatio ipsius B huic novissimæ deinceps subsequens, est $\frac{bed}{n^3}$.

4°. Et expectatio ipsius A huic subsequens, est $\frac{aeed}{n^4}$.

5°. Et expectatio ipsius A huic demum subsequens est $\frac{aeedd}{n^5}$. Et sic deinceps de cæteris; adeoque erunt

Expectationes omnes ipsius A.

$$
\begin{array}{l}
\frac{a}{n} \\
+\frac{aeed}{n^4} + \frac{aeedd}{n^5} \\
+\frac{ae^4d^3}{n^8} + \frac{ae^4d^4}{n^9} \\
+\frac{ae^6d^5}{n^{12}} + \frac{ae^6d^6}{n^{13}} \\
\quad \&c.
\end{array}
$$

Jam seposito parumper primo termino $\frac{a}{n}$, columna prima perpendicularis constituit progressionem geometricam infinite decrescentem, cujus summa est $\frac{aeed}{n^4 - eedd}$. Resumatur primus terminus $\frac{a}{n}$, isque addatur summæ progressionis, & aggregatum erit $\frac{naeed + an^4 - aeedd}{n \times \overline{n^4 - eedd}}$.

Columna secunda constituit progressionem alteram Geometricam, cujus summa est $\frac{aeedd}{n \times \overline{n^4 - eedd}}$.

Summa

Summa igitur expectationum ipsius *A* est $\frac{aeed^4 + an^3}{n^4 - eedd}$.

Expectationes omnes ipsius B.

$$\frac{bd}{nn} + \frac{bed}{n^3}$$
$$\frac{beed^3}{n^6} + \frac{be^3d^3}{n^7}$$
$$\frac{be^4d^5}{n^{10}} + \frac{be^5d^5}{n^{11}}$$
&c.

Summa primæ columnæ est $\frac{bdnn}{n^4 - eedd}$:

Summa secundæ columnæ est $\frac{bden}{n^4 - eedd}$:

Adeoque summa expectationum ipsius *B* erit $\frac{bdnn + bden}{n^4 - eedd}$.

Ergo ratio expectationum erit, ut $aeed + an^3$ ad $bdnn + bden$.

Si pro a, b, n, d, e, scribantur 5, 6, 36, 31, 30, respective, exprimetur ratio quæsita in numeris, nempe ut 10355 ad 12276.

Corollarium. Si numerus casuum, quibus collusores vincere possunt, numquam exhauriatur, adeo ut ludus possit in infinitum continuari, ita tamen ut collusores, propter istam continuationem, ponantur aliquando in iisdem circumstantiis in quibus antea fuerunt; expressiones sortium finitæ erunt.

Prob. XIV. *Assumptis* 12 *calculis* 4 *albis, &* 8 *nigris, certet* A *cum* B *fore ut velatis oculis, si* 7 *calculos exemerit, eorum* 3 *albi sint futuri: Quæritur ratio expectationis ipsius* A *ad expectationem ipsius* B.

Solutio. 1°. Inveniantur casus omnes quibus 7 calculi ex 12 eximi possint; casus erunt 792, ut patet ex Doctrina combinationum.

$$\frac{12}{1} \times \frac{11}{2} \times \frac{10}{3} \times \frac{9}{4} \times \frac{8}{5} \times \frac{7}{6} \times \frac{6}{7} = 792.$$

2°. Seponantur 3 albi, & inveniantur casus omnes quibus 4 nigri ex 8 iis adjungi possint; casus illi erunt 70.

$$\frac{8}{1} \times \frac{7}{2} \times \frac{6}{3} \times \frac{5}{4} = 70.$$

Quoniam autem 4 sunt casus quibus 3 albi ex 4 possint eligi, multiplicetur 70 per 4, adeoque casus erunt 280, quibus 3 albi cum 4 nigris possint eximi.

3°. Ex

3°. Ex lege ludorum, ille qui in se suscipit ut effectum aliquem producat, etiamnum victor censetur, si effectum pluries produxerit quam in se susceperit, nisi contrarium expresse sit cautum, adeoque si 4 albi cum 3 nigris eximantur, *A* victor censendus erit; Igitur seponantur 4 albi, & inveniantur casus omnes quibus 3 nigri, ex 8, 4 albis adjungi possint; casus illi erunt 56.

$$\frac{8}{1} \times \frac{7}{2} \times \frac{6}{3} = 56.$$

4°. Igitur *A* casus habet 280 + 56 = 336, quibus victor evadat: Subtrahantur casus illi ex 792, & casus residui erunt 456 quibus *B* victor evadere possit: Ergo ratio sortis ipsius *A*, ad sortem ipsius *B*, erit ut 336 ad 456, sive ut 14 ad 19.

Generaliter. Sit n numerus calculorum omnium, a numerus alborum, b numerus nigrorum, c numerus quem *A* eximat; & erit

Numerus Casuum omnium $\frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} \times \frac{n-4}{5} \times \frac{n-5}{6}$ &c. quæ series continuari debet ad tot terminos quot sunt unitates in c.

Numerus casuum quibus *A* calculos c eximere potest absque ullo albo

$$\frac{b}{1} \times \frac{b-1}{2} \times \frac{b-2}{3} \times \frac{b-3}{4} \times \frac{b-4}{5} \times \frac{b-5}{6} \text{ \&c.}$$

Numerus casuum quibus *A* calculum unum album eximere potest.

$$\frac{b}{1} \times \frac{b-1}{2} \times \frac{b-2}{3} \times \frac{b-3}{4} \times \text{\&c.} \times \frac{a}{1}.$$

Numerus casuum quibus *A* calculos duos albos eximere potest

$$\frac{b}{1} \times \frac{b-1}{2} \times \frac{b-2}{3} \times \frac{b-3}{4} \text{ \&c.} \times \frac{a}{1} \times \frac{a-1}{2}.$$

Numerus casuum quibus *A* calculos tres albos eximere potest

$$\frac{b}{1} \times \frac{b-1}{2} \times \frac{b-2}{3} \text{ \&c.} \times \frac{a}{1} \times \frac{a-1}{2} \times \frac{a-2}{3}.$$

Numerus casuum quibus *A* calculos quatuor albos eximere potest

$$\frac{b}{1} \times \frac{b-1}{2} \text{ \&c.} \times \frac{a}{1} \times \frac{a-1}{2} \times \frac{a-2}{3} \times \frac{a-3}{4}.$$

Et sic deinceps.

Prob. XV. A, B, C, *tres collusores, quorum dexteritates sint æquales, deponant singuli* 1, *& istis conditionibus certent;* 1°. *Ut illorum duo ludum incipiant;* 2°. *Ut victus locum suum tertio cedat, ita ut ille tertius jam cum victore contendat, quæ conditio in posterum semper sit observanda;* 3°. *Ut victus semper multetur summa* q *quæ deposito augendo inserviat;* 4°. *Ut ille depositum sic gradatim auctum, totum obtineat, qui alteros duos suc-*

ſucceſſive vicerit. Quæritur quanto melior vel deterior ſit ſors ipſorum A *&* B, *quos ludum incipere ponimus, quam ipſius* C.

Solutio. Ponatur ludum in infinitum continuari poſſe, hoc pacto.

$$\left.\begin{array}{l} A \text{ vincit } B \\ C \text{ vincit } A \\ B \text{ vincit } C \\ \\ A \text{ vincit } B \\ C \text{ vincit } A \\ B \text{ vincit } C \\ \\ A \text{ vincit } B \\ C \text{ vincit } A \\ B \text{ vincit } C \\ \quad \&c. \end{array}\right\} \text{Depoſitum} \left\{\begin{array}{l} 3+p \\ 3+2p \\ 3+3p \\ \\ 3+4p \\ 3+5p \\ 3+6p \\ \\ 3+7p \\ 3+8p \\ 3+9p \\ \quad \&c. \end{array}\right.$$

Sit R ſpectator aliquis, qui poſtquam A vicerit B ſemel, roget A an velit ſummas quas ſe obtenturum ſperat ipſi vendere, & quanti illas æſtimet cui A annuens reſpondeat.

Cum jam vicerim B, eſt mihi æqua ſors utrum obtineam vel non obtineam $3+2p$, adeoque ſumma iſta valet $\frac{3+2p}{2}$.

Si jam acciderit ut C me vincat, ſed tamen vices meæ certandi cum C revertantur, erit tunc mihi ſors æqua utrum obtineam, vel non obtineam $3+5p$, adeoque expectatio vincendi ipſum C tunc temporis valebit $\frac{3+5p}{2}$. Sed cum ſint 7 adverſus 1 fore ut vices illæ non revertantur (etenim C vincere me debet, B vincere C, ego B rurſus,) ſumma iſta quam me obtenturum ſpero valet $\frac{3+5p}{2\times 8}$.

Ad eundem modum, A computatione rurſus inita deprehendet, valorem deinceps ſummæ quam ſe obtenturum ſperat, eſſe $\frac{3+8p}{2\times 8\times 8}$.

Et ſequentis $\frac{3+11p}{2\times 8\times 8\times 8}$. Et ſic in infinitum.

R computationem hanc juſtam eſſe comperiens, pendat ipſi A ſummas, $\frac{3+2p}{2}$, $\frac{3+5p}{2\times 8}$, $\frac{3+8p}{2\times 8\times 8}$, $\frac{3+11p}{2\times 8\times 8\times 8}$, *&c.* quæ ope ſequentis Theorematis in ſummam unam redigantur.

Theorema. $\frac{n}{b}+\frac{n+d}{bb}+\frac{n+2d}{b^3}+\frac{n+3d}{b^4}$ *&c.* ad inf. $=\frac{n}{b-1}+\frac{d}{\overline{b-1}|^2}$.

Diſtin-

Distinguatur series $\frac{3+2p}{2} + \frac{3+5p}{2\times 8}$ &c. in partes duas

$$\frac{3}{2}\times 1 + \frac{1}{8} + \frac{1}{8\times 8} + \frac{1}{8\times 8\times 8} + \frac{1}{8\times 8\times 8\times 8} \text{ \&c.}$$

$$+\frac{p}{2}\times 2 + \frac{5}{8} + \frac{8}{8\times 8} + \frac{11}{8\times 8\times 8} + \frac{14}{8\times 8\times 8\times 8} \text{ \&c.}$$

Pars 1ª constituit progressionem geometricam, cujus summa est $\frac{12}{7}$.

Pars 2ª sepositis communi multiplicatore $\frac{p}{2}$, & termino primo 2, summatur per Theorema præmissum, & fit $\frac{5}{7} + \frac{3}{49} = \frac{38}{49}$, cui jam addito primo 2, summa erit $\frac{136}{49}$, qua multiplicata per $\frac{p}{2}$, productum $\frac{68}{49}p$, exhibebit summam secundæ seriei. Ergo R pendet ipsi A $\frac{12}{7} + \frac{68}{49}p$.

Eodem modo R ad B se convertens, illum roget utrum velit summas quas ille se obtenturum sperat, ipsi vendere, cui *B* assentiens, & eadem innixus ratione qua ipse A, requirat summam $\frac{3}{7} + \frac{31}{49}p$, quam R justam esse deprehendens, ipsi B pendat.

Denique R eodem cum C pacto inito, pendat ipsi pro summis quas ille se obtenturum sperat, $\frac{6}{7} + \frac{48}{49}p$.

Sit S spectator alius, quem A roget (postquam vicerit B semel) utrum velit ipsius jacturas sustinere, hoc est utrum velit multari summis p, pro ipso A, quoties acciderit ut ipse sit multandus, & quanto pretio velit hanc in se sortem suscipere, cui S respondeat.

Quoniam tibi sors est æqua utrum vincas C vel non, adeoque utrum multeris summa p, vel non, hujus multæ sortem, si in manum mihi dederis $\frac{1}{2}p$, sustinebo.

Quod si illud evenerit ut C te vincat, & B vincat C, adeo ut secunda vice tibi cum C certandum sit, tunc multæ ejusdem sortem si dederis mihi $\frac{1}{2}p$, pariter sustinebo: Verum cum sint 3 adversus 1 fore ut illud non eveniat, hujus multæ sortem, nunc si mihi in manum dederis $\frac{1}{8}p$, sustinebo.

Et eodem argumentandi modo, huic proximam sortem si mihi dederis $\frac{1}{16}p$.

Et huic deinceps proximam, si dederis $\frac{1}{64}p$, &c.

Jam A ipsi S assentiens, tradat ipsi S summas, $\frac{1}{2}p * + \frac{1}{8}p + \frac{1}{16}p * + \frac{1}{64}p + \frac{1}{128}p * + \frac{1}{512}p + \frac{1}{1024}p$, &c. quæ summæ in unam redactæ fiunt $\frac{5}{7}p$.

Et eodem modo B & C pacto inito cum S, ipsi tradant $\frac{3}{7}p$ & $\frac{6}{7}p$, respective, ut suas multarum sortes sustineat.

A recipit

$$\begin{array}{ll} \text{A recipit ab R} & \frac{12}{7} + \frac{68}{49}p. \\ \text{A tradit ipſi S} & \frac{35}{49}p. \\ \hline \text{Ipſi A ſupereſt} & \frac{12}{7} + \frac{33}{49}p. \end{array}$$

Sed A depoſuerat 1, priuſquam ludus inciperetur: Ergo lucratur A $\frac{5}{7} + \frac{33}{49}p$.

$$\begin{array}{ll} \text{B recipit ab R} & \frac{3}{7} + \frac{31}{49}p. \\ \text{B tradit ipſi S} & \frac{21}{49}p = \frac{3}{7}p. \\ \hline \text{Ipſi B ſupereſt} & \frac{3}{7} + \frac{10}{49}p. \end{array}$$

Sed B depoſuerat $1+p$, (videlicet 1 priuſquam ludus inciperetur, & p poſtquam ſemel victus fuerat ab A,) ergo B lucratur $-\frac{4}{7} - \frac{39}{49}p$.

Summa igitur lucrorum ipſorum A & B eſt $\frac{1}{7} - \frac{6}{49}p$.

Jam poſueramus A viciſſe ipſum B ſemel priuſquam colluſores pacta inirent cum R & S; ſed priuſquam ludus inchoaretur, B poterat æqua ſorte expectare ut vinceret ipſum A; adeoque ſumma lucrorum $\frac{1}{7} - \frac{6}{49}p$ in duas partes æquales dividenda, adeo ut utriuſque lucrum cenſendum ſit $\frac{1}{14} - \frac{3}{49}p$.

Ergo concludere jam licet, jacturam ipſius C, eſſe $\frac{1}{7} - \frac{6}{49}p$, ſive lucrum $-\frac{1}{7} + \frac{6}{49}p$.

Sed ut corroboretur computatio noſtra, videamus quale futurum ſit lucrum ipſius C, eadem methodo qua uſi fuimus pro inveniendis lucris ipſorum A & B.

$$\begin{array}{ll} \text{C recipit ab R} & \frac{6}{7} + \frac{48}{49}p. \\ \text{C tradit ipſi B} & \frac{42}{49}p. \\ \hline \text{Ipſi C ſupereſt} & \frac{6}{7} + \frac{6}{49}p. \\ \text{Sed C depoſuerat} & \frac{7}{7} \\ \text{Ergo C lucratur} & -\frac{1}{7} + \frac{6}{49}p. \end{array}$$

Jam

Jam fiat $\frac{1}{7} - \frac{6}{49}p = 0$, & invenietur $p = \frac{7}{6}$, ergo ſi multa ad ſummam quam ſinguli deponunt ſit ut 7 ad 6, colluſores æquali conditione certant.

Si multa ſit ad ſummam quam ſinguli deponunt in minori ratione quam 7 ad 6, *A* & *B* potiori conditione certabunt, *C* deteriori.

Si multa ſit ad ſummam quam ſinguli deponunt in majori ratione quam 7 ad 6, *A* & *B* deteriori conditione certant, *C* potiori.

Corol. I. Poſtquam *A* vicerit *B* ſemel, probabilitates vincendi erunt ut $\frac{12}{7}$, $\frac{6}{7}$, $\frac{3}{7}$, ſive ut 4, 2, 1; ita ut maxima probabilitas ſit ipſius *A*, proxima ipſius *C*, minima ipſius *B*.

Corol. II. Spectator *R* priuſquam ludus inchoetur, id ſuſcipere in ſe poterit, ut ſummam 3 de qua colluſores contendunt, & multas omnes pendat, ſi ſibi initio in manus datum ſit $3 + 3p$.

Corol. III. Si dexteritates colluſorum ſint in ratione data, ſortes colluſorum eadem ratiocinatione determinabuntur.

Corol. IV. Si multa ſit negativa, ita ut victus portiunculam depoſiti 3 ſumat, v. g. $\frac{3}{10}$, & ludus ſit finiendus ſtatim atque depoſitum exhauſtum fuerit, ſortes colluſorum eadem ratiocinatione determinabuntur.

Corol. V. Si plures ſint colluſores, *A*, *B*, *C*, *D*, *&c.* & non prius ludo deſiſtant quam illorum unus alios omnes ſucceſſive vicerit, ratio ſortium etiam invenietur.

Corol. VI. Si multa non ſit definita, ſed continuo creſcat vel decreſcat, qua libuerit lege, ratio ſortium etiam determinabitur, ſi non per expreſſiones finitas, at ſaltem per ſeries ad verum perpetuo convergentes.

Prob. XVI. A & B, *quorum dexteritates ſint æquales inter ſe, dato Globorum numero certent; jam poſt ludos aliquot peractos, deſit ipſi* A *ludus* 1 *quominus victor evadat, ipſi* B *vero* 2: *Quæritur ratio illorum ſortium.*

Solutio. Sit m numerus globorum omnium, ita ut uterque habeat $\frac{1}{2}m$; ſit p numerus caſuum quibus duo vel plures ex globis ipſius B propius ad metam accidere poſſint; ſit q numerus caſuum quibus unus vel plures ex globis ipſius B propius ad metam accidere poſſint, adeo ut $q - p$ ſit numerus caſuum quibus unus ex globis ipſius B (excluſive pluribus) poſſit ad metam propius accidere; ſit s numerus variationum omnium quas globi omnes ſubire poſſint; ſit 1 depoſitum totum.

Patet B habere caſus p quibus obtineat 1, & caſus $q - p$ quibus obtineat $\frac{1}{2}$, adeoque illius expectationem eſſe $\frac{p + \frac{1}{2}q - \frac{1}{2}p}{S} = \frac{\frac{1}{2}p + \frac{1}{2}q}{S}$.

Jam conſtat ex Doctrina combinationum, globos omnes m variari poſſe vicibus, $m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3}$, &c. quæ ſeries continuari debet, donec

nec ultimus terminus fiat æqualis unitati, adeoque eſſe $s = m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3}$ &c.

Conſtat ex eadem Doctrina globos numero $\frac{1}{2}m$, poſſe permutari binos, vicibus $\frac{1}{2}m \times \frac{1}{2}\overline{m-1}$, dum globi reliqui omnes ipſorum A & B, quorum numerus $m-2$, poſſunt variari vicibus $\overline{m-2} \times \overline{m-3} \times \overline{m-4}$, &c. adeoque eſſe $p = \frac{1}{2}m \times \overline{\frac{1}{2}m-1} \times \overline{m-2} \times \overline{m-3} \times \overline{m-4}$, &c. Igitur $s : p :: m \times \overline{m-1} : \frac{1}{2}m \times \overline{\frac{1}{2}m-1} :: m-1 : \frac{1}{4}m - \frac{1}{2}$, & $p = \dfrac{\frac{1}{4}ms - \frac{1}{2}s}{m-1}$.

Liquet globos numero $\frac{1}{2}m$, poſſe ſumi ſigillatim vicibus $\frac{1}{2}m$, dum globi reliqui omnes ipſorum A & B quorum numerus $m-1$, variari poſſunt vicibus $\overline{m-1} \times \overline{m-2} \times \overline{m-3} \times \overline{m-4}$, &c. adeoque eſſe $s : q :: m : \frac{1}{2}m :: 1 : \frac{1}{2}$; Eſt igitur $q = \frac{1}{2}s = \dfrac{\frac{1}{2}ms - \frac{1}{2}s}{m-1}$.

Ergo $\dfrac{\frac{1}{2}p + \frac{1}{2}q}{s} = \dfrac{\frac{3}{8}m - \frac{1}{2}}{m-1}$, ſubtrahatur hoc ex 1, & reſiduum $\dfrac{\frac{5}{8}m - \frac{1}{2}}{m-1}$ erit expectatio ipſius A, adeoque ratio ſortium ipſorum A & B erit ut $\frac{5}{8}m - \frac{1}{2}$ ad $\frac{3}{8}m - \frac{1}{2}$, ſive ut $5m-4$ ad $3m-4$.

Corol. I. Si numerus globorum eſſet infinitus, ratio ſortium fieret tandem ut 5 ad 3.

Corol. II. Si dexteritates ſint in ratione data, ratio ſortium eadem ratiocinatione invenietur.

Prob. XVII. A *&* B *quorum dexteritates ſint æquales inter ſe, dato globorum numero certent; jam poſt ludos aliquot peractos, deſit ipſi* A *ludus* 1 *quominus victor evadat, ipſi vero* B 3*: Requiritur ratio ſortium ipſorum* A *&* B.

Solutio. Sit ut in præcedenti Problemate m numerus globorum omnium; ſit r numerus caſuum quibus 3 vel plures ex globis ipſius B ad metam propius accidere poſſint, p numerus caſuum quibus 2 vel plures, q numerus caſuum quibus 1 vel plures propius ad metam poſſint accidere; ſit s numerus variationum omnium quas globi omnes poſſint ſubire.

Ergo B caſus habet r quibus obtineat 1, caſus $p-r$ quibus obtineat $\frac{1}{2}$, & caſus $q-p$ quibus obtineat $\frac{3m-4}{8m-8}$, ut patet ex præcedenti, adeoque ſumma illius expectationum erit $\dfrac{r \times 1 + \overline{p-r} \times \frac{1}{2} + \overline{q-p} \times \frac{3m-4}{8m-8}}{s}$

$$= \frac{\frac{1}{2}r + \frac{1}{2}p + \overline{q-p} \times \frac{3m-4}{8m-8}}{s}.$$

Jam globi numero $\frac{1}{2}m$ poſſunt permutari terni, vicibus $\frac{1}{2}m \times \overline{\frac{1}{2}m-1} \times \overline{\frac{1}{2}m-2}$, dum globi omnes reliqui ipſorum *A* & *B* quorum numerus $m-3$, poſſunt variari vicibus $\overline{m-3} \times \overline{m-4}$, *&c.* Igitur eſt $r = \frac{1}{2}m \times \overline{\frac{1}{2}m-1}$

$\overline{\tfrac{1}{2}m - 1} \times \overline{\tfrac{1}{2}m - 2} \times \overline{m-3} \times \overline{m-4}$, &c. Sed est $s = m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3} \times \overline{m-4}$, &c. ergo $r = \dfrac{\tfrac{1}{8}ms - \tfrac{1}{2}s}{m-1}$.

Sed ex præcedenti Problemate est $p = \dfrac{\tfrac{1}{4}ms - \tfrac{1}{2}s}{m-1}$, & $q = \dfrac{\tfrac{1}{2}ms - \tfrac{1}{2}s}{m-1}$.

Substitutis igitur valoribus istis pro r, p, q, fiet expectatio ipsius $B = \dfrac{9mm - 26m + 16}{32mm - 64m + 32}$. Subtrahatur hæc ab 1, & erit expectatio ipsius $A = \dfrac{23mm - 38m + 16}{32mm - 64m + 32}$; adeoque ratio sortium ipsorum A & B, erit ut $23mm - 38m + 16$ ad $9mm - 26m + 16$, quæ convenit numero globorum cuicunque, binario excepto.

Verum ut ratio sortium ipsorum A & B quum singulis globis certant, sive quum numerus globorum est 2, inveniatur; resumatur expressio generalis expectationis ipsius B, videlicet $\dfrac{\tfrac{1}{2}r + \tfrac{1}{2}p + \overline{q-p} \times \dfrac{3m-4}{8m-8}}{s}$, & ponantur r & $p = 0$, & erit expectatio ipsius $B = \dfrac{q \times \dfrac{3m-4}{8m-8}}{s} = \dfrac{\tfrac{1}{2}m - \tfrac{1}{2}}{m-1} \times \dfrac{3m-4}{8m-8} = \tfrac{1}{2} \times \tfrac{2}{8} = \tfrac{1}{8}$, qua substracta ex 1, erit expectatio ipsius $A = \tfrac{7}{8}$, ergo ratio sortium ipsorum A & B hoc in casu erit ut 7 ad 1, quod aliunde constat ex principiis jamdudum expositis.

Corol. I. Si numerus globorum esset infinitus, ratio sortium fieret tandem ut 23 ad 9.

Corol. II. Si desint ipsi A ludi quotvis quominus victor evadat, & ipsi B ludi itidem quotvis, ratio sortium eadem ratiocinatione invenietur.

Corol. III. Si dexteritates sint in ratione data, ratio sortium etiam invenietur.

Prob. XVIII. *Certet* A *cum* B, *fore ut ipse, dato tentaminum numero, tessera dato facierum numero constante, facies quascunque datas jecerit: Quæritur expectatio ipsius* A.

Solutio. Sit $p+1$ numerus facierum in tessera, n numerus tentaminum datus, f numerus facierum quas jaci oporteat.

Numerus casuum quibus A monada semel vel pluries, tentaminibus numero n, jacere possit, est $\overline{p+1}|^n - p^n$, ut patet ex jam demonstratis.

Expungatur binarius e numero facierum, ita ut numerus facierum reducatur ad p; & erit numerus casuum quibus A monada semel vel pluries tentaminibus numero n, jacere possit $p^n - \overline{p-1}|^n$.

Ergo jam restituto binario, numerus casuum quibus A monada & binarium jacere possit, est differentia istorum casuum, videlicet $\overline{p+1}|^n - 2p^n + \overline{p-1}|^n$.

Expungatur nunc ternarius, & erit numerus casuum quibus *A* monada & binarium jacere possit, $p^n - 2 \times \overline{p-1}|^n + \overline{p-2}|^n$.

Ergo, jam restituto ternario, numerus casuum quibus *A* monada, binarium, & ternarium jacere possit, est $\overline{p+1}|^n - 3 \times p^n + 3 \times \overline{p-1}|^n - \overline{p-2}|^n$. Et sic deinceps de cæteris.

Scribantur ergo ordine potestates omnes, (mutatis alternatim signis) $\overline{p+1}|^n - p^n + \overline{p-1}|^n - \overline{p-2}|^n + \overline{p-3}|^n$ *&c.* Et præfigantur illis coefficientes potestatis designatæ per f, & summa terminorum erit numerator expectationis ipsius *A*, cujus denominator erit $\overline{p+1}|^n$.

Exemp. I. Sit 6 numerus facierum in tessera, & 2 numerus facierum datarum quas jaci oporteat tentaminibus 8, & erit expectatio ipsius *A*,

$$\frac{6^8 - 2 \times 5^8 + 4^8}{6^8}.$$

Exemp. II. Sit 6 numerus facierum in tessera, & 6 numerus facierum quas jaci oporteat tentaminibus 12, & erit expectatio ipsius *A*,

$$\frac{6^{12} - 6 \times 5^{12} + 15 \times 4^{12} - 20 \times 3^{12} + 15 \times 2^{12} - 6 \times 1^{12}}{6^{12}}.$$

Exemp. III. Contendat *A* cum *B* fore ut ipse, tentaminibus numero 43, tessera faciebus 36 constante, facies duas datas jecerit, sive ut binis tesseris vulgaribus jecerit duas monadas simul, atque etiam duos binarios simul; & erit expectatio ipsius *A* $\dfrac{36^{43} - 2 \times 35^{43} + 34^{43}}{36^{43}}$.

N. B. Facilis erit additio & subtractio partium ex quibus expectationes istæ componuntur, ope Tabulæ Logarithmorum.

Prob. XIX. *Invenire quotenis tentaminibus futurum sit probabile ut collusorum alter* A *facies quascunque datas jaciat, tessera constante dato facierum numero.*

Solutio. Sit ut prius $p + 1$ numerus facierum in tessera, n numerus tentaminum datus, f numerus facierum quæsitus. Ponatur Log. $\dfrac{1}{1 - \sqrt[f]{\frac{1}{2}}} = \alpha$, & Log. $\dfrac{p+1}{p} = \beta$; & erit $n = \dfrac{\alpha}{\beta}$ prope.

Demonstratio. Si numerus facierum quas jaci oporteat sit 6, expectatio ipsius *A* erit

$$\frac{\overline{p+1}|^n - 6p^n + 15 \times \overline{p-1}|^n - 20 \times \overline{p-2}|^n + 15 \times \overline{p-3}|^n - 6 \times \overline{p-4}|^n + \overline{p-5}|^n}{\overline{p+1}|^n}.$$

Fingatur terminos $p+1$, p, $p-1$, $p-2$, *&c.* esse in progressione Geometrica, quæ suppositio non multum a vero aberrabit, si præsertim p ad 1 habuerit rationem satis magnam, & ponatur $\dfrac{p^n}{\overline{p+1}|^n} = \dfrac{1}{r}$; ergo expectatio

pectatio ipſius A erit $1 - \frac{6}{r^n} + \frac{15}{r^{2n}} - \frac{20}{r^{3n}} + \frac{15}{r^{4n}} - \frac{6}{r^{5n}} + \frac{1}{r^{6n}} = \frac{1}{2}$.

Extrahatur utrinque radix ſexta, & fiet $1 - \frac{1}{r^n} = \sqrt[6]{\frac{1}{2}}$, ergo $r^n = \frac{1}{1 - \sqrt[6]{\frac{1}{2}}}$

ponatur jam Log. $r = \beta$, & Log. $\frac{1}{1 - \sqrt[6]{\frac{1}{2}}} = \alpha$, & erit $n\beta = \alpha$, adeoque $n = \frac{\alpha}{\beta}$, & eadem erit demonſtratio de cæteris caſibus.

Si ſit aliqua ſuſpicio ne valor indicis n ſic inventus non ſit ſatis accuratus, tunc ſubſtituatur valor iſte pro n, & notetur error, tunc mutetur aliquantulum valor iſte, & notetur novus error, & ope duorum errorum valor indicis n ſatis accurate corrigetur, ſi Regula falſi adhibeatur.

Poteſt valor indicis n ſic inventus corrigi per ſeriem infinitam, ex natura Problematis depromptam, talem ut primus terminus hujus ſeriei ſit valor iſte quem aſſignavimus; ſed correctio per differentiam errorum ſufficit ad uſus practicos.

Exemp. I. Invenire quotenis jactibus vulgaris teſſeræ, probabile ſit ut A facies omnes jaciat.

Log. $\frac{1}{1 - \sqrt[6]{\frac{1}{2}}} = 0.9621753$, Log. $\frac{6}{5} = 0.0791812$, ergo $n = \frac{0.9621753}{0.0791812} = 12 +$. Ergo concludere jam licet numerum jactuum quæſitum fore 12 circiter, ſi vero 12 ſubſtituatur pro n in æquatione caſui huic competente, invenietur expectatio ipſius A .437 prope, quæ aliquanto debita nempe .5 minor eſt; ergo ponatur 13 pro n, & invenietur expectatio ipſius A .513, quæ eſt debita major; ergo poterit A in ſe ſuſcipere ut facies omnes tentaminibus 13 jaciat, idque potiori conditione.

Exemp. II. Invenire quotenis tentaminibus futurum ſit probabile ut A teſſera faciebus 216 conſtante, facies ſex datas jaciat, ſive ut tribus teſſeris vulgaribus * *Triadas* omnes jaciat. * *Raffles.*

Log. $\frac{1}{1 - \sqrt[6]{\frac{1}{2}}} = 0.9621753$, Log. $\frac{216}{215} = 0.0020152$, ergo $n = \frac{0.9621753}{0.0020152} = 477$ prope.

Prob. XX. A & B *quorum dexteritates ſint in ratione data, videlicet, ut* a *ad* b, *ea conditione ludant, ut quoties* A *ludum unum vicerit,* B *tradat ipſi nummum unum; quoties vero* B *vicerit,* A *ipſi tradat nummum unum: & non prius ludo deſiſtant, quam eorum alter nummos omnes alterius lucratus fuerit. Adſlent vero ſpectatores duo* R & S, *quorum* R *affirmet certamen finitum iri intra datum ludorum numerum,* S *neget: Quæritur expectatio ipſius* S. *De Duratione Ludorum.*

Solutio. Caſus I. Sit 2 numerus nummorum quos uterque colluſorum habeat; ſit etiam 2, numerus de quo R & S contendant: Jam propter 2,

numerum ludorum de quo contenditur, elevetur $a+b$ ad poteſtatem 2, quæ erit $aa+2ab+bb$: terminus $2ab$ ipſi S favet, reliqui adverſantur, adeoque illius expectatio erit $\frac{2ab}{\overline{a+b}|^2}$.

Caſus II. Sit 2 numerus nummorum quos uterque colluſorum habeat, & ſit 3 numerus ludorum de quo R & S contendant; elevetur itaque $a+b$ ad poteſtatem 3^{am}, quæ erit $a^3+3aab+3abb+b^3$. Jam termini duo $+a^3+b^3$, omnino ipſi S adverſantur, reliqui duo $3aab+3abb$, partim favent, partim adverſantur; dividantur ergo termini iſti in partes ſuas, videlicet $3aab$ in aab, aba, baa, atque $3abb$ in abb, bab, bba, & partes $aba+baa+abb+bab$, ſive $2aab+2abb$ ipſi S favent, reliquæ adverſantur. Adeoque expectatio ipſius S erit $\frac{2aab+2abb}{\overline{a+b}|^3}$, ſive (diviſis numeratore & denominatore per $a+b$) $\frac{2ab}{\overline{a+b}|^2}$, quæ eadem eſt ac in caſu præcedenti.

Caſus III. Sit 2 numerus nummorum quos uterque colluſorum habeat, & 4 numerus ludorum de quo ſpectatores contendant; elevetur itaque $a+b$ ad poteſtatem 4^{am}, quæ erit $a^4+4a^3b+6aabb+4ab^3+b^4$; termini $a^4+4a^3b+4ab^3+b^4$ omnino ipſi S adverſantur, terminus unicus $6aabb$ partim favet, partim adverſatur: dividatur ergo terminus iſte in partes ſuas, $aabb$, $abab$, $abba$, $baab$, $baba$, $bbaa$, & partes quatuor, $abab$, $abba$, $baab$, $baba$, ſive $4aabb$, ipſi S favent; adeoque illius expectatio erit $\frac{4aabb}{\overline{a+b}|^2}$.

Caſus IV. Sit 2 numerus nummorum quos uterque colluſorum habeat, & 5 numerus ludorum de quo ſpectatores contendant, & expectatio ipſius S invenietur eadem ac in præcedenti caſu.

Caſus V. Sit 2 numerus nummorum quos uterque colluſorum habeat, & 6 numerus ludorum de quo ſpectatores contendant, & expectatio ipſius S invenietur $\frac{8a^3b^3}{\overline{a+b}|^6}$.

Generalius. Sit 2 numerus nummorum quos uterque colluſorum habeat, & $2+d$ numerus ludorum de quo ſpectatores contendant, erit

$$\frac{\overline{2ab}|^{1+\frac{1}{2}d}}{\overline{a+b}|^{2+d}}$$ expectatio ipſius S.

Ubi nota d numerum eſſe parem; quod ſi d ſit numerus impar, expectatio ipſius S eadem erit ac ſi numerus ille unitate eſſet diminutus.

Caſus VI. Sit 3 numerus nummorum quos uterque colluſorum habeat, & $3+d$ numerus ludorum de quo ſpectatores contendant, & invenietur expectatio ipſius $S=\frac{\overline{3ab}|^{1+\frac{1}{2}d}}{\overline{a+b}|^{2+d}}$.

Ubi nota d numerum eſſe parem; quod ſi d ſit numerus impar, expectatio ipſius S eadem erit ac ſi numerus ille unitate eſſet diminutus.

Caſus VII. Sit 4 numerus nummorum quos uterque colluſorum habeat, & 4 numerus ludorum de quo ſpectatores contendant, & invenietur expectatio ipſius $S\ \dfrac{4a^3b + 6aabb + 4ab^3}{\overline{a+b}|^4}$.

Caſus VIII. Sit 4 numerus nummorum quos uterque colluſorum habeat, & 6 numerus ludorum de quo ſpectatores contendant, & invenietur expectatio ipſius $S\ \dfrac{14a^4bb + 20a^3b^3 + 14aab^4}{\overline{a+b}|^6}$.

Tabula expectationum ipſius S, pro numero nummorum 4.

4.	$\dfrac{4a^3b + 6aabb + 4ab^3}{\overline{a+b}|^4}$
6.	$\dfrac{14a^4bb + 20a^3b^3 + 14aab^4}{\overline{a+b}|^6}$
8.	$\dfrac{48a^5b^3 + 68a^4b^4 + 48a^3b^4}{\overline{a+b}|^8}$
10.	$\dfrac{164a^6b^4 + 232a^5b^5 + 164a^4b^6}{\overline{a+b}|^{10}}$
12.	$\dfrac{560a^7b^5 + 792a^6b^6 + 560a^5b^5}{\overline{a+b}|^{12}}$

&c.

Tabula iſta facile continuabitur, ſi ſequentia adnotentur.

1°. Coefficientem termini primi in quolibet numeratore eſſe ſummam coefficientem terminorum omnium in numeratore præcedenti. 2°. Coefficientem termini ſecundi eſſe aggregatum ſummæ iſtius, & coefficientis termini ſecundi præcedentis. 3°. Coefficientem termini tertii eundem eſſe, ac coefficientem termini primi. 4°. Producta literalia, ex præcedentibus, prima ex primis, ſecunda ex ſecundis, formari, multiplicatis præcedentibus per ab. 5°. Denominatores omnes eſſe poteſtatem illam binomii $a + b$, quæ deſignatur per numerum ludorum de quo R & S contendunt.

Hic obiter venit obſervandum coefficientes omnes, primi ex primis, ſecundi ex ſecundis, generari poſſe. Etenim ſi ex ultimo præcedente, quadruplicato, ſubtrahatur penultimus duplicatus, orietur coefficiens quæſitus.

Regula Generalis. Sit n numerus nummorum quos uterque colluſorum habeat, $n + d$ numerus ludorum de quo ſpectatores contendant.

Elevetur $a + b$ ad poteſtatem n, & reſecentur termini duo extremi; multiplicetur reſiduum per $aa + 2ab + bb$, & rejiciantur termini extremi; fiat rurſus multiplicatio reſidui per $aa + 2ab + bb$, & rejiciantur extremi, & ſic deinceps fiant tot multiplicationes quot ſunt unitates in $\frac{1}{2}d$; & productum ultimum erit numerator expectationis ipſius S; denominator vero ſemper erit $\overline{a+b}|^{n+d}$.

N. B. Si d ſit numerus impar, ſubſtituatur $d - 1$ pro d.

Sit

Si n sit numerus impar, dividi poterunt numerator & denominator expectationis per $a+b$, & fiet expectatio simplicior.

Exemp. I. Sit 4 numerus nummorum quos uterque collusorum habeat, & 10 numerus ludorum de quo spectatores contendant, sint autem dexteritates in ratione æqualitatis; quæritur expectatio ipsius S.

Est $n=4$, & $n+d=10$; igitur est $d=6$, & $\frac{1}{2}d=3$. Elevetur itaque $a+b$ ad potestatem 4^{am}, & resectis semper extremis, fiant 3 multiplicationes per $aa+2ab+bb$.

$$
\begin{array}{llllll}
a^4 \mid & +4a^3b & +6aabb & +4ab^3 \mid & +b^4 & \\
 & aa & +2ab & +bb & & \\
\hline
4a^5b \mid & +6a^4bb & +4a^3b^3 & & & \\
 & +8a^4bb & +12a^3b^3 & +8aab^4 & & \\
 & & +4a^3b^3 & +6aab^4 \mid & +4ab^5 & \\
\hline
 & 14a^4bb & +20a^3b^3 & +14aab^4 & & \\
 & aa & +2ab & +bb & & \\
\hline
14a^6bb \mid & +20a^5b^3 & +14a^4b^4 & & & \\
 & +28a^5b^3 & +40a^4b^4 & +28a^3b^5 & & \\
 & & +14a^4b^4 & +20a^3b^5 \mid & 14aab^6 & \\
\hline
 & 48a^5b^3 & +68a^4b^4 & +48a^3b^5 & & \\
 & aa & +2ab & +bb & & \\
\hline
48a^7b^3 \mid & +68a^6b^4 & +48a^5b^5 & & & \\
 & +96a^6b^4 & +136a^5b^5 & +96a^4b^6 & & \\
 & & +48a^5b^5 & +68a^4b^6 \mid & +48a^3b^7 & \\
\hline
 & 164a^6b^4 & +232a^5b^5 & +164a^4b^6 & &
\end{array}
$$

Et erit expectatio ipsius $S = \dfrac{164a^6b^4+232a^5b^5+164a^4b^6}{\overline{a+b}|^{10}}$, & propter a & b æquales erit ista expectatio $\dfrac{164+232+164}{2^{10}} = \dfrac{560}{1024} = \dfrac{35}{64}$.

Exemp. II. Sit 5 numerus nummorum quos uterque collusorum habeat, & 10 numerus ludorum de quo spectatores contendant, ita ut S neget certamen finitum iri intra ludos 10; sit autem dexteritas ipsius A ad dexteritatem ipsius B ut 2 ad 1.

Est $n=5$, & $n+d=10$; est igitur $d=5$. Et propter d imparem, fingatur $d=4$, ergo $\frac{1}{2}d=2$. Elevetur itaque $a+b$ ad potestatem 5^{am}, & resectis semper extremis, fiant 2 multiplicationes per $aa+2ab+bb$.

as

$$\begin{array}{l}
a^5 \mid + 5a^4b + 10a^3bb + 10a^2b^3 + 5ab^4 \mid + b^5 \\
\qquad\qquad aa + 2ab + bb \\
\hline
5a^6b \mid + 10a^5bb + 10a^4b^3 + 5a^3b^4 \\
\qquad + 10a^5bb + 20a^4b^3 + 20a^3b^4 + 10aab^5 \\
\qquad\qquad + 5a^4b^3 + 10a^3b^4 + 10aab^5 \mid + 5ab^6 \\
\hline
\quad 20a^5bb + 35a^4b^3 + 35a^3b^4 + 20aab^5 \\
\qquad\qquad aa + 2ab + bb \\
\hline
\quad 20a^7bb \mid + 35a^6b^3 + 35a^5b^4 + 20a^4b^5 \\
\qquad\qquad + 40a^6b^3 + 70a^5b^4 + 70a^4b^5 + 40a^3b^6 \\
\qquad\qquad\qquad + 20a^5b^4 + 35a^4b^5 + 35a^3b^6 \mid + 20a^2b^7 \\
\hline
\qquad\qquad 75a^6b^3 + 125a^5b^4 + 125a^4b^5 + 75a^3b^6
\end{array}$$

Ergo expectatio ipsius S erit $\dfrac{75a^6b^3 + 125a^5b^4 + 125a^4b^5 + 75a^3b^6}{\overline{a+b}|^9}$.

Sive divisis numeratore & denominatore per $a+b$, propter numerum n imparem, fiet expectatio $= \dfrac{75a^5b^3 + 50a^4b^4 + 75a^3b^5}{\overline{a+b}|^8} = 25a^3b^3 \times \dfrac{3aa + 2ab + 3bb}{\overline{a+b}|^8}$.

Et positis 2 & 1 pro a & b respective, fiet expectatio $= \dfrac{8 \times 25 \times 19}{6561} = \dfrac{3800}{6561}$.

Prob. XXI. *Sit* 4 *numerus nummorum quos uterque collusorum habeat; Requiritur ratio dexteritatum quæ faciat ut* R *possit æqua sorte affirmare certamen finitum iri intra ludos* 4, S *negare.*

Solutio. Expectatio ipsius S, jam inventa, est $\dfrac{4a^3b + 6aabb + 4ab^3}{\overline{a+b}|^4}$, & quoniam, ex Hypothesi, R & S æqua sorte contendunt, ponatur $\dfrac{4a^3b + 6aabb + 4ab^3}{\overline{a+b}|^4} = \frac{1}{2}$, sive $a^4 - 4a^3b - 6aabb - 4ab^3 + b^4 = 0$. Addatur $12aabb$ utrobique, & fiet $a^4 - 4a^3b + 6aabb - 4ab^3 + b^4 = 12aabb$. Extrahatur hinc inde radix quadratica, & erit $aa - 2ab + bb = ab\sqrt{12}$, sive facto $a : b :: z : 1$, $zz - 2z + 1 = z\sqrt{12}$, ubi invenietur radix duplex $z = 5.274$, & $\dfrac{1}{5.274}$. Ergo sive ratio dexteritatis ipsius A ad dexteritatem ipsius B sit ut 5.274 ad 1, vel ut 1 ad 5.274, R & S æqua sorte contendent.

Prob. XXII. *Sit* 4 *numerus nummorum quos uterque collusorum habeat; Requiritur ratio dexteritatum talis, ut possit* R *affirmare finitum iri certamen*

men intra 4 *ludos*, S *negare, atque sint sortes ipsorum* R & S *in ratione data, videlicet ut* 3 *ad* 1.

Solutio. Expectatio ipsius S ex numero ludorum 4, & ratione dexteritatum oriunda est $\frac{4a^3b + 6aabb + 4ab^3}{\overline{a+b}|^4}$. Eadem expectatio propter datam rationem sortium est $\frac{1}{4}$. Ergo sit $\frac{4a^3b + 6aabb + 4ab^3}{\overline{a+b}|^4} = \frac{1}{4}$; sive $a^4 - 12a^3b - 18aabb - 12ab^3 + b^4 = 0$. Jam facto $a : b :: z : 1$, erit $z^4 - 12z^3 - 18zz - 12z^1 + 1 = 0$. Supponatur hæc æquatio ex binis istis quadraticis formari, $zz + yz + 1 = 0$. Et $z^2 + pz + 1 = 0$.

$$\text{Ergo } z^4 \begin{matrix} +y \\ +p \end{matrix} z^3 \begin{matrix} +py \\ +2 \end{matrix} zz \begin{matrix} +y \\ +p \end{matrix} z + 1 = 0.$$

Comparentur coefficientes terminorum Homologorum, & erit $y + p = -12$, & $py + 2 = -18$, sive $py = -20$; unde orietur æquatio $yy + 12y = 20$, cujus radix negativa erit $= -13.483$. Substituatur valor iste in locum ipsius y, & erit $zz - 13.483z + 1 = 0$, cujus æquationis radix duplex invenietur 13.407, & $\frac{1}{13.407}$ prope, ergo sive a ad b sit ut 13.407 ad 1, sive ut 1 ad 13.407, ratio sortium ipsorum R & S erit ut 3 ad 1,

Prob. XXIII. *Sit* 4 *numerus nummorum quos uterque collusorum habeat; Requiritur ratio dexteritatum quæ faciat ut* R *possit æqua sorte affirmare certamen finitum iri intra ludos 6*, S *negare.*

Solutio. Expectatio ipsius S ex numero ludorum, & ratione dexteritatum oriunda, erit $\frac{14a^4bb + 20a^3b^3 + 14aab^4}{\overline{a+b}|^6}$. Ejusdem expectatio propter datam sortium æqualitatem erit $= \frac{1}{2}$. Ergo erit $\frac{14a^4bb + 20a^3b^3 + 14aab^4}{\overline{a+b}|^6} = \frac{1}{2}$, sive $a^6 + 6a^5b - 13a^4bb - 20a^3b^3 - 13aab^4 + 6ab^5 + b^6 = 0$, & facto $a : b :: z : 1$.

$$z^6 + 6z^5 - 13z^4 - 20z^3 - 13zz + 6z + 1 = 0.$$

Ponatur hæc æquatio ex binis istis formari.

$$z^2 + yz + 1 = 0.$$
$$\& \; z^4 + pz^3 + qz^2 + pz + 1 = 0.$$

$$\begin{array}{llllll} \text{Ergo } z^6 & + yz^5 & + z^4 & & & \\ & + pz^5 & + pyz^4 & + pz^3 & & \\ & & + qz^4 & + qyz^3 & + qzz & \\ & & & + pz^3 & + pyzz & + pz \\ & & & & + zz & + yz + 1. \\ \hline \end{array}$$

$$\text{Sive } z^6 \begin{matrix} +y \\ +p \end{matrix} z^5 \begin{matrix} +1 \\ +py \\ +q \end{matrix} z^4 \begin{matrix} +2p \\ +qy \end{matrix} z^3 \begin{matrix} +1 \\ +py \\ +q \end{matrix} zz \begin{matrix} +p \\ +y \end{matrix} z + 1 = 0.$$

Et

Et comparatis coefficientibus erit $y+p=6$, $1+py+q=-13$, seu $py+q=-14$, $2p+qy=-20$. Unde orietur æquatio $y^3-6yy-16y+32=0$, cujus una radicum erit -2.9644, qua substituta in locum ipsius y, in æquatione $z^2+yz+1=0$, habebitur æquatio nova $z^2-2.9644z+1=0$. Ubi invenietur radix duplex 2.576, & $\frac{1}{2.576}$; ergo sive dexteritas ipsius *A* ad dexteritatem ipsius *B* sit ut 2.576 ad 1, seu ut 1 ad 2.576, *R* & *S* æqua sorte contendent.

Corollarium. Omnes hujus generis æquationes, in quibus ratio dexteritatum determinanda venit ex datis numero nummorum & numero ludorum, ad dimensiones dimidio saltem pauciores, quam sit numerus ludorum datus, semper reducentur; etenim coefficientes terminorum hinc inde ab extremis æqualiter distantium semper iidem erunt, adeoque si fingatur æquationes istas formari ex $yy+yz+1=0$, & æquatione altera cujus coefficientes hinc inde ab extremis æqualiter distantes sint iidem, comparationes terminorum homologorum non erunt plures quam est dimidius ludorum numerus, adeoque dimensiones quantitatis y dimidio saltem pauciores erunt quam dimensiones quantitatis z.

Prob. XXIV. *Positis iisdem ac in* Prob. 20. *habeat* A *nummos* p, B *vero nummos* q: *Quæritur expectatio ipsius* S.

Solutio. Sumatur Binomium $a+b$, & rejectis semper terminis in quibus dimensiones quantitatis a excedunt dimensiones quantitatis b per q, & terminis in quibus dimensiones quantitatis b excedunt dimensiones quantitatis a per p, multiplicentur continuo termini residui per $a+b$, & fiant tot multiplicationes quot sunt unitates in dato ludorum numero unitate diminuto, & habebitur numerator expectationis ipsius *S*, cujus denominator erit potestas binomii $a+b$ designata per numerum ludorum.

Exemplum. Sit $p=3$, & $q=2$; numerus ludorum 7.

$$\begin{array}{l}
a+b \\
a+b \\
\hline
aa \mid +2ab+bb \\
\qquad a+b \\
\hline
2aab+3abb \mid +b^3 \\
\qquad a+b \\
\hline
2a^3b \mid +5aabb+3ab^3 \\
\qquad\qquad a+b \\
\hline
5a^3bb+8aab^3 \mid +3ab^4 \\
\qquad\qquad a+b \\
\hline
5a^4bb \mid +13a^3b^3+8aab^4 \\
\qquad a+b \\
\hline
13a^4b^3+21a^3b^4 \mid +8aab^5
\end{array}$$

Ergo

Ergo erit expectatio ipsius $S = \frac{13a^4b^3 + 21a^3b^4}{\overline{a+b}|^7}$.

Prob. XXV. A *&* B *collusores duo, quorum dexteritates sint in ratione data, hoc pactum ineant, ut non prius ludo desistant quam datus numerus ludorum sit transactus; sint* R *&* S *spectatores duo, quorum* R *contendat fore ut aliquando ante conclusum certamen vel expirante certamine,* A *victorem se præstiterit pluries quam* B *dato ludorum numero; Quæritur expectatio ipsius* R.

Solutio. Sit n numerus ludorum transigendus priusquam A & B ludo desistant, sit $n-d$ numerus ludorum de quo R & S contendant, sit ratio dexteritatum ut a ad b. Elevetur $a+b$ ad potestatem n, tunc si d sit numerus impar, sumantur tot termini istius potestatis quot sunt unitates in $\frac{d+1}{2}$; sumantur etiam tot termini sequentes quot jam sumpti fuerunt, sed mutentur illorum coefficientes, iisque præfigantur coefficientes terminorum præcedentium ordine retrogrado: Si vero d sit numerus par, sumantur tot termini potestatis $\overline{a+b}|^n$ quot sunt unitates in $\frac{d+2}{2}$, sumantur etiam tot termini sequentes quot sunt unitates in $\frac{1}{2}d$, sed præfigantur illis coefficientes terminorum præcedentium ordine retrogrado, omisso ultimo præcedentium, & habebitur numerator expectationis ipsius R, quorum denominator erit $\overline{a+b}|^n$.

Exemp. I. Sit 10 numerus ludorum transigendus priusquam A & B ludo desistant, sit 3 numerus ludorum quibus aliquando A superaturus est ipsum B, sit ratio dexteritatum ut 1 ad 1: Elevetur $a+b$ ad potestatem 10^{am}, videlicet, $a^{10} + 10a^9b + 45a^8bb + 120a^7b^3 + 210a^6b^4 + 252a^5b^5 + 210a^4b^6 + 120a^3b^7 + 45aab^8 + 10ab^9 + b^{10}$.

1°. Est $n = 10$; 2°. $n - d = 3$; ergo est $d = 7$, & $\frac{d+1}{2} = 4$. Sumantur ergo 4 termini istius potestatis, videlicet $a^{10} + 10a^9b + 45a^8bb + 120a^7b^3$; sumantur etiam 4 termini sequentes, illisque præfigantur coefficientes terminorum præcedentium ordine retrogrado, & termini sequentes evadent $120a^6b^4 + 45a^5b^5 + 10a^4b^6 + a^3b^7$. Ergo erit expectatio ipsius $R = \frac{a^{10} + 10a^9b + 45a^8bb + 120a^7b^3 + 120a^6b^4 + 45a^5b^5 + 10a^4b^6 + 1a^3b^7}{\overline{a+b}|^{10}}$ $= \frac{352}{1024}$.

Exemp. II. Sit $n = 6$, & $n - d = 4$; ergo est $d = 2$, & $\frac{d+2}{2} = 2$. Ergo expectatio ipsius R erit $\frac{a^6 + 6a^5b + a^4bb}{\overline{a+b}|^6}$.

N. B. Si d sit numerus impar, poterunt numerator & denominator expectationis ipsius R dividi per $a+b$.

Prob.

Prob. XXVI. A & B, *quorum dexteritates ſint in ratione data, videlicet ut* a *ad* b, *hoc pactum ineant, ut non prius ludo deſiſtant quam datus ludorum numerus ſit tranſactus: Adſint ſpectatores duo* R & S, *quorum* R *affirmet,* S *neget, fore ut aliquando ante finitum certamen, vel expirante certamine,* A *ſit ſuperaturus ipſum* B *dato ludorum numero* q; & *fore etiam ut aliquando* B *ſit ſuperaturus ipſum* A *dato ludorum numero* p: *Quæritur expectatio ipſius* R.

Solutio. Inveniatur numerus caſuum quibus A ſuperare poſſit ipſum B dato ludorum numero q, per *Prob.* 25.

Inveniatur numerus caſuum quibus B ſuperare poſſit ipſum A dato ludorum numero p, per idem.

Inveniatur denique numerus caſuum quibus neuter ſuperare poſſit alterum datis ludorum numeris, per *Prob.* 24.

Addantur hi caſus ſimul, & ex eorum aggregato ſubtrahatur $\overline{a+b}|^n$, & habebitur numerator expectationis ipſius R, cujus denominator erit $\overline{a+b}^n$.

Exemplum. Contendat R fore ut aliquando A ſit ſuperaturus ipſum B 2 ludis, & fore etiam ut aliquando B ſit ſuperaturus ipſum A 3 ludis, & ſit numerus ludorum tranſigendus 7.

Numerus caſuum quibus poſſit A ſuperare ipſum B 2 ludis, eſt $a^7+7a^6b+21a^5bb+21a^4b^3+7a^3b^4+aab^5$.

Numerus caſuum quibus poſſit B ſuperare ipſum A 3 ludis, eſt $14a^4b^3+7a^3b^4+21aab^5+7ab^6+b^7$.

Numerus caſuum quibus neuter alterum ſuperare poſſit datis ludorum numeris, eſt $13a^4b^3+21a^3b^4$.

Summa omnium iſtorum caſuum erit

$$a^7+7a^6b+21a^5bb+35a^4b^3+35a^3b^4+22aab^5+7ab^6+b^7.$$

Subtrahatur $\overline{a+b}|^7$ ſeu

$$a^7+7a^6b+21a^5bb+35a^4b^3+35a^3b^4+21aab^5+7ab^6+b^7$$

Reſiduum erit $1aab^5$.

Ergo expectatio ipſius R erit $\dfrac{aab^5}{\overline{a+b}|^7}$.

2. Cum Methodus ſynthetica, qua uſus eſt *D. de Moivre* ad inveniendam cujuſque Colluſoris ſortem, in uſum verti nequeat tunc quando plures quam tres ſunt colluſores, ob vix perſpiciendam legem progreſſionis ſerierum quæ ſe offerunt; oſtendam hic quo modo Analyſis in ejuſmodi Problematibus, ubi depoſitum continuo augetur, adhiberi queat: eumque in finem demonſtrationem dabo analyticam trium Theorematum, quæ inveni, & quidem diu ante viſum D. *Moyvræi* libellum de Menſura Sortis, occaſione triplicis quæſtionis mihi ab amico circa ludum hunc, quem Galli vocant *le Jeu de la Poule*, propoſitæ, pro inveniendis ſcil. probabilitate vincendi, lucro item vel damno cujuſque Colluſoris, & duratione certaminis.

A Solution of the XV. Problem, by M. Nic Bernoulli, n. 341. p. 133.

 Theo-

Theorema 1. Si Collusores aliquot A, B, C, D, E, &c. quorum numerus est $n+1$ & dexteritates sunt æquales, deponant singuli 1, & istis conditionibus certent. 1°. Ut illorum duo A & B ludum incipiant. 2°. Ut victus locum suum tertio C cedat, ita ut ille tertius C jam cum victore contendat, quique ex hoc certamine victor evaserit cum quarto D ludat, & ita deinceps. 3°. Ut ille depositum totum obtineat, qui omnes collusores successive vicerit. Dico probabilitates vincendi duorum quorumlibet collusorum sese immediate in ordine ludendi sequentium esse in ratione $1+2^n$ ad 2^n, adeoque expectationes lusorum A (B), C, D, E, &c. esse in progressione Geometrica.

Demonstratio. Ponantur expectationes vincendi ipsius A vel $B=a$, ipsius $C=c$, ipsius $D=d$, ipsius $E=e$, &c. Porro cum accidere possit, ut collusor aliquis prima vice in ludum intrans inveniat adversarium qui vel nondum, vel semel, vel bis, vel ter, &c. jam successive victor extitit, vocetur expectatio lusoris illius primo casu $=z$, secundo $=y$, tertio $=x$, quarto $=u$, quinto $=t$, &c. Item cum collusor aliquis vinci possit ab adversario qui antea jam vel nullum, vel unum, vel duos, vel tres, &c. collusores successive vicit, ita ut exiens e ludo relinquat adversarium qui vel semel, vel bis, vel ter, vel quater &c. victor extitit, vocetur expectatio seu probabilitas vincendi ejus qui exit e ludo primo casu $=h$, secundo $=k$, tertio $=l$, quarto $=m$, &c. Hisce omnibus positis habebuntur sequentes novem series æquationum signatæ N°. 1. N°. 2. N°. 3, &c. usque ad N°. 9. *Tab.* I. Ratio eas inveniendi breviter hæc est. Inter æquationes N. 1°. reperitur *ex. gr.* $f=\frac{1}{8}t+\frac{1}{8}u+\frac{1}{4}x+\frac{1}{2}y$. Nam collusor F certabit vel cum collusore A, vel B, vel C, vel D, vel E: ut primum vel secundum contingat, oportet ut vel A vel B quater successive victor existat, cujus eventus probabilitas est $\frac{2}{16}$ seu $\frac{1}{8}$: Ut tertium contingat oportet ut C ter victor existat, cujus eventus probabilitas est etiam $\frac{1}{8}$: Ut quartum contingat oportet ut D bis successive vincat, quod probabilitatem habet $\frac{1}{4}$; Ut quintum contingat, oportet ut E semel vincat, cujus eventus probabilitas est $\frac{1}{2}$; ergo lusoris F probabilitas vincendi est $=\frac{1}{8}t+\frac{1}{8}u+\frac{1}{4}x+\frac{1}{2}y$. Sic inter æquationes N°. 2. est *ex. gr.* $x=\frac{1}{2}l+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots\frac{1}{2^n}\times h+\frac{1}{2^n}\times 1$. Collusor enim qui certat cum adversario qui jam bis successive victor extitit, vincere potest vel omnes collusores, vel aliquos, vel nullum. Prioris eventus probabilitas est $\frac{1}{2^n}$, secundi $\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\frac{1}{2^n}$, & tertii $\frac{1}{2}$;

si

si primus eventus contingat, probabilitas vincendi evadit certitudo integra seu 1; si secundus, exit e ludo relinquens collusorem qui semel vicit; si tertius, exit e ludo relinquens collusorem qui ter successive vicit; adeoque sors ejus totalis est $\frac{1}{2} \times l + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \ldots \frac{1}{2^n} \times b + \frac{1}{2^n} \times 1$.
Simili ratiocinio inveniuntur æquationes N°. 3. Collusor enim qui victus ab adversario exit e ludo, relinquens *ex. gr.* collusorem unius tantum ludi victorem, acquirit sortem vel ipsius *C*, vel ipsius *D*, vel ipsius *E*, vel ipsius *F*, *&c.* prout adversarius a quo victus est vincit vel omnes collusores præter unum, vel omnes præter duos, vel omnes præter tres, *&c.* unde sequitur quod $b = \frac{1}{2^{n-1}} \times c + \frac{1}{2^{n-2}} \times d + \frac{1}{2^{n-3}} \times e + \frac{1}{2^{n-4}} \times f +$ *&c.* Æquationes N°. 4. inveniuntur subtrahendo æquationes N°. 2. ab invicem: & æquationes N°. 5. subtrahendo æquationes N°. 3. ab invicem. Æquationes N°. 6. inveniuntur substituendo in æquationibus N°. 4. valores inventos per æquationes N°. 5. Æquationes No. 7. inveniuntur quærendo valores ipsarum $z, y, x, u,$ *&c* per æquationes N°. 1. Et his valoribus substitutis in æquationibus N° 4. habebuntur æquationes N°. 8. quæ comparatæ cum æquationibus N°. 6. dant æquationes N°. 9. ex quibus sequitur quod $1 + 2^n : 2^n :: a : c :: c : d :: d : e,$ *&c. Q. E. D.*

Corollarium. Hinc facile inveniuntur probabilitates vincendi singulorum Collusorum, quas habent tum ante ludum inceptum, tum in quolibet statu in quem ludum prosequendo pervenire possunt. Si sint, *ex. gr.* tres collusores *A*, *B*, *C*, erit $n = 2$, & $1 + 2^n : 2^n :: 5 : 4 :: a : c$: id est, probabilitates vincendi ipsorum *A*, *B*, *C*, antequam *A* vicerit *B*, vel *B* vicerit *A*, se habent ut numeri 5, 5, 4, adeoque ipsæ probabilitates sunt $\frac{5}{14}, \frac{5}{14}, \frac{4}{14}$: omnes enim simul sumptæ facere debent 1 seu certitudinem integram. Postquam *A* vicerit *B*, probabilitates vincendi ipsorum *B*, *C*, *A*, erunt *b*, *y* seu *c*, & (quia *A* æqualem habet expectationem ad victoriam, & ad sortem ipsius *B* obtinendam) $\frac{1+b}{2}$ respective, *hoc est*, quia per æq. 1. N°. 3. $b = \frac{1}{2^{n-1}} \times c = \frac{1}{2} c$, & $c = \frac{4}{14} = \frac{2}{7}$ ut modo inventum, hæ probabilitates erunt $\frac{1}{7}, \frac{2}{7}, \frac{4}{7}$, ut D. *de Moivre* invenit Coroll. 1. Prob 15. * pag. 242. * *supr.* p. 207.

Si sint quatuor collusores *A*, *B*, *C*, *D*, erit $n = 3$, & $1 + 2^n. 2^n :: 9, 8$, adeoque probabilitates collusorum ab initio ludi erunt ut 9, 9, 8, $\frac{8 \times 8}{9}$, sive ut 81, 81, 72, 64, *hoc est*, ipsæ *a*, *a*, *c*, *d*, erunt $\frac{81}{298}, \frac{81}{298}, \frac{72}{298}$ & $\frac{64}{298}$. Postquam *A* vicerit *B*, probabilitates ipsorum *B*, *D*, *C*, *A*, erunt *b*, *d*, *c*, $1 + 3b$

$\frac{1+3b}{4}$, eſt autem per æq. 1. No. 3. $b = \frac{1}{2^{n-1}} \times c + \frac{1}{2^{n-2}} \times d = \frac{1}{4}c + \frac{1}{2}d$, $c = \frac{72}{298} = \frac{36}{149}$, & $d = \frac{64}{298} = \frac{32}{149}$, ut modo inventum: ergo hæ probabilitates erunt $\frac{25}{149}$, $\frac{32}{149}$, $\frac{36}{149}$, $\frac{56}{149}$ reſpectivè. Poſtquam *A* vicerit *B* & *C*, probabilitates vincendi ipſorum *C*, *B*, *D*, *A*, erunt k, $\frac{c}{2}$, x, $\frac{1+b}{2}$, ſeu (quia per æq. 2. No. 3. $k = \frac{1}{2^{n-2}} \times d = \frac{1}{2}d$, & per æq. 3. No. 7. $x = 2d - c$) $\frac{16}{149}$, $\frac{18}{149}$, $\frac{28}{149}$, $\frac{87}{149}$. Et nota quod calculi bonitas confirmetur ex eo, quod ſummæ harum probabilitatum, *hoc eſt*, $\frac{1}{7} + \frac{2}{7} + \frac{4}{7}$ in priori exemplo, & $\frac{25}{149} + \frac{32}{149} + \frac{36}{149} + \frac{56}{149}$, nec non $\frac{16}{149} + \frac{18}{149} + \frac{28}{149} + \frac{87}{149}$ in poſteriori exemplo, ſingulæ ſint $= 1$ ſeu certitudini integræ.

Theorema 2. Poſitis quæ prius & inſuper hac conditione, ut victus ſemper mulctetur ſumma p, quæ depoſito augendo inſerviat; quod depoſitum ſic gradatim auctum illi ſoli cedat, qui omnium ſucceſſive colluſorum victor extiterit; denotatis etiam ut antea per literas minuſculas a, c, d, e, &c. probabilitatibus vincendi ipſorum *A* (vel *B*), *C*, *D*, *E*, &c. reſpectivè: per eaſdem vero literas majuſculas *A*, *C*, *D*, *E*, &c. ipſorum *A* (vel *B*), *C*, *D*, *E*, &c. expectationibus, *hoc eſt*, portionibus depoſiti quas ſinguli expectant: Dico, fore ſemper $C = \frac{\overline{A+ap} \times 2^n - ncp}{1+2^n}$, $D = \frac{\overline{C+cp} \times 2^n - ndp}{1+2^n}$, $E = \frac{\overline{D+dp} \times 2^n - nep}{1+2^n}$, &c.

Demonſtratio. Denotetur ut prius per literas minuſculas z, y, x, u, t, &c. probabilitas vincendi ludentis cum adverſario, qui jam vel nullum, vel unum, vel duos, &c. colluſores ſucceſſive vicit; per eaſdem vero literas majuſculas *Z*, *Y*, *X*, *U*, *T*, &c. ejus expectatio, quam ſcil. habet diverſis illis caſibus, depoſito exiſtente $n+1$, $n+1+p$, $n+1+2p$, $n+1+3p$, &c. reſpectivè. Sic etiam per literas minuſculas h, k, l, m, &c. denotetur probabilitas vincendi luſoris victi ab adverſario, qui antea vel nullum, vel unum, vel duos, &c. colluſores ſucceſſive vicerat; quemadmodum per literas majuſculas *H*, *K*, *L*, *M*, &c. ejuſdem expectatio diverſis illis caſibus, depoſito exiſtente $n+1+p$, $n+1+2p$, $n+1+3p$, &c. reſpectivè. His poſitis, iiſdem quibus antea ratiociniis inveniuntur ſequentes duodecim æquationum ſeries in *Tab.* II. ſignatæ No. 1. No. 2.

No. 3.

N°. 3. &c. Inter æquationes N°. 1. *ex. gr.* eſt $E = \frac{U}{4} + \frac{X + xp}{4} + \frac{Y + 2yp}{2}$. Luſor enim E ludet vel cum luſore A, vel luſore B, vel C, vel D. Si ludit cum A vel B, expectatio ejus erit $= U$, quia ludit cum adverſario qui jam tres adverſarios vicit, depoſito exiſtente $n + 1 + 3p$. Si ludit cum luſore C, expectatio ejus erit $= X + xp$, ludit enim cum adverſario qui jam duos colluſores vicit, adeoque ſi depoſitum eſſet $n + 1 + 2p$ ejus expectatio eſſet $= X$: verum quia ludente E depoſitum eſt $= n + 1 + 3p$, ob tres colluſores victos & ſumma p mulctatos, addenda eſt expectationi X portio illa mulctæ unius p, quam luſor E ſperare poteſt: eſt autem hæc portio (quia probabilitas ejus vincendi eſt x) $= xp$, ejus igitur expectatio totalis tunc erit $= X + xp$. Sic ſi ludit cum luſore D, expectatio ejus erit $= Y + 2yp$: additur ad Y (quæ eſſet ejus expectatio depoſito exiſtente $n + 1 + p$) portio $2yp$, quæ ipſi debetur de duabus mulctis $2p$, quibus depoſitum $n + 1 + 3p$ majus eſt quam $n + 1 + p$. Simili modo habentur æquationes N°. 2, 3, 4 & 5. Subſtituendo autem primam æquationem N°. 2 *Tab.* I. in æquationibus N°. 4. habentur æquationes N°. 6. Et ſubſtituendo primam æquationem N°. 3. *Tab.* I. in æquationibus N°. 5. habentur æquationes N°. 7. quibus dein in æquationibus N°. 6. ſubſtitutis habentur æquationes N°. 8. Æquationes N°. 9. inveniuntur quærendo valores ipſarum Z, Y, X, U, &c. per æquationes N°. 1. *Tab.* I. & II. vel N°. 2. *Tab.* II. & N°. 7. *Tab.* I. Et his valoribus ſubſtitutis in æquationibus N°. 4. habentur æquationes N°. 10. Quæ comparatæ cum æquationibus N°. 8. (in quibus pro z ſubſtitnatur a, per 1. æq. *Tab.* I.) dant æquationes N°. 11. Et hæ æquationes No. 11. comparatæ cum æquationibus N°. 9. *Tab.* I. dant æquationes N°. 12. quæ conſtituunt Theorema, quod demonſtrandum erat.

Corollarium. Hinc quoque facile inveniuntur ſingulorum Colluſorum ſortes ſeu expectationes, ipſorumque adeo lucra vel damna. Sint *ex. gr.* tres colluſores A, B, C: erit $C = \frac{\overline{A + ap} \times 2^n - ncp}{1 + 2^n} =$ (ob $n = 2$) $\frac{4A + 4ap - 2cp}{5} =$ (ob $a = \frac{5}{14}$ & $c = \frac{2}{7}$ per Corollarium Theor. 1.) $\frac{4A + \frac{6}{7}p}{5}$. Unde cum omnium trium expectationes ſimul ſumptæ, *id eſt*, $A + A + C$ æquare debeant id quod ab initio depoſitum fuit, *id eſt* 3, erit $2A + \frac{4A + \frac{6}{7}p}{5} = \frac{14A + \frac{6}{7}p}{5} = 3$, & $14A = 15 - \frac{6}{7}p$, & $A = 15 - \frac{6}{7}p$, & $A = \frac{15}{14} - \frac{3}{49}p =$ expectationi luſoris A vel B: proinde C expectatio luſoris tertii $C = \frac{4A + \frac{6}{7}p}{5} = \frac{6}{7} + \frac{6}{49}p$. A quibus expectationibus ſi ſubtrahatur 1, id quod ab initio ſinguli depoſuerunt, remanebit

bit ibi $\frac{1}{14} - \frac{3}{49}p$, hic $\frac{6}{49}p - \frac{1}{7}$; quemadmodum D. *de Moivre* invenit.

Exempl. 2. Sint collusores 4, A, B, C, D, erit $C = \frac{\overline{A+ap} \times 2^n - ncp}{1+2^n}$ $= (\text{ob } n = 3)\ \frac{8A + 8ap - 3cp}{9} = (\text{ob } a = \frac{81}{298}\ \&\ c = \frac{36}{149}$, per Coroll. Theorema 1.) $\frac{8A + \frac{216}{149}p}{9}$; item $D = \frac{\overline{C+cp} \times 2^n - ndp}{1+2^n} =$ $\frac{8C + 8cp - 3dp}{9} = (\text{ob } d = \frac{32}{149}$ per id. corr.) $\frac{8C + \frac{192}{149}p}{9} =$ $\frac{64A + \frac{3456}{149}p}{81}$: unde habebitur æquatio $2A + C + D = 2A +$ $\frac{8A + \frac{216}{149}p}{9} + \frac{64A + \frac{3456}{149}p}{9} = \frac{298A + \frac{5400}{149}p}{81} = 4$, sive $149A +$ $\frac{2700}{149}p = 162$, & $A = \frac{162}{149} - \frac{2700}{22201}p$. Hinc $C = \frac{8A + \frac{216}{149}p}{9} = \frac{144}{149}$ $+ \frac{1176}{22201}p$, & $D = \frac{64A + \frac{3456}{149}p}{81} = \frac{128}{149} + \frac{4224}{22201}p$. Subtracta autem unitate 1, quam singuli ab initio ludi deposuerunt, remanebit $\frac{13}{149}$ $- \frac{2700}{22201}p$ pro lusore A vel B, $\frac{1176}{22201}p - \frac{5}{149}$ pro C, & $\frac{4224}{22201}p - \frac{21}{149}$ pro D; quæ singula indigitabunt lucrum vel damnum, prout pars affirmata præpollet negatæ, vel contra. Simili ratione habebuntur etiam sortes quas acquirunt in quolibet statu in quem ludum prosequendo pervenire possunt.

Theorema 3. Positis quæ prius, si adsint spectatores Q, R, S, T, U, *&c.* quorum numerus sit n unitate minor quam numerus collusorum, quorumque prior Q affirmet certamen finitum iri post $n+p$ ludos peractos, R post $n+p-1$, S post $n+p-2$, T post $n+p-3$, U post $n+p-4$, *&c.* præcise, non antea; sintque q, r, s, t, u, *&c.* sortes ipsorum Q, R, S, T, U, *&c.* Dico fore $q = \frac{1}{2}r + \frac{1}{4}s + \frac{1}{8}t + \frac{1}{16}u +$ *&c.*

Demonstratio. Vocetur A collusor ille, qui post $n+p$ ludos vincere supponitur: hic intrare debet in ludum post p ludos peractos, & tum ludet contra adversarium, qui jam vel unum vel duos, vel tres, *&c.* collusores successive vicit. Jam cum, ut primus casus contingat, & ut collusor A omnes suos collusores præter unum, *id est*, $n-1$ collusores successive vincat, æque probabile sit quam ut adversarius ejus vincat $n-1$ collusores, *id est*, (quia jam unius collusoris victor fuit) ut certamen finiat post $n+p-1$ ludos peractos, hujusque eventus probabilitas sit $= r$: erit probabilitas ut collusor A unum adhuc collusorem vincat, *id est*, certamen

tamen finiat poſt $n+p$ ludos $= \frac{1}{2}r$. Sic, ut ſecundus caſus exiſtat, & ut *A* omnes colluſores præter duos vincat, æque probabile eſt quam ut certamen finiatur poſt $n+p-2$ ludos, adeoque ut tunc *A* vincat adhuc duos colluſores, *id eſt*, ut certamen finiat poſt $n+p$ ludos, probabilitas erit $= \frac{1}{4}s$. Eodem modo ut, tertio caſu exiſtente, *A* vincat omnes colluſores, probabilitas eſt $= \frac{1}{8}t$; ut quarto $= \frac{1}{16}u$, *&c.* Quare ut indifferenter certamen finiatur poſt $n+p$ ludos, probabilitas eſt $\frac{1}{2}r + \frac{1}{4}s + \frac{1}{8}t + \frac{1}{16}u +$ *&c.* $= q$. *Q. E. D.*

Corollarium 1. Facile hinc invenitur quænam ſit probabilitas ut certamen finiatur intra datum quemvis ludorum numerum. Series enim fractionum incipientium a fractione $\frac{1}{2^{n-1}}$, quarum denominatores creſcant in continua proportione dupla, numerator autem cujuſque fractionis ſit ſumma numeratorum tot fractionum immediate præcedentium quot ſunt unitates in $n-1$, dabit omnes ſucceſſive probabilitates, ut certamen finiatur peractis præciſe n, $n+1$, $n+2$, $n+3$, *&c.* ludis: & per conſequens ſi addantur tot termini hujus ſeriei quot ſunt unitates in $p+1$, ſumma ipſorum exprimet probabilitatem ut certamen finiatur ad minimum ludis $n+p$ peractis. *Ex. gr.* Si ſint colluſores 4, adeoque $n=3$, habebitur hæc ſeries $\frac{1}{4}$, $\frac{1}{8}$, $\frac{2}{16}$, $\frac{3}{32}$, $\frac{5}{64}$, $\frac{8}{128}$, $\frac{13}{256}$, $\frac{21}{512}$ *&c.* E qua ſi fiat alia $\frac{1}{4}$, $\frac{3}{8}$, $\frac{8}{16}$, $\frac{19}{32}$, $\frac{43}{64}$, $\frac{94}{128}$, $\frac{201}{256}$, *&c.* cujus termini ſint ſummæ terminorum præcedentis ſeriei, denotabunt iidem termini qualis ſit probabilitas ut certamen finiatur ad minimum 3, 4, 5, 6, *&c.* ludis.

Corollarium 2. Poteſt terminus quicunque prioris ſeriei (excepto primo termino,) ut & ſumma omnium terminorum, *id eſt*, terminus quicunque poſterioris ſeriei, per formulam generalem exprimi hoc modo. Si $n+1$ ſit numerus colluſorum, & p ſit numerus terminorum, erit ultimus terminus prioris ſeriei $\frac{1}{2^n} - \frac{p-n+1}{1 \times 2^{2n}} + \frac{\overline{p-2n} \times \overline{p-2n+3}}{1 \times 2 \times 2^{3n}} -$

$$\frac{\overline{p-3n} \times \overline{p-3n+1} \times \overline{p-3n+5}}{1 \times 2 \times 3 \times 2^{4n}} +$$

$$\frac{\overline{p-4n} \times \overline{p-4n+1} \times \overline{p-4n+2} \times \overline{p-4n+7}}{1 \times 2 \times 3 \times 4 \times 2^{5n}}, - \text{\&c.}$$ Et ſumma omnium terminorum ſive ultimus terminus poſterioris ſeriei $= \frac{p+1}{1 \times 2^n} -$

 $p-n$

$$\frac{\overline{p-n}\times\overline{p-n+3}}{1\times2\times2^{2n}}+\frac{\overline{p-2n}\times\overline{p-2n+1}\times\overline{p-2n+5}}{1\times2\times3\times2^{3n}}-$$
$$\frac{\overline{p-3n}\times\overline{p-3n+1}\times\overline{p-3n+2}\times\overline{p-3n+7}}{1\times2\times3\times4\times2^{4n}}+\&c.$$

Tabula I.

Intrat.		Exit.		No. 1.
	Sors		Sors	$a=z$
0	z	1	b	$c=y$
1	y	2	k	$d=\frac{1}{2}x+\frac{1}{2}y$
2	x	3	l	$e=\frac{1}{4}u+\frac{1}{4}x+\frac{1}{2}y$
3	u	4	m	$f=\frac{1}{8}t+\frac{1}{8}u+\frac{1}{4}x+\frac{1}{2}y$
4	t			

No. 2.

$$z=\frac{1}{2}+\overline{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\frac{1}{2^n}}\times b+\frac{1}{2^n}\times 1$$

$$y=\frac{1}{2}k+\overline{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\frac{1}{2^n}}\times b+\frac{1}{2^n}\times 1.$$

$$x=\frac{1}{2}l+\overline{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\frac{1}{2^n}}\times b+\frac{1}{2^n}\times 1$$

$$u=\frac{1}{2}m+\overline{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\frac{1}{2^n}}\times b+\frac{1}{2^n}\times 1.$$

No. 3.

$$b=\frac{1}{2^{n-1}}\times c+\frac{1}{2^{n-2}}\times d+\frac{1}{2^{n-3}}\times e+\frac{1}{2^{n-4}}\times f+\ldots$$

$$k=\frac{1}{2^{n-2}}\times d+\frac{1}{2^{n-3}}\times e+\frac{1}{2^{n-4}}\times f+\ldots$$

$$l=\frac{1}{2^{n-3}}\times e+\frac{1}{2^{n-4}}\times f+\ldots$$

$$m=\frac{1}{2^{n-4}}\times f+\ldots$$

No. 4.	No. 6.	No. 8.
$z-y=\frac{1}{2}b-\frac{1}{2}k$	$=\frac{1}{2^n}\times c$	$=a-c$
$y-x=\frac{1}{2}k-\frac{1}{2}l$	$=\frac{1}{2^{n-1}}\times d$	$=2c-2d$
$x-u=\frac{1}{2}l-\frac{1}{2}m$	$=\frac{1}{2^{n-2}}\times e$	$=4d-4e$

No. 5.

Tabula II

Intrat. Depositum		Sors	*Exit.* Depositum		Sors	Deposit.
$n+1$	0	Z	$n+1+p$	1	H	$n+1+$
$n+1+p$	1	Y	$n+1+2p$	2	K	$n+1+p$
$n+1+2p$	2	X	$n+1+3p$	3	L	$n+1+2p$
$n+1+3p$	3	V	$n+1+4p$	4	M	$n+1+3p$
$n+1+4p$	4	T				$n+1+4p$

N° 1

$A=Z$

$C=Y$

$D=\frac{1}{2}X+\frac{1}{2}\times\overline{Y+yp}$

$E=\frac{1}{4}V+\frac{1}{4}\times\overline{X+xp}+\frac{1}{2}\times\overline{Y+2yp}$

$F=\frac{1}{8}T+\frac{1}{8}\times\overline{V+up}+\frac{1}{4}\times\overline{X+2xp}+\frac{1}{2}\times\overline{Y+3\ldots}$

N° 2

$$Z=\tfrac{1}{2}\times\overline{H-p}+\tfrac{1}{4}\times\overline{H-p+hp}+\tfrac{1}{8}\times\overline{H-p+2hp}+\tfrac{1}{16}\times\overline{H-p+3hp}+\cdots\tfrac{1}{2^n}\times\overline{H-p+nhp-hp}+\tfrac{1}{2^n}\times\overline{np+n+1}$$

$$Y=\tfrac{1}{2}\times\overline{K-p}+\tfrac{1}{4}\times\overline{H-p+2hp}+\tfrac{1}{8}\times\overline{H-p+3hp}+\tfrac{1}{16}\times\overline{H-p+4hp}+\cdots\tfrac{1}{2^n}\times\overline{H-p+nhp}+\tfrac{1}{2^n}\times\overline{np+p+n+1}$$

$$X=\tfrac{1}{2}\times\overline{L-p}+\tfrac{1}{4}\times\overline{H-p+3hp}+\tfrac{1}{8}\times\overline{H-p+4hp}+\tfrac{1}{16}\times\overline{H-p+5hp}+\cdots\tfrac{1}{2^n}\times\overline{H-p+nhp+hp}+\tfrac{1}{2^n}\times\overline{np+2p+n+1}$$

$$V=\tfrac{1}{2}\times\overline{M-p}+\tfrac{1}{4}\times\overline{H-p+4hp}+\tfrac{1}{8}\times\overline{H-p+5hp}+\tfrac{1}{16}\times\overline{H-p+6hp}+\cdots\tfrac{1}{2^n}\times\overline{H-p+nhp+hp}+\tfrac{1}{2^n}\times\overline{np+3p+n+1}$$

N° 3

$$H=\tfrac{1}{2^{n-1}}\times\overline{C+ncp-cp}+\tfrac{1}{2^{n-2}}\times\overline{D+ndp-2dp}+\tfrac{1}{2^{n-3}}\times\overline{E+nep-3ep}+\tfrac{1}{2^{n-4}}\times\overline{F+nfp-4fp}+$$

$$K=\tfrac{1}{2^{n-2}}\times\overline{D+ndp-dp}+\tfrac{1}{2^{n-3}}\times\overline{E+nep-2ep}+\tfrac{1}{2^{n-4}}\times\overline{F+nfp-3fp}+\cdots$$

$$L=\tfrac{1}{2^{n-3}}\times\overline{E+nep-ep}+\tfrac{1}{2^{n-4}}\times\overline{F+nfp-2fp}+\cdots$$

$$M=\tfrac{1}{2^{n-4}}\times\overline{F+nfp-fp}+\cdots$$

N° 4 — N° 6 — N° 8

$$Y-Z=\tfrac{1}{2}K-\tfrac{1}{2}H+\tfrac{1}{4}hp+\tfrac{1}{8}hp+\tfrac{1}{16}hp+\cdots\tfrac{1}{2^n}\times hp=\tfrac{1}{2}K-\tfrac{1}{2}H+zp-\tfrac{1}{2}hp=-\tfrac{1}{2^{n-0}}\times C-\tfrac{ncp}{2^n}+zp$$

$$X-Y=\tfrac{1}{2}L-\tfrac{1}{2}K+\tfrac{1}{4}hp+\tfrac{1}{8}hp+\tfrac{1}{16}hp+\cdots\tfrac{1}{2^n}\times hp=\tfrac{1}{2}L-\tfrac{1}{2}K+zp-\tfrac{1}{2}hp=-\tfrac{1}{2^{n-1}}\times D-\tfrac{ndp}{2^{n-1}}-\tfrac{cp}{2^n}+\ldots$$

$$V-X=\tfrac{1}{2}M-\tfrac{1}{2}L+\tfrac{1}{4}hp+\tfrac{1}{8}hp+\tfrac{1}{16}hp+\cdots\tfrac{1}{2^n}\times hp=\tfrac{1}{2}M-\tfrac{1}{2}L+zp-\tfrac{1}{2}hp=-\tfrac{1}{2^{n-2}}\times E-\tfrac{nep}{2^{n-2}}-\tfrac{dp}{2^{n-1}}\ldots$$

N° 5 — N° 7

$$K-H=-\tfrac{1}{2^{n-1}}\times\overline{C+ncp-cp}+\tfrac{1}{2^{n-2}}\times dp+\tfrac{1}{2^{n-3}}\times ep+\tfrac{1}{2^{n-4}}\times fp+\cdots=-\tfrac{1}{2^{n-1}}\times C-\tfrac{ncp}{2^{n-1}}+hp$$

$$L-K=-\tfrac{1}{2^{n-2}}\times\overline{D+ndp-dp}+\tfrac{1}{2^{n-3}}\times ep+\tfrac{1}{2^{n-4}}\times fp+\cdots=-\tfrac{1}{2^{n-2}}\times D-\tfrac{ndp}{2^{n-2}}-\tfrac{cp}{2^{n-1}}+hp$$

$$M-L=-\tfrac{1}{2^{n-3}}\times\overline{E+nep-ep}+\tfrac{1}{2^{n-4}}\times fp+\cdots=-\tfrac{1}{2^{n-3}}\times E-\tfrac{nep}{2^{n-3}}-\tfrac{dp}{2^{n-2}}-\tfrac{cp}{2^{n-1}}+hp$$

N° 9.

$Z=A$

$Y=C$

$X=2D-C-cp$

$V=4E-2D-C-2dp-2cp$

N° 10 — N° 8

$$C-A=Y-Z=-\tfrac{1}{2^n}\times C-\tfrac{ncp}{2^n}+zp$$

$$2D-2C-cp=X-Y=-\tfrac{1}{2^{n-1}}\times D-\tfrac{ndp}{2^{n-1}}-\tfrac{cp}{2^n}+zp$$

$$4E-4D-2dp-cp=V-X=-\tfrac{1}{2^{n-2}}\times E-\tfrac{nep}{2^{n-2}}-\tfrac{dp}{2^{n-1}}-\tfrac{cp}{2^n}+zp$$

N° 11 — N° 12

$$C=\frac{A\times 2^n+ap\times 2^n-ncp}{1+2^n}=\frac{\overline{A+ap}\times 2^n-ncp}{1+2^n}$$

$$D=\frac{C\times 2^n+cp\times\overline{2^{n-1}-\tfrac{1}{2}}+ap\times 2^{n-1}-ndp}{1+2^n}=\frac{\overline{C+cp}\times 2^n-ndp}{1+2^n}$$

$$E=\frac{D\times 2^n+dp\times\overline{2^{n-1}-\tfrac{1}{2}}+cp\times\overline{2^{n-2}-\tfrac{1}{4}}+ap\times 2^{n-2}-nep}{1+2^n}=\frac{\overline{D+dp}\times 2^n-nep}{1+2^n}$$

I. Senex sc.

No. 5.	No. 7.	No. 9.
$h - k = \frac{1}{2^n - 1} \times c$	$z = a$ $y = c$	$c = a \times \frac{2^n}{1 + 2^n}$
$k - l = \frac{1}{2^n - 2} \times d$	$x = 2d - y = 2d - c$ $u = 4e - x - 2y = 4e - 2d - e$	$d = c \times \frac{2^n}{1 + 2^n}$
$l - m = \frac{1}{2^n - 3} \times e$		$e = d \times \frac{2^n}{1 + 2^n}$

Corollarium 3. Poteſt quis, priuſquam ludus inchoetur, in ſe ſuſcipere, ut ſummam $n+1$ de qua colluſores contendunt, & mulctas omnes pendat, ſi ſibi initio in manus datum ſit $n + 1 + 2^n - 1 \times p$.

Demonſtrationem duorum præcedentium corollariorum curioſis indagandam relinquo.

3. *Deſignationes.* Si B & C colluſores duo ſimul certent; ad deſignandum B victorem eſſe, C victum, ſcribatur BC; atque viciſſim ad deſignandum C victorem eſſe, B victum; ſcribatur CB: & ſic de cæteris.

Another general Solution by Combinations and infinite Series's. by Mr. De Moivre, *ib. p.* 145.

Ponatur 1° B vincere A, certamenque concludi tribus ludis

$$\left.\begin{array}{l} BA \\ \hline BC \\ BD \end{array}\right\} \text{Sic patet } B \text{ victorem neceſſario evadere.}$$

Ponatur 2° B vincere A, certamenque concludi quatuor ludis

$$\left.\begin{array}{l} BA \\ \hline CB \\ CD \\ CA \end{array}\right\} \text{Sic patet } C \text{ victorem neceſſario evadere.}$$

Ponatur 3° B vincere A, certamenque concludi quinque ludis

$$\left.\begin{array}{ll} BA & BA \\ \hline CB* & BC \\ DC & DB \\ DA & DA \\ DB & DC \end{array}\right\} \text{Sic patet } D \text{ victorem neceſſario evadere, idque duplici modo.}$$

Ponatur 4° B prima vice vincere A, certamenque concludi ſex ludis

$$\left.\begin{array}{lll} BA & BA & BA \\ \hline CB & CB* & BC \\ DC* & CD* & DB \\ AD & AC & AC \\ AB & AB & AD \\ AC & AD & AB \end{array}\right| \text{Sic patet } A \text{ victorem neceſſario evadere, idque triplici modo.}$$

Pona-

Ponatur 5° certamen concludi ſeptem ludis, ponaturque ſemper *B* prima vice vincere ipſum *A*.

B A	*B A*	*B A*	*B A*	*B A*	
C B	*C B*	*C B**	*B C*	*B C*	
D C	*D C**	*C D*	*D B*	*D B*	
*A D**	*D A*	*A C*	*A D**	*D A*	Sic patet *B* vel *C* neceſſario victores evadere, *B* triplici modo, *C* duplici.
B A	*B D*	*B A*	*C A*	*C D*	
B C	*B C*	*B D*	*C B*	*C B*	
B D	*B A*	*B C*	*C D*	*C A*	

Ponatur 6° certamen concludi octo ludis,

B A	*B A*	*B A*	*B A*	*B A*	*B A*	*B A*	*B A*
C B	*C B*	*C B*	*C B*	*C B**	*B C*	*B C*	*B C*
D C	*D C*	*D C**	*C D*	*C D*	*D B*	*D B*	*D B*
A D	*A D**	*D A*	*A C*	*A C*	*A D*	*A D**	*D A*
*B A**	*A B*	*B D*	*B A**	*A B*	*C A**	*A C*	*C D*
C B	*C A*	*C B*	*D B*	*D A*	*B C*	*B A*	*B C*
C D	*C D*	*C A*	*D C*	*D B*	*B D*	*B D*	*B A*
C A	*C B*	*C D*	*D A*	*D C*	*B A*	*B C*	*B D*

Sic patet *C* victorem evadere triplici, *D* duplici, *B* triplici modo, *&c.*

Nunc ordine ſcribantur literæ quibus victores deſignantur.

3, | 1 *B*
4, | 1 *C*
5, | 2 *D*
6, | 3 *A*
7, | 3 *B* + 2 *C*
8, | 3 *C* + 2 *D* + 3 *B*
9, | 3 *D* + 2 *A* + 3 *C* + 3 *D* + 2 *A*
10, | 3 *A* + 2 *B* + 3 *D* + 3 *A* + 2 *B* + 3 *A* + 2 *C* + 3 *D*.
&c.

Perſpecta illarum formatione, patebit 1° literam *B* in ordine aliquo ſemper toties reperiri, quoties *A* in ordine ultimo & penultimo reperitur: 2° *C* in ordine aliquo toties reperiri quoties *B* in ordine ultimo & *D* in penultimo reperiuntur: 3° *D* in ordine aliquo toties reperiri quoties *C* in ultimo & *B* in penultimo: 4° *A* in ordine aliquo ſemper toties reperiri quoties *D* in ordine ultimo & *C* in penultimo reperiuntur.

Sed numerus variationum dato cuilibet ludorum numero competens, duplus eſt numeri variationum omnium dato ludorum numero unitate diminuto competentis: adeoque Probabilitas quam habet Colluſor *B* ut vincat dato ludorum numero, eſt ſubdupla probabilitatis quam habebat *A* ut vinceret dato ludorum numero minus uno; atque etiam ſubquadrupla probabilitatis quam habebat idem *A*, ut vinceret dato ludorum numero minus duobus: & ſic de cæteris.

Probabilitas

Probabilitas quam habet *C*, ut vincat dato ludorum numero, eſt ſubdupla probabilitatis quam habebat *B*, ut vinceret dato ludorum numero minus uno; atque etiam ſubquadrupla probabilitatis quam habebat *D*, ut vinceret dato ludorum numero minus duobus.

Probabilitas quam habet *D* ut vincat dato ludorum numero, eſt ſubdupla probabilitatis quam habebat *C*, ut vinceret dato ludorum numero minus uno; atque etiam ſubquadrupla probabilitatis quam habebat *B*, ut vinceret dato ludorum numero minus duobus.

Probabilitas quam habet *A* ut vincat dato ludorum numero, eſt ſubdupla probabilitatis quam habebat *D*, ut vinceret dato ludorum numero minus uno; atque etiam ſubquadrupla probabilitatis quam habebat *C* ut vinceret dato ludorum numero minus duobus.

Ex jam obſervatis facile eſt componere Tabulam Probabilitatum, quas *B*, *C*, *D*, *A* habent ut victores evadant dato ludorum numero, atque etiam illorum ſortium ſeu expectationum.

Tabula Probabilitatum, &c.

		B	C	D	A
′	3	$\frac{1}{4} \times \overline{4+3p}$			
″	4		$\frac{1}{8} \times \overline{4+4p}$		
‴	5			$\frac{2}{16} \times \overline{4+5p}$	
⁗	6				$\frac{3}{32} \times \overline{4+6p}$
v	7	$\frac{3}{64} \times \overline{4+7p}$	$\frac{2}{64} \times \overline{4+7p}$		
v′	8	$\frac{3}{128} \times \overline{4+8p}$	$\frac{3}{128} \times \overline{4+8p}$	$\frac{2}{128} \times \overline{4+8p}$	
v″	9		$\frac{3}{256} \times \overline{4+9p}$	$\frac{6}{256} \times \overline{4+9p}$	$\frac{4}{256} \times \overline{4+9p}$
v‴	10	$\frac{4}{512} \times \overline{4+10p}$	$\frac{2}{512} \times \overline{4+10p}$	$\frac{6}{512} \times \overline{4+10p}$	$\frac{9}{512} \times \overline{4+10p}$
′x	11	$\frac{13}{1024} \times \overline{4+11p}$	$\frac{10}{1024} \times \overline{4+11p}$	$\frac{2}{1024} \times \overline{4+11p}$	$\frac{9}{1024} \times \overline{4+11p}$
x	12	$\frac{18}{2048} \times \overline{4+12p}$	$\frac{19}{2048} \times \overline{4+12p}$	$\frac{14}{2048} \times \overline{4+12p}$	$\frac{4}{2048} \times \overline{4+12p}$
		&c.		&c.	&c.

Jam vero Series iſtæ ſunt convergentes, adeoque ſingulæ ſummari poſſunt per vulgarem Arithmeticam; & obtinebuntur vel ſummæ accuratæ ſi poſſint, vel ſaltem approximatæ, ſi non liceat terminos multos adhibere.

Invenire

Invenire summas Probabilitatum ad infinitum usque pergentium, quas Collusores habent ut victores evadant.

Sint Probabilitates omnes ipsius B ad infinitum, nempe

$$B' + B'' + B''' + B'''' + B^{v} + B^{vi}, \text{ \&c.} = y$$

Probabilitates ipsius C

$$C' + C'' + C''' + C'''' + C^{v} + C^{vi}, \text{ \&c.} = z$$

Probabilitates ipsius D

$$D' + D'' + D''' + D'''' + D^{v} + D^{vi}, \text{ \&c.} = v$$

Probabilitates ipsius A

$$A' + A'' + A''' + A'''' + A^{v} + A^{vi}, \text{ \&c.} = x$$

Scribantur autem in Scala perpendiculariter descendente, ad hunc modum.

$$\begin{aligned} B' &= B' \\ B'' &= B'' \\ B''' &= \tfrac{1}{2}A'' + \tfrac{1}{4}A' \\ B'''' &= \tfrac{1}{2}A''' + \tfrac{1}{4}A'' \\ B^{v} &= \tfrac{1}{2}A'''' + \tfrac{1}{4}A''' \qquad \text{Proinde } y = \tfrac{1}{4} + \tfrac{1}{4}x. \\ B^{vi} &= \tfrac{1}{2}A^{v} + \tfrac{1}{4}A'''' \end{aligned}$$

Ergo $y = \frac{1}{4} + \frac{1}{2}x + \frac{1}{4}x$.

Demonstratio. Etenim prima columna perpendicularis $= y$, ex Hypoth.

Est vero $A' + A'' + A''' + A'''' + A^{v}$ &c. $= x$, ex hypothesi;

Ergo $\frac{1}{2}A' + \frac{1}{2}A'' + \frac{1}{2}A''' + \frac{1}{2}A'''' + \frac{1}{2}A^{v}$ &c. $= \frac{1}{2}x$.

Proinde $\frac{1}{2}A'' + \frac{1}{2}A''' + \frac{1}{2}A'''' + \frac{1}{2}A^{v}$ &c. $= \frac{1}{2}x - \frac{1}{2}A'$.

Et $B' + B'' + \frac{1}{2}A'' + \frac{1}{2}A''' + \frac{1}{2}A''''$ &c. $= \frac{1}{2}x - \frac{1}{2}A' + B' + B''$.

Sed $\frac{1}{2}A' = 0$, $B'' = 0$ & $B' = \frac{1}{4}$, ut patet ex Tabula.

Ergo secunda columna perpendicularis $= \frac{1}{4} + \frac{1}{2}x$.

Sed tertia columna perpendicularis $= \frac{1}{4}x$.

erit igitur $y = \frac{1}{4} + \frac{3}{4}x$.

Simili modo scribantur

$$\begin{aligned} C' &= C' \\ C'' &= C'' \\ C''' &= \tfrac{1}{2}B'' + \tfrac{1}{4}D' \\ C'''' &= \tfrac{1}{2}B''' + \tfrac{1}{4}D'' \qquad \text{hoc est } z = \tfrac{1}{2}y + \tfrac{1}{4}v. \\ C^{v} &= \tfrac{1}{2}B'''' + \tfrac{1}{4}D''' \\ C^{vi} &= \tfrac{1}{2}B^{v} + \tfrac{1}{4}D'''' \\ &\text{\&c.} \end{aligned}$$

Ergo $z = \frac{1}{8} + \frac{1}{2}y - \frac{1}{8} + \frac{1}{4}v$.

Scribantur

Scribantur etiam
$$\begin{aligned}
D' &= D' \\
D'' &= D'' \\
D''' &= \tfrac{1}{2} C' + \tfrac{1}{4} B' \\
D'''' &= \tfrac{1}{2} C'' + \tfrac{1}{4} B'' \\
D^{v} &= \tfrac{1}{2} C''' + \tfrac{1}{4} B''' \\
D^{vi} &= \tfrac{1}{2} C^{v} + \tfrac{1}{4} B'''' \\
&\&c.
\end{aligned}$$
& pari Argumento patebit $v + \frac{1}{2} z + \frac{1}{4} y$

Scribantur denique
$$\begin{aligned}
A' &= A' \\
A'' &= A'' \\
A''' &= \tfrac{1}{2} D'' + \tfrac{1}{4} C' \\
A'''' &= \tfrac{1}{2} D''' + \tfrac{1}{4} C'' \\
A^{v} &= \tfrac{1}{2} D'''' + \tfrac{1}{4} C'' \\
A^{vi} &= \tfrac{1}{2} D^{v} + \tfrac{1}{4} C''' \\
&\&c.
\end{aligned}$$
Unde concludetur $x = \frac{1}{2} v + \frac{1}{4} z$.

Resolutis autem quatuor istis æquationibus, reperietur

$$B' + B'' + B''' + B'''' \ \&c. = y = \frac{56}{149}$$

$$C' + C'' + C''' + C'''' \ \&c. = z = \frac{36}{149}$$

$$D' + D'' + D''' + D'''' \ \&c. = v = \frac{32}{149}$$

$$A' + A'' + A''' + A'''' \ \&c. = x = \frac{25}{149}$$

Valoribus istis inventis, ponatur jam $\frac{56}{149} = b$, $\frac{36}{149} = c$, $\frac{32}{149} = d$, $\frac{25}{149} = a$.

Iterum sit,
$$\begin{aligned}
3B'p + 4B''p + 5B'''p + 6B''''p \ \&c. &= py. \\
3C'p + 4C''p + 5C'''p + 6C''''p \ \&c. &= pz. \\
3D'p + 4D''p + 5D'''p + 6D''''p \ \&c. &= pv. \\
3A'p + 4A''p + 5A'''p + 6A''''p \ \&c. &= px.
\end{aligned}$$

$$\begin{aligned}
3B' &= 3B' \\
4B'' &= 4B'' \\
5B''' &= \tfrac{5}{2} A'' + \tfrac{5}{4} A' \\
6B'''' &= \tfrac{6}{2} A''' + \tfrac{6}{4} A'' \\
7B^{v} &= \tfrac{7}{2} A'''' + \tfrac{7}{4} A''' \\
8B^{vi} &= \tfrac{8}{2} A^{v} + \tfrac{8}{4} A'''' \\
\text{Ergo } y &= \tfrac{3}{4} + \tfrac{3}{4} x + a
\end{aligned}$$

Etenim prima Columna perpendicularis $= y$, ex Hypothesi:

$3B' + 4B'' = \frac{3}{4}$: Nam est $B' = \frac{1}{4}$, & $B'' = 0$.

$3A' + 4A'' + 5A'''$ &c. $= x$ ex Hypothesi.

$A' + A'' + A'''$ &c. $= a$, ut repertum est.

Est

Eſt igitur $4\ A' + 5\ A'' + 6\ A''' + 7\ A''''$ &c. $= x + a$

Et $\frac{4}{2}\ A' + \frac{5}{2}\ A'' + \frac{6}{2}\ A''' + \frac{7}{2}\ A''''$ &c. $= \frac{1}{2}x + \frac{1}{2}a$.

Sed A' o

Ergo ſecunda Columna perpendicularis $= \frac{1}{4} + \frac{1}{2}x + \frac{1}{2}a$.

$$3\ A' + 4\ A'' + 5\ A''' + 6\ A'''' \text{ \&c.} = x$$
$$2\ A' + 2\ A'' + 2\ A''' + 2\ A'''' \text{ \&c.} = 2a$$

Eſt igitur $5\ A' + 6\ A'' + 7\ A''' + 8\ A''''$ &c. $= x + 2\,a$.

Et $\frac{5}{4}\ A' + \frac{6}{4}\ A'' + \frac{7}{4}\ A''' + \frac{8}{4}\ A''''$ &c. $= \frac{1}{4}x + \frac{1}{2}a$.

Eſt igitur tertia Columna perpendicularis $= \frac{1}{4}x + \frac{1}{2}a$.

Erit igitur $y = \frac{3}{4} + \frac{1}{2}x + \frac{1}{2}a + \frac{1}{4}x + \frac{1}{2}a$

ſive $y = \frac{3}{4} + \frac{3}{4}x + a$, quod erat probandum.

$$\begin{aligned}
3\ C' &= 3\ C' \\
4\ C'' &= 4\ C'' \\
5\ C''' &= \tfrac{5}{2}\ B'' + \tfrac{5}{4}\ D' \\
6\ C'''' &= \tfrac{6}{2}\ B''' + \tfrac{6}{4}\ D'' \\
7\ C^{V} &= \tfrac{7}{2}\ B'''' + \tfrac{7}{4}\ D''' \\
8\ C^{VI} &= \tfrac{8}{2}\ B^{V} + \tfrac{8}{4}\ D''''
\end{aligned}$$

&c.

Ergo $z = \frac{1}{2}y + \frac{1}{2}b + \frac{1}{4}v + \frac{1}{2}d$.

Etenim prima Columna perpendicularis $= z$, ex Hypotheſi.

$$3\ C' + 4\ C'' = \tfrac{1}{2}.$$
$$3\ B' + 4\ B'' + 5\ B''' + 6\ B'''' \text{ \&c.} = y.$$
$$B' + B'' + B''' + B'''' \text{ \&c.} = b.$$

Eſt igitur $4\ B' + 5\ B'' + 6\ B''' + 7\ B''''$ &c. $= y + b$.

Sed $4\ B' = 1$.

Ergo $5\ B'' + 6\ B''' + 7\ B''''$ &c. $= y + b - 1$.

$\frac{5}{2}\ B'' + \frac{6}{2}\ B''' + \frac{7}{2}\ B''''$ &c. $= \frac{1}{2}y + \frac{1}{2}b - \frac{1}{2}$.

Ergo ſecunda Columna perpendicularis $= \frac{1}{2} + \frac{1}{2}y + \frac{1}{2}b - \frac{1}{2} = \frac{1}{2}y + \frac{1}{2}b$.

Iterum, $3\ D' + 4\ D'' + 5\ D''' + 6\ D''''$ &c. $= v$

$2\ D' + 2\ D'' + 2\ D''' + 2\ D''''$ &c. $= 2d$

Eſt igitur $5\ D' + 6\ D'' + 7\ D''' + 8\ D''''$ &c. $= v + 2\,d$.

Et $\frac{5}{4}\ D' + \frac{6}{4}\ D'' + \frac{7}{4}\ D''' + \frac{8}{4}\ D''''$ &c. $= \frac{1}{4}v + \frac{1}{2}d$.

Ergo tertia Columna perpendicularis $= \frac{1}{4}v + \frac{1}{2}d$.

Eſt igitur $z = \frac{1}{2}y + \frac{1}{2}b + \frac{1}{4}v + \frac{1}{2}d$, quod erat probandum.

Eodem prorſus ordine ſcribantur.

$$\begin{aligned}
3\ D' &= 3\ D' \\
4\ D'' &= 4\ D'' \\
5\ D''' &= \tfrac{5}{2}\ C'' + \tfrac{5}{4}\ B' \\
6\ D'''' &= \tfrac{6}{2}\ C''' + \tfrac{6}{4}\ B'' \\
7\ D^{V} &= \tfrac{7}{2}\ C'''' + \tfrac{7}{4}\ B''' \\
8\ D^{VI} &= \tfrac{8}{2}\ C^{V} + \tfrac{8}{4}\ B''''
\end{aligned}$$

&c.

Unde $v = \frac{1}{2}z + \frac{1}{2}c + \frac{1}{4}y + \frac{1}{2}b$.

$$\begin{aligned}
3\ A' &= 3\ A' \\
4\ A'' &= 4\ A'' \\
5\ A''' &= \tfrac{5}{2}\ D'' + \tfrac{5}{4}\ C' \\
6\ A'''' &= \tfrac{6}{2}\ D''' + \tfrac{6}{4}\ C'' \\
7\ A^{V} &= \tfrac{7}{2}\ D'''' + \tfrac{7}{4}\ C''' \\
8\ A^{VI} &= \tfrac{8}{2}\ D^{V} + \tfrac{8}{4}\ C''''
\end{aligned}$$

&c.

Et $x = \frac{1}{2}v + \frac{1}{2}d + \frac{1}{4}z + \frac{1}{2}c$.

Quæ

Quæ quidem Conclufiones eodem modo demonftrantur ac fuperiores. Solutis autem quatuor iftis æquationibus, elicietur

$$y = \frac{45536}{149|^2},\ z = \frac{38724}{149|^2},\ v = \frac{37600}{149|^2},\ x = \frac{33547}{149|^2} = \frac{33547}{22201}.$$

Ergo, fi velint B, C, D, A vendere Spectatori cuidam R fummas quas finguli obtinere fperant, æquum erit ut emptor R pendat

ipfi B $\quad 4 \times \frac{56}{149} + \frac{45536}{22201}p$, $\qquad$ ipfi C $\quad 4 \times \frac{36}{149} + \frac{38724}{22201}p$.

ipfi D $\quad 4 \times \frac{32}{149} + \frac{37600}{22201}p$, $\qquad$ ipfi A $\quad 4 \times \frac{25}{149} + \frac{33547}{22201}p$.

Invenire Probabilitates quas habent B, C, D, A, *ut mulctentur, dato ludorum numero.*

Si Ludi duo tantum fint, erunt hoc modo,

$$\left.\begin{array}{cc} \underline{B\ A} & \underline{B\ A} \\ C\ B & B\ C \end{array}\right\}$$ Unde patet B vel C neceffario mulctari.

Si Ludi tres fuerint, hoc modo fe res habet,

$$\left.\begin{array}{cccc} \underline{B\ A} & \underline{B\ A} & \underline{B\ A} & \underline{B\ A} \\ C\ B & C\ B & B\ C & B\ C \\ D\ C & C\ D & D\ B & B\ D \end{array}\right\}$$ Hinc patet C, vel D vel B neceffario mulctari.

Si vero quatuor Ludi fuerint,

$$\left.\begin{array}{cccccc} \underline{B\ A} & \underline{B\ A} & \underline{B\ A} & \underline{B\ A} & \underline{B\ A} & \underline{B\ A} \\ C\ B & C\ B & C\ B & C\ B & B\ C & B\ C \\ D\ C & D\ C & C\ D & C\ D & D\ B & D\ B \\ A\ D & D\ A & A\ C & C\ A & A\ D & D\ A \end{array}\right\}$$ Debet igitur A triplici modo, D duplici, C fimplici, mulctari,

Et fic de cæteris. Ex quibus manifefta eft Compofitio Tabulæ fubjunctæ Probabilitatum quas B,C,D,A, habent ut mulctentur, dato ludorum numero.

	Num. Lud.	B	C	D	A
'	2	$\frac{1}{2}$	$\frac{1}{2}$		
''	3	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	
'''	4		$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$
''''	5	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{3}{16}$
v	6	$\frac{6}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
v'	7	$\frac{6}{64}$ &c.	$\frac{8}{64}$	$\frac{8}{64}$	$\frac{4}{64}$

 Sint

Sint autem y, z, v, x summæ omnium Probabilitatum quas B, C, D, A habent respective ut mulctentur.

Scribantur eodem ordine ac in præcedentibus.

$$\begin{array}{ll}
B' = B' & C' = C' \\
B'' = B'' & C'' = C'' \\
B''' = \frac{1}{2}A'' + \frac{1}{4}A' & C''' = \frac{1}{2}B'' + \frac{1}{4}D' \\
B'''' = \frac{1}{2}A''' + \frac{1}{4}A'' & C'''' = \frac{1}{2}B''' + \frac{1}{4}D'' \\
B^{v} = \frac{1}{2}A'''' + \frac{1}{4}A''' & C^{v} = \frac{1}{2}B'''' + \frac{1}{4}D''' \\
B^{vi} = \frac{1}{2}A^{v} + \frac{1}{4}A'''' & C^{vi} = \frac{1}{2}B^{v} + \frac{1}{4}D'''' \\
\&c. & \&c.
\end{array}$$

Ergo $y = \frac{1}{4} + \frac{1}{2}x + \frac{1}{4}x$. Ergo $z = \frac{1}{2} + \frac{1}{2}y + \frac{1}{4}v$.
$= \frac{1}{4} + \frac{3}{4}x$.

Scribantur deinde

$$\begin{array}{ll}
D' = D' & A' = A' \\
D'' = D'' & A'' = A'' \\
D''' = \frac{1}{2}C'' + \frac{1}{4}B' & A''' = \frac{1}{2}D'' + \frac{1}{4}C' \\
D'''' = \frac{1}{2}C''' + \frac{1}{4}B'' & A'''' = \frac{1}{2}D''' + \frac{1}{4}C'' \\
D^{v} = \frac{1}{2}C'''' + \frac{1}{4}B''' & A^{v} = \frac{1}{2}D'''' + \frac{1}{4}C''' \\
D^{vi} = \frac{1}{2}C^{v} + \frac{1}{4}B'''' & A^{vi} = \frac{1}{2}D^{v} + \frac{1}{4}C'''' \\
\&c. & \&c.
\end{array}$$

Ergo $v = \frac{1}{4} + \frac{1}{2}z + \frac{1}{4}y$. Ergo $x = \frac{1}{2}v + \frac{1}{4}z$.

Resolutis autem quatuor istis æquationibus, invenietur

$$y = \frac{243}{149} \qquad z = \frac{252}{149} \qquad v = \frac{224}{149} \qquad \& \ x = \frac{175}{149}$$

Ergo si velit Spectator aliquis S mulctas omnes sustinere, æquum erit ut ipsi S

$$B \text{ tradat } \frac{243}{149}p \qquad C\,\frac{252}{149}p \qquad D\,\frac{224}{149}p \qquad \& \ A\,\frac{175}{149}p.$$

Sublatis itaque summis probabilitatum quas singuli Collusores habent ut mulctentur, è summis expectationum quas habent iidem si victores abeant, restabunt sortes eorum respective: nempe

$$B \text{ recipit ab } R\,\frac{4\times 56}{149} + \frac{45536}{22201}p$$

$$B \text{ tradit ipsi } S \qquad \frac{243}{149}p$$

$$\text{Ergo ipsi } B \text{ superest } \frac{224}{149} + \frac{9329}{22201}p$$

Sed B deposuerat 1 priusquam ludus inciperetur.

$$\text{Ergo } B \text{ lucratur } \frac{75}{149} + \frac{9329}{22201}p.$$

C Re-

C recipit ab R $\quad \frac{4 \times 36}{149} + \frac{38724}{22201}p$

C tradit ipsi S $\quad \frac{252}{149}p$

Ergo ipsi C superest $\frac{144}{149} + \frac{1176}{22201}p$

Sed C deposuerat 1.

Ergo C lucratur $-\frac{5}{149} + \frac{1176}{22201}p$.

D recipit ab R $\quad \frac{4 \times 32}{149} + \frac{37600}{22201}p$

D tradit ipsi S $\quad \frac{224}{149}p$

Ergo ipsi D superest $\frac{128}{149} + \frac{4224}{22201}p$

Sed D deposuerat 1.

Ergo D lucratur $-\frac{21}{149} + \frac{4224}{22201}p$.

A recipit ab R $\quad \frac{4 \times 25}{149} + \frac{33547}{22201}p$

A tradit ipsi S $\quad \frac{175}{149}p$

Ergo ipsi A superest $\frac{100}{149} + \frac{7472}{22201}p$

Sed A deposuerat $1 + p$, nempe 1 priusquam ludus inchoaretur, & p postquam semel victus fuerat à B:

Ergo A lucratur $-\frac{49}{149} - \frac{14729}{22201}p$.

$$
\begin{aligned}
\text{Lucrum ipsius } B &= +\frac{75}{149} + \frac{9329}{22201}p \\
\text{ipsius } C &= -\frac{5}{149} + \frac{1176}{22201}p \\
\text{ipsius } D &= -\frac{21}{149} + \frac{4224}{22201}p \\
\text{ipsius } A &= -\frac{49}{149} - \frac{14729}{22201}p \\
\text{Summa Lucrorum} &= \quad 0 \qquad\qquad 0
\end{aligned}
$$

Summa autem lucrorum ipsorum B & $A = \frac{26}{149} - \frac{5400}{22201}p$; sed posueramus B vicisse ipsum A semel, priusquam Collusores pacta inirent cum R & S. Priusquam vero ludus inchoaretur, A poterat æqua sorte expectare

 ut

ut vinceret ipſum *B*, adeoque ſumma lucrorum $\frac{26}{149} - \frac{5400}{22201}$ in duas partes æquales eſt dividenda, ita ut utriuſque lucrum cenſendum ſit $\frac{13}{149} - \frac{2700}{22201}p$.

Ponatur $\frac{13}{149} - \frac{2700}{22201}p = 0$, & erit $p = \frac{1937}{2700}$.

Ergo ſi ſit mulcta p ad ſummam quam ſinguli deponunt ut 1937 ad 2700, *A* & *B* nihil lucrantur, nihil perdunt. Verum hoc in Caſu *C* lucratur $\frac{1}{225}$, quam *D* perdit.

Coroll. 1. Spectator *R*, priuſquam ludus inchoetur, id ſuſcipere in ſe poterit, ut ſummam 4 de qua Colluſores contendunt, & mulctas omnes pendat, ſi ſibi initio in manus darentur $4 + 7p$.

Coroll. 2. Si dexteritates Colluſorum ſint in ratione data, ſortes Colluſorum eadem ratiocinatione determinabuntur.

Coroll. 3. Si Series aliqua ita ſit conſtituta, ut continuò decreſcat, & terminus quivis ad præcedentes quoſlibet habeat rationes datas, ſive eaſdem ſive diverſas, ſeries iſta accurate ſummabitur. Inſuper ſi termini omnes hujus Seriei multiplicentur per terminos progreſſionis Arithmeticæ, ſinguli per ſingulos, Series nova reſultans accurate ſummabitur.

Coroll. 4. Si ſint Series plures collaterales, ita relatæ ut terminus quilibet cujuſque Seriei ad præcedentes quoſlibet aliarum Serierum habeat rationes datas, ſive eaſdem ſive diverſas, ita ut Series iſtæ collaterales ſe decuſſent data qualibet lege conſtanti, Series iſtæ accurate ſummabuntur. Inſuper ſi termini omnes harum Serierum multiplicentur ordinatim per terminos Progreſſionis Arithmeticæ, ſinguli per ſingulos, Series novæ ex hac multiplicatione reſultantes etiamnum accurate ſummabuntur.

Clavis ad Problema generale.

Si ſint Colluſores quotcunque *v. g.* Sex, *B*, *C*, *D*, *E*, *F*, *A* & Probabilitates quas habent ut victores evadant, ſive ut mulctentur, dato Ludorum numero, denotentur reſpective $B_{,}$ $C_{,}$ $D_{,}$ $E_{,}$ $F_{,}$ & $A_{,}$; & Probabilitates dato Ludorum numero his proximo & minori competentes, per $B_{,,}$, $C_{,,}$, $D_{,,}$, $E_{,,}$, $F_{,,}$, $A_{,,}$; & Probabilitates dato Ludorum numero his itidem noviſſimis proximo & minori competentes, per $B_{,,,}$, $C_{,,,}$, $D_{,,,}$, $E_{,,,}$, $F_{,,,}$, $A_{,,,}$, & ſic deinceps, erit ſemper,

$$
\begin{aligned}
B_{,} &= \tfrac{1}{2} A_{,,} + \tfrac{1}{4} A_{,,,} + \tfrac{1}{8} A_{,,,,} + \tfrac{1}{16} A^{v} \\
C_{,} &= \tfrac{1}{2} B_{,,} + \tfrac{1}{4} F_{,,,} + \tfrac{1}{8} E_{,,,,} + \tfrac{1}{16} D^{v} \\
D_{,} &= \tfrac{1}{2} C_{,,} + \tfrac{1}{4} B_{,,,} + \tfrac{1}{8} F_{,,,,} + \tfrac{1}{16} E^{v} \\
E_{,} &= \tfrac{1}{2} D_{,,} + \tfrac{1}{4} C_{,,,} + \tfrac{1}{8} B_{,,,,} + \tfrac{5}{16} F^{v} \\
F_{,} &= \tfrac{1}{2} E_{,,} + \tfrac{1}{4} D_{,,,} + \tfrac{1}{8} C_{,,,,} + \tfrac{1}{16} B^{v} \\
A_{,} &= \tfrac{1}{2} F_{,,} + \tfrac{1}{4} E_{,,,} + \tfrac{1}{8} D_{,,,,} + \tfrac{1}{16} C^{v}
\end{aligned}
$$

Et fiat ſemper retrogreſſus ordinatim ad tot literas minus duobus quot ſunt Colluſores, omittaturque ſemper litera *A*; prima æquatione excepta ubi litera *A* terminos omnes præter primam occupat.

XXXIII.

XXXIII. *Papers omitted.*

1. Solutio Problematis à Cl. Viro D. Johan. Bernoullio in Diario Gallico, Feb. 1703. propositi; Quam D. G. Cheynæo communicavit Jo Craig. *n.289. p.1527.*

2. Vindiciæ Matheseos universalis Gregorianæ contra secundos Abbatis Galloysii impetus in Historia Acad. Scient. *Ann.* 1703. *n.308. p.2336.*

3. A Letter from Monf. *l'Abbé Conti*, to the late Mr. *Leibnitz*, concerning the Dispute about the Invention of the Method of Fluxions, or Differential Method; with Mr. *Leibnitz*'s Answer. *n.359. p. 923.*

4. Apologia D. Brook Taylor, J. U. D. & R. S. S. contra V. C. J. Bernoullium, Math. Prof. Basileæ, de contumeliis quibusdam ab illo in Actis Lipsiensibus insertis. *n.360. p 955.*

XXXIV. *Accounts of Books omitted,*

1. Apollonii Pergæi Conicorum Libri Octo, & Sereni Antissensis de Sectione Cylindri & Coni Libb. duo. Fol. Reg. E Theatro Oxon. 1710. *n.354. p.732.*

2. Euclidis quæ supersunt omnia, Gr. Lat. ex recensione Davidis Gregorii, M. D. Astr. Prof. Savil. Fol. Oxon. 1703. *n.289. p.1558.*

3. Methodus Incrementorum. Auctore Brook Taylor, LLD. R. S. Secr. Lond. 1715. wherein he corrects a Mistake in the 25th Proposition of the Second Part. *n.345. p.339.*

4. De Locis solidis secunda Divinatio Geometrica, in quinque Libros injuria temporis amissos, Aristæi Senioris Geometræ. Autore Vincentio Viviani, Magni Ducis Etruriæ Mathematico primario, & R. S. S. Fol. 1701. *n.291. p.1607.*

5. Commercium Epistolicum Collinii & aliorum De Analysi promota; publish'd by order of the Royal Society, in relation to the Dispute between Mr. *Leibnitz* and Mr. *Keill*, about the Right of Invention of the Method of Fluxions, by some call'd the Differential Method. *n.342. p.173.*

CHAP. II.

Opticks.

1. *A universal Spherico-Catoptrick Theorem, by Mr.* Ditton. *n.295. p.1810.*

Sit DEF Speculi Spherici concavi portio, cujus Centrum B, semidiameter BE vel BD: Sit etiam A punctum radians in axe collocatum, a quo profluat radiosa linea AD, quæ ad punctum D reflectatur in DC. Investiganda jam est Foci C, a speculi vertice *E* distantia.

Notandum vero, quod punctum *D* ipsi *E* proximum supponimus. Radii enim remotiores oculum (quem in axe AE constituimus) præterlabuntur, nec ad imaginis visionem aliquid faciunt. Porro, propter arcum *Plate 3. Fig. 36.*

DE

DE indefinite parvum, anguli D A B, A D B (ut & ipsorum summa D B C) sunt quam minimi, ac idcirco eandem habebunt inter se rationem, quam ipsis latera opposita: quo ratiocinii principio posito, ad Theorema Dioptricum pervenit D. *Halleius*, Geometriæ Professor apud Oxonienses.

Abr. Vol. I. *p.* 183.

Hisce præmissis, sit $AB = b$. $BD = BE = r$. $BC = z$. CE ($= r - z$, sed brevitatis causa ponatur) $= f$. Quantitates b & r cognitæ sunt (dantur enim semidiameter speculi, ac puncti lucidi a vertice distantia) z vero & f quæsitæ ac incognitæ. Jam in Triangulo D A B, erit $\angle DAB : \angle ADB :: r : b$. Item in Triangulo D B C, $\angle BDC = \angle ADB$, ex natura Reflexionis, & $\angle DBC = \angle DAB + \angle ADB$, ex *Elem. Eucl.* Ergo cum $\angle DBC$ sit ut $r + b$, & $\angle BDC$ ut b; erit etiam $\angle DBC : \angle BDC :: r + b : b$, & (quod ex principio supra memorato consequitur) $DC : BC :: r + b : b$. Sed quoniam punctum D ipsi E proximum est, erit D C ipsi C E æqualis æstimanda, ergo $CE : BC :: r + b : b$; hoc est $f : z :: r + b : b$, & (comparando Antecedentium & Consequentium summas ad Antecedentes) $f + z : f :: r + 2b : r + b$; sed $f + z = r$, ergo $r : f :: r + 2b : r + b$, ergo $f = \frac{rr \times rb}{r \times rb}$. Q: E: I.

Si ponatur $r + b$ ($=$ A E) $= d$, Theorema in formam contractiorem redigetur, & sic stabit $f = \frac{rd}{2d - r}$. Sed utrovis modo, focorum inventioni, quæcunque tandem sit, vel Speculi forma, vel radiorum conditio, aptum evadet.

Coroll. I. Erit $zd = df - rf$, sive $AE \times BC = AB \times CE$, vel quod idem est, linea A E harmonice dividitur in punctis A, B, C, E; nam prædicta Rectangulorum æqualitas, lineæ secundum proportionem harmonicam sectæ, propria est. Patet hæc veritas: Est enim $f = \frac{dr}{2d - r}$, & $z = r - f = r - \frac{dr}{2d - r}$, unde valores hosce substituendo, Æquatio manifesta fiet. Adeo ut in omni Speculo Spherico, lineæ D A, D B, D C, D E, sunt Harmonicales; & Punctum radians, Centrum, Focus, Vertex sunt puncta divisionem Harmonicam efficientia.

Coroll. II. 1ma Posito $d > r$; erit ex calculo f, sive $\frac{rd}{2d - r} > \frac{r}{2}$ semper. Hoc est, si puncti radiantis distantia major sit Semidiametro Speculi, foci distantia semper major erit quarta parte Diametri.

Item, erit $\frac{rd}{2d - r} < r$ semper. Hoc est, distantia foci semper erit minor speculi semidiametro.

2d. Si ponatur $d = r$, erit $\frac{rd}{2d - r}$, sive $f = r$. Hoc est, si punctum radians in centro speculi constituatur, Imago ejus ibi cum eo unietur.

3tio. Si ponatur $d = r$, tum ipsius f expressio erit vel positiva vel negativa vel infinita, prout quantitas $2d$ quantitate r vel major est vel minor, vel ei æqualis.

Si

Si $2d > r$, hoc eſt, ſi $d > \frac{r}{2}$, tum punctum radians & focus ad eaſdem partes ſpeculi jacent.

Si $2d < r$, vel $d < \frac{r}{2}$, tum Imago, in axe ultra ſpeculi verticem producto, ſita eſt.

Si $2d = r$, vel $d = \frac{r}{2}$, Imago infinite diſtat, ſive radius reflexus axi parallelus evadit.

Coroll. III. Calculi hujus ope expedite determinari poteſt, quomodo objecti radiantis (ſpeculi reſpectu) motui, ipſius Imaginis motus reſpondet. Sit (ut antea) Imaginis a ſpeculo diſtantia $= \frac{dr}{2d-r}$, quando objecti diſtantia eſt d. Mutetur jam utcunque objecti diſtantia, & ex d, fiat nd, quantitate n Numerum vel integrum vel fractum deſignante: & ſic loco prioris Æquationis, $f = \frac{dr}{2d-r}$, habebimus pro Novo Foco aliam Æquationem, $F = \frac{ndr}{2nd-r}$. Et quidem ſi n Numerum integrum exprimere ſupponatur, ſecunda hæc objecti diſtantia primâ major erit, ſi vero ſit fractus, tum minor erit prima.

Hiſce poſitis, ſi $d > r$, & n ſit integer, erit $F < f$, id eſt, erit $\frac{ndr}{2nd-r} < \frac{dr}{2d-r}$ ſive $2nddr - ndrr < 2nddr - drr$, quod manifeſtum eſt. Hoc eſt, ſi in ſpeculo concavo objecti diſtantia major ſit ſemidiametro, tum recedente objecto a ſpeculo, Imago verſus ſpeculum accedet. Rurſus, deſignet n Numerum fractum, & tunc reperietur $2nddr - ndrr > 2nddr - drr$, ſive $F > f$. Hoc eſt, accedente objecto ad ſpeculum recedet Imago.

Supponatur jam $d < \frac{r}{2}$; ut & alia quæcunque ſit objecti diſtantia nd intelligatur ea ſemper minor eſſe quam $\frac{r}{2}$. Tum erunt $2nddr - ndrr$, & $2nddr - drr$, quantitates negativæ; ſive $ndrr - 2nddr$, & $drr - 2nddr$ quantitates poſitivæ. Et quidem ſi n numero integro æquetur, erit $ndrr - 2nddr > drr - 2nddr$, ſive $F > f$; ſi vero n fractio ſit, tum erit $ndrr - 2nddr < drr - 2nddr$, ſive $F < f$. Hoc eſt, ſi in ſpeculo concavo objecti diſtantia minor ſit ſpeculi Diametri quarta parte, tum recedente objecto a ſpeculo, recedet & Imago; vel accedente objecto verſus ſpeculum, Imago etiam accedet.

Et hæc omnia (quæ calculi veſtigia premendo deduximus) Scholio unico concluſit, & in ſua Catoptrica tradidit D. *Gregorius* apud Oxonienſes Aſtronomiæ Profeſſor.

Coroll.

Coroll. IV. In Æquatione $f = \frac{dr}{2d - r}$, si ponatur d infinita, erit $f = \frac{r}{2}$; quæ regula eſt pro Radiis parallelis, ſive pro objecto radiante ad diſtantiam infinitam remoto. Idem ſequetur, poſito b infinito in Æquatione $f = \frac{rr + rb}{r + 2b}$.

Coroll. V. In Æquatione $\frac{dr}{2d - r}$, mutato quantitatis r ſigno negativo in poſitivum, erit $f = \frac{dr}{2d + r}$; vel in æquatione $f = \frac{rb + br}{r + 2b}$, mutato ſigno poſitivo in negativum, erit tunc $f = \frac{rb - rr}{2b - r}$, quæ regulam exhibet pro ſpeculo verſus objectum radians convexo. Patet hæc mutatio ſigni; nam ſicut in ſpeculo concavo $d = r + b$, ſic in convexo $d = b - r$.

Coroll. VI. In ſpeculo convexo (ſtantibus quæ ad *Cor.* III. annotavimus de Concavo) patebit quod (ſi n ſit numerus integer) $2rndd + ndrr > 2rndd + drr$; & ($n$ fractione exiſtente) quod $2rndd + ndrr < 2rndd + drr$. Hoc eſt, quod recedente objecto a ſpeculo vel verſus idem accedente, Imago ſimiliter recedet vel accedet.

Patet etiam in ſpeculo convexo, objecto ad immenſam uſque diſtantiam retrocedente, Imaginem tamen illius non ultra Diametri partem quartam abire a vertice, ſed ibi, in puncto, centrum inter & verticem medio, ſe ſiſtere. Poſito enim d vel b infinito, erit $f = \frac{dr}{2d}$ vel $\frac{br}{2b}$, id eſt (utrovis modo) $= \frac{r}{2}$.

Hiſce adjungi poteſt & Problematis Catoptrici ſolutio, *Radiantis poſitionem reſpectu ſpeculi dati talem invenire, ut radians ad ipſius Imaginem a ſpeculo factam, datam habeat rationem.* Sit Ratio data $r : q$, & ſymbolo O deſignetur Objectum, I Imago, d diſtantia objecti, & f imaginis a ſpeculo. Jam (quod demonſtravit D. *Greg.*) erit $O : I :: d : f$, (hoc eſt Objectum & Imago ſunt diſtantiis ſuis a ſpeculi vertice directe proportionales) & quoniam requiritur ut ſit $O : I :: r : q$, debet etiam eſſe $d : f :: r : q$, vel (ipſius f expreſſionem ſcribendo) $d : \frac{dr}{2d - r} :: r : q$, unde $2ddq - rdq = rdr$, & $2dq = rr + qr$, & $d = \frac{rr + rq}{2q}$. Unde quoniam $dr = \frac{rrr + rrq}{2q}$, & $2d - r = \frac{rr}{q}$, erit etiam f ſive $\frac{dr}{2d - r} = \frac{rrr + rrq}{2q} : \frac{rr}{q} = \frac{qrrr + qqrr}{2qrr} = \frac{r + q}{2}$, quæ eſt ipſius f, ſive imaginis a ſpeculo diſtantia, huic objecti diſtantiæ congrua. Ergo ſi ſtatuatur objectum ad diſtantiam $\frac{rr \times rq}{2q}$, ipſius Imago facta ad diſtantiam $\frac{r + q}{2}$ ei compa-

rata,

ata, eandem habebit rationem, quam $q : r$, five erit $O : I :: r : q$. Nam $O : I :: d : f :: \frac{rr + rq}{2q} : \frac{r + q}{2} :: r : q$. Q. E. I.

Objectum Radians & Imaginem hic tanquam lineas consideravimus. Si enim Superficies sunt, tum erit $O : I :: d^2 : f^2$, & $d^2 : f^2 :: r : q$, sic ut ultimo deveniatur ad Æquationem $4dd - 4qdr = r^3 - qrr$, e qua radicis d valor, Methodis vulgaribus facillime inveniri potest.

II. *Exp.* 1. Having sew'd together end-wise two Pieces of Ribbon four Inches long each, the one blew, the other red, whose common Breadth was $\frac{1}{4}$ of an Inch; I caus'd it to be held in such manner, that the Light which fell from the Clouds thro' the Window was so reflected, that the Angle made by the Rays of Light, which came in at the Middle of the Window, with the Plane of the Ribbon produced, was equal to the Angle made by a Line drawn from the Ribbon to my Eye and the said Plain of the Ribbon. My Eye was plac'd as far behind the Ribbon as the Window was before it, the Distance from which to me was about 12 Feet. Then looking thro' a Prism at the Ribbon, it appear'd broken asunder in the Place where the blew and red Half joyn'd. If the Prism was held with the refracting Angle downwards (or laid with one of its Planes flat upon the Nose) the blew Half of the Ribbon appear'd to be carried down lower than the red, as at *B R*, but if the refracting Angle of the Prism was turn'd upwards (as when the Prism has one of its Planes laid flat to the Forehead) then the blew Half of the Ribbon was lifted up, as at *b ρ*.

Some Experiments of Light and Colours, mentioned by Sir Isaac Newton in his Opticks, lately repeated by Dr. Desaguliers. n. 348. p. 433.

Fig. 1.

The Prism was of white Glass, having every Angle of 60 Degrees: but when instead of it, one of a greenish sort of Glass, such as Object-Glasses of Telescopes are made of, was used, having the refracting Angle which I look'd thro' of about 48 Degrees: the same Phænomenon was more distinct, this Glass having no Veins, but the Red and Blue were nearer to a streight Line: in such manner that if *A* represent the Ribbon seen through the first Prism, *B* will represent the Ribbon seen thro' the second Prism, *Fig.* 2. If the refracting Angle of the last Prism had been as great as that of the first, the Light being transmitted thro' too great a Body of greenish Glass, the Phænomenon would not have succeeded so well.

The blue Ribbon being somewhat too pale, and the red a little dull; I repeated the Experiment with a Skeen of blue, and one of red Worsted join'd together in the Middle as the Ribbons were before; and, the Colours of both being very intense, the Experiment succeeded better with both Prisms. All that were present trying the Experiment found it to succeed, and that every Circumstance answer'd to the Account given in *Prop.* 1. *Theor.* 1. *Book* 1. of Sir *Isaac Newton*'s *Optics*, as far as the Directions there given were followed. So that it appear'd that the Blue being carried lower than the Red in the first case, and lifted higher in the second, was owing to the greater Refraction of the

blue Ray: for tho' each Part of the Ribbon or Worsted reflected all manner of Rays, yet the Phænomenon was very apparent; as also that the blue Ribbon or Worsted reflected the blue Rays more copiously than the red Rays, and that the red Ribbon or Worsted reflected the red Rays more than the blue ones, because the Red of the blue Half seen thro' the Prism was less intense than that of the red Half, and the Blue or Purple of the red Half seen thro' the Prism was less intense than that of the blue Half.

N. B. If the Ribbon or Worsted is laid upon any enlightned Body, the Phænomenon will not succeed so well; the Colours of the Body seen thro' the Prism mixing with those of the Ribbon or Worsted. Even a black Body will not do, if Light falls upon it: but there must be a black Cloath behind, in such manner that no Light falling upon it can be reflected so as to disturb the Phænomenon. And if a short-sighted Person looks through the Prism, a concave Lens between his Eye and the Prism will render the Phænomenon more distinct than it wou'd otherwise be.

Exp. II. Some Days after, the Sun shining, I made two Holes *H,h*, in the Window Shut *S s*, of a darkned Room; thro' which letting the Suns Beams pass, by means of two Prisms *A, B*, (one near each Hole) I open'd the Rays coming from the Sun into the two colour'd *Spectra* α β, where the following Colours were very distinct, *viz.* Red, Orange, Yellow, Green, Blue, Purple and Violet. Now the Reason of their being more distinct than ordinary, was, that the Prisms which I made use of were made of the greenish Glass mentioned before; which is very free from those Veins by which the Colours are too much thrown into one another, by the best white Prisms of the common sort.

The forementioned colour'd *Spectra* being thrown into the Room, to the Distance of about 20 Feet from the Window where the Sun's Light came in, I caus'd a Piece of white Paper π, ½ Inch broad and 5 Inches long, to be held within the refracted Rays, (at the Distance of 10 Feet from the Windows,) which produc'd these Colours in such manner, that by turning the Prisms round their Axes, I cou'd make the red Ray of the *Spectrum*, made by the one Prism, fall upon one half of the Paper, and the purple Ray of the *Spectrum* made by the other Prism fall upon the other Half; for the *Spectra* were both vertical, the Lines which terminated the long Sides of them towards each other just touching, as appears in *Fig.* 3. Then at the Distance of 9 Foot, looking thro' the Prism C at the Paper thus colour'd, the red Half appear'd very much separated from the Purple, the one seeming lifted up from the other; the Red or the Purple appearing the highest, according as the refracting Angle of the Prism was either held upwards or downwards. The Phænomenon is much more distinct this way than any other; for the Paper not only seems divided into two, when it is coloured by a red and a purple Ray, but also by a Red and Blue, (*Fig.* 4.) by a red and a green Ray (*Fig.* 5.) or indeed by any two Colours that are different how near soever

ver their Places in the *Spectra* be to each other. The Halves of the Paper appear, when view'd thro' the Prifm, to be farther from each other, when the Paper is ting'd with fuch Colours as are farther from each other in the Series of Colours in the *Spectrum* : and neareft, tho' ftill divided, when neighbouring Colours fall upon the Paper, as Yellow and Green, or a light and a deep Green. But the Paper appears no way divided, when colour'd with the Red of the two *Spectra*, (*Fig.* 6.) if thofe Reds are equally intenfe : and fo of the other Colours.

Exp. III. I held a Lens of about 3 Foot Radius at the Diftance of Six Feet from the oblong Paper (on which a red and purple Ray falling, made it look half Red and half Purple) and I projected the Image of the faid colour'd Paper at the Diftance of about Six Foot on the other Side of the Lens, on a white Sheet of Paper; where it was obfervable, that when the red Half was diftinctly painted on the white Paper (which was known by the Edges of the Image being regularly terminated) then the blue Half of the Image was confus'd : but if the white Paper was brought about two Inches nearer to the Lens, the Image of the Blue Half became diftinct, and that of the red Half confus'd.

I try'd the Experiment with a Paper colour'd half red and half blue, the red with Carmine and the blew with Smalt, making the Candle to enlighten the Paper (the Room being otherwife dark) and the Experiment fucceeded in the fame manner. The Experiment thus made is the fame that Sir *Ifaac Newton* gives an Account of, *Book* 1. *Part* 1. *Theor.* 1. *of his Optics.* Only it is to be obferv'd that when the oblong Paper is colour'd with red and blue from the Prifms, the focal place, where the red part of the Image is diftinct, is more diftant from the place where the blue part of the Image is diftinct, than when the Paper is colour'd with the Painter's Powders, and much more vivid.

The 7*th* Figure fhews the Projection of the Paper ting'd with the Rays; and *Fig.* 8. the Projection of it when painted : where a black Thread is wrapp'd round the red and the blue part, that the Diftinctnefs of the Image of the Thread may fhew when the red or when the blue part of the Image of the Paper is moft diftinct.

N. B. When the Candle enlightens the painted Paper, fet an opaque Body as *B* between the Candle and Lens; left the Image of the Candle being alfo projected fhould difturb the Experiment.

Exp. IV. Having made an Hole of $\frac{1}{4}$ Inch Diameter in the Window-Shut of the darkned Room, I fuffer'd a Sun-Beam to come into the Room, which I intercepted with a Prifm at the Diftance of 5 Inches from the Hole; and after its Refraction in paffing thro' the Prifm, I receiv'd it upon a Sheet of white Paper, where it was colour'd, making an oblong Image of the Sun or *Spectrum* of about 9 Inches in length and 2 in breadth, which Breadth was nearly equal to the Diameter of the round Image of the Sun received upon a Paper at the fame Diftance from the Hole, which here was 18 Foot. Or if the fun be too high, a Looking-Glafs being put in the room of the Prifm will throw a white round *Spectrum* upon the Pa-

per, which held at the said Distance of 18 Foot, will have its Diameter equal to the Breadth of the colour'd *Spectrum*.

The Colours of the *Spectrum* were these; Red, Orange, Yellow, Green, Blue, Purple and Violet, tho' the Violet was so faint in this as to be scarce perceivable. *See Fig.* 9.

N. B. The Axis of the Prism in this, and all the other Experiments hereafter mention'd must be perpendicular to the Ray that falls on it; and the Plane into which the Ray enters must be held in such a Position, that the Angle which such a Ray makes with that Plane when it enters, may be equal to the Angle made by the middle Line of those Rays which emerge after Refraction, on the other Side of the refracting Angle of the Prism, with the Plane out of which they emerge. That is $\angle BDG = \angle AEH$.

If the Plane *AC*, on which the Sun-Beam falls, be turned nearer to a perpendicular to the Sun Beam than before, the *Spectrum* will be much longer: if it be more inclin'd to the said Beam, the *Spectrum* will be shorter, and in both cases less distinct. See the *Spectrum DE* and the *Spectrum de* in *Fig.* 10. and 11. where *H*, *h*, represents the Hole in the Window Shut in each Case; *AC*, *ac* the Plane of the Prism on which the Rays enter; *BC*, *bc*, that out of which they emerge; *P*, *p* the perpendicular, and *C*, *c* the refracting Angle.

If the Plane *AC* be still more oblique to *HF*, all the Light will be reflected, and there will be no colour'd Image or *Spectrum* made by Refraction at all. *Fig.* 12.

But if it be held so as to be more nearly perpendicular to the Sun Beam than in *Fig.* 10. the whole Beam will indeed enter the Prism; but meeting with *BC* the lower Surface of the Prism, or rather the Surface of the Air contiguous to it, some of the Light will by the Plane *BC* be reflected to *de*, passing almost perpendicularly thro' *AB*; and the rest will emerge thro' *BC*, and by Refraction make the Imperfect Spectrum *DE*. See *Fig.* 10.

If the Sun Beam enter *AC* perpendicularly and in the middle of it, the Light will be all reflected as in *Fig.* 13. some of it by the Plane *BC* to *R*, and the rest by the Plane *AB* to *ρ*. But if the Beam fall nearer to *A* (still perpendicularly) it will all be reflected by the Plane *AB*; if nearer to *B*, it will be all reflected by the Plane *BC*.

In order therefore to have the colour'd *Spectrum* as it ought to be, care must be taken that the emerging coloured Light may make the same Angle with the Plane *BC*, as the immerging Light does with the Plane *AC*; that is, the Angle *AEH* must be equal to *BDG*, as was said before, *Fig.* 9: which may also be seen on the enlightned Dust in the Air. But the best way is to turn the Prism on its Axis, and at the same time look at the colour'd *Spectrum*, which will rise and fall, and become longer or shorter, as you turn the Prism; and between the Ascent and Descent of the Image, it will appear Stationary: there stop the Prism, and the Reflection will be such as is required for all the Experiments hereafter mention'd.

In

In order to have the Prifm move freely on its Axis, and ftop any where, I fix'd each End of it into a triangular Collar of Tin, from the End of which came a Wire, which was the Axis of the Prifm produc'd; and fo I laid it on two wooden Pillars, with a Notch on the Top to receive the Wires, and fix'd it to a fmall Board juft broad enough to ftand faft, See *Fig.* 14.

Exp. V. I took the Prifm *C D*, and through it look'd at the coloured *Spectrum R P*, which appear'd then round and white as at *S*, juft as if it had been the Sun's Light received on a Paper from the Hole *H*, and feen with the naked Eye. In this Cafe the Prifm *C D* muft be held *in directum* with *A B*, and the refracting Angles in the two Prifms muft be equal. This *Spectrum* appearing white but juft in one Point, is not fo readily found; but the beft way is to look through the fame Prifm *A B* which makes the *Spectrum*, which may eafily be done if it be pretty long, and then *R P* will be feen white and round, and as at *S*, as if coming directly from *H*. *See* Fig. 15.

Exp. VI. I held a broad *Lens L l*, ground to a Radius of $2\frac{1}{2}$ Feet, in fuch manner that the whole colour'd *Spectrum* fell upon it; and after Refraction all the Colours appear'd to converge, if receiv'd on a Paper at *pp*; but when the Paper was held in the Focus at *F* in the Pofition *π F π*, the *Spectrum* was round and perfectly white by the Union of all the colour'd Rays. If the Paper was held at Π Π, the Colours appear'd to diverge from each other, but then the Red was uppermoft, which before us'd to be the loweft, and fo on in an inverted Order.

I try'd the fame Experiment with a *Lens* of one Foot Radius, with one of 9 Inches, and with another of 7, and the Succefs was the fame. See the 16th Figure, where the *R*, *O*, *Y*, *G*, *B*, *P*, *V*, exprefs the Colours.

N. B. Care muft be taken that the very end of the Red, and the Extremity of the Violet be taken in by the *Lens*; otherwife the *Spectrum* will not be perfectly white at the Glafs's Focus.

There is no fix'd Diftance of the Prifm from the Lens, but it ought to be brought fo near the Prifm that the two Ends of the *Spectrum* may fall nearer the Axis of the Lens than the Edges of the Lens; becaufe there the Refraction is not fo regular.

Behind the Lens *L*, which made the Colours converge into White at the diftinct Bafe or Focus *F*, I plac'd the Lens *l*, which made the White be at *f* the diftinct Bafe of the two Glaffes combin'd; and the Experiment fucceeded as before. Fig. 17.

When the Paper was held in the Focus of the Lens, fo as to receive the white Image of the colour'd *Spectrum* projected by the Lens; if with a Card I intercepted the red Ray, the white appear'd ting'd with Purple, and if I intercepted the Violet or purple Ray, or both, the white appear'd ting'd with red; and if the red was intercepted at the fame time, the *Spectrum* appear'd to be a mixture of yellow, green and blue. If any fingle colour was fuffer'd to fall upon the Lens, the reft being intercepted, that colour wou'd continue the fame; only it wou'd be more intenfe in the Focus of the Lens.

Exp.

Exp. VII. I took a Board (*Fig.* 18.) *q b s* which ſtood reclining on a Prop *t*, having an Hole of a Quarter of an Inch Diameter at *b*, and behind it a Priſm *B* ſupported on two Props, as above-mention'd, ſo as to turn eaſily about its Axis; and having ſet this Board on the Ground with the Priſm behind it at *B*; by turning the Priſm *AC* about its Axis, I firſt made the red Ray of the colour'd *Spectrum* paſs through the Hole *b*, and fall obliquely upon the ſecond Priſm *B*. This Ray after its Refraction in paſſing through the ſecond Priſm, was carried up to the Ceiling of the Room at the place mark'd *R*: then I made the purple Ray fall upon the Board, and paſs through the Hole *b*, as the red had done before; and after Refraction through the Priſm *B* it was carried up to the Ceiling at *P*. And the green Ray being afterwards made to paſs the ſecond Priſm in the ſame manner, went up to *G*: and ſo of all the intermediate Rays, which were by this ſecond Refraction thrown to the intermediate places on the Ceiling between *R* and *P*.

Care is to be taken that the ſecond Priſm be plac'd oblique to the Rays which come through the Hole *b*; leaſt they be reflected, as they wou'd be, if the Board being in the Poſition *QS*, and the ſecond Priſm in the Poſition *LNM*, the Ray from the firſt Priſm be *ρ b*: for then it will be reflected upwards to *σ* inſtead of being refracted (*Fig.* 19.) Neither muſt the Plane of Immerſion be too oblique, leaſt the Incident Ray be reflected downwards by it, as the Ray *R b* is by the Priſm *B* thrown to *E*, in *Fig.* 20. Several have confeſs'd to me that they at firſt us'd to fail in this Experiment, for want of ſetting the ſecond Priſm in a due Inclination.

Tho' the Colours by the ſecond Refraction on the Ceiling appear'd unchang'd, when ſeen by the naked Eye, yet if view'd through a Priſm, they afforded new Colours (except ſome part of the red, and ſome part of the Violet) which was owing to their not being fully ſeparated; for which reaſon I made the following Experiment, to prove that if the Colours be well ſeparated, they are truly homogeneal and unchangeable.

N. B. When the Priſms are good, and no Clouds are near the Sun, the Extremity of the red or Violet will afford unmix'd Colours in this Experiment; otherwiſe not.

Exp. VIII. Having made a Hole in the Window-Shut 2 Inches wide (*Fig.* 21.) I applied to it a Tin Plate, which ſliding up and down hid all this Hole in the Wood, and only tranſmitted a ſmall Beam through it's own Hole *H*, whoſe Diameter was $= \frac{1}{16}$ Inch. This Beam, by means of the Looking Glaſs *L*, plac'd on the Board of the Window *XW*, I reflected horizontally to the other end of the Room. But to correct the Irregularity of the Reflection of the Looking-Glaſs, I made uſe of the Frame of Paſt-Board *Pp*, which had an Hole in it *b* of $\frac{1}{16}$ Inch likewiſe; and placing it at *Pp* I ſuffer'd ſome of the reflected Beams to paſs through it, ſo as to fall upon the Lens *FE* (convex on both Sides, and ground to a Radius of $4\frac{1}{2}$ Feet) at the Diſtance of 9 Feet, ſo that the Image of the Hole *b* was projected to *f* on the other ſide of the Glaſs, at the Di-

ſtance

ſtance of 9 Feet more. Juſt behind the Lens, which by a Screw in the Stand *S* might be rais'd or let down, ſo as always to receive the Beam along its Axis, I plac'd a Priſm *A* (upright on one of its Ends and eaſily moveable about its Axis, by reaſon of its Wire turning freely in an Hole in the ſolid piece of Wood *T*, which ſtood on another Stand behind the Lens) as near as I cou'd to the Lens *E F*, ſo that the Image of *b* inſtead of being round, white, and projected to *f*, was caſt ſidewiſe on a white Paper ſtretch'd on a Frame, and appear'd colour'd, and 30 or 40 times its Breadth, as at *M N*. The Colours in this Caſe were very vivid and well ſeparated, only the Violet had ſome pale Light darting from its End, upon account of ſome Veins in the Priſm *A*, and the Light not coming directly from the Sun, but reflected; which ought not to have been, if the Sun had been low enough to have thrown the Rays a good way into the Room without the help of a Looking Glaſs.

To ſhew that the Colours in this *Spectrum* were ſimple and homogeneal Lights, I made the following Experiments.

Exp. IX. Having made an Hole *b* in the Paper which receiv'd the colour'd *Spectrum*, I ſuffer'd the red Light to paſs; which being refracted by a ſecond Priſm fell upon another Paper at *T*, where it appear'd ſtill red whether ſeen with the naked Eye or Priſms of different refracting Angles. To the Eye which ſaw it through the Priſm *V*, it appear'd indeed lower as at *t*, but red, round and unchang'd. I made the Experiment upon all the Colours, which by this means appear'd to be ſimple and homogeneal. See *Fig.* 22. where the ſame Letters denote the Lens, Priſm and firſt Paper.

Through the ſame Lens and Priſm the *Spectrum* was made to fall on a Book; then through the Priſm *F* it appear'd unchang'd; and the Letters in the Book, which croſs'd the *Spectrum*, were as diſtinct as when ſeen with the naked Eye. See *Fig.* 23.

N. B. The Axis of the Priſm *F* ought to be perpendicular to the long Axis of the *Spectrum s m* thrown on the Book, which will appear as at σ μ; and the Priſm in the Poſition repreſented at *F*, with its flat Side towards the Noſe: for that is the moſt convenient Poſition for looking at the *Spectrum* in theſe Experiments.

I ſuffer'd the purple Ray only to paſs through the Hole *b*, and fall upon a Book at *P*, the Letters of which appear'd at π, and were as diſtinct through the Priſm *Q* as when ſeen with the naked Eye: and I had the ſame Succeſs with all the other Rays. See *Fig.* 24.

But if a Sun-Beam as *r* comes through the Hole *H* directly upon the Book at *W*, an Eye looking at it through a Priſm at *X* will ſee this Beam at *Y* oblong and colour'd; and the Letters on which it falls, confus'd. See *Fig.* 24.

N. B. The Lens ought to be very good, without Veins or Blebs, and ground to no leſs a Radius than I mention'd in the Experiment; though a Radius of a Foot or two longer is not amiſs. The Priſm ought to be of the ſame Glaſs as the Object-Glaſſes of Teleſcopes, the white Glaſs, of which

which Prifms are ufually made, being commonly full of Veins. And the Room in thefe laft Experiments ought to be very dark.

A few Days after, having got very good Prifms made for the purpofe of the above-mention'd Glafs, I made all the Experiments over again with better Succefs; and had the *Spectrum* very regularly terminated, without any pale Light darting from the ends of it.

Sir Ifaac Newton's Doctrine of the different Refrangibility of the Rays of Light confirm'd, by the fame, ib. p. 448.

2. After the *Experimentum Crucis* made by two Prifms, I fhou'd not give the following Experiment, but that it is fo eafy to be made, that by it thofe who want the *Apparatus* (or are unwilling to be at the pains) to make the *Experimentum Crucis*, may at any time fatisfy themfelves of the Truth of the fore-mention'd Doctrine.

Let the Candle A be fet before the Bar of a Chimney Looking-Glafs, fuch as is reprefented by HH (*Fig.* 25.) which is a piece of Looking-Glafs Plate confifting of four Planes, feen in the Section of it $\alpha\, f\, d\, \beta$, *viz.* $d\, \beta$ which is quick-filver'd behind, $f\, \alpha$ a Plane parallel to it, $f d$ one of the Side-planes bezell'd towards $d\, \beta$, or inclin'd to it in an Angle of about 40 Degrees (tho' from 30 to 40 will do, but the greater the Angle the better, if it does not exceed 45°). $\alpha\, \beta$ the other Side Plane inclin'd in the fame Angle to $\beta\, d$.

The Rays of the Candle which come from A to γ fall obliquely on the Plane $\alpha\, \beta$, fo that inftead of going on to a, they are by Refraction made to incline more towards the Perpendicular $p\; p$, namely to go on in the Line $\gamma\, c$ and then are reflected from the Point c on the quick-filver'd Surface, in the Direction $c\, \kappa$, fo as to make the Angle $\kappa\, c\, d = \gamma\, c\, \beta$. Now as the Rays which would go to κ, if not refracted, emerge obliquely from the Plain $\alpha\, \beta$, they leave the Direction $c\; \kappa$, and decline from the perpendicular $\pi\; \pi$, and, being differently refracted, open into four differently colour'd Rays; *viz.* $b\, R$ a red Ray, $b\, TO$ a Ray made up of Orange and Yellow; $b\, G\, B$ a Ray made up of Green and Blue or a Sea-Green, and $b\, P$ a purple Ray.

If from the place $E\, e$ you look full upon the point b, the *Spectrum* or Image of the Candle at b will appear doube, abut not mix'd; that is, there will appear a Sea-green Spot and a red Spot, s it were, one upon another; but not fo as to produce a mix'd or intermediate Colour. Then if the right Eye or Eye at E be fhut, there will appear only a green Spot to the Eye at e; if the Eye at e be fhut, the Eye at E will fee only a red Spot.

If you come nearer to b, fo that the Eyes at ϵ 1, ϵ 2 receive the moft and the leaft refrangible Rays, there will be a double *Spectrum*, *viz.* a red and purple one juft touching, or upon one another: and the Phænomenon will anfwer as before. *Fig.* 25.

If keeping both Eyes open, you direct their Axes towards O a point nearer than the ufual place of the compound *Spectrum* S, (*Fig.* 26.) which point is in a Line from the Nofe N to the point S; or in other Words, if you look full at O, or at the End of your Finger held in O; the red and the blue (or purple Spot) will appear to be divided from each other after the manner reprefented at $p\, r$ (in *Fig.* 27) where the red will appear to be on the Right-hand, and the blue on the Left. To

To make plain what is meant by *seeing* the *Spectra p* and *r* whilst we look full at O, I beg leave to explain the Distinction between *looking* and *seeing*; that I may the better shew how this Phænomenon proves that the Sensation of different Colours is caus'd by Rays differently refracted.

Definition I. The *Optic Axis* is a Line which going through the Center of the Convexity of all the Coats and Humours of the Eye, falls upon the middle of the *Retina*, as *α a* or *A a Fig.* 28.

Def. II. To *look* at any point, is to turn both Eyes towards it in such manner, that the *Optic Axes* making an Angle at the said point as *a*, the Rays from *a* may have the *Optic Axis* for their Axis, and (by their Convergence upon the *Retina* after Refraction in the *Eye*) may paint the Image of the said point upon the middle of the *Retina* of each Eye, where the Optic Axis in each Eye falls.

Def III. To *see* without *looking*, is to direct the Optic Axes to some other place than to the point which is then seen; and in such a Case, the Image of the point seen will be projected upon a part of the *Retina* of each Eye, where the Optic Axis does not fall, namely either nearer to the Nose *N* as in (*Fig.* 26.) at the points of the *Retina* mark'd *n n*; or farther from the Nose than the middle of the *Retina*, as at *o o* in *Fig.* 29.

Whatever is *seen*, by being *look'd* at with both Eyes, always appears single, by reason of the Communication between the middle of the *Retina* in one Eye, and the middle of the *Retina* of the other: there being no such Communication between any other part of the *Retina* in one Eye, and the Correspondent part of the *Retina* in the other, when these Correspondent parts are equally distant from the Nose.

There is indeed a Communication between the Nervous Fibres on the Right-side of the Retina *of one Eye, and the nervous Fibres on the Right-side of the* Retina *of the other Eye, and so of those on the Left: but no single Object can be so painted in each Eye, as to have its Image on the Right or Left part of one* Retina *that communicates with the Right or Left part of the other, of the same bigness and at the same time as in the other; because in whatever Position the Object is, it must be nearer to one Eye than to the other, except it be just in a Line from the Nose betwixt the two Eyes streight forward.*

Hence it is that if there be two Candles set before any one, the first at the distance of one Foot, and the second at the distance of two Feet, from the Eyes; he that looks at the second Candle at *B* will see it single, but see the first Candle or the Candle *A* double; one Appearance being in the Line *A D γ*, the other in *o A E*, because it paints it self upon *o o* in the Retina of each Eye, which points are not the middle points, but farther from the Nose than the middles *m m*.

So if *B* be the first Candle, and *C* the second, he that looks at *B* will see *C* double, because it is painted in the *Retina* at the points *n n* nearer the Nose than *m m*; and so will appear to be in the same Position as *p r*. in *Fig.* 27

If *γ ε* be two Candles so dispos'd, *Fig.* 30. that by the Interposition of a perforated Board *F F*, *γ* can paint it self only in the Eye *R*, and *ε* in the

 the

the Eye *L.* Upon making the Optic Axes meet at *B* and to tend towards ρ and γ, ρ and γ will each paint an Image on the middle of the *Retina* of each Eye, by crossing their Rays at *B* : and thus the two Candles will appear to be but one, or rather to be in one Place, upon the account of the Communication of the middle of each *Retina*. But if instead of the Candles, ρ be a piece of red Silk, and γ a piece of green Silk, the same Position of the Eyes will make an Image at *B*, appearing like a red and green Spot together without a Mixture of the Colours. If ρ be a red hot Iron, and γ a Candle of Sulphur, the Phænomenon will be more distinct. If the Optic Axes be turn'd directly towards γ and ρ, as if there was no Board *F F* in the way, there will appear two Holes in the Board, the one having the red hot Iron in it, the other the Candle.

Now if, of the refracted Rays of the Candle in the first Case (*Fig.* 25.) those which diverge from each other, so as to fall into each Eye, cause the same Sensations respectively, as the Rays which come from a red hot Iron and those which come from a blue Candle; it is evident that the Candle in the first Case affords red-making and blue-making Rays after Refraction, and that those Rays are differently refrangible; the red *b R* (*Fig.* 25,) the least refrangible, as declining less from the perpendicular $\pi\pi$; and the Purple as *b P* declining most from the said perpendicular.

The same will (*cæteris paribus*) be found true in the intermediate Rays; and to be certain that the Experiment is as I have related it, the Planes $a f$ and $f d$ of the Bar may be covered with Paper.

Of Microscopes by Mr. Leewenhoeck n. 273. p. 903.

III. It has often happen'd that in the bursting of Glass, or of Sparks flying out of Wood-Coals (my Eye being a little too near) small particles of the Glass or Fire came into my Eye, and caused it to smart, upon which I used to arm my Eyes with Spectacles, against the like Accidents for the future.

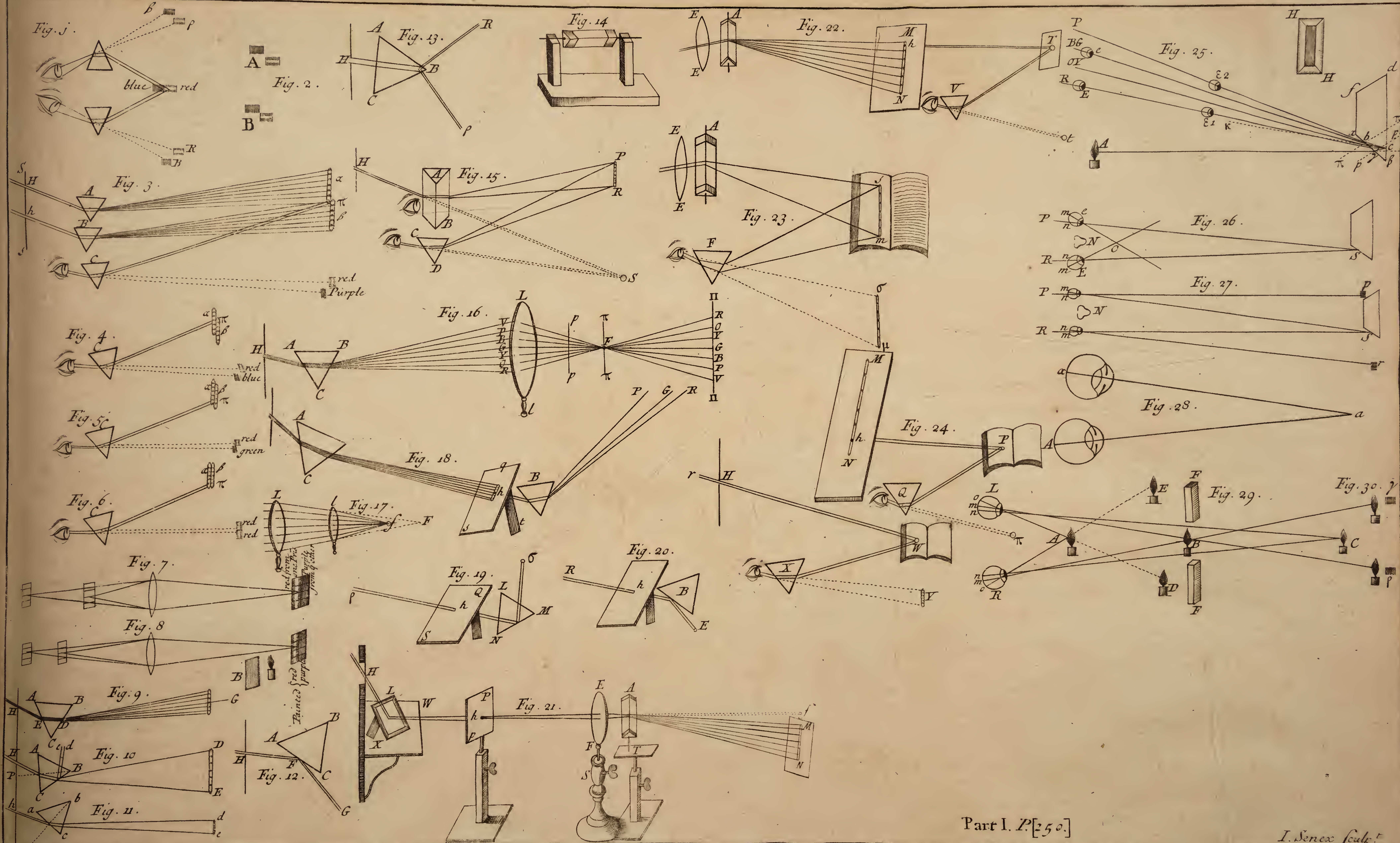
Now I observe when I look through one of my Glasses by Candle-Light, that near the upper part of my Eye in the *Tunica cornea*, there appears a fine small Flame of a Candle inversed, no bigger than the common Letters we use in writing, and opposite to it appear two round clear Lights, so very small that the Flame of the Candle is not be perceived therein.

From hence I conclude, that the *Tunica Cornea* of the Eye, by the Wound it receiv'd from those Particles of Glass, has lost something of its Roundness, which occasion'd those Appearances, and that when the wounded part stood just before the Sight it obstructed it, &c.

I observed also, that in several places of the *Tunica Cornea*, there lay Veins no longer than the Breadth of 2 or 3 Hairs put together, wherein I could perceive the Globules of Blood very distinctly; these Vessels were so small, that they could contain but one Globule in the Diameter of them, and the Blood had no manner of Motion.

The Vessels seem to me to be broken off from other Blood-Vessels, and when the Particles of Blood are a little crowded together, or when one of those longish Vessels are somewhat bended, it appears to the Sight, as if one saw a thick Cloud.

With



Part I. P.[250.]

I. Senex sculp.t

With this Cloud of ſmall Particles, the Eyes are ſurrounded but more one time than another, for ſome are diſperſt, and then others ariſe in their places; when we view theſe Veſſels with their Globules of Blood, thro' one of my Glaſſes againſt a Candle or other ſtrong Light, they ſeem to be in a continual Motion, whereby thoſe Particles that are in the *Tunica Cornea*, be their Motion never ſo ſmall, ſeem to us, as if they were moving in the Air, but by ſtrict Examination we ſhall find, that they are one and the ſame Particles, which ſometime appear Aſcending, and at other times deſcending.

Beſides the aforemention'd Blood Veſſels, we find in the *Tunica Cornea* round Particles that lye ſcatter'd about, which Particles I judge to be Globules of Blood; an ignorant perſon ſeeing theſe Particles in continual Motion, for as I ſaid before, ſometimes they appear aſcending, and ſometimes deſcending, would be apt to ſay, that theſe Particles were not in his Eye, but in the Air before the Glaſs; and perhaps too, that the deſcending Particles were the Influxes of the Star, and thoſe that ſeem'd aſcending the Exhalations of the Earth, or other Bodies.

It has often befallen too, when I lookt againſt a ſtrong Light, through my Microſcopes, that I ſaw an infinite number of exceeding ſmall Particles, that had all a glittering Motion.

I never imagined that theſe Particles were in the Air, as others would, but rather that they were in the Chriſtalline humor of the Eye between the *Tunica Cornea* and the *Criſtalina*, the Motion of which ſmall particles is occaſioned by preſſing the *Tunica Cornea*, when we ſhut our Eye cloſe together.

IV. I had not an opportunity of examining Mr. *Leeuwenhoeck*'s Glaſſes particularly, which is a Favour he allows to none; therefore I am not capable at this diſtance to deſcribe either their Make or Uſe, any further than that to me they appear'd to be Spherules lodg'd between two Plates of Gold or Braſs, in a hole whoſe Diameter might not be bigger than that of a ſmall Pins head, and the Objects I ſaw through them were pretty and diverting; but ſtill their Make and Truth are unknown.

Of the manner of making Microſcopes, by Dr. Arch. Adams, n. 325. p. 24.

Mr. *Butterfield* is very curious in melting his Glaſs, but I ſuppoſe unſucceſsful in caſting his Spheres; for beſides that a ſufficient quantity of beaten Glaſs cannot ſtick to the moiſtned point of a fine Needle; ſo neither can it run equally, hold the Needle how you will, nor the *Globule* when run ſtick to the Needle, but muſt unavoidably drop; and whereſoever it happens to fall, it muſt in that almoſt liquid State receive Impreſſions ſufficient to ſpoil the Figure of a Sphere.

Mr. *Gray* has ſhown the defect of his Method, which he us'd to recover by grinding and poliſhing his Glaſſes on a Braſs Plane, and ſo reduce 'em to Hemiſpherules; but how far ſhort poliſh'd Glaſſes (I ſpeak of ſmall ones) come of thoſe which are caſt, I leave to any one to judge who has ſeen both. His Water and Quickſilver Microſcopes I never ſaw, ſo can ſay little to them.

After what manner Mr. *Wilson*'s Glasses are made I know not, but sure his greatest Magnifyers are ill plac'd, they being sunk to so great a distance from the Eye, the Object cannot appear to that Advantage it otherwise would; if therefore instead of a hollow Cap he would contrive a plain Plate of any Metal for the Reception of the Glass, then the Eye and the Object might come to their due distance; neither ought there to be any Calx or Glass between the Object and the Spherule, when we use the greatest Magnifyers, because if the *Focus* of a Sphere be upon the Extremity of its Circumference, any small distance from that must spoil the Truth of the Objects appearance.

I cannot say, that the Glasses I have made are without fault, but I think they magnify more than any I have yet seen; and were they plac'd to the best advantage, they would magnify much more than they do: They are made thus.

I take a piece of fine Window Glass, and I rase it with a Diamond into as many lengths as I think needful, not exceeding an eighth of an Inch in breadth; then holding one of these lengths between the Fore-Finger and Thumb of each Hand, over a very fine Flame, until the Glass begin to soften, I draw it out till it be as fine as a Hair and break: Then inuring each of the ends into the purest part of the Flame, I have two Spheres presently, which I can make larger or less as I please; if they stay long in the Flame, they'll have spots, so I draw 'em out presently after they turn round. As for the Stem, I break it off as near the Ball as I can, and lodging the remainder of this Stem between the Plates, and by drilling the Hole exactly round, all this Protuberance is bury'd between the Plates, and the Microscope performs to Admiration; insomuch, that the same Thread of very fine Muslin appeared 3 or 4 times bigger in one of these, than it did in the first or second of Mr. *Wilson*'s. I thought I saw Animals in fine old Brandy, but they were so nimble in their Motion, that I can give no particular Description of them.

A Way for Myopes to use Telescopes without Eye-Glasses, by Dr. Desaguliers, *n.* 361. p. 1017.

V. *Lemma* 1. What is requir'd of a Telescope is to give large, and distinct Vision; that is, to make the Object (as in *Galilæo*'s Telescope) or its Image (as in the Telescopes made up of Convex *Lentes*) appear under a great Angle, and to have all the Rays of those Pencils that enter the Eye, meet in a point upon the *Retina* of the Eye, on their respective Axes.

The first Figure represents the Combination of two Convex *Lentes* for the Astronomical or inverting Telescope; where the above-mentioned Requisites are obtain'd. *A B* is the Object suppos'd at a vast distance from the Objective *Lens L L*, so that Rays coming from the Extremity *A* of the Object, will fall upon the *Lens L L*, in the same manner as if they were parallel to their Axis *A X*; and after passing the Glass unite at *a*, where they project the Image of the point *A*; from whence diverging, they fall on the Eye Glass *l l*, and having pass'd through it, go on parallel to each other, and enter the *Cornea* of a common Eye *E*, which unites those parallel Rays upon its *Retina R R R* at *α*, where the Image

Image of *a* is projected: The same may be said of the Rays that come from *B*, and after their several Refractions through the two Glasses and the Coats and Humours of the Eye, meet upon the *Retina* at *β*, where they project the distinct Image of the Point *b*. The Rays that come from all the Points of the Object *A B* being affected after the same manner, give a distinct Image of those points upon the *Retina*, and therefore the Object doth appear distinct.

The Object will also appear magnified in the same proportion as the Angle *l C l* = to *b M a* (under which its Image is seen,) is greater than the Angle *A C B* under which the Object *A B* would be seen by the naked Eye; as is more at large demonstrated by Dioptrical Writers.

Lemma 2. If parallel Rays fall upon the *Cornea* of a *Myops*, or short-sighted Person, they will unite in the Eye, before they come to the *Retina*, the farther from it the more Convex the Eye is; but if the Rays which fall upon the *Cornea* diverge in proportion to the too great Convexity of the Eye, as from *D*, such Rays will be so refracted by the Coats and Humours of the Eye as to meet in one point upon the *Retina* *R R*, see *Fig.* 2 and 3. Where I have in the Scheme neglected the Refraction of the Rays passing out of the *Crystalline K* into the *Vitreous* Humour *V*, as I do in the other Cases.

This *Lemma* is also demonstrated by Dioptrical Writers.

Lemma 3. If two Pencils of Rays (in each whereof all the Rays are parallel to the Axis, as *a C*) fall upon different parts of the *Cornea*, at the greatest distance from one another that can be allow'd for those Rays to enter the Pupil *P P*, their *Axes* will, after entring the *Aqueous* Humour, converge, and meet either in the Vitreous or Crystalline Humour, according to the Convexity of the *Cornea* through which they pass'd, and diverge again before they come to the *Retina*; the Rays of each Pencil converging upon their respective Axes, to the place where the said Axes cross one another, *Fig.* 4.

Demonstration. The Axes *a Ca*, *α Cα*, falling obliquely upon the *Cornea* at *C C*: and entring from *Air* into the *Aqueous* Humour, will be refracted towards the Perpendicular to *K*: where striking more directly upon the *Crystalline*, they will go on to *α*, *a*, upon the *Retina R R R R*, decussating at *V* within the *Vitreous* Humour. The other Rays *r*, *r*; *ρ*, *ρ*, after their Refraction in the *Aqueous* Humour, fall more obliquely on the *Crystalline*, and therefore are refracted again so as to meet at *V*, where the Axes also meet, and thence go on to the *Retina R R R R*, *Fig.* 4.

Lemma 4. But if the Axes of the above-mention'd Pencils are Parallel, the Rays that accompany them diverging from a Point so near the Eye, that the divergence may be proportionable to the too great Convexity of the Eye; then only the Axes will meet in the Eye before they come to the *Retina* (by *Lemma* 3.) but the other Rays will not unite upon their respective Axes, till they come to the *Retina*, (by *Lemma* 2.)

Prop. I suppose the Eye of the *Myops* so Convex that he can see no farther than a common Eye, with the Eye-Glass of a Telescope before

it;

it: then the Eye of the *Myops* being in the place of the Eye-Glaſs, will receive the Rays diverging from the ſeveral points of the Image (projected by the Object-Glaſs in its *Focus*,) in ſuch manner, that they will after their ſeveral Refractions meet in reſpective Points on the *Retina*; and the Axes of the Pencils which come from the Extremities of the Object, will, in the Eye, make the Angle *B V A* = to *b c a*, under which the Image *a b* is ſeen, by *Lemma* 4. The *Cornea* and *Aqueous* Humour here ſupply the place of the Eye-Glaſs, and the *Cryſtalline* and *Vitreous* Humours that of a common Eye, See the 5th *Fig.* wherein *R* is the *Retina*, *V* the *Vitreous* Humour, and *K K* the *Cryſtalline* Humour; and the Image *b a* is ſuppos'd to be brought down from the *firſt Fig.* which repreſents the *Aſtronomick Teleſcope*: the too great Convexity of the Eye here being in the place of an Eye-Glaſs.

An Objection may be made to this, *viz.* that *P P* the Pupil of the Eye being ſmall, will take in but a very little Image, or a ſmall part of the Object: But then if the Eye be mov'd ſucceſſively to all the parts of the Space where the Eye-Glaſs was, it can take any part of the Object; and if the Object-Glaſs be large, which may more eaſily be made than a large Eye-Glaſs, and the Tube a Foot wide or wider, as much may ſucceſſively be taken in, as if an Eye-Glaſs might be had of a Foot Diameter. A little practice may make any *Myops* ſo ready, as to keep an Object when once found, though the place where he ſtands be ſhaken. It would not be amiſs to hold a *Lens* in one's Hand (for an Eye-Glaſs) to find the Object at firſt, till Cuſtom has made it eaſy without it: when once the Object is found, it may be eaſily kept.

An Eye more ſhort-ſighted than I have ſuppos'd, will perform the Office of a more Convex Eye-Glaſs, being brought nearer to the diſtinct Baſe of the Object Glaſs; and an Eye leſs Convex, the Office of a leſs Convex Eye-Glaſs: but with this difference, the more Convex the Eye is, the eaſier may the Object be found, and the larger and more lucid will it appear.

I have ſeen *Saturn*'s Ring very plain with an Object Glaſs of little more than 6 Foot *Radius*, without an Eye-Glaſs.

I have alſo found out a new way for the *Presbytæ* to make uſe of an Object-Glaſs, by placing the Eye nearer the *Lens* than its *Focus*, by ſo much as their Eye is flatter than a common Eye, ſo as to make, as it were, the Teleſcope of *Galilæo*: the flat Eye ſerving as a common Eye arm'd with a Concave *Lens*. I have ſo fixt the Teleſcope as to make a *Presbyta* read at a great Diſtance a ſmall Print.

If this Experiment be made at Sea with a very large Tube, big enough to put in the Head, and move it about, and the Object Glaſs large, it may poſſibly enable one to obſerve the Eclipſes of *Jupiter*'s Satellits.

Of the Invention of Teleſcopick Sights, by Mr. Derham, *n.* 352 *p.* 603.

VI. In *Monſieur de la Hire's* firſt Part of his *Tabulæ Aſtron.* publiſh'd in 1687. I find theſe Words, *Paucis abhinc annis D. Picard inſignis Aſtronomus,*

atque

Fig: I.

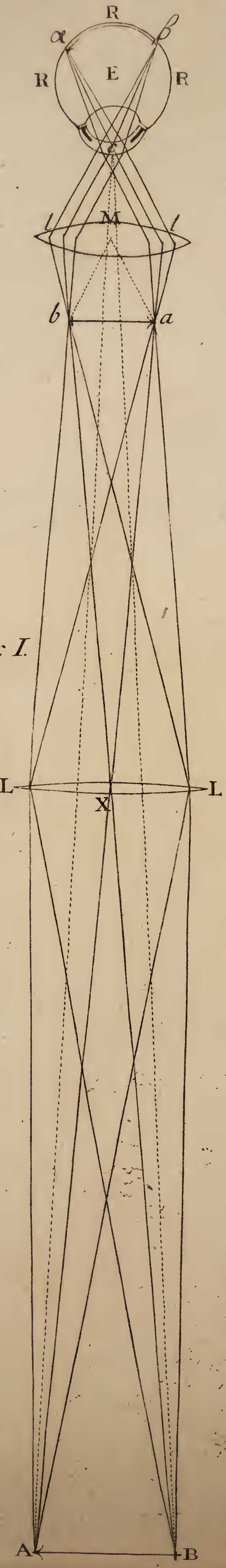


Fig: II.

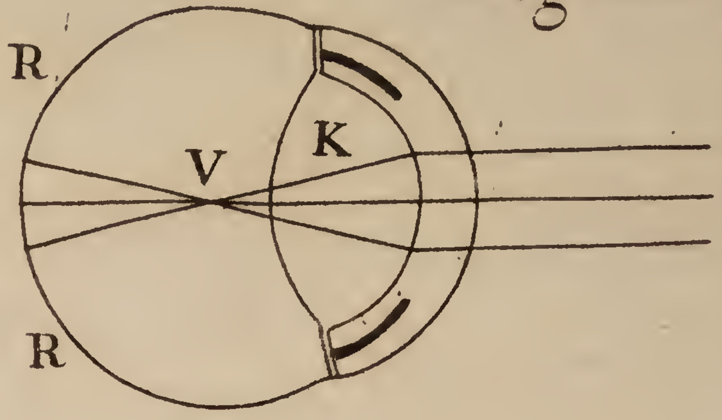


Fig: III.

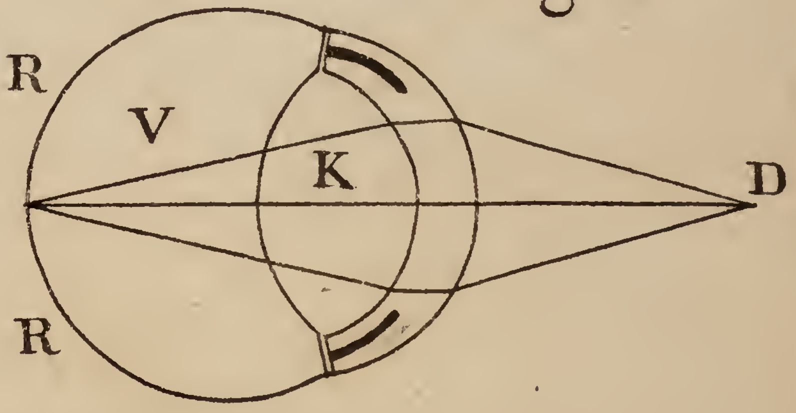


Fig: IV.

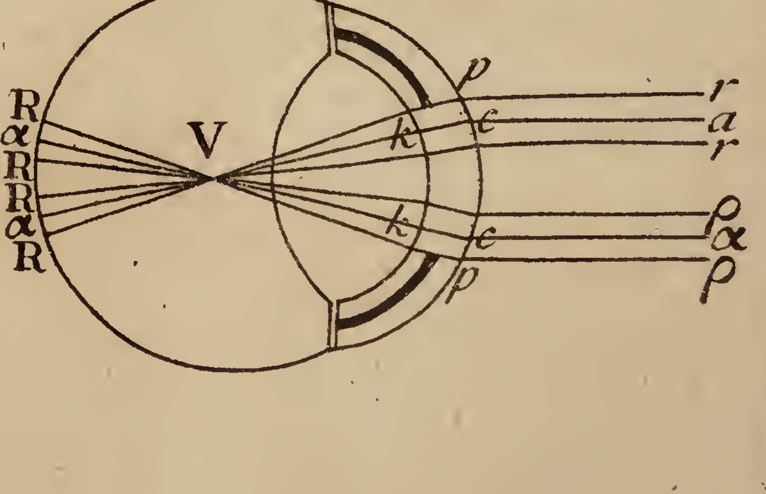
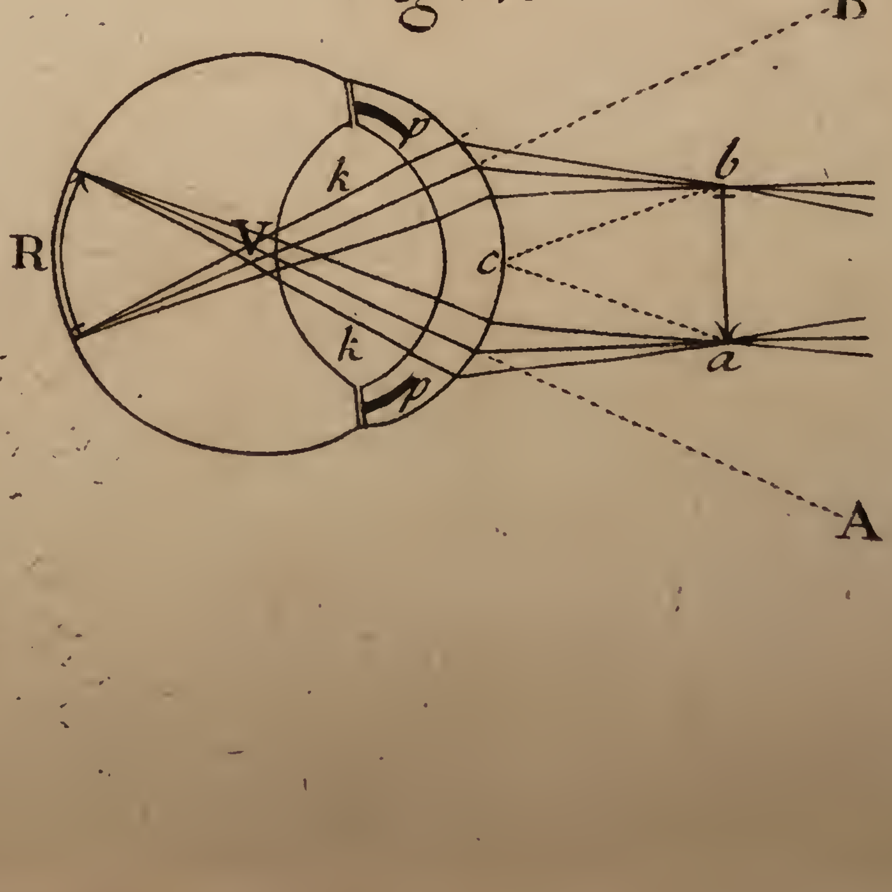


Fig: V.



atque in eadem Academia [*Regia Scientiarum*] *Socius, Dioptrarum crenas ab inſtrumentis ſuſtulit, eorumque loco ſubſtituit Teleſcopia; quæ res Presbytis & Myopibus*, &c. In which it is not indeed expreſly ſaid that Mr. *Picard* was the Inventor of this way, but only that he applied Teleſcopes. But by reaſon it implies that it was Mr. *Picard*'s Invention, I think my ſelf bound to do Mr. *Gaſcoigne* the Juſtice to aſſert his Invention to him.

That Mr. *Gaſcoigne* was the firſt that applied Teleſcopick Sights to Aſtronomical Inſtruments, appears from a Letter of his now in my Hands, to Mr. *Crabtrie, Jan.* 25. 164$\frac{0}{1}$. wherein he ſaith, *If here,* (that is in the Diſtinct-Baſe) *you place the Scale that meaſures—, or if here an Hair be ſet, that it appear perfectly through the Glaſs—, you may uſe it in a Quadrant, for the finding of the Altitude of the leaſt Star viſible by the Perſpective wherein it is. If the Night be ſo dark, that the Hair or Pointers of the Scale be not to be ſeen, I place a Candle in a Lanthorn, ſo as it caſt Light ſufficient into the Glaſs; which I find very helpful when the Moon appeareth not, or it is not otherwiſe light enough.*

In another Letter, dated on *Chriſtmas-Eve*, 1641. he tells Mr. *Crabtrie* how he had applied his Teleſcopick Sights to a Sextant. Saith he, *Mr.* Horrox's *Theory of the Moon I ſhall be ſhortly furniſhed to try. For I am fitting my Sextant for all manner of Obſervations, by two Perſpicills with Threads. And alſo I am conſulting my Workman about the making of Wheels like* β, γ, δ, ε, *of* † *Diagr.* 3, *to uſe two Glaſſes like a Sector. If I once have my Tools in readineſs to my Deſire, I ſhall uſe them every Night. I have fitted my Sextant by the Help of the Cane, two Glaſſes in it, and a Thread, ſo as to be a pleaſant Inſtrument, could Wood and a Country-Joiner or Workman pleaſe me.*

† *This Diagram is wanting in the Letter.*

In another Letter, (the Date of which is worn out, but is, in Mr. *Crabtrie*'s Hand, called his 10th Letter to him) he ſaith, *I have given order for an Iron Quadrant of Five Foot, which will give me the* 1000*th part of one Degree, which ſhall be furniſhed like my firſt Scale; only my Workman is ſo* * *throng for my Father, that I fear it will not be finiſhed before the Eclipſe. I have cauſed a very ſtrong Ruler to be exactly made, and intend to fit it with Curſors of Iron, with Glaſſes in them and a Thread, for my Sextant.*

* A Yorkſhire *Phraſe for fully employ'd.*

To theſe I could have added many other Paſſages of the like Nature: but theſe may be ſufficient, to ſhew that Mr *Gaſcoigne*, as early as 1640, made uſe of Teleſcopes on Quadrants and Sextants, as well as in his Invention of the Micrometer.

In Mr. *Crabtrie*'s ſecond Letter, *October* 30. 1640. after a Demonſtration that the Solar Spots are not Planets at a Diſtance from the Sun, but ſomething adhering to, or very near the Sun's Body: He ſaith, *Something I am ſure you were telling me concerning a way of obſerving the Places of the Planets by your Glaſſes. But I have not a little lamented that my Time cut me ſo ſhort, when I was with you, that I could not more fully ruminate and digeſt thoſe ſtrange Inventions which you ſhewed me, and told*

me

me of. You told me (as I remember) you doubted not in time to be able to make Observations to Seconds. I cannot but admire it and yet, by what I saw, believe it: but long to have some farther Hints of your Conceit for that Purpose. One Means, I think, you told me was, by a single Glass in a Cane, upon the Index of your Sextant, by which (as I remember) you find the exact Point of the Sun's Rays. But the way how, I have quite forgotten, and much desire. Your Device for the exact Division of a Quadrant, by dividing 1½ Degrees into 10 Parts, I did then understand, but do not now fully remember. If it might not be too much Trouble to you, I should intreat you to give me such a Paper-Demonstration thereof as you shewed me, and two or three Lines plainly of the Use thereof, how to find those small Parts.

I cannot conceal how much I am transported beyond my self with the Remembrance (of that little I do remember) of those admirable Inventions which you shewed me when I was with you. I should not have believed the World could have afforded such exquisite Rarities, and I know not how to stint my longing Desires, without some further Taste of these selected Dainties. Mr. *Crabtrie* in his Letter *of Dec.* 28. 1640. hath these Words, *My Friend Mr.* Horrox *professeth, that little Touch I gave him of your Inventions, hath ravished his Mind quite from it self, and left him in an Extasie between Admiration and Amazement.*

Thus, among many, I have related some of the Passages of Mr. *Gascoigne*'s and Mr. *Crabtrie*'s Letters relating to *Telescopick Sights.* From whence it is very manifest, that long before the *French* Gentleman's Claims, our Countryman Mr. *Gascoigne* had made use of those Sights in his Astronomical Instruments; particularly in two or more Sorts of *Micrometers* (as I plainly find) and in his *Quadrant* and *Sextant.* He was scarce 20 Years of Age when he held these Correspondencies with Mr. *Crabtrie.* And at the Age of 23. he was killed at *Marston-Moor-Battle*, on *July* 2. 1644. fighting for King *Charles* I.

VII. *A Paper Omitted.*

n. 281. *p.* 1241. The Description and Manner of using a late invented Set of small Pocket-Microscopes, made by Mr. *James Wilson*; which with great ease are apply'd in viewing Opake, Transparent and Liquid Objects: as the *Farina* of the Flowers of Plants, *&c.* The Circulation of the Blood in Living Creatures, *&c.* The *Animalcula* in *Semine*, &c.

CHAP.

CHAP. III.

Aſtronomy.

I. *A new way to draw the* Meridian *Line, by Mr.* Stephen Gray, *n.* 268. *p.* 639.

I Have thought of a new contriv'd Inſtrument for drawing the Meridian Line, which is eaſy in its Uſe, and ſufficiently exact. Take the *Gnomon* of an Horizontal Dial for the Latitude of the Place, and to the Hypotenuſa fix two Sights, whoſe Centers may be parallel to the ſame: let the Eye-ſight be a ſmall Hole, but the others Diameter muſt be equal to the Tangent of the double Diſtance of the North Star, from the Pole; the Diſtance of the Sights being made Radius, let the Stile be rivetted to the end of a ſtreight Ruler; then when you would make uſe of it, lay the Ruler on an Horizontal Plane, ſo that the end to which the Stile is fixt may over hang, then look through the Eye-ſight, moving the Inſtrument till the North Star appears to touch the Circumference of the whole in the other Sight, on the ſame Hand with the Girdle of *Caſſiopeia*, or on the oppoſite Side to that whereon the Star in the Great *Bear's* Rump is at that time, then draw a Line by the Edge of the Ruler, and it will be a true Meridian Line.

Of Drawing one by the Pole-Star, by the ſame, n. 270. *p.* 815.

II. Let there be taken a Teleſcope of about 16 Foot, or longer if you pleaſe, in the Plane of its Focus place a Ring of Braſs at right Angles to the Axis of the Glaſs, the Diameter of the inward Circle equal to the double Tangent of the Pole-Star from the Pole, the focal length of the Object Glaſs being made Radius, as was ſaid in the Deſcription of the Meridian Inſtrument; let the Ring be divided into 24 Hours, with their Minutes number'd from the Right-hand towards the Left, as in our common Nocturnals; the Eye Glaſs muſt be equal in its Diameter to the horary Ring: But becauſe this may be too chargeable, this Contrivance will ſerve; the Eye Glaſs muſt lye in a broad Index towards one end, this is to turn on a Center Pin, that lies in the Center of the Glaſs, and conſequently over the Center of the horary Ring, from which it muſt be equal to the diſtance of the focus of the Eye Glaſs, then let the Tube be elevated to the height of the Pole, and directed to the Pole Star, till by turning the Index through the Eye Glaſs, you perceive the Star to touch the Horary Ring on that ſide the Star in the great Bears Rump lies, or on the oppoſite to that in the Hip of *Caſſiopeia*; whereas had not the Glaſs inverted the Object, it ſhould have been the contrary; then bring one of the 12s to be in a perpendicular to the other by a Plum Line, ſo will the Star ſtand at its Horary diſtance from the Meridian; or if the Latitude of the Place be unknown, by the Right Aſcenſion of the Sun and Star the Line of its coming to the Meridian will be eaſily obtained, and then the Hour of the Night found, will as eaſily

easily give the Stars horary distance from the Meridian; then elevate the Tube towards the Star, bringing the Meridian, or 12 and 12 into the plain of the perpendicular, turn the Glass about till you see the Pole Star stand at its horary distance from the Meridian; so will the Instrument when fixed, shew the horary distance throughout the whole Day, or as long as it remains in this Position, by the apparent Motion of the Star in the Ring. The best time to fix the Instrument, will be when this, or any of the other two Stars above-mentioned, are about 6 Hours from the Meridian. The Latitude of the place is now given with the utmost Preciseness, for the Axis of the Glass lies now in the Axis of the World, and if one of the sides of the Tube be parallel thereto, as it ought to be at the upper end, hang a Line and Plummet from the point of the Suspension; find another point equal in distance to the length of the Line, or a knot towards the lower end, the distance from this knot to the former point will be but the Chord of the Latitude, and if from the same edge of the Index, another Line and Plummet be hung towards the lower end of the Tube, these two Lines, when at rest, will be in the plain of the Meridian.

This Instrument may be made to shew the Hour with as much Facility, as a Clock or Sun-Dial, if the Horary Ring be made to move within a larger fixed one, and the outward Circle of the former be divided into the Days of the Month, respect being had to the right Ascension of the Sun and Star; then by bringing the 2 opposite Points in the fixed Circle to the Perpendicular, which is done at the fixing the Instrument, move the Circle till the Day of the Month come to any of these, and the Ring is rectify'd for that Day, and if the Air be clear, you will see the Star stand at the true time of the Day or Night.

If when by the annual increase of its Declination, the Pole-Star by moving in a lesser Circle be brought too far from the edge of the Ring, that the exact Hour and Minute cannot well be distinguished; this Inconveniency may be easily remedied by making a lesser Ring, or by extending a fine thread of Silk cross the Ring, till it cuts the Star, and at the same time it gives the Hour, or which will yet make this Instrument commodious for other Purposes, there may be made an Index to move on the Center of the Hour-wheel, which being brought to cut the Star with the edge that proceeds from the Center, it will at the same time cut the Hour, and now we need not be sollicitous about the exact Diameter of the Ring, provided it do but a little exceed the distance of the Pole-Star from the Pole, the focal length of the Glass being made Radius.

Mr. *John Flamsteed*, Math. Reg. and F. R. S. has lately discovered, that there is a Parallaxis of the Earth's annual Orbit at the Pole-Star of about 40 or 45 Seconds, whereby the Diameter of the Stars parallel is greater in *June* than in *December* by about one Minute, two Seconds, which he has evinced from 7 Years successive Observations, whereby the Earths Motion is indubitably demonstrated.

Now

Now if on the edge of this Index there be drawn a Scale of Degrees, Minutes and Seconds, to the Radius of the Glass, we shall not only have a very accurate Instrument for the Hour, but be furnished with one whereby we shall see the truth of the Earth's Motion, confirmed by the access and recess of our Star, towards and from the Pole, according to the Earth's place in the Ecliptick, as that learned Person above-mentioned has discovered; and that not only when the Star transits the Meridian, but in clear Air at any time of the Day; one shall likewise observe that annual increase of the Pole-Stars Declination, caused by the Precession of the Equinox. The Pole-Star may be seen in the Day time with a Telescope of 16 Foot, for with one of this length I saw that Star *April* 26. 1701. from 4 in the Morning till 7, and cou'd have seen it longer, had not Clouds interposed; and again the 1st of *May*, when the Sun had been up more than half an Hour, *viz.* at 5 in the Morning, I soon found it, and saw it afterwards as oft as I pleased, till half an Hour after 9 the same Morning.

III. This Instrument I have made use of for several Years, and would recommend it, upon my own Experience, for a very nice way to find the Meridian of any place, and to see the *Transits* of the Celestial Bodies over it, either Northward or Southward. *An Instrument to observe the Sun, &c. pass the Meridian, by Mr. Derham, n. 291. p. 1579.*

The Instrument is thus made of Wood, or rather Iron, or Brass, to indure the Weather, without swelling or contracting, *viz.* Prepare a small flat Iron Bar, as in *Fig.* 8. *C. C.* at each end of which rivet on two upright Sights, to turn stiffly, at the Joynts, *I. I.* Let one of the Sights *c.d.* have a Perforation big enough to see the *Pole-Star* through it, *Plate* 4. the other Sight *a. b.* a very small Perforation, to see the *Sun* through. Just behind the Joynts fix two upright Arms, *C.D.* and *C. D*, but to bend off, so as to be out of the way of the Sights, when you look through them. These Arms ought to be long enough for the Plumb-lines to reach the Polar-Star, on one side; and the Sun at his greatest height, on the other, when you look through either of the Sights. The Plumb-lines therefore are Tangents to their opposite Sights, and their lengths may be found by a Table of natural Tangents, and making the distance of the two Sights Radius. Thus in the Latitude of *London*, if the Instrument be two Feet from Sight to Sight, the Southern Plumb-line hath need to be near four Feet, and the Northern Plumb-line about two Feet ten Inches. On the tops of these two Arms place two small cross pieces *D. E.* and *D. E.* to turn with a Joynt at *D.* which cross-pieces are to hold the Plumb-lines *E. F.* and *E. F.* and to turn off and on, so as to bring the Plumb-lines to the Sights exactly. Place this Instrument on a Pedestal *H.* to turn round on it stiffly at the Pin *G.*

Your Instrument being thus prepared, the way to *Set and Use it* is thus; Plant it in a convenient place, where the Polar Star may be seen by Night, and the Sun by Day. When that Star is on the Meridian, is the time to set this Instrument, which is thus to be done; *viz.*

Through the Sight with the larger Hole *c. d.* look at the Pole-Star, and turn the whole Inftrument about, until you fee the oppofite Plumb-line nicely to interfect the Pole-Star. Or when you have brought the Plumb-line near the Star, you may more eafily bring the Plumb-line to interfect, by moving the Sight *c. d.* backward or forward, at the Joynt *I*, inftead of moving the whole Inftrument. And that you may more eafily fee the Pole-Star through the Sight, let the Plumb-line be a very fine Cats-gut ftring or Horfe-hair, *&c.* And if it be white, or fome fuch light colour, it will be the better feen, with the help of a Candle fhining on it by Night; which is neceffary.

The Sight *c. d.* and oppofite Plumb-line being thus fet in a direct line with the Polar-Star on the Meridian, it is manifeft, that the Inftrument lyeth exactly in the Meridian, fo as to fee any Star on the Meridian to the North. And that you may fee the fame Southerly; the next Day, or when you pleafe, you may hang up the Plumb-line *E. F.* upon the Southern Arm *C. D.* fo as that the Plumb-line may exactly interfect the Perforation *c. d.* This may eafily be done by moving the top Joynt, with the Plumb-line on its crofspiece backward and forward, till the Plumb-line hangeth to your Mind. If the Sight with the leffer Perforation *a. b.* be not exactly under the Northern Plumb-line, it muft be brought to be fo by turning the Sight, by help of its Joynt at *I.* And then all the Inftrument is fet right, fo as to fee the Sun, Moon or Stars come on the Meridian towards the South.

But to fee the Sun tranfit the Meridian, it is neceffary to guard the Eye, with a coloured Glafs, or a Glafs darkened with the fmoak of a Lamp or Candle. Which for the fake of thofe who do not know the way of it, it may be neceffary to defcribe.

A Glafs to look upon the Sun. Chufe two pieces of Glafs cut into the fame fize and figure; but take care they do not refract vitioufly. Which may be known by moving the Glafs before the Eye. If the Objects you look on, feem to dance about, the Glaffes are falfe and refract; but true if all feems fteady. Smoak one of thefe Glaffes over the Flame of a Lamp or Candle, until it be obfcured enough to take off the Sun's Rays fufficiently, but not fo as to darken it too much. This may be feen by looking upon the Sun with it, or upon the Candle. One of the Glaffes being thus darkened, lodge them both together, and faften them in a little Cafe fit for the Purpofe, with the fmoaked fide innermoft, and an edging of Card between, to keep the Glaffes afunder, fo as that the Soot may not be rubb'd off, or difordered.

'Tis good to have two Glaffes thus prepared, one for a ftrong Sun; the other lefs darkened, for the Sun behind a thin Cloud, or Mift, *&c.*

With one of thefe Glaffes held behind, or before the Sight *a. b.* you may plainly fee the Sun pafs.

To imitate the aforefaid Inftrument in a Journey, or where-ever you come. Inftead of an intire Inftrument, you may prepare only two Sights with Perforations as before. Let thefe Sights be nailed or fcrewed

ſcrewed down, upon the tops of two Stakes at *I. I*, ſo as to turn ſtiffly upon them. The Plumb-lines (one at leaſt) may be hung up at the end of an Houſe or on the Bough of a Tree (if the Wind would not ſhake it) or any where you ſee fit. And the Sights muſt be ſtuck up, ſo as to bring the Pole-Star to interſect, and all be performed, as hath been before directed.

This, although in a manner the ſame with the Inſtrument before, yet is more convenient in ſome reſpects, chiefly becauſe the Plumb-lines may be made longer, and the Sights ſet farther aſunder, than in the Inſtrument before can conveniently be done; which is ſome, although no great advantage for ſeeing the Tranſits. Alſo, theſe Sights may be made ſo light, as to be eaſily carried about; or they may be eaſily made, or imitated in any place where-ever you come.

To know when the Polar-Star cometh on the Meridian: The way is this, Subſtract the *Right Aſcenſion* of the *Sun* from the *Right Aſcenſion* of the *Pole-Star*, the *Remainder* giveth the Degrees, Minutes and Seconds when the Pole Star tranſits the Meridian above the Pole. Divide theſe Degrees by 15, it giveth the Hours, and every Degree under 15 multiplied by 4, giveth the Minntes; and every Minute multiplied by 4, giveth the Seconds, of Apparent time of the Pole Stars Southing. I ſcarce need to ſay that it cometh under the Pole at 12 Hours diſtance, only making ſome ſmall allowance for the Alteration of the Sun's Right Aſcenſion in that 12 Hours time. But you may ſhorten your Labour, by uſing Tables of the Sun's R. Aſc. in *Time*, inſtead of his R. Aſcenſion in *Degrees*, &c. which may be found in Sir *Jon. Moor*'s *Math. Compend.* and in divers other Books.

If the Sun's R. Aſc. exceedeth the Pole-Stars, add 360 Degrees, or 24 Hours, and then ſubſtract. The R. Aſc. of the Pole-Star is determin'd by Mr. *Flamſteed* to be $0^h\ 33'.\ 4''$ of *Time*, Anno 1690, and the increaſe of its R. Aſc. in 10 Years $1'\ 16''$. of *Time*. Therefore in 1703, the R. Aſc. of the Pole-Star was $0^h\ 35'.\ 22''$. of *Time*. Or you may ſee when the Pole-Star cometh to the Meridian, by hanging up a Plumb-line, and obſerving when the *Thill-Horſe* in *Charles*'s *Wain*, called *Alioth*, comes near the Line, together with the Pole-Star, on one ſide the Pole; or the bright Star of the third Magnitude in *Caſſiopeia*'s *Thigh* on the other ſide.

The foregoing Inſtruments may be ſet by any Star; but the Pole-Star, in our Northern Hemiſphere, is moſt convenient, becauſe it maketh but a ſmall Circle round the Pole, and therefore moveth ſlower, and conſequently is longer in tranſiting the Meridian. And therefore a ſmall Error in Calculation, or a little Expence of Time in ſetting the Inſtrument, may be admitted, as little deſerving regard.

1ſt, You may ſee with all imaginable Exactneſs, when it is Noon even to 1, 2, or at moſt 3 Seconds of Time. For you may ſee when the very Limb of the Sun toucheth the Meridian, and whilſt all his Disk is paſſing it. So that by much it exceedeth all Sun-Dials: ſo far that if you *The Uſes of theſe Meridian Inſtruments.*

once

once uſe this Inſtrument, you will be ready to lay aſide all Sun-Dials; the beſt of which (unleſs we except Mr. *Molineux*'s) can never ſhew the Time to one or many Seconds.

But beſides all this, another vaſt Conveniency is, That it will *fit moſt Latitudes.* So that there is no need of having a ſtrict regard to the Elevation of the Pole, nor any danger of Error in making and ſetting, as is in moſt other Inſtruments, but all is with Eaſe and Certainty performed. Therefore,

2dly, Into whatſoever place you come, you may eaſily ſee the Errors of the Sun-Dials there, and which go trueſt, and which falſe.

3dly, As the Sun, ſo alſo the fixt Stars may be ſeen to tranſit the Meridian, whereby the *Hour of the Night* may as exactly be known, as of the Day by the Sun, knowing the R. Aſc. of the Star that tranſits. For (as before for the Pole-Star) ſubſtract the R. Aſc. of the Sun from the R. Aſc. of the Star, the Remainder converted into Time, is the Time of that Star's Culmination or Southing. And if 12 Hours be added or ſubſtracted (making due Allowance for the Alteration of the Sun's R. Aſc. in that time) it ſheweth the exact time of that Star's coming to the Meridian Northward.

4thly, The Hour of the Day and Night being thus to 1, 2 or 3 Seconds diſcoverable by the aforeſaid Inſtruments, I doubt not but that they may be uſeful in finding the exact *Differences of Meridians*, either by the *Eclipſes* of *Jupiter*'s *Satellits*, or the Occultations of the Fixt-Stars by the Moon.

Theſe Meridian Inſtruments may be of Service where-ever long Teleſcopes are us'd, for ſeeing the Appulſes of the Moon to the Fixt-Stars or the Eclipſes of *Jupiter*'s *Satellits*; which is only on Land. Unleſs (which I have thought feaſible) a convenient ſtanding for a Man, and a Teleſcope might be hung penduloully in a Ship, which (eſpecially in a calm Sea) may be as little ſubject to diſturbance as the Pendula of Watches are, which will retain their Motion at Sea.

5thly, You may with all Exactneſs continue a Meridian Line for 20 Miles, by looking through either Sight, and ſeeing what Objects are interſected by the Plumb-Lines.

6thly, Theſe Inſtruments are prepar'd with little coſt or trouble, and eaſily carried about or imitable in any place, the latter eſpecially, which may be made pocketable, or ſoon gotten.

Of the Maxima, & Minima, *occurring in the Celeſtial Motions, by* Mr. De Moivre, *n.* 360. *p.* 952.

IV. Sit ABP Orbis Planetæ Ellipticus, AP Axis Tranſverſus, CB Semiaxis conjugatus, S Sol, Q Focus alter Ellipſeos. Per S ducatur SM ipſi CB parallela: & erit punctum M in quo *Maxima* cum velocitate creſcit vel decreſcit diſtantia a Sole, & $SM = AC - \frac{SC^2}{AC}$.

Plate 3. *Fig.* 37.

Si vero capiatur SL media proportionalis inter Semiaxes AC, CB, erit punctum L in quo *Maxima* fit æquatio Centri, ut vocant; ſive ubi motus

motus angularis fit æqualis medio Motui. Quod si Eccentricitas non major sit quam in plerisque Planetis, $BL = \frac{1}{2} BM$ quamproxime: Est vero $SL = \sqrt{\sqrt{AC^4 - AC^2 SC^2}}$.

Si quæratur punctum N, in quo fit *Maxima* mutatio Velocitatis motus realis in Curva, Problema Solidum est. Est enim $2NS = 4AC - 2NQ$ ad $3NQ - AC$ ut $AC^2 - CS^2 = CB^2$ ad NQ^2; adeoque si ponatur $AC = a$, $CB = c$ & $NQ = y$, habebitur æquatio $y^3 - 2ayy + \frac{1}{2}ccy - \frac{1}{2}acc = 0$. Qua resoluta erit y sive NQ distantia puncti quæsiti N ab altero Ellipseos foco. In Orbibus autem parum Eccentricis, quales sunt Planetarum, si fiat $CD = SQ$, & junctæ AD æqualis ponatur AK, erit reliqua pars Axis $KP = NS$ distantiæ puncti N a Sole quamproxime. Si vero Orbis fuerit Parabolica, erit SN ad SP ut 5 ad 4, angulusque NSP erit $53^\circ.\ 8'$ fere, cujus Sinus est $\frac{4}{5}$ Radii.

At Punctum O, in quo motus apparens sive angularis acceleratio Planetæ descendentis, vel retardatio ascendentis *Maxima* fit, hoc modo obtinebitur. In AC capiatur $CG = \frac{1}{6} AC$, ac fiat angulus CSF 30 *gr.* ductæque SF æqualis ponatur CE, ipsique GE sit GH æqualis. Dico, si distantia SO fiat æqualis ipsi PH, quod in puncto O proveniet *Maxima* mutatio motus angularis Planetæ in Orbe Elliptico $ABOP$ gyrantis; eo scilicet in Orbis loco secundæ differentiæ æquationum centri Planetæ reperientur *Maximæ*. Est autem $SO = \frac{7}{6}AC - \sqrt{\frac{1}{3}\frac{1}{6}AC^2 + \frac{1}{3}SQ^2}$. Quod si Orbis Parabolica fuerit, ut in Cometis, fiet SO ad SP ut 8 ad 7, angulusque OSP fiet $41^\circ.\ 24'\frac{1}{2}$, sive cujus Sinus sit ad Radium ut $\frac{1}{4}\sqrt{7}$ ad 1.

Denique *Minima* cum Velocitate mutatur directio Tangentis Orbitæ in puncto R, si fiat SR æqualis duabus tertiis Axis majoris AB. Quod si Eccentricitas SC minor fuerit quam $\frac{1}{3} PC$, *Minimum* hoc non locum habet, sed decrescit semper hæc Velocitas quacum revolvitur Tangens, usque in ipsum Aphelion; quemadmodum se res habet in omnium Planetarum motibus. Neque etiam in orbe Parabolico obtinet, ob Axem ejus in infinitum protensum *.

* v. supra Chap. I. p. 1.

V. Ut mutabilem apparentiam stellæ χ in collo Cygni deprehenderem, occasionem dedit stella capiti Cygni vicina, quam *Hevelius*, Astronomus solertissimus, Anno 1670 & 71. observavit. Cum enim spem conciperem fore ut eadem stella nunc iterum sæpius esset apparitura, non secus ac stella in collo Ceti, quam *Hevelio* post primam disparitionem mox iterum apparuisse constabat, quærebam eam 1° & 6 to (11° & 16°) Julii 1686. noctibus serenis, non autem reperiebam; sed potius animadvertebam stellam illam 5 æ magnitudinis in collo Cygni, a *Bayero* græca litera χ signatam, desiderari. Die vero 9 (19.) Octobris deprehendi eam nudis oculis distincte omnino. Et quia facile adducebar ut existimarem eandem nudis nostris oculis iterum disparituram, delineavi aliquot ipsam circumstantes stellulas, ope bipedalis magnæque capacitatis Tubi, ut ex harum cum illa comparatione magnitudinem ejus, cum decresceret, expenderem, uti Fig. *A.* exhibetur.

A new Star in the Swan's Neck, by Mr. Kirch, n. 343. p. 226.

Plate 3.

Deprehen-

Deprehenſum eſt quoque, ſtellam iſtam paulatim decreviſſe, donec eamTubo 8 pedum nonpotuerim amplius aſſequi, cum tamen aliam illam, in collo Ceti, quando nudo oculo non amplius patet, per tubum 4 pedum ſemper dignoſcere poſſim.

Ab illo tempore quærebam ſtellam iſtam variis noctibus fruſtra, tandem vero tamen eam reperi 6to (16°) Aug. 1687, iterum ope octopedalis Tubi, at vero exiguam valde. Inde de die in diem creviſſe deprehendebatur, & factum eſt ut die 23 Octob. (Novemb.) iterum prima vice, nudo oculo, ſeſe conſpiciendam præberet, valde licet adhuc dum exiguam. Die 2do (12°) Nov. optime erat conſpicua, etiam poſt 26tum Nov. (6. Dec.) ut hoc ultimo die jam iterum in ſtatu decreſcendi exiſteret. Poſtmodum non niſi per Tubos dignoſci potuit, tandemque adeo exigua evaſit, ut iterum Tubo 8 pedum eam deprehendere non potuerim. Atque ita animadverſum eſt hac vice, ab una diſparitione uſque ad alteram, annum unum, menſem unum, unamque hebdomadam circiter effluxiſſe. Sequentes quoque obſervationes docuerunt hancce ſtellam tempus ſatis conſtans in ſua apparitione ſervare, non tamen quavis vice ad æqualem magnitudinem pervenire. Imo aliquando accidit, ut nudo oculo plane inviſibilis maneat, dum per Tubum eſt conſpicua, & maximam ſuam magnitudinem aſſecuta eſt; prout Anno 1688 in fine, & 1689 in principio anni accidit. E contrario, ſequente Anno 1690, ſtella hæc eo melius videri poterat, & quidem notabiliter major quam ſua vicina, quam *Bayerus* juxta χ collocavit extra collum Cygni, nullaque litera notavit, quam ego ſolius memoriæ juvandæ gratia, Hebraica litera ג notavi. Et poſtquam hujus ſtellæ apparentiam & diſparentiam ſæpius obſervavi, comperi eam valde eſſe regularem, revolutionemque 404½ dierum ſervare.

A Remark by Dr. Ed. Halley, *ib. p.* 228.

Cum Miſcellanea Berolinenſia *ſerius ad nos perlata ſint, non ante annum ultimo elapſum hanc Stellam novam ſecundam D.* Kirchii *monitum conſpeximus; idque juxta Idus Julii,* ſt. vet. *cum multo clarior quam vicina* ג, *ac fere æqualis mediæ in collo Cygni* Bayero *η) apparuit: ſed poſt menſem nudis oculis inconſpicua facta, tandem etiam Teleſcopio evanuit.*

A Hiſtory of new Stars, by Dr. Halley, *n.*346. *p.* 354.

VI. The firſt new Star in the *Chair* of *Caſſiopeia*, was not ſeen by *Cornelius Gemma* on the Eighth of *November* 1572, who ſays, he that Night conſidered that part of Heaven in a very ſerene Sky, and ſaw it not: but that the next Night, *November* 9°. it appeared with a Splendour ſurpaſſing all the fixt Stars, and ſcarce leſs bright than *Venus*. This was not ſeen by *Tycho Brahe* before the 11th of the ſame Month, but from thence he aſſures us that it gradually decreaſed and died away, ſo as in *March* 1574, after ſixteen Months, to be no longer viſible; and at this Day no Signs of it remain. The Place thereof in the Sphere of Fixt Stars, by the accurate Obſervations of the ſame *Tycho*, was 0s 9°. 17'. *a* 1ma ✶ ♈is, with 53°. 45'. North Latitude.

Such another Star was ſeen and obſerved by the Scholars of *Kepler*, to begin to appear on *Sept.* 30°. *ſt. vet. anno* 1604, which was not to be ſeen the Day before: but it broke out at once with a Luſtre ſurpaſſing that

that of *Jupiter*; and like the former it died away gradually, and in much about the ſame time diſappeared totally, there remaning no Foot-ſteps thereof in *January* 160$\frac{5}{6}$. This was near the Ecliptick, following the Right Leg of *Serpentarius*; and by the Obſervations of *Kepler* and others, was in $7^s\ 20^{\circ}\ 00'$ a 1^{ma} ✳. ♈, with North Latitude $1^{\circ}\ 56'$. Theſe two ſeem to be of a diſtinct *Species* from the reſt, and nothing like them has appear'd ſince.

But between them, *viz.* in the Year 1596, we have the firſt Account of the wonderful Star *in Collo Ceti*, ſeen by *David Fabricius* on the third of *Auguſt*, ſt. vet. as bright as a Star of the third Magnitude, which has been ſince found to appear and diſappear periodically: its Period being preciſely enough 7 Revolutions in Six Years, though it return not always with the ſame Luſtre. Nor is it ever totally extinguiſh'd, but may at all times be ſeen with a Six-Foot-Tube. This was ſingular in its Kind, till that in *Collo Cygni* was diſcovered. It precedes the firſt Star of *Aries* $1^{\circ}.\ 40'$. with $15^{\circ}.\ 57'$ South Latitude.

Another New Star was firſt obſerved by *Will. Janſonius* in the Year 1600, *in Pectore* or rather *in eductione Colli Cygni*, which exceeded not the third Magnitude. This having continued ſome Years, became at length ſo ſmall, as to be thought by ſome to diſappear entirely: but in the Years 1657, 58 and 59, it again aroſe to the third Magnitude, tho' ſoon after it decay'd by Degrees to the fifth or ſixth Magnitude, and at this Day is to be ſeen as ſuch in $9^s\ 18^{\circ}.\ 38'$. a 1^{ma} ✳. ♈, with $55^{\circ}.\ 29'$. North Latitude.

A fifth New Star was firſt ſeen by *Hevelius* in the Year 1670, on *July* 15. *ſt. vet.* as a Star of the third Magnitude, but by the Beginning of *October* was ſcarce to be perceived by the naked Eye. In *April* following it was again as bright as before, or rather greater than of the third Magnitude, yet wholly diſappeared about the middle of *Auguſt*. The next Year, in *March* 1672. it was ſeen again, but not exceeding the ſixth Magnitude: ſince when it has been no further viſible, though we have frequently ſought for its Return; its place is $9^s\ 3^{\circ}.\ 17'$. *a* 1^{ma} ✳. ♈. and has Lat. North. $47^{\circ}\ 28'$.

The Sixth and laſt is that we above deſcribed from the *Acta Berolinenſia*, of the Year 1710. diſcovered by Mr. *G. Kirch* in the Year 1686, and its Period determined to be of 404$\frac{1}{2}$ Days: and though it rarely exceed the fifth Magnitude, yet is it very regular in its Returns, as we found in the Year 1714. Since then we have watched, as the Abſence of the Moon and the Clearneſs of Weather would permit, to catch the firſt beginning of its Appearance in a Six-Foot-Tube, that bearing a very great Aperture diſcovers moſt minute Stars. And on *June* 15. laſt, it was firſt perceived like one of the very leaſt Teleſcopical Stars: but in the reſt of that Month and *July* it gradually encreaſed, ſo as to become in *Auguſt* viſible to the naked Eye; and ſo it continued all the Month of *September*. After that it again died away by degrees, and on the 8th of *December* at Night was ſcarce diſcernible by the Tube, and

as near as could be gueſſed, equal to what it was at its firſt Appearance on *June* 25th: ſo that this Year it has been ſeen in all near Six Months, which is but little leſs than half its Period: And the middle, and conſequently the greateſt Brightneſs, falls about the 10th of *September.*

Of Nebulæ *or lucid Spots among the Fixt Stars by* —— *n.* 347. *p.* 390.

VII. Not leſs wonderful are certain luminous Spots or Patches, which diſcover themſelves only by the Teleſcope, and appear to the naked Eye like ſmall fixt Stars; but in reality are nothing elſe but the Light coming from an extraordinary great Space in the Ether; through which a lucid *Medium* is diffuſed, that ſhines with its own proper Luſtre. This ſeems fully to reconcile that Difficulty which ſome have moved againſt the Deſcription *Moſes* gives of the Creation, alledging that Light could not be created without the Sun. But in the following Inſtances the contrary is manifeſt; for ſome of theſe bright Spots diſcover no ſign of a Star in the middle of them; and the irregular Form of thoſe that have, ſhews them not to proceed from the Illumination of a Central Body. Theſe are, as the aforeſaid New Stars, ſix in Number, all which we will deſcribe in the order of Time, as they were diſcovered; giving their Places in the Sphere of Fixt Stars, to enable the Curious, who are furniſhed with good Teleſcopes, to take the Satisfaction of contemplating them.

The firſt and moſt conſiderable is that in the middle of *Orion*'s Sword, marked with θ by *Bayer* in his *Uranometria*, as a ſingle Star of the third Magnitude; and is ſo accounted by *Ptolemy*, *Tycho Brahe* and *Hevelius*: but is in reality two very contiguous Stars environed with a very large tranſparent bright Spot, through which they appear with ſeveral others. Theſe are curiouſly deſcribed by *Hugenius* in his *Syſtema Saturninum* p. 8. who there calls this Brightneſs, *Portentum cui certe ſimile aliud nuſquam apud reliquas Fixas potuit animadvertere*: affirming that he found it by chance in the Year 1656. The middle of this is at preſent in ♊ 19°. 00, with South Lat. 28$\frac{3}{4}$.

About the Year 1661, another of this ſort was diſcovered (if I miſtake not) by *Bullialdus*, *in Cingulo Andromedæ*. This is neither in *Tycho* nor *Bayer*, having been omitted, as are many others, becauſe of its Smallneſs: But it is inſerted in the Catalogue of *Hevelius*, who has improperly call'd it *Nebuloſa* inſtead of *Nebula*; it has no ſign of a Star in it, but appears like a pale Cloud, and ſeems to emit a radiant Beam into the North Eaſt, as that in *Orion* does into the South Eaſt. It precedes in Right Aſcenſion the Northern in the Girdle, or ν *Bayero*, about a Degree and three Quarters, and has Longitude at this time ♈ 24°. 00'. with Lat. North 33$\frac{1}{3}$.

The third is near the Eclíptick between the *Head* and *Bow* of *Sagittary*, not far from the Point of the Winter Solſtice. This it ſeems was found in the Year 1665. by a *German* Gentleman *M. J. Abraham Ihle*, whilſt he attended the Motion of *Saturn* then near his *Aphelion*. This is

is ſmall but very luminous, and emits a Ray like the former. Its Place at this time is ♑ 4°½ with about half a Degree South Lat.

A fourth was found by Dr. *Edm. Halley* in the Year 1677, when he was making the Catalogue of the Southern Stars. It is in the *Centaur*, that which *Ptolemy* calls ὁ ἐπὶ τῆς τοῦ νώτου ἐκφύσεως, which he names *in dorſo Equino Nebula*, and is *Bayers* ω; it is in appearance between the fourth and fifth Magnitude, and emits but a ſmall Light for its Breadth, and is without a radiant Beam; this never riſes in *England*, but at this time its Place is ♏ 5°¾ with 35 ⅕ South Lat.

A fifth was diſcovered by Mr. *G. Kirch* in the Year 1681, preceding the Right Foot of *Antinous*: It is of its ſelf but a ſmall obſcure Spot, but has a Star that ſhines through it, which makes it the more luminous. The Longitude of this is at preſent ♑. 9°. *circiter*, with 17°⅙. North Latitude.

The ſixth and laſt was accidentally hit upon by Dr. *Edm. Halley* in the Conſtellation of *Hercules*, in the Year 1714. It is nearly in a right Line with ζ and η of *Bayer*, ſomewhat nearer to ζ than η: and by comparing its Situation among the Stars, its Place is ſufficiently near in ♏ 26 ½. with 57°. 00. North. Lat. This is but a little Patch, but it ſhews it ſelf to the naked Eye, when the Sky is ſerene and the Moon abſent.

There are undoubtedly more of theſe which have not yet come to our Knowledge, and ſome perhaps bigger, but though all theſe Spots are in Appearance but little, and moſt of them but of few Minutes in Diameter; yet ſince they are among the Fixt Stars, that is, ſince they have no annual Parallax, they cannot fail to occupy Spaces immenſely great, and perhaps not leſs than our whole Solar Syſtem. In all theſe ſo vaſt Spaces it ſhould ſeem that there is a perpetual uninterrupted Day, which may furniſh Matter of Speculation, as well to the curious Naturaliſt as to the Aſtronomer.

VIII. Having of late had occaſion to examine the quantity of the Preceſſion of the Equinoctial Points, I took the pains to compare the Declinations of the fixt Stars delivered by *Ptolomy*, in the 3d Chapter of the 7th Book of his *Almag.* as obſerved by *Timocharis* and *Ariſtyllus* near 300 Years before *Chriſt*, and by *Hipparchus* about 170 Years after them, that is about 130 before *Chriſt*, with what we now find: and by the reſult of very many Calculations, I concluded that the fixt Stars in 1800 Years were advanced ſomewhat more than 25 degrees in Longitude, or that the Preceſſion is ſomewhat more than 50″ *per Ann.* But that with ſo much uncertainty, by reaſon of the imperfect Obſervations of the Ancients, that I have choſen in my Tables to adhere to the even proportion of five Minutes in ſix Years, which from other Principles we are aſſured is very near the Truth. But while I was upon this Enquiry, I was ſurprized to find the Latitudes of three of the principal Stars in Heaven directly to contradict the ſuppoſed greater *Obliquity* of the *Ecliptick*, which ſeems confirmed by the Latitudes of moſt of the reſt: they

Of the Change of Latitude of ſome of the Fixt Stars, by Dr. Halley, n. 355. p. 736.

they being ſet down in the old Catalogue, as if the Plain of the Earth's Orb had chang'd its Situation, among the fixt Stars, about 20′ ſince the time of *Hipparchus*. Particularly all the Stars in *Gemini* are put down, thoſe to the *Northward* of the Ecliptick, with ſo much leſs Latitude than we find, and thoſe to the *Southward* with ſo much more *Southerly* Latitude. Yet the three Stars *Palilicium* or the *Bulls* Eye, *Sirius* and *Arcturus* do contradict this Rule directly: for by it, *Palilicium* being in the Days of *Hipparchus* in about 10 *gr.* of *Taurus* ought to be about 15 Min. more *Southerly* than at preſent, and *Sirius* being then in about 15 of *Gemini* ought to be 20 Min. more *Southerly* than now; yet *e contra* *Ptolomy* places the firſt 20 Min. and the other 22 more *Northerly* in Latitude than we now find them. Nor are theſe Errors of Tranſcription, but are proved to be right by the Declinations of them ſet down by *Ptolomy*, as obſerved by *Timocharis*, *Hipparchus* and himſelf, which ſhew that thoſe Latitudes are the ſame as thoſe Authors intended. As to *Arcturus*, he is too near the Equinoctial Colure, to argue from him concerning the change of the Obliquity of the Ecliptick; but *Ptolomy* gives him 33′ more *North* Latitude than he now has; and that greater Latitude is likewiſe confirmed by the Declinations delivered by the aforeſaid Obſervers. So then all theſe three Stars are found to be above half a degree more *Southerly* at this time than the Antients reckoned them. When on the contrary at the ſame time the bright Shoulder of *Orion* has in *Ptolomy* almoſt a degree more *Southerly* Latitude than at preſent. What ſhall we ſay then? It is ſcarce credible that the Antients could be deceived in ſo plain a matter, three Obſervers confirming each other. Again theſe Stars being the moſt conſpicuous in Heaven, are in all probability the neareſt to the Earth, and if they have any particular Motion of their own, it is moſt likely to be perceived in them, which in ſo long a time as 1800 Years may ſhew it ſelf by the Alteration of their places, though it be utterly imperceptible in the ſpace of a ſingle Century of Years. Yet as to *Sirius* it may be obſerved that *Tycho Brahe* makes him 2 Min. more *Northerly* than we now find him, whereas he ought to be above as much more *Southerly* from his Ecliptick, (whoſe Obliquity he makes $2\frac{1}{2}$ greater than we eſteem it at preſent) differing in the whole $4\frac{1}{2}$ Min. One half of this difference may perhaps be excuſed, if Refraction were not allowed in this Caſe by *Tycho*; yet two Minutes, in ſuch a Star as *Sirius*, is ſomewhat too much for him to be miſtaken.

But a more evident proof of this Change is drawn from the Obſervation of the Application of the Moon to *Palilicium* at *Athens Anno Chriſti* 509 *Mart.* 11°. when in the beginning of the Night the Moon was ſeen to follow that Star very near, and ſeem'd to have Eclipſed it.* Now from the undoubted Principles of Aſtronomy, it was impoſſible for this to be true at *Athens*, or near it, unleſs the Latitude of *Palilicium* were much leſs than we at this time find it.

* v. Bulliald. *Aſtr. Philolaica.* p. 172.

This

This Argument feems not unworthy of the *Royal Society's* Confideration, to whom I humbly offer the plain Fact as I find it, and would be glad to have their Opinion.

But whether it were really true, that the Obliquity of the Ecliptick was, in the time of *Hipparchus* and *Ptolomy*, really 22 Min. greater than now, may well be queftion'd; fince * *Pappus Alexandrinus*, who lived but about 200 Years after *Ptolomy*, makes it the very fame that we do. *v. Pappi *Coll. Lib.* VI. *Prop.* 35.

IX. Satis notum eft Orbitæ *Caffinianæ* hanc effe naturam. Si a datis duobus punctis *F* & *G* ad quodvis curvæ punctum *H* ducantur rectæ *F H*, *G H*; rectangulum fub *FH*, *G H* æquale eft dato fpatio. Recta *F G* hinc inde producta donec curvæ occurrat, oftendit Vertices *A* & *B*; & *AB* eft Axis principalis; mediumque inter vertices punctum *C* eft figuræ Centrum; & *D E*, per *C* ad *A B* normalis, Axis minor; punctaque *F* & *G* Foci. *Of the Orbit of* Caffini, *by Dr.* Gregory, *n.* 293. *p.* 1704. *Plate* 3. *Fig.* 38.

In hac figura fi axis minor excedat diftantiam focorum, Curva figuram terminans eft ubique verfus centrum cava, qualis vulgo habetur. Si, manente axe principali, diftantia focorum minuatur, augebitur axis minor, qui tamen minor manet axe Ellipfis eodem axe principali, iifdemque focis defcriptæ; donec tandem, coeuntibus focis, ille evadat æqualis axi majori, & figura abeat in Circulum. Si vero, e contra, diftantia focorum augeatur; minuetur axis minor, fietque æqualis dictæ diftantiæ cum hæc eft ad axem principalem ficut unitas ad medium proportionalem inter unitatem & ternarium.

Si ulterius augeatur diftantia focorum, minuetur adhuc axis minor, & Curva non erit amplius ad hujus extrema verfus centrum cava, fed convexa, ut in *Fig.* 39. donec aucta eoufque diftantia focorum, ut hæc fit ad axem majorem ficut latus quadrati ad ejufdem diametrum, axis minor fiat nullus, & curva pertingat ad centrum hinc inde. *Fig.* 39.

Si diftantia focorum fuerit major quam pro dicta ratione, axis minor fit impoffibilis, & figura in duas conjugatas abit, ut in *Fig.* 40. quæ aucta focorum diftantia minuentur, donec tandem figura in bina puncta conjugata abeat. *Fig.* 40.

Crefcente porro diftantia focorum, rurfus emergunt binæ figuræ conjugatæ, quæ fimiliter crefcunt atque prius decreverant a prioribus diverfæ in focorum verticumque ordine, augenturque donec infinitæ evadant. Pofteaque Syftema hoc iifdem gradibus ad circulum rurfus accedet quibus ab illo receffit.

Ex his vel primo intuitu fatis patet Figuram hanc ad conftituendam Planetæ Orbitam minime idoneam effe. Ut enim taceam cafus ubi in duas conjugatas abit, Orbitæque naturam deponit, nimirum ubicunque tanta fit ejus excentricitas, quantam Cometæ (fi circa Solem Planetarum inftar verfantur, quod verifimillimum eft) ad curfum fuum defcribendum poftulant: ut hos inquam cafus prætercam: funt etiam in iis cafibus, ubi in fe redit orbitamque perficit, quædam ejus excentricitates ita amplæ, ut curva prope *D* & *E* (*Fig.* 39.) verfus Solem convexa evadat; adeoque vi a Sole centrifuga Planetæ opus effet, ut hanc Orbitæ

fuæ

ſuæ partem percurreret, dum interim in locis propioribus & remotioribus *B* & *A* vis ad Solem centripeta requiritur. Id eſt corpora circumſolaria ea lege moveri poſſe concedendum eſſet, ut in paribus a Sole intervallis hic vis centripeta, illic centrifuga obtineret, quod quam ſit a naturæ legibus alienum facile omnes perſpicient. Et licet nullius e Planetis tanta ſit excentricitas; cum tamen Geometris notum ſit, figuræ, cujus ſpecies omnes ultra certum terminum muneri cuivis naturæ obeundo ineptæ fuerint, ejus ſpecies reliquas citra dictum terminum quaſi eidem muneri idoneas, admitti non poſſe: Neceſſe eſt *Curvam hanc Caſſinianam ex Aſtronomia rejicere*, non ſolum ob rationes *Prop.* VIII. *Lib.* III. *Elem. Aſtr.* adductas, nempe *quod neque obſervatis cæleſtibus congruat propter minoris axis brevitatem, neque rationes Phyſicæ reſpondeant, cum ad illam deſcribendam opus eſſet vi centripeta ad Solem abhorrente ab illa per rerum naturam uſurpata*, ſed etiam propter abſolutam impoſſibilitatem. Impoſſibile namque eſt hujus figuræ ſpeciem quamcunque poſſe a Planeta percurri, ita ut anguli ad focum a Sole diverſum proportionales ſint temporibus; ſic enim area per radium vectorem deſcripta non eſſet tempori proportionalis. Non enim aucto angulo ad focum unum æqualibus incrementis, areæ ad alterum incrementa ſimul facta etiam æqualia ſunt, uti perperam nuper ſentiebam.

In duobus ultimis ſchematibus maxima figuræ latitudo invenitur, ſi centro *C* per focos deſcribatur circulus; ſecabit namque hic Curvam in punctis *L*, *L* quæſitis. Eſtque maxima ordinata *K L* tertia proportionalis rectis *G F* & *F D* in horum primo, vel quarta proportionalis ipſis *G F*, *G A*, & *A F* in utroque.

Superſtite *D E*, ordinata ex foco *F P* æqualis eſt ſemi-axi minori *C D*, quando axis minor eſt ad diſtantiam focorum ut latus quadrati ad diametrum. Si diſtantia focorum fuerit major quam pro hac ratione, *F P* excedet *C D*.

Solution of Kepler's *Problem, by Dr.* John Keill, *n.* 337. *p.* 1.

X. Ob æquabilem Arearum deſcriptionem (a *Keplero* demonſtratam,) quo Planetarum loca in propriis Orbitis ad datum tempus determinentur, neceſſe eſt ut ſolvatur Problema quod ſequitur.

Invenire Poſitionem rectæ, quæ per datæ Ellipſeos focum alterutrum tranſiens, abſcindat Aream motu ſuo deſcriptam, quæ ſit ad Aream totius Ellipſeos in ratione data.

Fig. 41.

Sit nempe Ellipſis *A P B*, cujus focus alteruter *S*. Invenienda eſt poſitio rectæ *S P*, quæ abſcindat Aream trilineam *A S P*, ad quam Area totius Ellipſeos eandem rationem habet, quam tempus Periodicum Planetæ Ellipſim deſcribentis, ad aliud tempus datum; qua inventa dabitur punctum *P* ubi planeta ad tempus illud datum verſatur. Vel ſit *A Q B* ſemicirculus ſupra Ellipſeos axem majorem deſcriptus, ducenda eſt per *S* recta *S Q* abſcindens Aream *A S Q*, ad quam Area totius circuli eſt in eadem ratione: ſi enim ex *Q* demittatur in Axem perpendicularis *Q H*, Ellipſi occurrens in *P*, ducta *S P* dabit Aream Ellipticam quæſitam; & punctum *P* erit locus Planetæ ad datum tempus. Eſt enim

ſemiſeg-

femifegmentum Ellipticum APH ad femifegmentum circulare $A\,Q\,H$, ut HP ad $H\,Q$, hoc eft, ut area totius Ellipfeos ad aream totius Circuli; fed eft triangulum SPH ad triangulum $S\,Q\,H$ in eadem ratione PH ad $Q\,H$: Adeoque Area ASP eft ad Aream totius Ellipfeos, ut area $AS\,Q$ ad aream totius Circuli. Unde fi habeatur methodus fecandi in data ratione Aream circuli, recta ducta per datum punctum S, facile erit hac ipfa ratione fecare Aream Ellipticam.

Ipfi *Keplero*, qui primus Problema propofuit, nulla innotuit Methodus directa computandi locum Planetæ ex dato tempore; fed illi neceffe fuit, per fingulos gradus femicirculi $A\,Q\,B$ progrediendo, ex dato arcu $A\,Q$, quam vocat Anomaliam Excentri, tam tempus per aream $A\,S\,Q$, quæ Anomaliæ mediæ eft proportionalis, quam angulum $A\,S\,P$, hoc eft locum Planetæ, feu Anomaliam coæquatam huic tempori refpondentem, calculo eruere.

Cum itaque difficilis fuit hujus Problematis folutio, Aftronomi ad alias tranfiverunt Hypothefes, fingendo punctum aliquod circa quod motus foret æquabilis, feu tempori proportionalis, & exinde data Anomalia media, coæquatam determinabant. Sed computus hifce hypothefibus innixus, obfervationibus non congruere deprehenfus eft: Itaque Geometræ varias adhibuerunt approximationes, quibus ex data Area $A\,S\,Q$ tempori Analoga, angulus $A\,S\,P$, hoc eft Planetæ locus, quam proxime eliciatur. At horum omnium facillima, & ad Praxim maxime expedita, mihi videtur effe illa methodus, quam tradit Dominus *Newtonus* in *Principiis*, *pag.* 111 & 112. *Edit.* 1*ma.* quæ fere fimilis eft ei, qua ex æquationibus affectis extrahunt Radicem Analyftæ; & quidem tanto magis eft æftimanda, quod non folum exhibeat Planetarum loca, quorum orbitæ ad Circuli formam proxime accedunt, fed eadem fere facilitate infervit etiam Cometis, qui in orbitis maxime excentricis moveantur. Hanc itaque methodum in gratiam Artificum, qui Tabulas Aftronomicas fecundum veras motuum leges, & non ex fictis hypothefibus condere volunt, hic exponendam duxi.

Sit itaque $A\,Q\,B$ femicirculus fupra Axem majorem Ellipfeos defcriptus, cujus centrum C, & focus in quo Sol locatur fit S. Ducatur $C\,Q$, in quam (fi opus fit) productam cadat perpendicularis $S\,F$. Eft Area $A\,S\,Q$ = fectori $A\,C\,Q$ + Triang. $C\,S\,Q = \frac{1}{2}C\,Q \times A\,Q + \frac{1}{2}C\,Q \times S\,F$; adeoque ob datam $\frac{1}{2}C\,Q$ erit Area $A\,S\,Q$ femper proportionalis arcui $A\,Q$ + recta $S\,F$, cum fcil. motus fit ab Aphelio verfus Perihelium: at cum a Perihelio ad Aphelion tendit Planeta, ut in Fig. 43. fit Area BSQ = fectori $B\,C\,Q$ — Triang. $C\,S\,Q$, adeoque erit illa proportionalis arcui $B\,Q$ — recta $S\,F$. Hinc fi capiatur Arcus $A\,N$ in Fig. 42: & 44. & $B\,N$ in Fig. 43. temporibus proportionalis, erit $A\,Q + S\,F = A\,N$, & $B\,Q - SF = B\,N$: unde SF erit $= QN$, modo arcus AN vel $B\,N$ fint proportionales temporibus quibus defcribuntur areæ $A\,S\,Q$ vel $B\,S\,Q$. Ut vero inveniatur in gradibus eorumque partibus menfura arcus in peripheria $A\,Q\,B$, qui fit æqualis rectæ $S\,F$, Fiat ut $C\,Q$ ad CS, ita arcus graduum 57,29578 (qui æqualis eft $C\,Q$ radio) ad arcum quartum, qui æqualis Fig. 42, 43, 44.

lis erit CS. Sit arcus ille B. Eſt autem CS ad SF ut Radius ad ſinum anguli SCF vel $AC\mathcal{Q}$. Fiat itaque ut Radius ad ſinum anguli $AC\mathcal{Q}$ vel arcus $A\mathcal{Q}$, ita arcus B ad alium D; erit arcus ille D æqualis rectæ SF, adeoque ſi, ad datum tempus, Area $AS\mathcal{Q}$ eſſet tempori proportionalis, eſſet Arcus $D=N\mathcal{Q}$: & capiendo arcum $NP=D$, punctum P caderet in $\mathcal{Q}$. Si vero Area $AS\mathcal{Q}$ non exacte tempori reſpondeat, punctum P cadet ſupra vel infra $\mathcal{Q}$, prout Area $AS\mathcal{Q}$ major ſit vel minor vera Area quæ tempori reſpondeat. Sit ea ASq & in Cq cadat perpendicularis SH: erit per hactenus demonſtrata $SH=Nq$. At eſt $SF=NP$, unde erit $SH-SF$ vel $SF-SH$, hoc eſt fere $HE=qP$ $\mathcal{Q}P-\mathcal{Q}q$ vel $=\mathcal{Q}q-\mathcal{Q}P$: Et ſi angulus $\mathcal{Q}Cq$ ſit parvus, erit $CH : C\mathcal{Q} :: HE : \mathcal{Q}q :: \mathcal{Q}P-\mathcal{Q}q : \mathcal{Q}q$, unde $C\mathcal{Q}+CH : C\mathcal{Q} :: \mathcal{Q}P : \mathcal{Q}q$, cum arcus $A\mathcal{Q}$ eſt quadrante minor. At cum is eſt quadrante major, erit $C\mathcal{Q}-CH : C\mathcal{Q} :: \mathcal{Q}P : \mathcal{Q}q$. Et ſimiliter cum arcus $B\mathcal{Q}$ eſt quadrante minor, erit $C\mathcal{Q}-CH : C\mathcal{Q} :: \mathcal{Q}P : \mathcal{Q}q$.

Si angulus $AC\mathcal{Q}$ vel $BC\mathcal{Q}$ parvus ſit, *h. e.* ſi Planeta prope Apſides verſetur, erit ut $CA \pm CS : CA :: \mathcal{Q}P : \mathcal{Q}q$.

Fiat ut CS ad $C\mathcal{Q}$ ita Radius R ad longitudinem quandam L, erit $C\mathcal{Q}=\frac{CS \times L}{R}$. Eſt vero Radius ad coſinum anguli $AC\mathcal{Q}$ ut SC ad CF vel CH (ſunt enim CH & CF fere æquales) quare erit $CH=\frac{SC \times cof.AC\mathcal{Q}}{R}$, adeoque $\mathcal{Q}P : \mathcal{Q}q :: \frac{CS \times L + CS \times cof.\ AC\mathcal{Q}}{R} : \frac{CS \times L}{R} :: L + cof.\ AC\mathcal{Q} : L$, cum arcus $A\mathcal{Q}$ ſit quadrante minor. At ſi $A\mathcal{Q}$ ſit quadrante major, erit $\mathcal{Q}P : \mathcal{Q}q :: L - coſin.\ AC\mathcal{Q} : L$.

Atque hac ratione ſi capiatur utcunque arcus $A\mathcal{Q}$, qui aliquantiſper minor ſit aut major vero, invenietur exinde arcus $\mathcal{Q}q$ huic addendus aut demendus, qui facit ut Area ASq ſit quam proxime tempori proportionalis. Et ſi loco $A\mathcal{Q}$ capiatur arcus Aq, & inſtituatur proceſſus priori ſimilis, invenietur alius Aq, qui ſimiliter eundem repetendo proceſſum dabit alium Aq, atque ſic quantumvis proxime ad veritatem accedere licebit.

Invento angulo ACq, facile habebitur angulus ASq, cum in triang. qCS dentur latera Cq & CS & angulus qCS. Dabitur exinde angulus CSq cujus tangens diminuendus eſt in ratione axis minoris Ellipſeos ad majorem, ut tandem habeatur tangens anguli ASP. Vel ſic forte facilius inveſtigatur angulus ASP. Sit F numerus qui exprimit longitudinem CS in partibus qualium $C\mathcal{Q}$ eſt 100000: a puncto q ad axem demittatur perpendicularis qr, qui erit ſinus arcus dati Aq, & erit Cr ejuſdem coſinus & Sr = ſummæ vel differentiæ rectarum Cr, CS, hoc eſt $Sr = F \pm$ coſin. $AC\mathcal{Q}$: adeoque in rectangulo triangulo rSq, datis Sr, rq, invenietur angulus rSq. Hinc ſi in unam ſummam addantur ſinus Log. ang. ACq, complementum Arithmeticum Logarithmi Sr, & Logarithmus rationis axis minoris Ellipſeos ad majorem dabitur Tangens anguli ASP.

Tanta

Tanta autem eſt hujus methodi facilitas ut ea exemplis magis quam ulteriori explicatione indigeat; adeoque licebit eam in motibus Planetæ Martis experiri; in cujus orbita, ſecundum Tabulas *Carolinas*, Excentricitas eſt ad diſtantiam mediam ut 14100 ad 152369, adeoque Logarithmus arcus B, qui æqualis eſt rectæ SC, erit 0,7244451. Erit etiam in hoc exemplo L partium 1080631 qualium Radius eſt 100000: Inveniendus ſit angulus $AC\mathfrak{Q}$ cum motus medius, ſeu arcus tempori proportionalis ab Aphelio computatus, ſit unius Gradus. Quoniam CS ſit hic fere pars decima ipſius CA, pono Arcum $A\mathfrak{Q}$ eſſe 0,9 grad, decima ſcil. parte minorem motu medio. Addatur ſinus Logarithmicus arcus $A\mathfrak{Q}$ ad Log. B, & fit ſumma $8,9205471 =$ Log. numeri 0.083281 qui numerus exprimit arcum æqualem rectæ $SF = NP$. Et ſi arcus $A\mathfrak{Q}$ eſſet recte aſſumptus, foret $AN - NP = A\mathfrak{Q}$, & $\mathfrak{Q}P = 0$. At hic eſt $\mathfrak{Q}P = 0,016719$, a quo ſi auferatur ejus pars undecima, cum AS ſuperat AC undecima circiter ipſius parte, reſtabit $\mathfrak{Q}q = 0,0152$; qui additus ad $A\mathfrak{Q}$ dat $Aq = 0.9152$, qui ne milleſima gradus parte a vero Aq differt. Sit ſecundo Arcus AN ſeu motus medius $= 2$ gr. Pono $A\mathfrak{Q} = 1,83$ prioris Aq fere duplum, & ad ejus ſinum Log. addatur Log. B. erit ſumma $9.2286997 =$ Log. numeri 0,16931, unde erit $\mathfrak{Q}P$ 0,00069; a quo ſi ſubducatur ejus pars undecima, fit $\mathfrak{Q}q = 0,00063$, & $Aq = 1,83063$, qui ne decies milleſima gradus parte a vero Aq diſcrepat. Eodem modo ſit motus medius ſeu arcus tempori proportionalis grad. 3. Fiat arcus $A\mathfrak{Q}$ $2,745 = 1,83 + 0,915$, & ad ejus ſinum Log. addendo Log. B, habebitur Log. numeri $0,25392 = NP$, & $AN - NP = 2,74608$, adeoque $\mathfrak{Q}P = 0,00108$, unde $\mathfrak{Q}q$ fere $= 0,001$ & $Aq = 2,746$. Sic unica duorum Logarithmorum additione invenietur arcus Aq, qui erit verus ad gradus partes milleſimas.

Si jam non gradatim ſed per ſaltum pergendo, inveniendus ſit angulus ACq, cum motus medius eſt grad. 45. Pono arcum $A\mathfrak{Q}$ eſſe graduum 40, & ad ſinum ejus Logarithmicum addendo Log. B fit ſumma $0.5325125 =$ Log. numeri 3,4081; qui numerus a 45 ſubductus relinquit $AN - NP = 41,5919$, cujus exceſſus ſupra arcum $A\mathfrak{Q}$ eſt 1,5919. Unde ſi fiat ut $L +$ coſ. $AC\mathfrak{Q}$ ad L ita 1,5919 ad alium, invenietur arcus $\mathfrak{Q}q$ eſſe graduum 1,4865, adeoque $Aq = 41,4865$, qui non multum ſupra milleſimam gradus partem a vero differt. Verum abſque hac proportione inveniri poteſt Aq, capiendo novum arcum $A\mathfrak{Q}$, qui ſit aliquantulum minor quam $AN - NP$, eidem tamen fere æqualis; ſcil. ſit $A\mathfrak{Q} = 41.50$, & addendo Log. datum B ad ejus ſinum Log. habebitur alter $NP = 3.5131$, qui ab AN ſubductus dat 41,4869 pro novo Aq: & hic arcus minore labore eruitur, & aliquanto propius ad verum accedit quam prior Aq.

Poſt inventum Aq correſpondentem motui medio 45°, rurſus gradatim pergendo, unica duorum Logarithmorum additione, habebitur Aq, ad omnes motus medii gradus ſubſequentes. Nempe cum motus medius ſit grad. 46, pono $A\mathfrak{Q} = 42,40$. & addendo ejus ſinum Log. ad conſtantem B, fiet $AN - NP = 42,4249$; cui arcui ſi novus $A\mathfrak{Q}$ æqualis ponatur, habebitur Aq, qui ne milleſima gradus parte a vero Aq diſcrepabit. Sic cum motus medius ſit 47°, pono $A\mathfrak{Q} = 43,36 =$

priori Aq + incremento istius arcus pro uno gradu motus medii, & addendo ejus sinum Log. ad Log. B, fit summa = Log. numeri 3,6402, qui ab AN subductus relinquit $AN-NP=43,3598$ = novo Aq, qui circiter gradus parte decies millesima a vero Aq discrepat.

Si omissis gradibus intermediis inveniendus esset arcus Aq, cum motus medius sit gr. 100. Pono $A\mathfrak{Q}$ grad. 96, & addendo ejus sinum Log. ad Log. B, fit summa = Log. numeri 5,273 unde $AN-NP=94,727$. Itaque pono secundo $A\mathfrak{Q}=94,72$ & addendo ejus sinum ad Log. B, habebitur Log. numeri 5,285, qui ab AN subductus dat $AN-NP=94,715=Aq$ quam proxime. Similiter si motus medius sit grad. 101, pono $A\mathfrak{Q}$ esse 95,71, cujus sinus Log. ad Log. B additus dat Log. numeri 5,2756; quo numero ab 101 sublato, restabit $AN-NP=95,7244=Aq$. Atque hac ratione, dato motu medio, si gradatim fiat processus, habebitur angulus ad centrum per unicam tantum duorum Logarithmorum additionem; quorum unus, qui constans est, in charta seorsim servandus, quo labori sæpius eundem exscribendi parcatur.

Transeamus jam ad Orbitam alterius speciei, talem nempe ut distantia Aphelii sit ad distantiam Perihelii ut 70 ad 1; qualis fere est istius Cometæ Orbita quem Periodum suam annis $75\frac{1}{2}$ complere primus deprehendit Sagacissimus Astronomus & Geometra D. *Edmundus Halleius*, Geometriæ Professor *Savilianus*. In hac Orbita erit AC vel $C\mathfrak{Q}$ partium 35,5; & CS 34,5 qualium SB est una. Et inveniendus est arcus Bq, cum motus medius est gradus pars centesima. Quoniam media distantia trigesies & quinquies circiter superat distantiam minimam, pono $B\mathfrak{Q}=0,35$, cum motus medius est 0,01. In hac Orbita invenitur constans Log. $B=1.7457133$. Hic itaque Log. ad sinum Log. arcus 0,35 additus dat Log. numeri 0,34013, qui ad arcum 0,01 additus erit = 0,35013. Si hæc summa esset æqualis 0,35, arcus $B\mathfrak{Q}$ esset recte assumptus: sed differentia est 0,00013. Unde quoniam CB est ad SB ut 35,5 ad 1, multiplicetur differentia 0,00013 per 35,5 & prodibit $\mathfrak{Q}q=0,004615$; unde erit arcus $Bq=0,354615$, qui vix per partes tres decies millesimas a vero discrepat.

Sit secundo motus medius 0,02, & ponatur $B\mathfrak{Q}$ esse 0,71. Ad ejus sinum Log. addendo Log. B, fit summa = Log. numeri 0,68998, unde $BN+NP=0,70998$, adeoque arcus assumptus $B\mathfrak{Q}=0,71$ nimius fuit: & est differentia = 0,00002, quæ si per 35,5 multiplicetur, & productus a $B\mathfrak{Q}$ subducatur, restabit $Bq=0,7092$, vix gradus parte decies millesima a vero aberrans.

Sit motus medius 0,03. Ponatur $B\mathfrak{Q}$ esse 1°,06: addendo ejus Log. sin. ad Log. B, fit summa = Log. numeri 1,03008, cui si addatur $BN=0,03$, fit summa 1,06008, qui numerus major est quam $B\mathfrak{Q}$; quare si differentia 0,00008 per 35,5 multiplicetur & ad $B\mathfrak{Q}$ addatur, erit $Bq=1,06284$. Similiter cum motus medius sit 0,04, pono $B\mathfrak{Q}=1°,40$ & invenio $NP=1,3604$; ad quem numerum addendo $BN=0,04$ fit summa = 1,4004 qui superat 1,40 per 0,0004. Multiplicetur hæc differentia per 35,5 & productus 0,01420 erit æqualis $\mathfrak{Q}q$, unde $Bq=1,41420$. In hisce omnibus errores sunt admodum exigui, & raro millesimam gradus partem transcurrentes.

In-

Inveniendus sit jam arcus $B q$; cum motus medius sit æqualis uni gradui. Pono $B Q = 20°$, & addendo ejus sinum Log. ad Log. B, habebitur Log. numeri 19,045; cui addendo $B N = 1°$, summa 20,045 superat 20 per ,045: Et cum in hoc casu L—cosin. $B Q$ est ad L ut 1 ad 11.5 fere, multiplico differentiam ,045 per 11,5, & productus ,5175 ad $B Q$ additus facit 20,5175. Pono igitur secundo $B Q = 20,51$, & prodibit, similiter ut in præcedentibus, $N P = 19,5092$; cui addendo $B N$ fit summa 20,5092, quæ minor est quam $B Q$: unde si differentia 0,0008 multiplicetur per 11,5, & productus 0,0092 substrahatur a $B Q$, restabit $B q = 20,5008$.

Sit denique motus medius æqualis duobus grad. Pono $B Q$ grad. 30, & invenitur $N P = 27,84$, cui addendo gradus duos, summa, 29.84 minor est quam 30; & si multiplicetur differentia 0,16 per 6,3 (nam L—cos. $B Q$ est ad L ut 1 ad 6,3 fere) fiet 1,008 $= Q q$; adeoque hic arcus a $B Q$ subductus dat $B q = 28,982$. Ut vero corrigatur $B q$, assumo secundo $B Q = 29°$, & simili processu invenietur $B q = 28,9672$.

XI. Plurima sunt maxime quidem paradoxa, omnemque fidem apud vulgus superantia, quæ tamen adhibitis Mathematicarum Scientiarum principiis levi negotio enodantur. Ac sane nullum Problema magis arduum & difficile videbitur; quam est Solis a Terra distantiam vero proximam determinare: quod tamen obtentis accuratis quibusdam observationibus, ad electa & prævisa tempora peractis, non multo opere efficietur. Id quod inclytæ huic Societati, quam immortalem fore auguror, in hac dissertatione ob oculos ponere libet, ut junioribus nostris Astronomis, quibus forsan hæc observare ob minorem ætatem obtingere potest, viam præmonstrem, qua immensam Solis distantiam intra quingentesimam sui partem rite dimetiri poterint.

A new way to find the Sun's Parallax, by Dr. Halley, *n.* 348. *p.* 454.

Notum est Solis a Terra distantiam a diversis Astronomiæ authoribus diversam fingi, prout cuique ex conjectura probabile visum est; a *Ptolemæo* quidem ejusque asseclis, uti & *Copernico* & *Tychone Brahæo*, *Terræ* semidiametris mille & ducentis, *Keplero* ter mille quingentis fere. *Ricciolus* distantiam *Keplerianam* duplicat, quam tamen *Hevelius* dimidio tantum auget. At vero visis in *Solis* disco ope Telescopii Planetis *Venere* & *Mercurio* mutuato fulgore nudatis, tandem compertum est Planetarum diametros visibiles multo minores esse quam eatenus haberentur; *Venerisque* Semidiametrum e *Sole* visam, non nisi quartam minuti primi partem vel quindecim secunda subtendere; *Mercuriique* semidiametrum, ad mediam ipsius a *Sole* distantiam, sub angulo decem tantum secundorum conspici; atque sub eodem etiam *Saturni* semidiametrum e *Sole* videri. *Jovis* autem Planetarum maximi semidiametrum non nisi tertiam minuti primi partem apud *Solem* subtendere. Unde, servata analogia, nonnullis e modernis Astronomis visum est, *Terræ* quoque semidiametrum e *Sole* conspectam, medio loco inter *Jovis* majorem & *Saturni* & *Mercurii* minorem angulum subtendere, *Venerisque* æqualem, nempe quindecim secundorum: adeoque *Solem* a *Terra* quatuordecim fere millibus semidiametrorum *Terræ* distare. Iisdem autem Authoribus, aliud argumentum paulo ampliavit hanc distantiam: quoniam enim *Lunæ* diameter paulo major

eſt quarta parte diametri *Terræ*, ſi Parallaxis *Solis* ponatur quindecim minutorum ſecundorum, fieret *Lunæ* corpus corpore *Mercurii* majus, Planeta ſcil. ſecundarius primario major; quod concinnitati Syſtematis mundani contrariari videretur. E contra vero *Venerem* inferiorem & Satellitio deſtitutam, majorem eſſe *Terra* noſtra ſuperiori & tam inſignem comitem nacta, vix concedere videtur eadem concinnitas. Ut itaque medio loco incedamus, ponatur *Terræ* ſemidiameter e *Sole* viſa, ſeu quod idem eſt, *Solis* Parallaxis horizontalis, duodecim ſecundorum cum ſemiſſe: unde *Luna* minor erit *Mercurio* & *Terra Venere* major; ac proveniet *Solis* a *Terra* diſtantia ſedecies mille cum quingentis *Terræ* ſemidiametris proxime. Huic autem diſtantiæ in præſentiarum aſſenſum præbeo, uſq; dum Experimento quod proponimus quanta ſit certius conſtet. Nec moror authoritatem quantumvis gravem eorum qui *Solem* ultra hos terminos in immenſum evehunt, freti obſervationibus vibrantis Penduli, determinandis his angulorum minutiis, uti videtur, haud ſatis fidis: ſaltem hac methodo tentanti Parallaxis aliquando nulla, aliquando etiam negativa occurret; hoc eſt diſtantia vel infinita fiet, vel infinito major: quod abſurdum. Et, ut verum fatear, minuta ſecunda vel etiam dena ſecunda inſtrumentis quantumvis affabre factis certo diſtinguere vix homini datum eſt; atque adeo minime mirandum, ſi tantorum Artificum multos & ingenioſos conatus hactenus eluſerit rei ipſius maxima ſubtilitas.

Dum autem ante 40 fere annos, in Inſula *Sanctæ Helenæ*, ſyderum polum Auſtralem ambientium obſervationibus operam darem; contigit mihi *Mercurium* ſub *Solis* diſco tranſeuntem omni adhibita diligentia obſervare: quodque mihi præter ſpem feliciter ſucceſſit, momentum quo *Mercurius* ingrediens *Solis* limbum interius contingere viſus eſt, pariterque momentum quo egrediens limbum Solis ſtrinxit, facto angulo contactus interioris, Tubo optimo viginti quatuor pedum accuratiſſime obtinui. Unde pro comperto habui intervallum quo *Mercurius* totus intra *Solis* diſcum tum temporis apparuit, etiam abſque errore unius minuti ſecundi temporis: Nam filum luminis Solaris, inter limbum planetæ obſcurum & *Solis* lucidum interceptum, quantumvis tenue in oculos incurrere viſum eſt; & in ictu oculi, denticulus in limbo *Solis* a *Mercurio* ingrediente factus evaneſcere, uti ab egrediente factus quaſi momento incipere. Hoc autem perſpecto ſtatim intellexi *Solis* Parallaxin ex hujuſmodi obſervationibus rite concludi poſſe, ſi modo *Mercurius Terris* vicinior majorem haberet parallaxin a *Sole*; etenim hæc parallaxium differentia tantilla eſt, ut ſemper minor ſit ipſa Solari quam quærimus; proinde *Mercurius*, licet frequenter intra *Solem* videndus, huic noſtro negotio vix ſatis aptus habebitur.

Reſtat itaque *Veneris* tranſitus per *Solis* diſcum, cujus parallaxis quadruplo fere major Solari, maxime ſenſibiles efficiet differentias, inter ſpatia temporis quibus *Venus Solem* perambulare videbitur, in diverſis *Terræ* noſtræ regionibus. Ex his autem differentiis debito modo obſervatis, dico determinari poſſe *Solis* parallaxin etiam intra ſcrupuli ſecundi exiguam partem. Neque alia inſtrumenta poſtulamus præter *Teleſcopia*

teſcopia & *Horologia* vulgaria ſed bona: & in Obſervatoribus non niſi fides & diligentia, cum modica rerum Aſtronomicarum peritia deſiderantur. Non enim opus eſt ut Latitudo Loci ſcrupuloſe inquiratur, nec ut Horæ ipſæ reſpectu meridiani accurate determinentur: ſufficit, Horologiis ad Cœli revolutiones probe correctis, ſi numerentur tempora a totali Ingreſſu *Veneris* infra diſcum *Solis*, ad principium Egreſſus ex eodem; cum ſcilicet primum incipiat Globus *Veneris* opacus limbum *Solis* lucidum attingere; quæ quidem momenta, propria experientia novi, ad ipſum ſecundum temporis minutum obſervari poſſe.

Ob leges autem motuum admodum arctas, rariſſime intra *Solis* orbem conſpicitur *Venus*, ac per plus quam centum & viginti annorum decurſum, ne ſemel quidem ibidem videbitur; nempe ab anno 1639. (cum præclaro Juveni *Horroxio* noſtro, eique primo & ſoli a rerum conditu, jucundiſſimum hoc ſpectaculum obtigit,) uſque in annum 1761, quo juxta Theorias quas hactenus cœlo conformes experimur, Stella *Veneris* iterum ſubtercurret *Solem*, *Maii* 26. mane; * ita ut *Londini*, hora fere ſexta matutina in medio diſci Solaris expectanda ſit, nec niſi quatuor minutis centro *Solis* Auſtralior. Duratio autem hujus tranſitus erit octo fere Horarum, nempe a ſecunda uſque in decimam fere matutinam. Atque adeo ingreſſus minime *Anglis* conſpicuus erit: cum autem Sol tum temporis occupaturus ſit 16. *Geminorum* gradum, viginti tres ferme gradus in Boream declinans; per totam quaſi Zonam frigidam Septentrionalem inocciduus conſpicietur: ac proinde qui littus *Norwegiæ* incolunt ultra Urbem *Nidroſiam*, quam *Drontem* vocant, uſque ad Promontorium ejus *Boreale*, *Venerem* *Solis* diſcum ſubingredientem obſervare poterunt; ac fortaſſe *Scotis* Borealioribus & Inſulæ *Hetlandiæ*, olim *Thylen* dictæ, incolis, in oriente Sole ingreſſus ille conſpici poterit. Quo tempore vero *Venus* *Solis* centro proxima erit, *Sol* verticalis erit ſupra littora Borealia ſinus *Gangetici*, vel potius regni *Peguani*; ac proinde in Regionibus circumvicinis, cum *Sol* in ingreſſu *Veneris* quatuor fere horis diſtabit ad ortum, & in egreſſu totidem fere ad occaſum, accelerabitur motus apparens *Veneris* intra *Solem* duplo fere parallaxeos horizontalis *Veneris* a *Sole*; quia *Venus* tunc ab ortu in occaſum fertur retrograde, interea dum oculus ad *Terræ* ſuperficiem poſitus in contrarias partes ab occaſu in ortum gyratur.

* *Vide Phil. Tranſact.* No. 193. *Abr.* Vol. I. p. 436.

Poſita autem parallaxi *Solis*, uti diximus, duodecim ſecundorum cum ſemiſſe, erit parallaxis *Veneris* 43^um ſecundorum; & ſublata parallaxi *Solis*, reſtabit ſaltem ſemiminutum pro parallaxi Horizontali *Veneris* a *Sole*, ac proinde dodrante ſaltem minuti promovebitur *Veneris* motus a parallaxi illa, interea dum *Solis* diſcum percurrit, in iis ſcilicet Poli altitudinibus quæ Tropico vicinæ ſunt; atque adhuc amplius in vicinia Æquatoris. *Venus* autem tum temporis ſatis accurate quatuor minuta prima ſingulis horis intra *Solem* conficiet; ac propterea dodranti minuti undecim ſaltem temporis minuta prima competunt, quibus duratio *Eclipſeos* hujus *Venereæ* ob parallaxin contrahetur. Atque ex hac contractione ſola liceret de parallaxi quam quærimus tuto pronunciare, ſi

modo

modo darentur *Solis* diameter *Venerisque* Latitudo in minimis accuratæ; quas tamen ad computum postulare, in re tam subtili, haud integrum est.

Procuranda est igitur alia observatio, si fieri possit, in locis illis ubi medium *Solis* occupat *Venus* in ipso Medinoctio; nempe sub Meridiano priori opposito, *i. e.* sex quasi horis vel 90 gradibus *Londino* occidentaliore, & ubi *Venus* paulo ante occasum *Solem* subintrat, paulo post ortum exit; id quod fiet in dicto Meridiano, sub altitudine Poli Borei quinquaginta sex circiter graduum: hoc est, in eo Sinu qui *Hudsoni* dicitur, ad Portum ejus cui nomen *Nelsoni* inditum. In locis enim huic circumvicinis parallaxis *Veneris* durationem transitus protrahet, & sex saltem temporis minutis longiorem efficiet; quia dum *Sol* ab occasu in ortum sub Polo tendere videtur, ea loca in disco *Terræ* motu contrario in occasum ferri videbuntur, hoc est motu cum motu proprio *Veneris* conspirante; proinde tardius moveri videbitur *Venus* intra *Solem*, ac cum diuturniore mora discum ejus pertransire.

Si itaque in utroque loco hic transitus ab Artificibus idoneis contigerit debite observari, manifestum est totis septendecim minutis longiorem futuram esse moram in portu *Nelsoni* observabile, quam quæ apud *Indos* orientales expectanda est: nec multum refert an ad Fortalitium *Sancti Georgii* vulgo *Maderas* dictum, vel ad *Bencoulam* in litore occiduo Insulæ *Sumatræ* prope æquatorem capiatur observatio, si *Anglis* tum temporis hæc studia curæ fuerint. Si vero *Gallis* his rebus invigilare placuerit, non incommode apud *Poudechery* se sistet Observator in litore *Sinus Gangetici* occidentali, sub altitudine Poli duodecim fere graduum. *Batavis* autem celeberrimum *Bataviæ* suæ Emporium Observatorium huic negotio satis aptum ministrat, si modo illis etiam animus fuerit hac in parte cœlorum scientiam promovere. Ac sane vellem diversis in locis ejusdem Phænomeni observationes a pluribus institui, tum ad majorem adstruendam ex consensu fidem, tum ne Nubium interventu frustraretur singularis Spectator, eo spectaculo quod nescio an denuo visuri sunt hujus & subsequentis seculi Mortales; & a quo pendet Problematis nobilissimi & aliunde inaccessi solutio certa & adæquata. Curiosis igitur syderum scrutatoribus, quibus, nobis vita functis, hæc observanda reservantur, iterum iterumque commendamus ut, moniti hujus nostri memores, observationi peragendæ strenue totisque viribus incumbant; iisque fausta omnia exoptamus & vovemus, præprimis ne nubili cœli importuna obscuritate exoptatissimo spectaculo priventur; utque tandem Orbium cœlestium magnitudines intra arctiores limites coercitæ in eorum gloriam famamque sempiternam cedant.

Diximus autem hac ratione Solis Parallaxin intra quingentesimam sui partem investigari posse, id quod nonnullis mirum sine dubio videbitur. Veruntamen si in utroque e locis nuper designatis accurata habeatur observatio; jam monstravimus, totis septendecim minutis differre inter se durationes *Eclipseωn* harum *Venerearum*, ex Hypothesi scilicet quod Solis parallaxis fuerit duodecim cum dimidio minutorum secundorum. Quod si major vel minor reperiatur ex observatione hæc differentia,

tia, in eadem fere ratione major vel minor erit Solis parallaxis. Cumque 17 minuta prima temporis competant duodecim secundis cum dimidio parallaxeos Solaris; pro unoquoque parallaxeos minuto secundo, orietur differentia plusquam 80 secundorum minutorum temporis; adeoque si habeatur differentia hæc intra bina secunda vera & comprobata, intra quadragesimam partem unius secundi minuti constabit quanta sit Solis Parallaxis; ac proinde distantia ejus determinabitur intra quingentesimam sui partem, saltem si parallaxis non minor reperiatur ea quam supposuimus: quadragies enim duodecim cum dimidio fiunt quingenti.

Hactenus Astronomice doctis satis superque rem indicasse mihi videor, quos etiam monitos velim, me in hoc argumento, Latitudinis Planetæ rationem non habuisse, tum ad vitandas calculi intricatioris molestias, conclusionem etiam minus evidentem reddituras; tum ob motum Nodorum *Veneris* nondum compertum, nec nisi ex hujusmodi corporalibus Planetæ cum *Sole* Conjunctionibus rite determinandum. Non enim conclusum est *Venerem* quatuor minuta infra *Solis* centrum transituram, nisi ex Hypothesi quod Planum Orbitæ *Veneris*, in Sphæra stellarum fixarum immobile, Nodos suos iisdem in locis habiturum sit, ubi anno 1639 inventi sunt. Quod si tramite Australiori transeat anno 1761, liquido patebit Nodos regredi; si vero Borealiori, progredi inter Fixas; idque in ratione $5\frac{1}{2}$ *min.* in centum annis *Julianis*, pro unoquoque minuto, quo via *Veneris* tum temporis plus vel minus distabit a *Solis* centro quam dictis quatuor minutis. Differentia autem inter durationes harum Eclipsium paulo minor fiet septendecim minutis, ob Latitudinem *Veneris* Australem; major vero futura, si procedentibus Nodis, ad Boream centri *Solem* transierit.

In eorum autem gratiam, qui cum observandis syderibus oblectentur, nondum tamen integram Parallaxium doctrinam hauserint, libet Schemate simulque Calculo paulo accuratiore, rem plenius exponere.

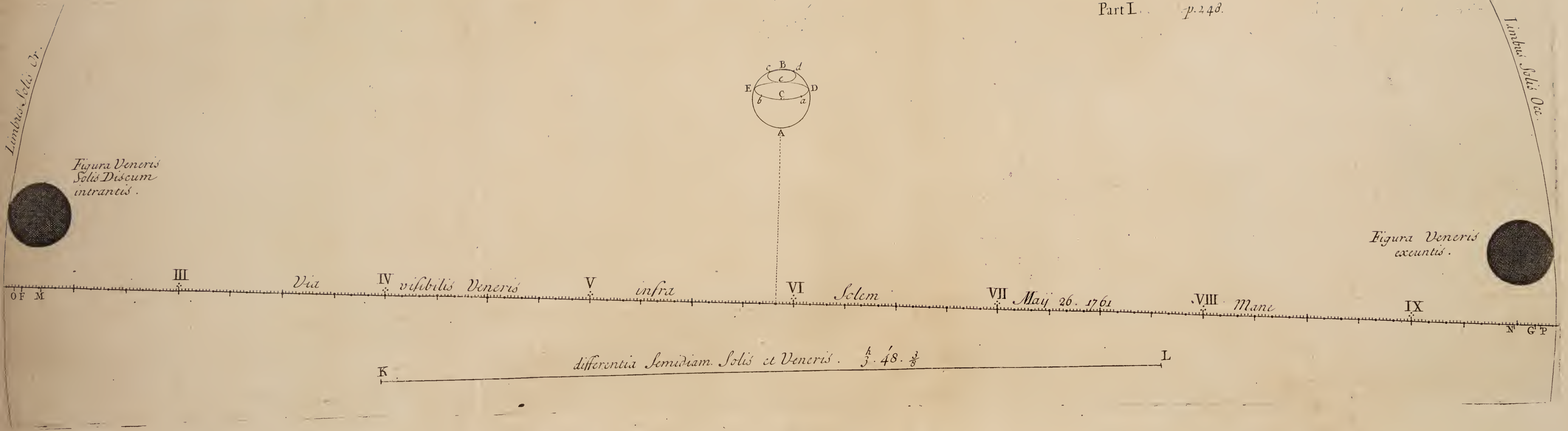
Ponamus igitur, anno 1761, *Maii* 25°. 17^h. 55′. *Londini*, *Solem* occupaturum ♊ 15°. 37′. ac proinde ad centrum ejus Eclipticam tendere in Boream angulo 6°. 10′. *Veneris* autem visibilem intra *Solis* discum Viam tum temporis descendere in Austrum, facto angulo cum Ecliptica 8°. 28′: proinde via *Veneris* tendet parum in Austrum respectu æquatoris, intersecans declinationis parallelas angulo 2°. 18′. Ponamus etiam *Venerem* ad dictum tempus *Solis* centro proximam fore, ac ab eodem quatuor minutis distare ad Austrum; singulisque horis etiam quatuor minuta prima intra *Solem* motu retrogrado describere. Erit autem *Solis* Semidiameter 15′. 51″. proxime, *Veneris* vero 0′. 37″$\frac{1}{2}$. Ac supponamus, experimenti gratia, differentiam parallaxium Horizontalium *Veneris* & *Solis*, quam quærimus, 0′. 31″. esse, qualis ex supposita *Solis* Parallaxi 0′. 12″$\frac{1}{2}$ elicitur. Describatur itaque centro *C* circellus *AEBD*, cujus semidiameter sit 0′. 31″. discum Terræ repræsentans, & in eo Ellipses parallelorum 22 & 56 *grad.* Latitudinis Borealis, modo jam ad construendas Eclipses Solares ab Astronomis usitato, ut *D a b E*, *c d e*: sit autem *B C A* Meridianus in quo *Sol*; ad quem inclinetur recta *F H G* Viam *Veneris* designans angulo 2°. 18′. quæque distet a centro *C* 240 partibus

tibus qualium *B C* eſt 31; & de *C* cadat recta *C H* ipſi *F G* perpendicularis. Plate 5. Ac poſito planeta in *H* ad $17^h. 55'$, vel $5^h. 55'$ mane, dividatur recta *F H G* in ſpatia Horaria III. IV, IV. V, V. VI, *&c.* ipſi *C H*, hoc eſt quatuor minutis æqualia. Fiat etiam recta *K L*, æqualis differentiæ apparentium Semidiametrorum *Solis* & *Veneris* ſive $15'. 13''\frac{1}{2}$. Et Circulus radio *K L*, centro vero quolibet puncto intra circellum Diſci *Terræ* deſcriptus, occurret rectæ *F G* in puncto denotante quota hora *Londini* numerabitur, cum in eo *Terræ* ſuperficiei loco, qui ſumpto in diſco puncto ſubjacet, *Venus* angulo contactus interioris *Solis* limbum continget. Ac ſi centro *C* radio *K L* deſcriptus circulus occurrat ipſi *F G* in punctis *F* & *G* erunt recte *F H*, *H G* $=14'. 41''$ id quod percurrere videbitur Venus tribus horis cum 40 Min. Cadet igitur *F* in $II^h. 15'$. *Londini*; *G* vero in $IX^h. 35'$ mane. Unde manifeſtum eſt quod, ſi Terræ magnitudo, ob immenſam diſtantiam, quaſi in punctum evaneſceret: vel ſi motu diurno deſtituta *Solem* haberet eidem puncto *C* ſemper verticalem, Eclipſeos hujus Mora integra per ſeptem horas cum triente duraret. Verum Terra interea motu motui *Veneris* contrario gyrata per 110 *grad.* Longitudinis ſuæ, ac proinde contracta dictæ moræ duratione, puta 12 Min. proveniet ea $7^h. 8'$. proxime, ſive 107 *grad.*

Jam in ipſo Meridiano *Venus Solis* centro proxima erit ad Oſtium orientale fluminis *Gangis*, ubi poli altitudo eſt 22 *grad.* circiter. Locus igitur ille utrinque æqualiter diſtabit a Sole, in momentis introitus & exitus planetæ, nempe $53\frac{1}{2}$ *grad.* ut ſunt puncta *a*, *b*, in parallelo majore *D a b E*. Erit autem Diameter *A B* ad diſtantiam *a b* ut quadratum Radii ad contentum ſub Sinubus $53\frac{1}{2}$ & 68 *grad.* hoc eſt, ut $1'.02''$ ad $0'.46''.13'''$; ac calculo rite inſtituto (quem ne Lectori tædio ſit, omittere præſtat) invenio quod circulus centro *a* & radio *K L* deſcriptus occurret rectæ *F H*, in puncto *M*, ad $II^h. 20'. 40''$; centro vero *b* deſcriptus occurret ipſi *H G* in *N*, ad $IX^h. 29' 22''$; horis ſcilicet *Londini* numeratis; proinde tota *Venus* intra *Solem* conſpicietur ad *Gangis* ripas, per $7^h. 8'.42''$. Recte igitur poſuimus durationem fore $7^h. 8'$; cum pars minuti hic nullius ſit momenti.

Aptato autem calculo ad *Portum Nelſoni*, invenio, quod Sole jamjam occaſuro, diſcum ejus ſubitura ſit Venus; ſtatim vero ab ortu ejus exitura ab eodem; Loco illo interea per Hemiſphærium a Sole averſum de *c* ad *d* tranſlato, motu motui *Veneris* conſpirante. Mora igitur *Veneris* intra *Solem* diuturnior fiet ob Parallaxin, puta quatuor minutis; ut ſit omnino $7^h. 24'$. ſive 111 *grad.* æquatoris. Cumque Latitudo Loci ſit 56 *gr.* erit ut Quadratum Radii ad contentum ſub Sinubus $55\frac{1}{2}$ & 34 *grad.* ita *A B* $= 1', 02''$ ad *c d* $=28''. 33'''$. Ac calculo rite peracto conſtabit, circulum centro *c* radio *K L* deſcriptum rectæ *F H* occurſurum in *O*, ad $II^h. 12' 45''$, centro vero *d* deſcriptum ipſi *H G* in *P*, ad $IX^h. 36'. 37''$. Quocirca duratio Moræ ad *Nelſoni* portum erit $7^h. 23'. 52''$; major ſcilicet quam ad oſtia *Gangis* totis $15'. 10''$ temporis, Quod ſi *Venus* abſque Latitudine tranſierit, fiet dicta differentia $18'. 40''$; Si vero quatuor minutis

Solis


Limbus Solis Or.
Limbus Solis Occ.
Figura Veneris Solis Discum intrantis.
Figura Veneris exeuntis.
c B d
E e D
b C a
A
O F M
III
Via
IV visibilis Veneris
V infra
VI Solem
VII May 26. 1761
VIII Mane
IX
N G P
K
differentia Semidiam. Solis et Veneris. 3h. 48'. 3/8
L

Solis centro fuerit Borealior, ad 21′. 40″ augebitur eadem differentia, multo major futura aucta Planetæ Latitudine Borea.

Londini autem, ex prædictis Hypothesibus, consequitur *Venerem* jam tum infra *Solem* ingressam orituram; & ad 9h. 37′ mane, in Egressu *Solis* limbum interius contacturam; ac denique non nisi hora 9h. 56′, orbem ejus integrum relicturam esse.

Iisdem etiam Hypothesibus constat *Venerem* extremum *Solis* limbum Boreum quasi centro suo stringere debere, Anno 1769. *Maii* 23°. 11h. 00′, ita ut, ob Parallaxin, in Borealibus *Norwegiæ* partibus, tota intra *Solem* inocciduum apparere poterit: dum in litoribus *Peruviæ* & *Chili*, vix exiguo sui segmento cadentis *Solis* disco quasi inequitare videbitur; uti in Insulis *Moluccis* earumque vicinia, oriente *Sole*. Quod si Nodi *Veneris* retrocedere reperiantur (ut ob nuperas quasdam observationes suspicio est) tum toto corpore intra orbem *Solis* ubique conspicua, maxima harum Eclipseon differentia argumentum Parallaxeos *Solaris* præbebit adhuc multo luculentius.

Quomodo autem ex observatis alicubi apud *Indos* Orientales, anno 1761. Ingressu & Egressu *Veneris*, & cum Exitu ejus apud Nos observabili collatis, eadem Parallaxis derivari poterit; aptando scilicet angulos Trianguli specie dati in trium Circulorum æqualium circumferentias, alia occasione docebitur.

XII. My Micrometer is not as usually to be put into a Tube, but is to measure the Species of the Sun on Paper (of any *Radius*) or to measure any part of it. By this means, I can easily, and very exactly, with the help of a fine Thread, take the Declination of a Spot at any time of the Day; and by my half Seconds Watch and a fine cross Hair, I can measure the distance of the Spots from the Sun's Eastern or Western Limb.

To use the Micrometer in Solar Observations, by Mr. Derham, *n.* 288. *p.* 1505.

This cross fine Hair should be set, not (as usually) at the exact focal distance from the Eye-Glass, but a little out of that distance, nearer the Object Glass, because the Shadow of the Star will thereby be much narrower, and appear more strong cross the Species of the Sun receiv'd on the Paper, which I observe, because it may be of good use in taking the Sun's Altitude, measuring his Diameter, *&c.*

XIII. *June* 15. 1703. between 4 and 5 *P. M.* I saw a Spot in the Sun, by placing a white Paper so far behind the Telescope of 6 Feet, as to give the Image of the Sun 9 Inches Diameter; the Spot was in the lower Right-hand Quadrant of the Sun's Disk, its form almost round, inclining to an Ellipsis, it was distant from the Limb of the Sun about 6 or 7 Minutes, its Diameter I judg'd to be about 10 or 12 Seconds; a little before the Sun set, I saw the Spot with a 16 Foot Telescope, and could perceive it environ'd with a Mistiness. The 16th I saw the Spot again about 2 in the Afternoon, and found it advanc'd nearer the Western Limb of the Sun. The 17th was cloudy. The 18th *P. M.* a little be-

Spots in the Sun, by Mr. St. Gray, *n.* 288. *p.* 1502.

fore 5, I ſaw the Spot with the 16 Foot Glaſs through thin Clouds, and found it very near the Limb of the Sun, little more then ½ a Minute, 'twas much contracted in breadth, ſo as to be 4 or 5 times longer than broad. The 19th I look'd for it, but could not ſee it, ſo I conclude it was either gone off the Disk of the Sun, or if it adhered to the Limb, the great Tremulation of the Atmoſphere hinder'd me from ſeeing it.

June the 26, 1703. in the Evening I look'd to ſee whether there were generated any new Spots in the Sun, but found none; but on the 27th about half an Hour after 8 in the Morning, by receiving the Sun's Image on white Paper from the 6 Foot Glaſs, I ſaw a Spot near the Vertical of the Sun towards the lower Limb; betwixt 9 and 10 I elevated the 16 Foot Tube, the Clouds now being of a convenient thickneſs to let me ſee the Sun without Prejudice to my Eyes, and found that this Spot was of a triangular form, and that it was accompanied with 2 other leſſer ones, as is expreſt in Fig. 45. the ſides of the great Spot were curvilinear, this with 2 leſſer ones made an Equicrural Triangle; at 4 in the Afternoon the Triangular Spot had a ſmall Fragment ſeparated from it, and it ſelf was now become Elliptical, the Spot *b. d.* was much augmented, but the Spot *e.* diminiſhed, and become longiſh, as in Fig. 46. At half an Hour after 5 the Fragment from the great Spot was it ſelf divided into 2, and the Spot *c.* was ſo narrow, as ſcarce to be ſeen, as at Fig. 47. At 6 a Clock, and 30 Minutes there was a ſmall Fragment ſeparated from the lower end of the great Spot, as at Fig. 48. At 7 a Clock the Spot *b.* was much encreas'd, but *c.* was vaniſh'd; the Obſervations made this Afternoon with the 16 Foot Glaſs, were when the Air was clear, and ſo to ſecure my Eye, the Eye Glaſs was ſmoak'd with a Wax-Candle.

Plate 3.

The 28th about 7 in the Morning, I ſaw that the great Spot was much augmented, but the leſſer ones that Yeſterday attended it, were vaniſh'd, and that there were two new ones generated at about 1½ Minutes diſtance from the great one below, and towards the Left hand of it the great one was a parallelogram, with a very black diagonal croſſing it, ſee Fig. 49. At 10 a Clock there was another diagonal croſſing the former, and the two leſſer Spots which before were longiſh, had now taken a round form, the Spot *c.* being much larger than the other at *b.* Fig. 50.

Solar Spots by Mr. Derham, *ib. p.* 1504.

XIV. The two Circles, Fig. 51, 52, repreſent the Sun's Disk, and N. the Northern part thereof, S. the Southern, E. the Eaſtern, and W. the Weſtern part. The place of the Spots, and the manner of their Appearance every Day, is repreſented with the Day of the Month on the Sun's Disk. But although the Figures of the Spots are done pretty exactly, yet their places on the Sun are not ſo; for being unprovided with convenient Inſtruments for the purpoſe, I could not exactly ſet off their Delineations, nor their diſtances from the Sun's Limb, but was forc'd to repreſent them only as well as I could, by taking the Species of the Sun upon Paper through a Teleſcope, and ſo marking out their places.

The

The Spots ſeemed ſtrong enough to have laſted another, or more Revolutions, but none have been viſible ſince the ſixth of this Month, on which Day I think I had a glimpſe of one on the Sun's Weſtern Limb, about 7 of the Clock in the Morning.

The Appearances of the Spots being in the Figures above ſet with every Day of the Month, I need ſay but little, only take notice of a few things that the Figures do not ſo well expreſs.

The Spot in Fig. 51. was as repreſented, *viz.* firſt round and ſtrong, afterwards long, and with a *Nucleus*. The very ſame Spot (I doubt not) I ſaw again on the Sun's Eaſtern ſide on *July* 5, but very faint, ſmall and long (as in Fig. 52.) ſo as to be but juſt diſcernible. On *July* 6 it quite diſappeared, both through my Tubes, and on Paper, which is better. *Plate* 3.

The Spots in Fig. 2. had theſe remarkable Appearances and Variations. On *June* 28. viewing the Sun towards Evening, I eſpy'd a large ſtrong dark Spot, with two or more glaring *Nubeculæ* behind it, ſomewhat like the Repreſentation in the Figure. Theſe the next Day were become four ſtrong dark Spots, the foremoſt with a Tail to it, conjoyning the little Spot next it, as in the Figure. On *June* 30. I ſaw Spots, but it being a cloudy Morning, and I abſent from my Tubes in the Afternoon, the Repreſentation of them in the Figure is not exactly as they were. *July* 4. between two long Spots appeared ſomething like a round *Nubecula*, as in the Figure. The reſt as in the Figures.

The ſingle Spot in *June* may be ſeen to have paſſed above half over the Disk before I had notice of it. And that or ſome others were, I hear, ſeen in *May*; but I ſaw them not.

XV. On *Saturday, May* 15. 1703. As I was obſerving the Setting of the Sun, in order to examine my Clocks, there appeared two Suns, the Mock Sun ſeemed above the real one, which was then only five degrees above the Horizon. Whereupon I took a good 7 Foot Teleſcope, with a ſmall Aperture, and ſoon diſcovered a Solar Spot near the Sun's Center, which I deſigned to obſerve more exactly the Day following, but it proved Cloudy. *May* the 16th, *Sunday* no Sun-ſhine. —— *by Captain* Stanyan. *n.* 294. *p.* 1756.

Monday the 17th, at Six a Clock in the Morning I took the ſame Teleſcope, armed with a clouded Eye-glaſs, and immediately perceived that the Spot was advanced conſiderably towards the Sun's Weſtern Limb; it ſeem'd of a ſtrong conſiſtence, very Compact, reſembling a Face, and was diſtant by Noon from the Anteriour Limb of the Sun's Disk 61 Seconds of Time. See Fig. 54.

Tueſday the 18th, at Noon I found the Spot diſtant from the preceeding Limb 46 Seconds of Time. Fig. 54.

Wedneſday the 19th. At Noon I obſerved the Solar Spot to be moved within 33 Seconds of Time of his Weſtern Limb. Fig. 54.

Thurſday the 20th, at Noon the Spot was arrived within 21 Seconds of Time of the preceding Limb, and moving nearly in a ſtraight

Line: interfecting the Parallel of Declination paffing through the Sun's Center. Fig. 54.

Friday the 21ft, we had no Sun-fhine.

Saturday the 22d, at feven a Clock in the Morning I obfervedthe Solar Spot was advanced very near the Limb of the Snn's Disk. Fig. 54.

Sunday May 23d, at 6 in the Morning I faw the Spot, which by that time was gotten to the very edge of the Sun's Disk, refembling a barley Corn, lean and flender, and of a duskifh Colour, wanting only its own fhorteft Diameter of the Sun's Limb. At 8 a Clock I obferved it again: Alfo at 10, and at 12. At 2 I perceived it was flid into the very Circumference, and hardly Vifible, had I not had an Eye upon it all the Day long. At Four I examined the Sun's Body with my Eighteen Foot Glafs, which is a good one, but could not perceive the leaft Glimpfe of it; fo that about Three in the Afternoon it totally difappeared. Fig. 54.

On *Tuefday, June* 3. 1703. about fix in the Evening, I obferved with my Eighteen Foot Glafs four Spots in the Sun's Disk, environ'd with a Miftinefs, thicker on the Right-Hand than on the Left, fituated in the upper Left-Hand Quadrant, about the 12 part of the Sun's Diameter diftant from his neareft Limb. From the Cloud about them proceeded both ways five long curve Rays, of a yellower Colour than the Sun's Body. Thefe Spots I could never fee more, though I watch'd them for feveral Days together. Fig. 55.

On *Monday, June* the 7th. at three a Clock in the Afternoon I difcovered the fame Spot (to my thinking) that I faw go off the Sun's Disk on *May* the 23. re-entring the Sun's Face juft at the time and place that I expected it.

At four of the Clock, the Sun being extreamly clear, I mounted my Eighteen Foot Telefcope, through which the Spot appeared diftinct, but flender like a Spider, with an Elliptical Speckly Mift about it, and 5 or 6 light-coloured Streaks. It feem'd to me to be as it were divided near the Top, as in the Figure. Fig. 56.

Tuesday, June the 8th, at fix this Morning the Spot was very vifible, and I faw it trace again its former Path, coming in exactly where I expected; it kept its fhape, but thofe Lemon coloured Streaks difappeared, though it felf and the Mift about it grew bolder and broader vifibly, as it re-entred the Sun's Disk.

Wednefday, June the 9th, at five of the Clock this Evening, I obferved the Spot with the 18 Foot Glafs, but could not perceive it had altered its Shape, but advanc'd gradually over the Sun's Disk, as it had formerly done.

Thurfday, June the 10th, at Noon, the Sun fhining very bright, I had an Opportunity of being affured it was the fame Spot; I plainly faw it move over its former Path, and was then diftant from its neareft Limb 29 Seconds of Time. At five in the Evening I obferved its Shape (with my 18 Foot Tube) to be altered, appearing bigger and blacker than ever, as in the Scheme, Fig. 57.

Friday,

Friday, June the 11th was an ill Day for Obſervations. But I had a Sight on't with the 18 Foot Glaſs; it continued black and bold, as before.

Saturday, June the 12th, at 7 a Clock in the Morning, the Sun's Body being very clear, I ſaw the Spot through the 18 Foot Glaſs, retaining its former Shape.

Sunday, June the 13th, by this Day Noon the Spot was arrived at the ſame Point of the Sun's Disk that I found it in on *Monday* at Noon, *May* the 17th; which makes me inclinable to believe it was the very ſame Spot.

Monday, June the 14th, according to Rules received Yeſterday from Mr. *Flamſteed*, I meaſured the diſtance of the Spot from the next Limb of the Sun's Disk, which I found to be 45 Seconds of Time from the anteriour edge of the Sun's Body: And upon *Tueſday, May* the 18th, it was obſerved to be in the very ſame place of its Path, within a ſingle Second of Time. At 4 I obſerved it with my 18 Foot Glaſs, and perceived that it had altered its Shape, appearing as at Fig. 58. I received it on the Scheme, and it was diſtant from the preceding Limb 612 ſuch parts as the Sun's Semidiameter is 900.

Tueſday, June the 15th, at Noon the Solar Spot was diſtant 32 Seconds of Time from the leading Limb of the Sun's Disk, and covered the very place where the ſame Spot had been obſerved on *Wedneſday* the 19th of *May*.

Wedneſday, June the 16th, No Sun-ſhine.

Thurſday, June the 17th, No Sun-ſhine.

Friday, June the 18th, At Noon I obſerved the Solar Spot waxing very ſlender, but, notwithſtanding that, it was black and bold to appearance, the Miſtineſs about it on the Right-Hand perceivable, and that on the Left grown ſlender, in proportion with the Spot it ſelf, and found it diſtant 5 Seconds of Time.

Saturday, June the 19th, At 5 this Morning, it being clear Weather, I ſaw the Spot diſtinctly with my 7 Foot Tube: At 9 a Clock I mounted my 18 Foot Glaſs, obſerving once in half an Hour all the Morning: At 12 I perceived that all the Cloud or Miſty Matter that uſed to ſurround the Spot was inviſible, and the Spot it ſelf reduced to little or no Breadth, in Compariſon to what it had been towards the Sun's Center, and ſo cloſe to the Limb of the Disk, that I could only perceive a ſmall Streak of the Sun's Light between it and the Limb of the Sun's Body; at two a Clock I could juſt perceive it, but grown extreamly ſlender.

The firſt Revolution I ſaw the Spot half in the Circumfererence of the Sun's Limb at two a Clock on *Sunday, May* the 23d: And the ſecond Revolution I juſt perceived it with the eighteen Foot Glaſs, at half an Hour after two a Clock, on *Saturday* the 19th Day of *June*.

On *Sunday, June* the 27th, about ſix a Clock in the Evening, I obſerved ſeveral Spots in the Sun's Disk, but had not the Conveniency to uſe my longeſt Teleſcope, becauſe of ſome Trees that were in my way

way to Weſtward, ſo that I made no Obſervation till the *Tueſday* following.

Tueſday, *June* the 29th, about ſeven in the Morning I counted ſixteen remarkable Spots in the Sun's Body, and near his Center they appeared as in the Figure, through the eighteen Foot Glaſs; then I took my ſeven Foot Teleſcope and Frame, and obſerved that the foremoſt Center of ſix that looked on the Paper as one Spot, was diſtant from the Sun's Anteriour Limb eighty one Seconds of Time, and the laſt Cluſter 87.

Fig. 59.

This Day the foremoſt Spot was diſtant from the following Limb, according to the Path of the Spot, juſt fifty five Seconds of Time. The Sun's Diameter was always 136 Seconds in the Tranſit, and the Spot was 126: So that the Spots Path was ten Seconds ſhorter than the Sun's Diameter.

Wedneſday, *June* the 30th, At Eight a Clock this Morning, obſerving the Solar Spots with my eighteen Foot Teleſcope, I perceived very plainly that they had wonderfully increaſed in Number, and ſtrangely changed their places. The Cluſter of ſeven Spots ſeemed to me to move gradually, as the ſingle Solar Spot did in *May*, but the Cluſter 4 went too faſt forward, the twelve Spots without a Miſt about them ſtraggled all manner of ways, and the nine Spots and the five black little ones went backward, and unbent it ſelf at the ſame time as it were into a ſtreight Line. I am apt to believe it went backward, as that the other went too faſt, or faſter than ordinary forward, for in twenty four Hours the foremoſt Cluſter advanced twenty one Seconds of Time, which is more by ſix Seconds than ever the ſingle Spot moved in that Time, even when neareſt the Sun's Center; and the diſtance in time between the firſt and the laſt Cluſter this Day, was greater by three Seconds than the Day before.

Fig. 60.

The foremoſt Cluſter of four Spots was diſtant from the advancing Limb of the Sun ſixty Seconds of Time.

At half an Hour paſt four, the advancing Cluſter paſs'd the Interſection in fifty five Seconds of Time, after the Sun's foremoſt Limb had paſſed Conformable to the Spots Path; and the laſt Spot paſſed in ſixty three Seconds of Time, the laſt Limb paſſing the Interſection, according to the Path of the Spot, in a hundred and twenty ſix Seconds of Time, the Sun's largeſt Diameter paſſing in a hundred and thirty ſix Seconds, the Spots by this time appeared ſtrangely black, and of very odd Shapes, as in the upper part of the Circle.

Thurſday, *July* the Firſt, At eight a Clock in the Morning I obſerved the Solar Spots with my eighteen Foot Teleſcope, the Weather being good, and ſaw that they rang'd themſelves in reſpect of one another, as is repreſented in the upper part of the Scheme: The leading and largeſt Spot being diſtant from the anteriour Limb forty four Seconds of Time, the laſt Cluſter lying a little awry, paſs'd in fifty three Seconds: After the Anteriour Limb ſo done, the following Limb

Fig. 61.

alſo

alſo paſs'd the Interſection, according to the Path of the Spot, in 125 Seconds of Time.

Friday,
Saturday, } No Sun-ſhine.

Sunday, July the 4th, This Morning at Eight a Clock the Leading Spot was diſtant from the advancing Limb ten Seconds of Time, the Spots and Cluſters retaining nearly the ſame Shape, but beginning to Contract themſelves, the foremoſt methought look'd ſtrong enough to make another Revolution, and paſs'd in 127 Seconds.

Monday, July the 5th, At ſeven a Clock I found the Spots had quite alter'd their Shape, appearing dull and ſlender, as in the lower part of the Scheme, and diſtant about four Seconds, being all included in a Cloud.

Tueſday, July the 6th, At ten a Clock the Sun's Disk, view'd with my eighteen Foot Teleſcope, was found clear of all Spots.

On the ſeventeenth Day of *July*, about four a Clock in the Afternoon I obſerved ſome Spots in the Sun's Body, reſembling thoſe I ſaw on *Thurſday* the third of *June*, only with this difference, that theſe appeared to me as if they had been heated red hot; they ſeemed to be in the ſame part of the Sun's Disk. I obſerv'd them above an Hour together that Day, but could never afterwards ſet Eye on them, nor diſcover whether they were coming in, or going off his viſible Disk. I continued to obſerve the Sun, as often as was poſſible, with my eighteen Foot Glaſs, till the end of the Month, but without farther Succeſs.

A Table of all the Spots *and* Fæculæ *on the* Sun, *at* Upminſter, *ſince* July 1703.

1703. *Octob.* 9 10 11 *Nov.* 19 22 1704. *Jan.* 16 17 18 19 21 22 23 *Jan.* 30. *Feb.* 23 25 *Mar.* 7 8 9	*March* 10 11 13 *April* 11 12 13 *May* 1 * 11 *June* 23. *none* *Some* 24 *Vaniſhed* 25 *July* 18 20 21 * 23 *Sept.* * 10 *nothing* 14 18 * 21 *Nov.* 17 18	*Nov.* 21 *Dec.* * 2 1705. *Jan.* 1 2 3 5 * 25 *Feb.* * 19 *March* 14 16 *April* 1 *May* 5 6 7 8 * 10 * 22 * 23 *June* 22 23	*June* 24 26 27 28 30 *July* * 1 * 3 24 *Sept.* 30 *Octob.* 2 3 5 6 7 25 26 30 31 *Nov.* 2 4 * 21	1706. *Feb.* 7 *Mar.* * 7 27 30 *April* 2 *Vaniſhed* 3 *June* 7 8 *July* 24 25 *Sept.* 4 5 6 7 8 10 *Faint* 11 *Extinct* 12 *Oct.* * 29 *Nov.* 5	*Faint* 6 * 8 28 29 30 *Dec.* 1 2 3 4 5 * 22 * 31 1707. *Feb.* 14 15 16 18 24 *March* 6 9 11	*March* 12 14 18 21 *Jun.* * 29 *Extinct* 30 *July* 1 *Languid* 2 *Scarce viſible* } 4 *More viſible* } 5 *Extinct* 6 *Appears* 8 * 10 *Aug.* * 31 *Sept.* * 12 16 *Spot* & 17 19 21 27	*No Spot nor* * 28 *Octob.* 31 *Nov.* 1 4 8 10 15 16 17 18 19 20 22 N.B. *This* Nov. 15. *another Spot aroſe on the Eaſtern ſide of the Disk, whilſt this was on the Weſtern.*

XVI. *Spots in the Sun, by Mr.* Derham, *n.*330. *p* 270.

In this Table the *Faculæ* are noted with an Aſteriſk; and the Duration of every Appearance of the ſame Spots or *Faculæ*, or the time they diſappeared, with a Line.

As to the Figure of the Spots. They are well known to change frequently; and therefore I think it of little uſe to give their Figures every time I obſerv'd them. But it is ſomewhat remarkable, that the Spots generally appear longiſh near the extream Parts of the Disk. If they are never ſo round near the middle of the Disk, they become longer and longer towards the Extremes, till (at going off) they ſeem to be nearly a ſtraight Line, nearly parallel to the Sun's Limb. Which is a manifeſt Argument, that the Sun is a Globe, and that theſe Spots are on, or very near its Surface.

Another thing remarkable is, The Mutability of the Shape of the Spots. I have more than once manifeſtly perceived them to change in the very time I have been looking upon them. Thus *Nov.* 19. 1703. I ſaw three or more Spots not far off the middle of the Disk; and whilſt I was looking upon them, they ſeemed to vary, both as to their Shape and Strength; ſometimes ſeeming longer, ſometimes ſhorter; ſometimes ſpiſs, ſometimes languid. And this they ſeemed to do, not only

only through my 16 Feet Tube, (which I thought at firſt was from the different Diſpoſition of my Eye) but alſo when I received the Sun's Image through a ſix Feet Teleſcope, on a white Paper, in a darkened Room. Theſe mutable Spots the Weather hindred me from ſeeing again till *November* the 22d, following; and then they were become only like a thin Smoak, or *Nebula*.

So again *April* 11. 1704. there were divers Spots with *Umbræ* about them. Theſe *Umbræ* or *Nebulæ* I could plainly perceive, whilſt I was looking on them, to be ſometimes very faint and thin, and ſometimes much darker and thicker. Theſe *Maculæ* and *Umbræ* I obſerved ſuddenly brake out in the Sun: For, on *April* 9. the Disk was free. But this *April* 11, laſt mentioned, I perceived them advanced near a quarter part on the Disk: And conſequently they brake out in the Sun within 48 Hours before. On *April* 13. the *Spots* were become *Umbræ*, in the Morning; and at 4 of the Clock in the Afternoon, there were no Remains of either *Maculæ* or *Umbræ*.

From this ſhort Continuance of theſe Spots on the Sun, it is more than probable, they were in a perpetual Flux and Change; and that thoſe Mutations which I perceived in them, whilſt I was looking on them, were real, not imaginary.

Alſo it may be farther remarked, (which I have frequently obſerved, and which as I remember *Scheiner* obſerved long ago.) That thoſe *Spots* and *Umbræ* which ſuddenly ariſe, do as ſuddenly decay, and are ſoon extinct. And ſuch Spots, I have farther obſerved, do ſeldom turn to *Faculæ*, as they commonly do when longer on the Sun, as I ſhall obſerve by and by.

Again, *May* 5. 1705. I could perceive two Spurs or Branches (running from a Spot) to change, and be ſometimes darker, ſometimes thinner.

So *March* 30. 1706. I obſerv'd ſuch another Variation. This Day, or but little before, *Spots* with *Faculæ* aroſe in the Sun, which remained not above three Days on him. One of theſe Spots I could manifeſtly perceive to be ſometimes quite extinct, and then again immediately to appear: And the *Faculæ* alſo, in half an Hours time, had plainly alter'd their Shapes.

October 29. the ſame Year, I could plainly perceive the *Maculæ* and *Faculæ* both to change: And whilſt I was carefully viewing them, I ſaw a Spot ariſe in one of the brighteſt *Faculæ*, and again nearly diſappear; and then again appear ſtrong and ſpiſs. I ſhould have been glad to have ſeen how they appeared next Day; but the Weather was ſtormy, cloudy, and wet for ſeveral Days after.

Another thing I have obſerved is, That the *Maculæ* do generally, if not always, become *Nebulæ* or *Umbræ* before they quite vaniſh; and after that, very frequently turn to *Faculæ*, or bright golden Spots, more illuſtrious and fulgid than the other Parts of that glorious Globe. If the Spots are of ſhort Duration, *Faculæ* ſeldom enſue: Or if they do, they are commonly the Remains of ſome Spots that had before been on

the Sun, and vaniſh'd perhaps on the ſide oppoſite to us. But Spots that long continue, if they vaniſh before that part of the Sun revolveth out of our ſight, do very often become *Faculæ.* Of which the Table affordeth ſeveral Inſtances, particularly *July* 3. 1705.

From theſe preceeding Particulars, and their Congruity to what we perceive in our own Globe, I cannot forbear to gather, *That the Spots on the Sun are cauſed by the Eruption of ſome new Vulcano therein*; which at firſt, pouring out a prodigious quantity of Smoak, and other opacous Matter, cauſeth the Spots: And as that fuliginous Matter decayeth and ſpendeth it ſelf, and the Vulcano at laſt becomes more torrid and flaming, ſo the *Spots* decay and grow to *Umbræ*, and at laſt to *Faculæ*; which *Faculæ* I take to be no other than more flaming brighter Parts than any other Parts of the Sun. Theſe *Faculæ* I have obſerved never continue long on the Sun: And the reaſon I conceive is, becauſe the *Vulcano*, after its Smoak is over, doth not long emit its Flames; by reaſon the fiery *Pabulum* is then near ſpent, when once it begins to flame: After which the torrid *Vulcano* ſoon returneth to the natural Temperature of the Sun, ſo nearly at leaſt as to eſcape our ſight, at ſo vaſt a diſtance as the Sun is from us.

Another thing that may be accounted for, and indeed doth in ſome meaſure confirm alſo what I have ſaid, is the *Nuclei*, or darker part of the Spots; generally in moſt Spots, and towards the middle of them. Now it is very uſual in Culinary Fires in this our Globe, when they emit Smoak, that the middle is the darkeſt part. If for Inſtance, we were from aloft in the Air, to ſee a thick Smoak come tumbling out of a *Chimney*, or the Mouth of a *Vulcano* juſt kindled, we ſhould find the middle part, juſt over the Mouth of the *Chimney*, or *Vulcano*, to be the moſt ſpiſs and dark, and towards the Extremes clearer and thinner. And ſo I take it to be in the Eruptions of the Sun; that the *Nucleus* is juſt over the Mouth of the ignivomous Cavern, and that the miſty parts of the Spot are the thinner parts of the Smoak, ſwimming about in that Fluid, or *Atmoſphere*, which I ſuppoſe doth ſurround the Sun, as well as our Globe, and the Moon manifeſtly; yea, and in all probability, every Planet of this our Solar Syſtem.

From what hath been ſaid, we may give a reaſon why there are ſometimes Spots frequently on the Sun, and ſometimes none in many Years. One thing I believe there is in this, That there may be Spots, but not always ſeen. But there are doubtleſs great Intervals ſometimes when the Sun is free; as between the Years 1660 and 1671, 1676 and 1684. In which time Spots could hardly eſcape the ſight of ſo many curious Obſervers of the Sun, as were then perpetually looking at him with their Teleſcopes in *England*, *France*, *Germany*, *Italy*, and all the World over; whatever might be before, from *Scheiner*'s Time. The reaſon I ſay, of this long diſappearance of the Spots, I take to be from the want of extraordinary Eruptions in that fiery Globe. The ſulphureous or other Matter, or *Pabulum* of thoſe Eruptions, is ſpent or diſſipated, and

and that Globe continues in its natural ordinary burning State, till there happens to be a fresh Collection of smoaking, Dissplosive, and extraordinary Matter, that causeth a new Eruption. Which Eruptions generally happen between what we may call the Sun's Tropicks, or in his torrid Zone: For I never observed any Spots to be near the Sun's Poles. And if I misremember not, the Spots in *Scheiner's* Cuts are all about the middle Zone of the Disk. The greatest Evagation I ever observed of them was *March* 8. 170$\frac{3}{4}$. On which Day, besides the dark Spots in the usual Zone, I perceived some faint Spots, scarce visible, much nearer the Southern Pole than I ever had seen them. But this was, no doubt, in some measure owing to the Position of the Earth in respect of the Sun, as well as to the southerly Place of the Spots on him: For about the Equinoxes, the Spots seem to march pretty far towards the Poles of the Sun, as may be seen by the annexed Schemes. *Fig.* 62. 63.

Having thus observ'd what part of the Sun the Spots commonly possess, I shall next take notice of their *Stages* and *Path* over the Sun. That the Sun moveth round his own *Axis*, is manifest, beyond doubt, from the Motion of the Spots. And that the Spots seem to traverse the Sun, sometimes in straight Lines, sometimes in curve Lines, curved this way, and that way, is as manifest also, and well known to the Curious, and is set forth in the annexed two Figures: Which Figures shew the Stages of the Spots every Day that I observed them, and the Lines they describe in several Months of the Year. The daily Stages in both Figures are exect; or if they seem otherwise, it is by reason the Observations were made at different times of the Day; as one in the Morning, the other some following Day in the Evening, or Afternoon. But the Declinations of the Spots, or their distances from the Sun's Northern or Southern Limb, are less exact in Fig. 63. than in Fig. 62. in which latter they are very near the Truth.

And the Causes of the Defects in the 63d Scheme I shall mention, to prevent the same Errors in others I my self ran into.

1. The Diminution of the Sun's vertical Diameter by the Refractions was the principal cause of my Errors. This, although I was sufficiently aware of, yet I did not think had been so considerable, for want of experimenting, or well considering the Matter: For I have sometimes found the perpendicular, or vertical Diameter of the Sun diminished, from 32′ 21″ on the Meridian, to 36′ 3″ at the Horizon, in one and the same Day.

2. For the same reason I was not aware of the time being so long before the Sun goes round, as I found it.

3. Another Error was measuring the Sun's Image on the Scene of white Paper, with the Shade of the Micrometer; and not by looking through the Tube, and so clasping the Limb of the Disk with the parallel edges of the Micrometer. The former, although practised by some eminent Astronomers, is a far more easy and indulgent, than accurate way.

 Since

Since my foregoing Account was drawn up, I have seen other Spots on the Sun, whose times are expressed in this following Table.

1707.		1709.	
Decemb.	4	*Jan.*	15
	10		21
	* 29		22
	* 30	*August*	13
1708.			* 17
July	31	*Octob.*	8
August	1	*Novemb.*	1
	5		2
	6		4
	22		5
	23		6
	24	1710.	
	28	*Jan.*	22
Septemb.	1	*April*	* 6
Novemb.	5	*Octob.*	14
Dec.	14		* 18
	26		

From the Spots in this Table I had frequent Occasions to be assured of my Opinion, in the foregoing Paper: Particularly in viewing the Spots of *August* 1. 1708. (represented Fig. 64.) where some were large and dark, others less and thinner, and all encompass'd with *Nebulæ:* In viewing these, I say, I observ'd great Alterations at the very time I was looking on them. Sometimes the *Nuclei* were very dark and black, sometimes less so; and the same thing I observed also in the *Nebulæ* encompassing them. One of the lesser Spots *b.* in Fig. 64. which the Day before was sufficiently visible and strong, was this Day, now thick and strong, and anon languid and less visible. And from the two Spots *a.* and *d.* I could plainly see a Smoak issuing out to *c.* and *f.* sometimes visible for 5 or 6 Minutes, and then disappearing for a quarter of an Hour, or more; and then again smoaking out, and again disappearing, as before. All which Particulars, I saw over and over again repeated, for a good while together, till I was weary of the Observation.

These Spots I was hindered from viewing until *August* 5. following: And then I found the Spot *b.* quite extinct, (as I expected,) as also some of the other Spots; together with the *Nebulæ* grown less. But the great Spot *a.* continued dark and strong, only sometimes fainter, and then again stronger; and sometimes like a half, or horned Moon; sometimes roundish, or rather of an oval Figure; of which latter Figure they commonly are, when they are near the Sun's Limb, which this Spot was not far off at this time.

These Particulars are Confirmations of what I said, *That the Solar Spots are no other than a Smoak rising out of the Body of the Sun.* Of which Opinion I have been almost ever since I first observ'd them, and find that I am not singular in this Opinion, as I shall shew from a part of a Letter from Mr. *Crabtrie* to Mr. *Gascoigne*, the Inventor of the Micrometer.

'I writ also to Mr. *Townley* at that time my Opinion in brief of the 'Sun's Spots, (which you conceive to be Stars,) and it seems he, or 'Mr. *Kay*, writ to the same purpose to you, desiring your Opinion: 'Which you freely deliver; for which I cannot but commend you, and 'especially for preferring Reason before any Man's Authority. Yet 'give me leave (*pace tua Amice desideratissime*) to speak my mind like-

'wise

'wise freely concerning these Appearances. I have often observed these 'Spots; yet from all my Observations cannot find one Argument to 'prove them other than fading Bodies. But that they are no Stars, 'but unconstant (in regard of their Generation) and irregular Excres-'cences arising out of, or proceeding from the Sun's Body, many things 'seem to me to make it more than probable'.

'For *First*, For their Form; they are seldom round, but of irregular 'shapes, and, as I have often seen, one side, or end of the Spot more 'thin than the rest, like to a certain misty Darkness, and by degrees 'thicker, grosser and darker, nearer to the main body of the Spot; just 'as the Smoak of some pitchy Fire, which is in one part very gross, 'and in another more rare and thin, turning at last into meer Air: Or 'like a Cloud, Fog or Mist, more thick, dark, and gross in the midst; 'and more thin, fluid, penetrable, and transparent towards the sides; 'which I suppose is not compatible with any of the Stars.'

'*Secondly*, For their Colour: The lightness thereof differenceth them 'from Stars or Planets; they being never of such absolute Darkness as 'I observed *Venus* the 24th of *November* last: Tho' I have seen Spots 'sometimes little less than she, yet always of a far paler and whiter 'Colour, looking (at least in some parts) like some thin dissipated 'substance.'

'*Thirdly*, For the manner of their Appearance. I have seen many 'Spots, which in the middle of the Sun appear of a round body, but 'coming towards the side of the Sun, appear long. Which (if you 'rightly consider it) is a demonstrative Argument that they are not 'Globes, as all the Planets and Stars are: (for Globes always appear of 'one form (round) in every Position) but Exhalations, or such like 'fluid Substances, extended to a broad flat form like our Clouds, which 'being over our Heads, and so in their full breadth, appear large and 'broad; but driven with the Wind, till they turn one edge upon us, 'seem of a long shape. So these Solar Clouds, being turned about the 'Sun, may in the middle shew their full breadth to us, and about both 'edges of the Sun, turn their edges to us; which answereth to the ap-'pearance'.

'*Fourthly*, For their Continuance. Some of these Spots, arising at 'the East-side of the Sun, vanish before they come to the midst of the 'Sun. Others appear first in the middle of the Sun, and vanish before 'they come to the western Limb; and for the most part they vanish be-'fore they have made a full Revolution about the Sun. Which argues 'them to be but thin, vanishing, fading Substances, not like the per-'manent Bodies of the Stars'.

'But to take off these Reasons, you answer, That you conceive these 'Spots to be Stars moving regularly in their own Orbs, which are many, 'tho' none of greater extent than about $\frac{1}{10}$ of the ☉ Semidiameter from 'its Circumference; and that the swifter Movers in the lower Orbs, 'overtaking the slower in the higher Orbs, cause an appearance. You

'seem

'seem therefore to think, that they being so thin Bodies, the Sun's
' Rays pass through them, and so one cannot be seen alone, till more
' being together, one heaped behind another, they stop the light of the
' Sun's Rays, and so cause an appearance. This I conceive is your Mean-
' ing: or else (as you seem to insinuate afterwards) that the higher re-
' flects the Sun's Rays strongly enough upon the lower (when they
' come within the Angle of Reflection) to make the interjacent Planet
' indiscernable'. But to these I answer,

1. ' If it be by their coming within the Angle of Reflection, that
' the light of the Sun reflected from the outer Planet upon the inner,
' doth make it (as you speak) indiscernable, then that Light so reflect-
' ed, is reflected either upon all places, as the Moons and Planets
' Light; or but upon one, as is the Reflection of a plain Looking-Glass.
' If the first, there would never be many seen (seldom above one or
' two) because the outermost would continually make the inner undis-
' cernable. But *Gassendus* affirms, There are seen sometimes forty at
' once in the Sun's Body. If the 2d, there would always be many seen,
' because reflected Light would but occupy a little room, and that but
' for a small time, till the swifter were past the place of Reflection:
' Whereas many Days there are none at all seen in the Sun's Hemisphere:
' And in both these Cases, the outermost Planet of all would always in
' the space of 27 Days, be seen in the same place, being never ob-
' scured, none of the inferior being able to reflect Light upon it. Add
' hereunto, if any kind of Reflection should make them to appear bright
' like the Sun, and so not distinguishable from the Light of the Sun,
' what should hinder, but we should see them also bright Bodies by the
' side of the Sun, when they are passing either by the West, or East-side
' of the Sun's Body? The Light being then reflected upon them by the
' inferior Planets as well as at other times, and that also upon much of
' that side of them which we should behold'.

' But if you wave this Conceit, as insufficient, and fly to your for-
' mer, That the swifter Movers in the lower Orbs, overtaking the
' slower in the higher Orbs, cause an appearance. To this I answer.
' 1. The thing you suppose seems to me neither necessary nor probable,
' nor do I conceive why they should not be seen, being themselves
' alone, as well as conjoined, seeing all other Stars and Planets are so.
' 2. If it be because they are of a thin, transparent Substance, till many,
' being one behind another, make them to seem grosser; Then they are
' not of the nature of other Planets, as is proved in ☿ and ♀, who of
' themselves appear dark Bodies, when they come between us and the
' Sun; nay, they must be more thin than our Clouds, which will easily
' be seen between us and the Sun, and hides it from us. 3. If it be be-
' cause they are so little, that the Imperfection of our Glasses cannot
' discover one alone, there must be, without doubt, many Millions of
' them; which how they can be included within the compass of $\frac{1}{10}$ of
' the ☉ Semidiameter, we shall consider anon. I have seen one of an

' ordinary

‘ ordinary Darkneſs, (yea darker than many greater) yet not above 5″ ‘ Diameter. If this conſiſt of two or many, of themſelves inviſible, ‘ how many were in thoſe which *Gaſſendus* ſaw of 1′ $\frac{1}{2}$ Diameter? 4. The ‘ Figure of theſe great ones (being neceſſarily compoſed of Stars of ſuch ‘ different Orbs and Motions) would quickly vary, by reaſon of the ‘ diverſity of their Motions; like as we ſee in a Flock of ſmall Birds. ‘ But 5thly, you ſay the furtheſt of theſe Orbs is not above $\frac{1}{10}$ of the ‘ Sun's Semidiameter from its Circumference. But there would not, in ‘ that ſmall ſpace, be room enough for ſo many Orbs of Planets, as ‘ have been ſeen at once. Which I prove thus. 1. *Gaſſendus* affirms ‘ there are ſometimes ſome of about the $\frac{1}{10}$ part of the ☉ Semidiame- ‘ ter; which is the whole ſpace allowed by you for them all. And I ‘ my ſelf have ſeen of $\frac{1}{15}$ of the ☉ Semidiameter: And yet you muſt ‘ confeſs theſe great ones could only be the Conjunctions of ſome, not ‘ all. 2. There are many times ſeen in the ☉ Superficies, a great num- ‘ ber of Spots, whoſe Diameters added together, would do more than ‘ twice fill the ſpace you ſpeak of. I my ſelf have ſeen it, and ſo I ‘ believe have you. *Gaſſendus* affirms, There are ſometimes 40 ſeen at ‘ once: If this was by Conjunction of Planets, in every Appearance, ‘ there was at leaſt 80 Bodies at once on this ſide the ☉; it may be as ‘ many on the other ſide, beſides thoſe unſeen (by your Reflection or ‘ otherwiſe) which doubtleſs muſt be far more than ſeen. For it is a ‘ moſt rare, and I think unheard of thing to ſee but 3 (which is leſs than ‘ the half) of our Planets, conjoin'd in viſible ☌ at once: So that with- ‘ out queſtion, if they be Planets, they are many hundreds; which ‘ muſt have ſo many ſeveral Orbs, and which certainly cannot be done ‘ in ſo narrow a compaſs, as the $\frac{1}{10}$ of the ☉ Semidiameter. And that ‘ they cannot have any larger (I ſuppoſe not ſo large an) extent from ‘ the ☉ Superficies, may be proved by their Motion, through the viſible ‘ Hemiſphere of the Sun's ſpherical Body, by comparing the Swiftneſs ‘ of their Motion towards the middle and ſides together. 6. If one of ‘ theſe (imagined) Planets be ſwifter than another, as they muſt needs be; ‘ then the ☌ of 2 or 3 ſwifter ones would make a Spot of ſpeedier motion ‘ than the ☌ of 2 ſlower ones: But the motion of all about the ☉ Cen- ‘ ter, is always equal; yea, and the Spots retain the ſame Poſition ‘ one to another, (conſidering the Sun's Sphericity, and the Angle of ‘ their appearance to us) juſt like the Fixed-Stars. So affirms *Gaſſen-* ‘ *dus, Moveri omnes eodem & uniformi motu, adeo ut, cum plures fuerint,* ‘ *nulla antevertat aliam, ſed eundem tenorem in diſco & perinde ſervent* ‘ *inter ſe, ac ſervant Fixæ in firmamento*'.

‘ As for that other annual Motion of the Spots, you ſpeak of, from ‘ Weſt to Eaſt, upon their Axis inclined above 8 Degrees to the Eclip- ‘ tick; I ſuppoſe it is not any real Motion of the Orbs of thoſe ſolar ‘ Planets or Spots, but only a viſible Motion ſo appearing, cauſed (in ‘ *Kepler*'s Syſtem) by the Sun's rolling upon its own Center in the ‘ midſt of all the Orbs, not exactly in the way of the *Temporary* Eclip-

tick,

‘ tick, but in the *Via regia* (as *Kepler* calls it) inclined certain Degrees ‘ to the *Temporary*; thereby turning about with him, the ſame way, ‘ his adventitious, or excrementitious Parts, the Spots, by his *Magnetical* or *Sympathetical Rayes.* And hence may be demonſtrated the appearance of that annual Motion in the Sun's Spots you ſpeak of. See ‘ *Galilæus, Syſt. Coſm. p.* 339, *& ſeq.* So alſo in *Ptolemy*'s and *Tycho*'s ‘ Syſtem, the ſame appearance may be demonſtrated, ſuppoſing the ⊕ ‘ fixed in the middle of the Univerſe, and the ☉ rolling round upon the ‘ ſame Poles of that *Via regia* (or way of the Spots) and keeping his ‘ Axis in Parallelifm continually towards one and the ſame part of the ‘ Univerſe. This may be certainly demonſtrated, altho' *Galilæus* there ‘ affirms the contrary. Other *Hypotheſes* of that Motion may be feigned, ‘ as by the annual Converſion of the Poles of the *Via regia* about the ‘ Poles of the Ecliptick in the Sun's Body; But none I conceive ſo compendious, as the one of the former. For my part, I incline to the ‘ firſt: yet if when we ſee you, you ſhew us any more likely Theory, ‘ for my part I ſhall be ready to conſent to you in any thing with ‘ reaſon'.

‘ Thus you have, what for the preſent I conceive of theſe *Maculæ* ‘ *Solares. Fromundus* mentions one *Jo. Tarde Gallus*, who thinks them ‘ to be Secondary Planets; who hath written a Book of that Subject, ‘ and calls them *Aſtra Borbonia*: But I could never yet ſee it. What ‘ you, or he, or others may alledge for that Opinion, I know not. In ‘ the mean time it were too much Levity in me, againſt my Judgment, ‘ to acknowledge them Stars; unleſs I ſee at leaſt ſome poſſibility how ‘ they may be ſo, or ſome probability why they ſhould not rather be ‘ Spots. Which when you, or they do produce from better grounded ‘ Reaſons, Optical Experiments, or Demonſtrations, I ſhall willingly recant my Opinion'.

Plate 3.

Fig. 62. *Shews the Stages and Lines deſcribed by the Spots upon the Sun in* September *and* November 1706. *and in* February *and* March 170$\frac{6}{7}$. *and in* September *and* November 1707. Fig. 63. *Shews the Stages and Lines deſcribed by the Spots upon the Sun in* January 170$\frac{3}{4}$. *and in* May, June, *and* October 1705.

Eclipſe of the Sun at Cambridge, 4 *Miles from* Boſton *in* NewEngland, June 12. 1694, *by Mr.* Tho. Brattle, *n.*292.*p.*1630.

XVII. Obſervations made of the Sun's Altitude before the Eclipſe began, in order to rectify the Watch.

By the Watch h ' "		Comp. Altit.	Time by Calcul.	Differ.
at 8 26 37	Mane	49 31	8 16 40	9 57
31 27		48 26	21 40	9 47
38 26		47 20	28 32	9 54

The

The Eclipse was first perceived at 9h. 25′. by the Watch, at which time the Sun had scarcely been eclips'd 1 Minute, so that

By the Watch.		True Time.		
At 9h	24′	9h	14′	It began
9	32	9	22	about 1 digit eclipsed
9	48	9	38	full 3 digits
9	57½	9	48	about 4
10	06	9	56	near 5
10	15	10	05	full 6
10	33	10	23	about 8
10	43	10	33	full 9
10	47	10	37	full 9½
10	53	10	43	full 10
10	59	10	49	about 10½
11	03	10	53	better than 10½
11	06	10	56	much the same.
11	09	10	59	rather decreasing.
11	10½	11	00½	sensibly decreased near ¼ of a digit
11	14½	11	04½	nearest to 10 digits
11	25	11	15	full 9 digits, *i. e.* full 3 digits restored, or the Shadow rather within 9 digits.
11	29	11	19	8½ compleat
11	34½	11	24½	full 8 digits
11	44	11	34	full 7
11	48	11	38	full 6½ digits
11	52½	11	42½	just 6
0	02½ P.M.	11	52½	just 5
0	13	0	03 P. M.	full 4
0	26	0	16	full 2½
0	32	0	22	better than 2
0	41	0	31	better than 1
0	48	0	38	ended.

Observations made after the Eclipse was done, of the Sun's Altitude, in order to rectify the Watch.

Time by the Watch.			Comp. Altit.		True Time.			Differ.
At 3h	31′	30″	P. M. 45	52	3 21	36	9	54
	36	15	46	23	26	16	9	59
	38	10	46	45	28	16	9	54
	46	50	48	19	36	48	10	02
	48	10	48	30	38	20	9	50

Hence

Hence it appears, that the Watch went about 10 Minutes too faſt during the whole Eclipſe, as we have all the way allowed.

So that the Eclipſe

Began at	9h. 14′ Mane
Ended-- ——— --0	38 P. M.
Laſting in all	3 24

In the Calculation the Latitude of *Boſton* was allow'd to be 42° 25′.

2. *Another at the ſame place,* Nov. 27. 1703. *by the ſame, ib. p.* 1634.

At 10h	00m	The Sun was not touch'd.
	06	The Moon entred on the S. S. W. Point as near as I could judge.
	15	The Eclipſe was conſiderably advanced.
	20	Seem'd to be about half a Digit eclips'd, rather more than leſs, and the Section to be a ſmall Matter more Weſterly.
10	25	Much the ſame, and near the ſame Point.
	30	Seem'd to be leſs.
	33½	The middle of the Section nearer the S. W. and the Diameter of the Section leſs every way.
	37½	Much leſs and nearer the Weſt.
	44½	It ended, and was juſt over, going off near the S. W. ſo that all the while it was within a Point or two of the place where it firſt came on, or between the S. S. W. and S. W.

I judg'd when it was at the height, that the Chord of the eclips'd part, was neareſt equal to the ſide of an inſcrib'd Decagon, or ſubtended about 1/10 of the Periphery of the Sun's Disk.

Another May 1. 1706. *at* Greenwich, *by Mr.* Flamſtead, *n.* 306. *p.* 2237.

XVIII. 1. The Morning was cloudy and moiſt, till about Eight a Clock, when the Clouds began to break, and we had ſometimes a ſight of the Sun through the Spaces betwixt them. A ſeven Foot Teleſcope was fitted up with a Scene to receive the Species of the Sun caſt through it, and on which it was about ſeven Inches Diameter, divided into Digits by ſix concentrick Circles. But Clouds coming, the Sun frequently rendred this way of obſerving inconvenient, and therefore laying aſide the *Apparatus* of the Scene, I viewed him through the ſame Teleſcope with ſmoaked Glaſſes, to ſave my Eyes, and noted:

1706 *May* 1ſt *Mane.*	Time corr. by the Pend. Clock.	
	8h. 21′ 30″	A very ſmall part of the Sun's Diameter was eclips'd.
	28 00	The Chord of the Arch of the Sun's Periphery eclipſed was 14′. 40″. then followed frequent Clouds through the Spaces betwixt; then ſome Zenith Diſtances of the Sun were taken for correcting

recting

	recting the Clock, and afterwards near the middle of the Eclipſe.
9h. 21′ 46″	The parts of the Diameter remaining clear —5 00
26 20	— ——— ——— — 4 30
10 31 50	Frequent large Clouds again till the Sun appeared through the breaks, and we ſaw the Eclipſe was not ended. Clouds again till
10 33 50	When the Sun ſhone out again we ſaw his Limb entire, and the Eclipſe certainly over.

2. Mr. *Stephen Gray* had prepared a Scene placed behind his 7 Foot Glaſs, ſo that the Species of the Sun projected on it was ſeven Inches over; b.t having the ſame ſort of Weather we had at *Greenwich*, he ſaw not the beginning, by reaſon of Clouds, but other Phaſes with the end he noted as follows. *At* Canterbury, *ib.p.*2238.

Correct Time by the Pend. Clock.	
8h. 53′	Digits 5 ½ darkned
9 08	———7
31	———10 or more
36	——— The Sun ſhining for a ſhort time, the Eclipſe ſeem'd to decreaſe.
55	———7 ½ a little clearer.
57	——— 6¾
10 02	———6.
4	——— 5¾
14	———4
16	———3¾
20	——— 2½
30	———1.
31	———0¾
10 36½	The end accurately with a Tube of 16 Foot.

3. Mr. *Abr. Sharp* caſt the Species of the Sun on a Scene-plate, behind his ſeven Foot Glaſs, ſo as it appeared ſeven Inches over. By reaſon of cloudy Weather, he ſaw neither the beginning nor end: But other Phaſes near the middle, as follows. *At* Horton, Yorkſhire, *ib. p.* 2239.

Times correct by the Pend. Clock.			
8h. 35′	00″	digits dark 3	By Ocular Eſtimation.
9 01	00	——— ———7	By Ocular Eſtimation.
4	54	——— — 8 $\frac{3}{10}$	Eclipſed on the Scene
6	33	——— —8 ½	
7	53	——— ———8 $\frac{6}{10}$	
12	50	——— —9	
16	08	——— ———9 $\frac{4}{10}$	

	18^h. 48′	——— $9\frac{1}{2}$	exactly, the Sun ſhining out clear.
	20 45	——— $9\frac{1}{2}$	the Sun ſtill ſhining clearly. Greateſt Obſcurity.
	21 48	——— $9\frac{1}{2}$	ſtill clear.
	28 46	——— 9	
	44 45	——— 7	
	54 42	——— $5\frac{1}{2}$	
10	06 10	——— $3\frac{1}{2}$	
	19 55	——— 1	preciſely.
	24 00		The Sun ſeen through the Clouds, the Eclipſe not ended.
	30 00		The Sun ſeen again perfectly round and entire.

At Bern, *ib.* p. 2240.

4. Captain *Stannyan*, who was there with his Kinſman, Her Majeſty's Envoy, writes the ſame Day to me, 'That the Sun was totally darkned 'there for $4\frac{1}{2}$ Minutes of Time; that a fixt Star and a Planet appeared 'very bright; and that *his getting out of the Eclipſe was preceded by a* '*Blood red ſtreak of Light, from its left Limb; which continued not long-* '*er than 6 or 7 Seconds of Time*; then part of the Sun's Disk appear'd, 'all of a ſudden, bright as *Venus* was ever ſeen in the Night; nay, 'brighter; and in that very inſtant gave a Light and Shadow to things, 'as ſtrong as Moon-light uſes to do'.

The Captain is the firſt Man I ever heard of that took notice of a red ſtreak of light preceding the Emerſion of the Sun's Body from a total Eclipſe. And I take notice of it to you, becauſe it infers that *the Moon has an Atmoſphere*; and its ſhort continuance of only 6 or 7 Seconds of Time, tells us that *its height is not more than the 5 or 6 hundredth part of her diameter*.

At Geneva, *by Mr.* Facio Duiller, *ib.* p. 2241,

5. The total Eclipſe of the Sun, which happened on the 12th of *May*, 1706. N. S. did preſent to the Inhabitants of *Geneva* a magnificent and ſurprizing Sight. The more learned did obſerve that Eclipſe with much Satisfaction: But it did ſtrike many of the Common People with a great deal of Terror. A little after the Sun's riſing, the Sky did ſeem clear; though the Air was thick already with ſome Vapours. Many little Clouds did afterwards ariſe here and there, and the Vapours did much encreaſe. For want of a Pendulum Clock, in a convenient place, the Moment of the total Immerſion, the Moment of the firſt Emerſion, and that of the end of the Eclipſe, could not be accurately obſerved. Tho' the Sky was ſomewhat overcaſt, the heat of the Sun was already felt, when the Eclipſe did begin: But a very ſenſible Coldneſs took place, as the Moon did, by degrees, cover a greater and greater part of the Sun, and the light decreaſe. The Eclipſe was obſerved only with ſome Glaſſes, either darkned with Smoak, or but little tranſparent; and by receiving the Sun's Image, through a ſix Foot Teleſcope, which repreſented the Objects inverted, upon a white Paper, placed at ſome diſtance from the Eye-Glaſs. When the Sun was near being totally dark, the bright Creſcent which did remain, was ſeen to diminiſh more and

more

more, upon the Paper, where its Image was received. And when that Creſcent was reduced to a very narrow Breadth, and to a very little Length, it was ſeen of a ſudden to diſappear: And in that Moment the whole Sun was eclipſed. At the ſame inſtant of Time, the Darkneſs, which was already very conſiderable, did become much greater. The Clouds did change of a ſudden their Colour, and became red, and then of a pale Violet. There was ſeen, during the whole Time of the total Immerſion, a whiteneſs, which did ſeem to break out from behind the Moon, and to encompaſs it on all ſides equally. The ſame whiteneſs was but little determined, in its outward ſide, and was not broad the twelfth part of the Diameter of the Moon. This Planet did appear very black, and her Disk very well defined, within the whiteneſs, which encompaſſed it about, and whoſe Colour was the ſame with that of a white Crown or *Halo*, of about four or five degrees in Diameter, which accompanied it, and had the Moon for its Center. The Star of *Venus* was ſeen, at the ſame time, at ſome diſtance, without that Crown, between the Eaſt and N. E in Reference to the Sun. The Planets of *Saturn* and *Mercury* were ſeen alſo by many, Eaſtward from the Sun's place. And if the Sky had been clear, many more Stars might have been ſeen, and with them the Planets of *Jupiter* and *Mars*; that towards the Eaſt, and this toward the Weſt: And ſo the ſeven Planets might have been ſeen, almoſt all at once. Accordingly ſome Gentlewomen, being in the Country, did tell, as is ſaid, more than ſixteen Stars. And many People, which were on the neighbouring Mountains, did ſee the Sky ſtarry, in ſome places, where it was not overcaſt, as during the Night, in the time of the full Moon. The total Immerſion did begin about three Quarters paſt Nine. The Duration of the total Darkneſs was preciſely three Minutes, or 180 Seconds, to the Moment that the firſt Ray of the Sun did begin to appear again, with much Brightneſs. And this time was obſerved with a ſimple Pendulum; which was afterwards compared with a Pendulum Clock, ſhewing the Seconds, and regulated upon the mean Motion of the Sun. The Council was met during the time of the Eclipſe; but they did riſe from their Seats, a little before the total Obſcuration; becauſe one could neither read nor write. They perceived as they came down the Stair-caſe of the Town-Houſe, ſome Swallows amazed, looking for a reſting place; and many Bats flying out. In other places the Hens and Pigeons would make haſte towards their Houſes.

A little time after the Sun had began to appear again, the whiteneſs and the Crown, which did encompaſs the Moon, did entirely vaniſh. The Sun did then ſhew it ſelf more and more; appearing at firſt as a little Creſcent, which did ſtill increaſe; and whoſe Concave ſide did ſeem terminated, as by an Arch, deſcribed with the Compaſs. A little before the total Obſcuration, the Country on the Weſt ſide did already ſeem overcaſt with Darkneſs; and afrer the total Obſcuration,

the

the Darkneſs was ſeen to leave us more and more, and to fly Eaſtward. According to Mr. Profeſſor *Gautier*'s Obſervations, from the firſt Emerſion of the Sun, to the end of the Eclipſe, there was $1^h\ 9'\ 30''$. As to the accurate times, they are uncertain, the Pendulum Clock having been ſet only by a ſmall Sun Dial. I ſend you alſo the following Account, which the ſame Gentleman did communicate to me.

‘ Obſervations on the Eclipſe of the Sun, of the 12th of *May*, 1706, ‘ made at *Marſeilles*, in the Obſervatory of the *Jeſuits* of St. *Croix*; ‘ by Monſieur *Chazelles*, Ingineer of the Galleys, and by Father *Laval*, ‘ *Jeſuit*, Royal Profeſſor of Hydrography.

‘ The Eclipſe did begin at	8^h.	28′	40″
‘ It did reach the Sun's Center at	9	6	11
‘ It was total at	9	34	15
‘ The Sun did begin to appear again at	9	37	9
‘ The Eclipſe did come again to the Center at	10	12	23
‘ It did entirely end at	10	47	50

‘ Three Stars were diſtinctly ſeen; and during three Minutes it was ‘ not poſſible to read. And there did remain one bright Digit, all about ‘ the Globe of the Moon'.

The Mannor Houſe of *Duillier* is in the Latitude of 46° 24′. in Longitude it is 4° 13′ 45″ to the Eaſtward of the *Royal Obſervatory* at *Paris*. And St. *Peter*'s Church at *Geneva*, is in Latitude 0° 12′ to the Southward, and in Longitude 0° 5′ 2″ to the Weſtward of *Duillier*. But of this another time.

According to theſe Obſervations, the Altitude of the Moon's Atmoſphere cannot well be ſuppoſed leſs than of 130 Miles, in perpendicular height: of which Miles 60 go to one Degree of the Earth. Neither could that Atmoſphere be diſcovered, before the time of this Eclipſe, by any Refraction of the Stars: Probably becauſe of this Refraction's ſmallneſs; and for want of proper Obſervations. And though it was very plain that the Atmoſphere of the Moon muſt needs ſhew it ſelf, in the time of a total Eclipſe of the Sun; yet I do not know that any body did think of this, till in the laſt Month of *May*, many Perſons did actually ſee it.

Some particular Obſervations, which are intended to be made publick, do evince that our Atmoſphere is ſometimes viſible, all along, from the Surface of the Earth to the perpendicular height of one Semidiameter of the Terreſtrial Globe. And the continued appearance of a Crown, of only four or five degrees Diameter, about the Sun, during the whole time of the total Obſcuration, does ſhew that the Æthereal Matter, in which that Crown was produced, muſt be at a very great height above the Surface of the Earth. But if that Crown was to be ſeen, ſo far as the Weather did permit, in all the Places where the Eclipſe was total, it muſt be concluded, that the Cauſe of it was not in our Air, but in ſome

ſome Vapours incompaſſing the Sun: and probably, in thoſe very Vapours, which produce that pointed Light, that has been obſerved lying in a manner along the Ecliptick, and that has the Sun for Center. Now either of theſe Concluſions, *viz.* concerning the great height of the parts of our Atmoſphere, capable of producing that Crown, or elſe concerning a Meteor obſerved, not in our Air, but in the Vapours that incompaſs the Sun, is very ſingular, and deſerves a great deal of Attention. If ever ſuch another appearance ſhould be ſeen, in the time of a total Eclipſe, it would be proper to obſerve accurately the leaſt Diameter of the Crown, from inſide to inſide: And to take notice whether, during the whole time of the total Immerſion, the inward Circle be every where continued, and of an uniform Figure. The leſs the ſaid Diameter, and the greater the Exceſs of the Moon's apparent Diameter above that of the Sun; as alſo the greater the apparent Altitude of the Sun is above the Horizon; the higher the Cauſe which produces the Crown muſt be above the Surface of the Earth. And the Poſition, upon the Moon's Disk, in reference to the *Zenith*, of the Points of Contact, where the Sun diſappears, or begins to ſhew it ſelf again, is here alſo of ſome Conſideration.

6. Habuimus die 12. *Maii* Eclipſin Solis totalem ſimul & annularem; totalem, quoniam Sol integer a Luna fuit obtectus; annularem autem non proprie ita dictam, ſed per Refractionem, quandoquidem circa Lunam fulgor apparuit rutilans, a radiis per Atmoſphæram Lunæ refractis ortus. *At* Zurich, *by Dr.* Scheuchzer, *ib. p.* 2246.

Initium Eclipſeos fuit mane hora	8. 54′.
Medium hora	9. 58′.
Finis hora	11. 12′.
Mora mediæ & plenæ obſcurationis	4′.

XIX. *Another at* Upminſter, Sept. 3. 1708. *by Mr.* Derham, *n.* 320. *p.* 312.

The correct apparent Time.	
	The Beginning we could not ſee for Clouds.
6h. 44′ 15″	The Sun peep'd out of the Clouds, and I judg'd by my Eye, that about one Tenth of a Digit was eclips'd.
	Then Clouds nearly all the time of the Eclipſe.
8 31 15	A little Obſcuration appear'd through the Teleſcope.
8 32 45	A very little Obſcuration through the Teleſcope.
	Then Clouds. And at
8 35 45	We could diſcern no Remains of the Eclipſe through the Teleſcope.

From theſe Obſervations I imagine the end was much about 8h. 33′ in the Morning.

XX. Though it be certain from the Principles of Aſtronomy, that there happens neceſſarily a Central Eclipſe of the Sun in ſome part or other of the Terraqueous Globe, about twenty eight times in each Period of eighteen Years; and that of theſe no leſs than eight do paſs *Another* April 22. 1715. *by* Dr. Halley. *n.* 343. *p.* 245.

over

over the Parallel of *London*, three of which eight are total with continuance: yet from the great variety of the Elements whereof the *Calculus* of Eclipses consists, it has so happened that since the 20th of *March*, *Anno Christi* 1140, I cannot find that there has been such a thing as a total Eclipse of the Sun seen at *London*, though in the mean time the shade of the Moon has often past over other Parts of *Great Britain*.

I received the Orders of the Society to provide for the Observation of this Eclipse to be made at their House in *Crane-Court*, and accordingly I procured a *Quadrant* of near 30 Inches *Radius*, exceedingly well fixt with Telescope Sights, and moved with Screws so as to follow the Sun with great Nicety; as also a very good *Pendulum Clock* well adjusted to the mean Time, and several Telescopes to accommodate the more Observers.

In order to examine both Clock and Quadrant, I, on the 20th of *April*, observed the distance of the upper Limb of the Sun from the *Zenith* 36°. 16′, and the next Day 35°. 58′; by which it appeared that the distances from the *Zenith* taken by this Quadrant ought to be encreased by about one Minute: and that Allowance being made, by several Observations taken before and after Noon on the said 21st Day, the Clock was found to answer the apparent Time or Hour of the Sun with sufficient Exactness, as not going above 10″ too fast. The next Day *April* 22°, just before the Eclipse began, we took three Distances of the Sun from the *Zenith*, *viz.* at 7h. 42′. 52″. A. M. the correct distance of the Sun's Center *a vertice* was 62°. 1′. 40″. at 7h. 45′. 48″. it was 61°. 34′. 40″. And again at 7h. 48′. 55″. it was 61°. 6′. 40″: which with the given Declination of the Sun and Latitude of the Place shew the true Times respectively to have been 7h. 42′. 38″, 7h. 45′. 35″. and 7h. 48′. 39″: all concurring that the Clock was only 14 Seconds too fast, and had gained scarce any thing sensible in a Day's time: so that it might be entirely depended upon during the continuance of the Eclipse.

Having computed that the Eclipse would begin at 8h. 7′, I attended soon after eight with a very good Telescope of about six Foot, without stirring my Eye from that part of the Sun whereat the Eclipse was to begin: and at 8h. 6′. 20″. by the Clock, I began to perceive a small Depression made in the Sun's Western Limb, which immediately became more conspicuous; so that I concluded the just Beginning not to have been above 5″ sooner; that is, exactly at 8h. 6′. 00″ correct Time.

From this time the Eclipse advanced, and by nine of the Clock was about ten Digits, when the Face and Colour of the Sky began to change from perfect serene azure blew, to a more dusky livid Colour, having an Eye of Purple intermixt, and grew darker and darker till the total Immersion of the Sun, which happened at 9h. 9′. 17″. by the Clock, or 9h. 9′ 3″. true time. This Moment was determinable with great nicety, the Sun's Light being extinguished at once; and yet more so was that of the Emersion, for the Sun came out in an Instant with so much Lustre that it surprized the Beholders, and in a Moment restored the Day,

viz.

viz. at $9^h. 12'. 26''$. true time, after he had been totally obſcured for $3'. 23''$ of Time. And as near as I could eſtimate the Points on the Moon's Limb; where the laſt Particle of the Sun vaniſhed was about the middle of the *South Eaſt* Quadrant of her Limb, or about 45 Degrees from her *Nadir* to the Left-hand: And the firſt Emerſion was about ten Degrees below the Horizontal Line through the Moon's Center on the Weſt ſide; and at 14 Minutes paſt nine, correct Time, I judg'd the Horns of the Eclipſe to have been exactly perpendicular, and by Conſequence, the Centers of the Sun and Moon to be in equal Altitude.

It was univerſally remarked, that when the laſt part of the Sun remained on his Eaſt ſide, it grew very faint, and was eaſily ſupportable to the naked Eye, even through the Teleſcope, for above a Minute of Time before the total Darkneſs; whereas on the contrary, my Eye could not endure the Splendor of the emerging Beams in the Teleſcope from the firſt Moment. To this perhaps two Cauſes concurred; the one, that the Pupil of the Eye did neceſſarily dilate it ſelf during the Darkneſs, which before had been much contracted by looking on the Sun. The other, that the Eaſtern parts of the Moon, having been heated with a Day near as long as thirty of ours, could not fail of having that part of its Atmoſphere replete with Vapours, raiſed by the ſo long continued Action of the Sun; and by conſequence it was more denſe near the Moon's Surface, and more capable of obſtructing the Luſtre of the Sun's Beams. Whereas at the ſame time the Weſtern Edge of the Moon had ſuffered as long a Night, during which there might fall in Dews all the Vapours that were raiſed in the preceeding long Day; and for that reaſon, that part of its Atmoſphere might be ſeen much more pure and tranſparent. But from whatever Cauſe it proceeded, the thing it ſelf was very manifeſt and noted by every one.

About two Minutes before the total Immerſion, the remaining part of the Sun was reduced to a very fine Horn, whoſe Extremities ſeemed to loſe their Acuteneſs, and to become round like Stars. And for the ſpace of about a quarter of a Minute, a ſmall Piece of the Southern Horn of the Eclipſe ſeemed to be cut off from the reſt by a good interval, and appeared like an oblong Star rounded at both Ends in this Form.  which appearance could proceed from no other Cauſe but the Inequalities of the Moon's Surface, there being ſome elevated parts thereof near the Moon's Southern Pole, by whoſe Interpoſition part of that exceedingly fine Filament of Light was intercepted.

A few Seconds before the Sun was all hid, there diſcovered it ſelf round the Moon a luminous Ring, about a Digit, or perhaps a tenth Part of the Moon's Diameter in Breadth. It was of a pale whiteneſs, or rather Pearl colour, ſeeming to me a little tinged with the Colours of the *Iris*, and to be concentrick with the Moon, whence I concluded it the Moon's Atmoſphere. But the great height thereof far exceeding that of our Earth's Atmoſphere; and the Obſervations of ſome, who found the Breadth of the Ring to encreaſe on the Weſt ſide of the Moon, as the

Emerſion approached; together with the contrary Sentiments of thoſe whoſe Judgment I ſhall always revere, makes me leſs confident, eſpecially in a Matter whereto I own I gave not all the Attention requiſite.

Whatever it was, this Ring appeared much brighter and whiter near the Body of the Moon than at a diſtance from it; and its outward Circumference, which was ill defined, ſeemed terminated only by the extream rarity of the Matter it was compoſed of; and in all Reſpects reſembled the Appearance of an enlightned Atmoſphere viewed from far: but whether it belonged to the Sun or Moon I ſhall not at preſent undertake to decide.

During the whole time of the total Eclipſe I kept my Teleſcope conſtantly fixt on the Moon, in order to obſerve what might occur in this uncommon appearance: and I found that there were perpetual Flaſhes or Coruſcations of Light, which ſeemed for a Moment to dart out from behind the Moon, now here, now there, on all ſides; but more eſpecially on the weſtern ſide before the Emerſion; and about two or three Seconds before it, on the ſame weſtern ſide where the Sun was juſt coming out, a long and very narrow Streak of a dusky but ſtrong red Light ſeemed to colour the dark Edge of the Moon; though nothing like it had been ſeen immediately after the Immerſion. But this inſtantly vaniſhed upon the firſt appearance of the Sun, as did alſo the aforeſaid luminous Ring.

As to the Degree of Darkneſs, it was ſuch that one might have expected to have ſeen many more Stars than I find were ſeen at *London*: The three Planets, *Jupiter*, *Mercury* and *Venus* were all that were ſeen by the Gentlemen of the Society from the top of their Houſe, where they had a free Horizon: and I do not hear that any one in Town ſaw more than *Capella* and *Aldebaran* of the fixed Stars. Nor was the Light of the Ring round the Moon capable of effacing the Luſtre of the Stars, for it was vaſtly inferiour to that of the Full Moon, and ſo weak that I did not obſerve that it caſt a Shade. But the under Parts of the Hemiſphere, eſpecially in the *South Eaſt*, under the Sun, had a crepuſcular Brightneſs: and all round us, ſo much of the Segment of our Atmoſphere as was above the Horizon, and was without the Cone of the Moon's Shadow, was more or leſs enlightened by the Sun's Beams: and its Reflection gave a diffuſed Light which made the Air ſeem hazy, and hindred the Appearance of the Stars. And that this was the real Cauſe thereof, appears by the Darkneſs being more perfect in thoſe Places near which the Center of the ſhade paſt, where many more Stars were ſeen, and in ſome not leſs than twenty; though the Light of the Ring was to all alike.

During the Time whilſt the Sun recovered his Light, ſeveral Altitudes were taken to examine the Regularity of the Clock's Motion; and though the Sun now roſe much ſlower than at the beginning, yet they all conſpired within a very few Seconds that the Clock went ſtill one Quarter of a Minute too faſt. And the end of the Eclipſe approaching,

I attended the Moment thereof with all the Care I could, and concluded the compleat Separation of the Sun and Moon at 10h. 20′. 15″. by the Clock, or exactly 10h. 20′. correct time. The Lord Chief Juſtice *Parker*, at the Moment of the Emerſion from total Darkneſs, obſerv'd the diſtance of the Planet *Jupiter* from the *Zenith* 48°. 29′. by which the Time thereof is verified.

Monſieur *le Chevalier de Louville* having ſeen the Beginning, applied himſelf to take Digits with his Micrometer, and to obſerve the Occultations of three Spots at that time ſeen in the Sun; and he was pleaſed to communicate the following Notes, *viz.*

At 8h.	28′	20″	Four Digits were Eclipſed.
8	32	57	The firſt and bigger Spot touched the Moon.
8	33	18	The ſame was wholly hid.
8	34	08	The firſt of the two leſſer Spots was hid.
8	34	58	The Second of them was hid.
9	36	01	Emerſion of the greater Spot.
9	38	26	Emerſion of the firſt leſſer Spot.
9	40	25	Emerſion of the ſecond leſſer Spot.
10	20	04	The end of the Eclipſe.

And he determined the time of the total Darkneſs 3′. 22″, or one Second leſs than by my Account.

The Heavens were all the while very propitious to us, and there was very little or no Wind, and not ſo much as one Cloud interrupted our View from the Beginning to the end; but no ſooner was the Eclipſe over, but a great Body of Clouds hid the Sun for many Hours after.

The Reverend Mr. *James Pound*, Rector of *Wanſted* in *Eſſex*, and R.S.S. gives the following Account of the principal Phænomena obſerved there; having rectifyed his Clock by ſeveral Altitudes of the Sun taken both before and after, *viz.*

At 8h.	6′.	37″	The Eclipſe firſt perceived.
9	9	28	The total Immerſion.
9	12	48	The Emerſion.
10	20	32	The juſt end of the Eclipſe.
0	3.	20	The Continuance of total Darkneſs.

The near Agreement of this Obſervation with our own (the difference being only what is due to the difference of our Meridians) makes us the leſs ſolicitous for what was noted at the *Royal Obſervatory* at *Greenwich*, from whence we can only learn that the Duration of total Darkneſs was 3′. 11″.

Mr. *William Derham* Rector of *Upminſter* in *Eſſex*, and R. S.S. made the following Obſervations there,

At 8h.	7′.	41″	The Eclipſe began.
8	33	46	The Moon touched the greater Spot.
8	34	36	She touched the middle Spot.
8	35	41	She touched the third Spot.

At 9h.	10′	58″	The total Darkneſs began on a ſudden, and *Aldebaran* appeared.
9	14	6	The Emerſion or end of total Darkneſs.
0	3	8	Continuance of total Darkneſs.
9	42	41	The third and laſt Spot diſcovered.
10	21	45	The End of the Eclipſe, by a 13½ Foot Glaſs.

And a little before the Beginning of the Eclipſe, he found the greater and preceeding Spot to be more Northerly than the Sun's Center 373½ ſuch parts as the Sun's Diameter was 1647, and that it followed his weſtern Limb 0′. 43″ of Time: by which *data* the Situation of that Spot is well determined.

My worthy Collegue Dr. *John Keill* by reaſon of Clouds ſaw nothing diſtinctly at *Oxford* but the end, which he obſerved at 10h. 15′. 10″. As to the total Darkneſs, he could only eſtimate it by the ſudden Change of the Light of the Sky; and reckoned its Continuance but 3′. 30″; which was certainly too little, the Center of the ſhadow having without doubt paſt very near *Oxford.* And Mr. *Cotes* at *Cambridge* had the misfortnne to be oppreſt by too much Company, ſo that he obſerved only the Occultations of the three Spots, *viz.* of the firſt and greateſt at 8h. 34′. 11″. of the ſecond at 8h. 35′. 15″, and of the laſt at 8h. 36′. 55″. He noted alſo the end of total Darkneſs at 9h. 14′. 37″, and the exact end of the Eclipſe at 10h. 21′. 57″.

We have received ſeveral Accounts from ſome Places which lay near the Track of the Center of the Shade, and which might have been very proper to determine the greateſt Continuance of the darkneſs; as from *Plymouth*, *Exeter*, *Weymouth*, *Daventry*, *Northampton* and *Lynn regis*, all agreeing that the whole Sun was obſcured at thoſe Places full four Minutes, and at ſome of them rather more. But theſe Obſervers give us no Account how they meaſured this Time, and therefore it may well be ſuppoſed they took it in a round Number, and perhaps from pocket Minute-Watches. What I think may beſt be relied on for this Purpoſe, are two correſponding Obſervations made, the one at *Barton* near *Kettering* in *Northamptonſhire*, where by the Obſervation of *John Bridges*, Eſq; Treaſurer of His Majeſty's Revenue of Exciſe, and R. S. S. with a good Pendulum-Clock and all due Care, the whole Sun was hid no more than 3′ 53″. The other was by Mr. *John Whiteſide*, A. M. Keeper of the *Aſhmolean Muſeum* at *Oxford*, and a skilful Mathematician, who obſerved after the ſame manner, at *King's-Walden* in *Hertfordſhire* near *Hitchin*, that the total Eclipſe continued but 3′. 52″. Hence it follows that the Center of the ſhade paſt near the middle between theſe two Places, which are but 30 Geographical Miles aſunder, and ſituate near at right Angles to the Way of the ſhade, and therefore that the total Obſcurity, where longeſt, could laſt but about 3′. 57″, or perhaps a Second or two more at *Lynn*, and leſs at *Plymouth*: the Velocity of the Progreſs of the ſhade gradually decreaſing, and its Diameter encreaſing as it paſt on to the Eaſtwards. And this Situation of the middle Line is confirmed by an

an Obſervation made at the Seat of the Right Honourable the Lord *Foley* at *Witley* eight Miles beyond *Worceſter*, by his Order, whereby it appears that the total Darkneſs laſted there 3′ 15″. Hence it follows that *Witley* was about three or four Miles farther from the Center of the Shade on the Northſide than *London* on the South; and *Witley* being by *Ogilby*'s Menſurations, 118 meaſured Miles from *London*, it is plain that the Center paſt over *Iſlip*, which is, by the ſame Admeaſurement, 57 ſuch Miles on that Road, and about five Miles almoſt due North from *Oxford*; ſo that the Center of the Shade left *Oxford* but very little upon the Right-hand. This Situation agrees perfectly well with the former between *Barton* and *King*'s-*Walden*, and as far as the Geography of our Country may be relied on, I conclude the Center to have entred upon *England* about *Plymouth*, and to have paſt over *Exeter*, the *Devizes*, *Iſlip*, *Buckingham* and *Huntington*, leaving *Oxford* and *Bedford* on the Right, and *Lynn* on the Left, and to have quitted the Coaſt of *Norfolk* about *Wells* and *Blakeney*.

As for the Limits of the Shade, both on the North and South ſide, we have by Enquiry gotten them with all the Exactneſs the thing is capable of, and we ſhould have been glad the *French Aſtronomers* had done the like for the total Eclipſe that paſt over *Languedoc*, *Provence* and *Dauphiny* on the 1ſt of *May* 1706. But as this is the firſt Eclipſe of this kind that has been obſerved with the Attention the Dignity of the Phænomenon requires, we hope thoſe which may happen for the future to traverſe *Europe*, may not paſs by ſo little regarded as hitherto.

As to the Southern Limit or Term where the Eclipſe ceaſed to be total on the South ſide of the Sun, we have received an Account of an Obſervation made at *Nortoncourt* about ten Miles on this ſide *Canterbury*, by Dr. *John Harris*, aſſiſted by Mr. *Stephen Gray*; by which we learn that the Eclipſe began there at 8h. 8′. 55″. and ended at 10h. 24′. 47″; and that the total Darkneſs continued but about one Minute or rather leſs, the middle thereof being at 9h. 13′. 52″. From this Duration it will follow that *Norton-court* was but about 3 or 4 Miles within the Shade. And that it was really ſo is confirmed by the Relation of the Inhabitants of *Boſton*, about Midway between *Norton-court* and *Canterbury*, who aſſured Mr. *Gray*, as he was returning home that ſame Day, that the Eclipſe was not total there, but as one of them expreſt it, before the Sun had quite loſt his Light on the Eaſt-ſide he recovered it on the Weſt: and that there was a ſmall Light left on the lower part of the Sun that appeared like a Star. And from *Cranbrook* in *Kent*, we are inform'd, by the Relation of *William Tempeſt*, Eſq; that he obſerved there the Sun to be extinguiſhed but for a Moment, and inſtantly to emerge again: So that the Limit paſt exactly over this Town, which is about 38 Geographical Miles from *London*, and very near the right Angle where the Perpendicular from *London* falls on the Line of the Limit, being 3′. 00 of Time to the Eaſtwards of *London* in the Latitude of 51°. 6′, as near as I can gather.

How-

How it past over *Sussex* we have not so authentick Relations, but have learnt it was total at *Wadhurst* beyond *Tunbridge-Wells*, as also for some short time at *Lewis*; but it was not so at *Brightling*, which Place being situated on an Eminence that has a commanding Prospect, all the Country to the Northward was seen in Darkness, whilst they there had some Benefit of a small Remainder of the Sun.

From these Observations we may conclude that this Limit came upon the Coast of *England*, about the middle, between *Newhaven* and *Brighthelmston* in *Sussex*, and passing by *Cranbrook* and *Boston*, left *Canterbury* about four Miles on the Right-hand, and quitted the Coast of *Kent*, not far from *Hern* toward the antient *Regulbium*, now called *Reculver*. So that it seems scarce one third part of *Kent*, and not so much of *Sussex*, out of all the South Coast of *Great Britain*, escaped being involved in this Darkness.

The Northern Limit, having past over a much greater Space, has had more Observers, and is not less curiously determined than the other. We find by the account given by Mr. *Roger Prosser*, Rector of *Haverford-West*, that the Eclipse was total there a Minute and half, whence it follows that *Haverford* was but about 6 Miles within the Limit; and therefore that it entred on *Pembrokeshire* about the middle of St. *Bride*'s *Bay*, leaving St. *David*'s and *Cardigan* on the Left-hand: and having traversed those two Counties and *Montgomeryshire*, it entred on *Shropshire*, leaving the Town of *Shrewsbury* 1'. 40''. in the shadow, as was observed there by Dr. *Hollings*: whereby it appears that *Shrewsbury* was about 8 Miles within the Limit. Thence it proceeded by the East-side of *Cheshire*, leaving *Whitchurch* and *Nantwich* a very little without; and passing by *Congleton* went over the *Peak* of *Darbyshire* into *Yorkshire*, and crost the great Northern Road between *Pontefract* and *Doncaster*, somewhat nearer the former than the latter. For by the Observations of *Theophilus Shelton*, Esq; at *Darrington*, about two Miles on this side *Pontefract*, (in Lat. 53°. 40' and Long. West from *London* 4'. 40''. of time, as may be concluded from *Norwood*'s Measure of a Degree) the Sun at 9h. 11'. was reduced almost to a Point, which both in Colour and Size resembled the Planet *Mars*; but whilst he watched for the total Eclipse, that Point grew bigger, and the darkness diminished; whence he argued the Limit to have been very little more Southerly; and since he has been informed that it was just total in *Barnsdale*, three Miles South from thence. And that it was so at *Badsworth* about the same distance from *Darrington*, we are told by a Letter of Mr. *Daubuz*, that he has a certain Account from that Place, that the luminous Ring round the Moon was seen there, which was no where visible but while the Eclipse was Total. From these *Data* we may securely determine the Remainder of this Track, and that the edge of the shadow having past over the rest of *Yorkshire*, went off to Sea about *Flamborough-head*.

So that of the forty Counties into which *England* is subdivided, only the five most Northerly have not had the Sun wholly hid from them; and

and ſix others have eſcaped but in part, *viz. Shropſhire*, *Cheſhire* and *Yorkſhire*, and the extream part of *Darbyſhire* on the *North*, and *Kent* and *Suſſex* on the *South*; all the reſt of the Kingdom having more or leſs ſuffered an Interval of total darkneſs.

I ſhall not at preſent conſider this Eclipſe as univerſal, but only as it related to *England*; and it ſhall ſuffice to ſay, that the ſhadow came out of the *Atlantick* Ocean, having paſt over the Iſlands *Azores*; and that the Southern Limit thereof reach'd the Iſle of *Uſhant*, and the North-weſt Coaſts of *Britanny* between *Breſt* and *Morlax*; and dividing our Iſlands of *Guernſey* and *Jerſey*, juſt touched upon the Promontory of *Normandy* called *Cape de Hague*. And that after it had quitted *England* and traverſed the *German Ocean*, it fell on *Jutland* on the Southſide, and *Norway* on the *North*; and thence proceeded to the Eaſtwards over *Sweden*, *Finland*, &c.

It remains now to conſider the Figure, Poſition, Direction, Velocity and Magnitude of the ſhadow as it paſt over us. And firſt as to the Figure, 'tis obvious that the ſhadow of the Moon being a Cone, and the Earth's Surface ſufficiently Spherical, the apparent ſhadow on the Earth will be the common Interſection of a Cone and Sphere, which is a Figure hitherto little conſidered by Geometers; and not being *in Plano* is not to be exactly deſcribed but in the Spherical or Conical Surface. How to find the Points of this Curve in all Caſes is taught by *P. Courſier*, in a very ſcarce Latin Book, printed at *Dijon* in *Burgundy*, and publiſhed at *Paris* in the Year 1663: nor do I hear of any other Author that has handled the ſame Subject ſince, though capable and worthy of further Improvement. By what he there delivers, *Prop.* 11. 12. Lib. I. it will be eaſily underſtood, that the Convexity of ſo ſmall a part of the Earth's Surface as the ſhadow commonly occupies, can produce but an inconſiderable Effect; ſo that without ſenſible Error we may take it for a Plain, and the Section for a true *Apollonian Ellipſis*, whoſe tranſverſe *Axis*, by reaſon of the ſmallneſs of the Angle of the Cone, will be to its Conjugate nearly as *Radius* to the *Sine* of the Sun's Altitude at its Center, eſpecially if he be conſiderably elevated. But when he is near the Horizon, it will be neceſſary to have regard to the true Figure, by reaſon of the great length to which the tranſverſe Axe is extended, and particularly when the ſhade is entring upon or leaving the Earth's Disk. Of theſe perhaps a fuller Account may be given upon a further occaſion.

As to the Poſition of the *Axis* of the ſhadow, it is manifeſt that it muſt always lie in the Plane of a great Circle of the Earth paſſing through the *Axis* of the Cone of the Shade: and therefore it will be only requiſite to obtain the Azimuth and Altitude of the Sun at the Place where the Center of the ſhade at any time is found, to determine the Situation of the Axe and *Species* of the Ellipſe required. Thus the middle of the Eclipſe at *London*, having been obſerved at 9h. 10'. 45", by the given Latitude and Declination we find his Azimuth about 59°. 00'. and

and Altitude 40°. 46′. that is juſt 40 Degrees high at the Center of the Shadow. Wherefore the tranſverſe Axe of the Ellipſe was to its Conjugate very near as *Rad.* to the *Sine* of 40°, or as 1000 to 643 *proxime*; and did make an Angle of 59°, or very little more, with the Meridian paſſing at that time through the Center of the Shade.

Next the Direction and the Velocity of the Motion wherewith the Center of the ſhade paſt over *England* comes to be conſidered, wherein the Reader is to be told that the ſhadow paſſes in a very compound Curve, which as the former, is not *in plano*, and only deſcribable on the Surface of the Sphere: nor is its Motion equable, but compounded of very many Elements producing a great Variety. By what Method its Points, and its Tangents in thoſe Points, are to be obtained, I reſerve to the next Opportunity; this Account being deſigned for the Curious in general: only I muſt acquaint them, that for ſo ſmall a part of the Curve as went over *England*, it may be eſteemed a right Line, with more Exactneſs than we uſually find in moſt of our Geographical Charts. And the like may be ſaid for the Velocity, which though in our preſent Inſtance it was continually decreaſing, may for ſo ſhort a time be ſuppoſed to have been the ſame without ſenſible Error.

By a careful Calculation I have determined the Velocity of the Motion, at the time of the middle of the Eclipſe at *London*, to have been 29 Geographical Miles in a Minute of Time *quam proxime*: and that its way made an Angle of 52°. 45′ with the Meridian towards the Eaſtwards of the North; wherefore the ſaid way made an Angle with the Axis of the Ellipſis of 68°. 15′. And the greateſt Duration of total Darkneſs having been 3′, 57″, (as was before ſhewn) it will follow, that that Diameter of the Elliptick Figure according to which the ſhade paſt, was no leſs than 114½ Geogr. Miles. And from the Elements of the Conicks 'tis eaſy to be proved, that ſuppoſing the Figure of the ſhade a true Ellipſe, whoſe Axes are as *Radius* to the *Sine* of 40 Degrees, the greater Axis would be 171 Geographical Miles, and the leſſer 110; and the neareſt diſtance between the Limits ſuppoſed Parallel 164 ſuch Miles.

And this Length of the *Axis* of the Shade, derived purely from the Continuance of total Darkneſs, is fully confirmed by the obſerved diſtance of the Parallel Limits; the one paſſing by *Badſworth* in *Yorkſhire*, the other by *Cranbrook* in *Kent*. For by the two Latitudes 53°. 37′ and 51°. 6′, with the difference of Longitude 7′ and 40″ of Time, or 1°. 55′, the diſtance of theſe two Places is given 166½ Geogr. Miles; with the mean Angle of Poſition 25 Degrees from the North Weſtwards; wherefore this Arch makes an Angle with the Track of the ſhade of 77°¼: and hence the neareſt diſtance of the Parallels becomes 163 ſuch Miles, which by the other way was found 164.

If therefore we conclude the *Axis* of the ſhadow, when the Sun was juſt 40 Degrees high, to have extended over 2°. 50′ of a great Circle, we may ſecurely determine the Difference of the Sun and Moon's Diameters at this time. For the difference of the Horizontal Parallaxes of the

the Sun and Moon being found to be 60′. 38″. (as ſhall be hereafter ſhewn, but is not required with extream exactneſs for this Purpoſe) the difference of the Parallaxes in Altitude at both ends of the Axis, will be found to be 1′. 56″; and by ſo much did the Diameter of the Moon when forty Degrees high exceed that of the Sun: Hence the horizontal diameter of the Moon in this Anomaly is found 33′. 27″, which may ſerve for a Rule in all other Caſes.

Laſtly, I have added the following *Synopſis* of ſuch Obſervations as have hitherto come to my Hands; acknowledging the Favour of all thoſe, who have been willing to promote our Endeavours to perfect the doctrine of Eclipſes.

Place	Obſerver	Beginn. h. ′ ″	Immerſ. h. ′ ″	Emerſ. h. ′ ″	Tot. ′ ″	End. h. ′ ″
Barton	M. *Bridges*				3.53	
Bell-bar	M. *Jones*	8. 6. 25	9. 9. 45	9. 13.27	3.42	
Broadway } *Carmarth.* }			8. 47.00	8. 49.30	2.30	
Cambridge	M. *Cotes*			9. 14.37		10. 21.57
Canterbury	M. *Gray*	8. 10. 00				10. 24.30
Cheſter	M. *Ward*	7. 57. 40				10. 6. 35
Crew	M. *Wright*		9. 2. 8		2.00	10. 9. 00
Dublin	L. *Arch.Biſh.*	7.42. 11				9. 49.40
Dublin	M. *Hawkins*	7.41. 30.				9. 48.45
Exon	L. *Biſhop*		8. 55. 0	8. 59. 0	4.00	10. 0. 00
Exon	M. *Hudſon*	7. 47.30			3.30	10. 0. 30
Greenwich	M. *Flamſteed*				3.11	
King's Wald.	M. *Whitſide*				3.52	
Llanidan { *Angleſey* {	M. *Rowland*	7.52. 30				
London	R. *Society*	8. 6. 00	9. 9. 3	9. 12.26	3.23	10. 20.00
Northampt.	M. *Hawkins*		9. 5. 22	9. 9. 24	4. 2	10. 15.35
Norton-court	D. *Harris*	8. 8. 55	9. 13.23	9. 14.22	0.59	10. 24.47
Oxon	D. *Keill*				3.30	10. 15.10
Paris	R. *Academy*	8. 11.00				10. 28.00
Plymouth	M. *Heines*	7. 41.00	8. 45.30	8. 50 00	4.30	9. 54.30
Portcheſter	C. *Candler*		9. 2. 25	9. 6. 15	3.50	
Salop	D. *Hollings*	7. 58. 0			1.40	10. 6. 00
Upminſter	M. *Derham*	8. 7. 41	9. 10.58	9. 14. 6	3. 8	10. 21.45
Wanſted	M. *Pound*	8. 6. 37	9. 9. 28	9. 12 48	3.20	10. 20.32
Weymouth	M. *Hobbs*		8. 54.00	8. 58. 00	4.00	
Witley	M. *Baxter*	7. 59. 0			3.15	10. 13.00

n. 345. *p.* 314. Mr. *John Edens*, being on his Voyage to *Teneriff*, observed the Eclipse at Sea, in Latitude, by Observation 34°. 20′, and Longitude 0h. 54′, West from *London*, as he concluded by their Distance and Position from the Island *Forte ventura*, which they soon after fell in with. He writes that it began at vih. 49′. and ended at viiih. 47′. this latter very exactly, though not quite so nice as to the beginning.

Had this Observer signified what difference of Meridians there was found between the Place of Observation, and the West end of *Forte ventura*, we might without sensible Error, have concluded the true Longitude, not only of that Island, but also of the Pike of *Teneriff*, where *Our Geographers* and the *Dutch* have fixed their first Meridian. But this Gentleman being both able and desirous to render the Publick this sort of Service, we hope from him such further Observations as may put the matter past Dispute. He adds, that the greatest Darkness was about $\frac{3}{4}$ of the Sun's Diameter, or nine Digits on the Northside.

From *Germany* we have received the following Accounts.

At *Nurenburg*, the beginning and greatest Obscurity could not be seen for Clouds, but the end happen'd at xih. 10′.$\frac{1}{5}$.

At *Hamburg*, the beginning was observed at viiih. 57′. The greatest Obscurity at xh. 5′. 30″, when xi$\frac{1}{2}$ *digg.* were darkned. The end could not be seen for Clouds.

At *Kiel* in *Holstein*, The beginning ixh. 14′. The greatest Obscurity xh. 19′. 20″, and the Quantity then eclipsed xi. *digg.* 20′. The end was at xih. 29′.

At *Berlin*, the beginning could not be seen for Clouds, but the greatest Obscurity was at 22 *min.* past ten, when xi *digg.* were eclipsed. The just end was at xih. 34′.

At *Franckfort* on the *Mein*, The Eclipse began at viiih. 50′. The greatest Darkness at xh. 11′, *but perhaps should be* xh. 1 *min.* the Digits being x. and 34 *min.* The end was observed at 10 *min.* past Eleven.

By whom these Observations were made, and with what Instruments, we are not as yet informed; but hope they may be exact enough to confirm the *Longitudes* of those several places, which are at present reasonably well known.

Since these there is lately come to Hand a *Dutch* Print entituled *Nouvelles Literaires*, publish'd at the *Hague*, wherein p. 404, 405. there is an Account of the Observation of this Eclipse at *Upsal* in *Sweden*, made by Mr. *Jo. Waller*, Professor of Mathematicks in that University, the Times being verified by three Clocks perfectly agreeing with one another, and with the Sun; but more especially by a Quadrant of 5 Foot Radius for taking the Sun's Altitude. By this Instrument he has determined the height of the Pole at *Upsall* 59°. 51′. 54″. And by the same, a little before the beginning of the Eclipse he found the height of the Sun 39°. 36′. 42″. his Clocks then shewing the Hour ixh. 47′. 50″, which proves that they were very near the true Time. At xh. 58′. 15″. the Altitude of the Sun being 44°. 17′ 29″, was the beginning of the total Darkness,

and

and at xi^h. 2'. 24''. was the end thereof, *alto sole* 44°. 29'. 13''. so that here the Duration of the total Eclipse was 4'. 9'', and the middle thereof but one third of a Minute after Eleven. And lastly the end is said to have happen'd about 4 Minutes before Noon, the Sun being 45°. 42'. 6''. high: But in this is a manifest Mistake, for it makes the Time of Emersion, or from the middle to the end, but 55' 20''; whereas being so near the Meridian, 'tis certain that this Emersion was the greater part of the Duration of the whole Eclipse, and consequently more than an Hour. Perhaps the Times might be deduced from the Altitudes only, and then the Mistake might be in supposing the end so much before Noon as it was really after it. However, to prevent all Doubts, we have compared this Observation with what we observed of this Eclipse at *London*, and find that in the Latitude of 59° 50', the Place where the middle of total Darkness was at xi^h. 0'. 20'', was near 19 Degrees more Easterly than *London* (that is exactly in the Meridian of *Dantzick*) and that the Eclipse began there at ix^h. 52' ½, and ended at xii^h. 10'. Wherefore the Duration could not be 2^h. 7'. 50'', as the *Editor* of the said *Nouvelles* has publish'd; not considering that the Beginning could not be seen for Clouds, as in the very next Words he assures us.

As to the Darkness, it was such that they could scarce distinguish one another: and besides *Jupiter*, *Mercury* and *Venus*; of the Fixt Stars *Cassiopea*, *Capella*, *Oculus Tauri* and *Orion*, (Sirius *not being yet risen*) were visible.

XXI. *Feb.* 19. 1717-8. Noribergæ Sol ortus est aliquantulum deficiens in limbo superiore, qui quidem defectus accrevit ad 3 plene Dig. Desiit Eclipsis 8^h. 8'. 48''. circa 60 *gr.* a Vertice Solis ad sinistras. *One at* Nuremburg, *by Mr.* Wurtzelbau.

2. Berolini Sol statim ab ortu cœpit deficere, Hora 6. 49'. vel 49'. ½. *n.* 357. *p.* 822. Circa medium Eclipsis nempe 7^h. 35'. erant partes lucidæ in Sole residuæ 24'. 40''. unde digiti obscurati 2 dig. 50'. Finis autem in 8^h. 28'. 10''. *At* Berlin, ib.

XXII. The Moon rose eclipsed, and the Horizon was so overcast, that I despair'd of having any Observation; but at ½ an Hour past 6 she came from under the Cloud, and at 6^h 25'. I had just a sight of her, and judge her eclips'd about 5 digits, at *An Eclipse of the Moon near* Boston, *by Mr.* Brattle, Feb. 11. 1700. *n.* 292. *p.* 1633.

6^h. 29'. The Section equidistant from M. Ætna and Horminius.

32. Palus Maræotis begins to be seen.

34½ Palus Maræotis and Mons Apollonius ½ out.

37½. Palus Maræotis quite free, and Palus Maræotis and Palus Mæotis in the perpendicular.

42¾. The shadow near an Inch from Palus Maræotis, Mons Horminius and Mons Hercules.

46¼. Palus Maræotis in the Nadir, and that part of Palus Mæotis to my Right-hand in the Prime Vertical.

57. The upper part of the Section is now, and has been for a long time in Insula Major in Mare Caspio (and the Section now

perpendicular) and the lower part wheeling about from Palus Maræotis.

7h. 20. Mount Sinai first appears at 22′. wholly free.

25 ½. Palus Maræotis and Mons Horminius near perpendicular.

43. The Eclipse over in the Telescope, and at 49′ to the naked Eye.

My Clock was set by my Ring-Dial about 9 a Clock in the Morning, as exactly as I could judge, and the Observation was made with my 4½ Foot Telescope, with all four Glasses in it.

XIII. 1. *Another* Dec. 12. 1703. *near* Boston *at* Cambridge, *by the same* n. 292. p. 1635.

11h. 45′. That part of the Moon's Disk near Alabastrinus, looks somewhat duskish, and the Eclipse beginning to enter between Palus Maræotis and M. Porphyritis.

11 53. The true Shadow was well entred.

58. M. Porphyritis just cover'd.

12 03 ½. Near 3 Digits darkned.

7 ½. Mount Ætna begins.

9 ½. Quite covered.

14 ½. Lacus Niger major and M. Sinai almost equidistant from the Section of the Shadow; Lacus Niger Major being somewhat the nearer of the two.

18 ¼. Lacus Niger Major begins. 19 ½ quite covered.

21 ½. Mount Sinai begins.

21 ¾. Quite covered, and the Moon about 6 Digits eclipsed.

12 24 ½. Besbicus begins.

26. Quite covered.

28 ¾. Bysantium begins.

29 ½ Covered, and Mount Horminius begins.

32. Apollonia begins.

33. Covered.

37. The Shadow equidistant from M. Corax and Mount Paropamisus, or somewhat nearer to M. Corax.

39 ½. Between 9 and 10 Digits eclipsed.

43. M. Corax begins.

44 ¾. Palus Mæotis begins, and at 45 ¼ the inner of M. Paropamisus begins.

50. Palus Mæotis quite covered.

51 ½. The Moon not quite eclipsed.

52. Nor yet.

53. Nor yet.

54. Scarce.

54 ½. Quite Immerg'd, and the Mora begins.

14 39. Precisely, she Emerg'd between Palus Maræotis and Mons Porphyritis.

42. Palus Maræotis begins.

43. Quite clear.

47. M. Porphyritis quite clear.

55. About

14 55. About 3 digits reſtor'd.
59. Mount Ætna begins.
15 02. That and Lacus Niger Major at the ſame time clear.
8½ Mount Sinai about half free.
9½ Quite free, and about 6 digits reſtored.
15. Besbicus free.
19¼. Byſantium free.
29½. About 9 digits ſeem'd to be reſtored.
30¼. Mons Herculis free.
32¾. Palus Mæotis begins.
38½. Quite free.
41½. Inſula Major in Mare Caſpio free, and in the middle of the Section.
42½. Not yet wholly clear.
45. Fully over in the Teleſcope, though a kind of a Smoak remain'd ſome little after to the naked Eye.

In order to the adjuſting of the time, I ſet my Clock with the greateſt exactneſs I could the Morning preceding, both from my Ring-Dial and the riſing of the Sun, which I very narrowly watch'd and obſerved, and found it to agree with the Sun's ſetting the following Evening; ſo that it went all the time the Eclipſe was, very ſteadily and regularly; but for the greater Certainty and Satisfaction, I took the Altitudes of the following Stars with the Braſs Quadrant with Teleſcope Sights out of my Chamber Window, the lowneſs whereof would not permit me to take them, when they were at all higher elevated.

* In dextro humero Orionis. By the Watch.		Comp. Alt.					Differ.		
6h	15′	78	18	6h	13′	40″	1′	20″	So that my
6	21½	77	03	6	20	28	1	02	Clock went
	26½	76	11	6	25	08	1	07	by theſe
Procyon									Obſervati-
8	9¼	77	20	8	08	04	1	11	ons near-
	14½	76	20	8	13	32	0	58	eſt 1′ too
	21	75	13	8	19	36	1	24	faſt.
Re- 10	8½	77	46	10	07	18	1	12	
gulus	17¼	76	11	10	15	58	1	17	

I had the good fortune to make ſome few Obſervations of the laſt of *December* the 11th, 1704. as follows. 2. *Some obſervations hereon, by* Mr. Hodgſon, *ib. p.* 1637.

The Heavens being cloudy moſt part of the Night, it was 35′ after 4 in the Morning following, before I could perceive that the Moon was eclipſed, and then as near as I could judge, ſhe had been ſo about three or four Minutes at moſt, from whence we may conclude it began at *London* about 31 or 32 Minutes after four the ſame Morning.

Mr.

Mr. *Brattle* found, that at 44 Minutes after 11 at Night, part of the Moon's Disk look'd somewhat duskish, and that at 52 Minutes, the Shadow was well entred, *so that from hence, as well as from a Comparison of the Ingress and Egress of the principal Spots, it probably began there about* 49 *Minutes after* 11, *whence it follows, that* Cambridge *in* New England *lies* $4^h\ 4'\ 2\frac{1}{2}$, *or* $70°\ 37'$ *to the Westward of the Meridian of* London.

I happen'd to see the Moon the same Morning at 35 Minutes after 5, when she wanted at most but 3 Minutes of being totally eclipsed; so that at *London* she immerg'd at 38 Minutes past 5.

Mr. *Brattle* saw her immerge exactly at 54 Minutes after 12, whence it follows, that the difference of the Meridians found by comparing these Observations, is $4^h\ 43'\ \frac{1}{2}$, or $70°\ 52'$ agreeing very well with the former; so that by taking a Mean between them, the difference of Longitude of the 2 Places is $4^h\ 43'$, or $70°\ 45'$.

XXIV. 1. *Another at* Zurich, Apr. 17. 1707. *N.S. by Dr.* Scheuchzer, *n.*310.*p.*2354.

H.	Min.	S.	
12	9 ad 18		Penumbra ex parte Maræotidis.
	18	40	Umbra vera intra discum.
	20	15	Palus Maræotis in Umbra.
	23		Maris Eoi principium.
	25	20	Mons Alabastrinus. Medium Maris Eoi.
	27	40	Principium sinus Sirbonii.
	29		Medium sinus Sirbonii, & Mare Ægyptiacum.
	29	20	Principium insulæ Cercinnæ.
	29	30	Lacus Meridionalis.
	30	30	Medium Cercinnæ.
	31	40	Finis Cercinnæ.
	33		Extrema protensio Montis Sopher.
	34		Insulæ inter Siciliam & Cercinnam.
	35		Principium Mauritaniæ, & sinus Hyperborei.
	36	40	Medius sinus Hyperboreus.
	37	20	Mare Pamphilium.
	37	30	Creta.
	37	40	Principium Ætnæ.
	38	40	Medium Ætnæ. Melos. Carpathos.
	39	30	Finis Ætnæ.
	41		Rhodus.
	43	40	Initium Sinai.
	45		Medium Maris Adriatici, & Sinai.
	45	20	Sinai Finis.
	46	30	Medium Adriatici.
	48	40	Principium Propont. & Maris Hyperborei.
	51	30	Medium Propontidis, & finis Adriatici.
	52	40	Principium Lacus nigri majoris.

H.	Min.	S.	
12	53	10	Lacus Thrasumenus.
	53	30	Medium Lacus nigri minoris.
	54	30	Principium Insulæ Besbyci.
	55	10	Principium Ponti Euxini in sinu Salmydesso.
	56		Finis Propontidis.
	57	30	Ponti Euxini inferioris principium.
	58		Byzantii principium.
	59	10	——finis.
	59	40	Promontorium Acherusium.
1	1		Borysthenis principium. Apollonia.
	1	30	Medium Ponti Euxini.
	3	20	Sinus Atheniensis Medius.
	4	40	Palus Byces.
	5	40	Promontorium Heracleum.
	7	40	Cochilis media. Finis sinus extremi Ponti.
	8	30	Lacus Corocondæ Medium.
	9	20	Promont. Hercul. & Maris Caspii initium.
	11	40	Initium Paludis Amadocæ.
	16	20	——Medium.
	18	10	——Finis.
	20		Paludes amaræ, & Lacus minor.
	20	40	Lacus major.
	22	10	Tenuissimus margo lucidus.
	23	20	Lunæ corpus totum in umbra.
	24	40	Discus Lunæ integer fere, excepto Mari Mediter. diluto quodam fulgore splendet, ut Maria distingui possint per Tubum.
	40		Alia macula præter paludem Mæotidem per tubum distingui nequit.
	45		Lunæ discus medius magis magisque obscuratur, ambitu manente lucidiore.
	12		Discus Lunæ rutilo colore nudo oculo refulgebat, nec per Telescopium macula ulla distingui potuit.
	15		Totus Discus obscurior magis, & magis, Peripheria manente lucidiuscula.
	28		Splendidior Discus e regione Paludis Maræotidis, umbraque densissima versus Paludem Mæotidem.
	33		Sensim lucidior redditur integer Discus, obscuritate majore tegente Paludem Mæotidem ejusque loca vicina.
	51	40	Redeunt sensim vestigia Marium.
	56	30	Pontus Euxinus, & Mare Caspium, in media obscuratione manent veluti nebula crassa perfusa.
	5		Distingui potest Mare Eoum, & vicina, ut ut Luna nondum ex umbra emerserit.
	9	40	Emersionis initium verum.

H.	Min.	S.	
3	11	30	Incipit emergere Palus Maræotis.
	13		Evasit:
	15	40	Incipit Mare Eoum.
	21	30	Evadit Sinus Sorbonius, & Mare Egyptium.
	26		Evasit Cassiotis Regio, & aliquot minuta ante Cercinna insula.
	27	40	Evasit Athos Mons, & Maltha.
	31		Emersit Mauritania.
	38		——— Corsica, & Sicilia.
	44		——— Mare Adriaticum.
	45	20	——— Media Propontis.
	49		——— Besbycus.
	52		——— Byzantium.
	57	30	——— Promontorium Acherusium.
4	5	20	Emersit Pontus Euxinus, & medium Caspium.
	6		Incipit Mæotis Palus.
	9		Emersit Caspium, & Media Mæotis.
	11	20	Emersit Mæotis.
	13	40	Penumbra.
	14	20	Luna tota integra.
			In Emersione videbatur mihi umbra distinctior, quam in immersione.
12	18	40	Eclipseos initium in umbra vera.
1	23	20	Obscuratio maxima.
3	9	40	Emersionis initium.
1	46	30	Duratio totalis obscurationis.
4	14	20	Emersionis finis.
3	55	50	Tota Duratio.
1	5	40	Ab initio ad Immersionem Lunæ totalem.
1	5	40	Ab emersione totalis Eclipseos ad finem.

2. *At* Boston, *by Mr.* Brattle, *n.* 312. *p.* 2471.

Tempus ex Alt. correct.			*Apr.* 5. O. S. Immersiones.
6h.	52′	″	Penumbra valde notabilis.
6	58	20	Palus Maræotis tegitur.
7	8	15	M. Porphyrites incipit.
7	9	20	Tegitur.
7	16		M. Ætna incipit.
7	17	15	Penitus tectus.
7	21	40	M. Sinaiincipit.
7	22	40	Plane tectus.
7	24		Insula Corsica tegitur.
7	31	40	Lacus niger major tegitur.
7	33		Insula Besbicus.
7	36	30	Bizantium.
7	37	20	M. Horminius.
7	40	30	M. Apollonii.

M.

		Tempus ex Alt.correct.		
M. Hercules		7h	44′	30″
M. Corax		7	51	30
Palus Mæotis incipit		7	52	45
Insul. Maj. in M. Caspio incipit		7	54	45
Tegitur		7	56	
Palus Mæotis omnino tegitur		7	57	30
Luna plena Immersa		8	1	15
Comp. Alt. Arcturi	53° 34′	8	28	
	51 30½	8	39	15
Comp. Alt. stellæ quæ sequitur lucidum in corona septentrionali				
Lat. 44° 33′	60° 2′	9	0	30
	56 57	9	17	15
Emersiones.				
Luna emergere plane incipit		9	46	30
M. Ætna tota illustrata		10	9	30
M. Sinai tota apparet		10	10	15
Insula Besbicus		10	25	
Bizantium		10	28	30
M. Apollonii		10	33	
M. Hercules		10	36	30
Palus Mæotis incipit		10	44	
Insula Major in M. Caspio restauratur		10	47	
Palus Mæotis tota retecta		10	49	
Luna plene illuminatur		10	54	

XXV. *Sept.* 18. 1708. I observ'd for half an Hour or more a thin Shade to possess that part of the Disk where the Eclipse began, which remain'd a good while after the Eclipse was over. *Another at* Upminster, *by Mr.* Derham, *n.* 320. *p.* 312.

The correct Appar. Time.			
7h	56′	30″	A thin Penumbra.
7	57	40	A darker Penumbra.
7	59	00	Yet darker, which may pass for the beginning of the Eclipse.
8	00	00	The Eclipse no doubt begun.
9	01	00	The Lucid parts of the Moon, not long before the middle of the Eclipse, were 925 parts of my Micrometer.
9	16	40	Diameter of the Moon 1634 parts of the Micrometer.
10	23	11	The end of the Eclipse draws nigh.
10	25	00	A little Obscuration.
10	26	00	Less.
10	28	15	A very little, except the Duskishness before mention'd.

The Moon's Eclipse at Streatham, *compar'd with the Calculation, by Mr.* Cressner, *n.* 325. *p.* 16.

XXVI. In the Lunar Eclipse *Feb.* 2. 1709-10. finding the time of the end to agree with the Calculation (with a very inconsiderable Difference) according to Sir *Isaac Newton*'s Theory; and there being no Examples of any Calculation according to that Theory, nor of the Theory's Agreement with the Observation; I thought proper to offer this one. I have added the Calculation from Mr. *Flamsteed*'s Tables, according to *Horrox*'s Theory, as I find them publish'd in Mr. *Whiston*'s Astronomical Lectures, with the *Radix*'s of the Mean Motions, corrected according to their first Author's later Observations, which are the same with those assum'd in Sir *Isaac Newton*'s Theory.

The Observation was made at *Streatham*, about six Miles near direct South of *London*, with a very good eight Foot Telescope. To correct the Clock, (for want of an Instrument,) I carried with me next Day two Watches, that were before adjusted to the Clock, and compar'd them with Mr. *Flamsteed*'s at the Royal Observatory, having first noted its Error by an Observation of the Sun's Transit of the Meridian, his Assistant communicated to me: Upon my return, I found my Watches still to agree together, and to my Clock, which proved them to have gone true, and gave me the exact Error of my Clock, and the true time at Observation.

17 09/10. *Feb.*	D.	H.	M.	Sec.
The Mean Time of the Mean Opposition	2	4	9	42
The Mean Time of the true Opposition	2	10	54	48
At which the true Place of the Sun is	10	24	55	50
And its Æquation to be added.				
The Place of the Moon at this Time, from Sir Isaac Newton's *Theory.*				
Mean Motion of the Moon	4	26	57	37
Annual Æquation Subtr.			8	34
The Correct Mean Motion	4	26	49	03
Mean Motion of Apog.	11	18	13	54
Annual Æquation of Apog. Ad.			14	31
Correct Mean Motion of Apog.	11	18	28	25
Second Æq. from the dist. of Ap. from Sun Ad.			2	57
Place of the Moon the 2d time Æquat.	4	26	52	00
Mean Motion of Node	11	01	34	25
Æquation of Node Subt.			06	54
Correct mean Motion of Node	11	01	27	31
The 3d Æquat. of the Moon from Nodes Aspect with the Sun Subt.				10
Place of the Moon the 3d time Æquated	4	26	51	50
Second Æquation of Apog. Subtr.		7	45	41
True Place of Apog.	11	10	42	44
Mean Anomaly	05	16	09	06
Æquation of Center Sub.		1	53	31
Moon's Place the 4th time Æquated	4	24	58	19
The Variation. Ad.				11

Moon's

Moons Place the 5th time Æquated	4	24	58	30
The 6th Æquation from the distance of the Luminaries and Apog. Ad.			1	20
Moons Place 6th time Æquated	4	24	59	50
The 7th Æquation. Ad.				34
True place of the Moon in its Orbit	4	25	00	24
True Place of the Sun	10	24	55	50
Moon beyond the Opposition			4	34
Which divided by the Horary Motion of Moon from Sun gives	*D.*	*H.*	7	42
The Mean Time therefore of Opposition.	*Feb.* 2	10	47	06
And the true Time	2	10	32	20

The Place of the Moon at the same time from the Tables in Mr. Whiston's *Astronomy, according to* Horrox's *Theory.*

Mean Motion of the Moon	4	26	57	37
Physical Parts Sub.			8	21
Correct Mean Motion	4	26	49	16
Mean Motion of Apog.	11	18	13	54
Æquation of Apog. Sub.		7	25	00
Mean Anomaly	5	16	00	22
Æquation of the Center Sub.		1	53	53
Place of the Moon in its Orbit	4	21	55	23
Distance from the Opposition				27
That is in time to be added				$45\frac{1}{3}$
The Mean time therefore of true Opposition is exactly	*D.* 2	*H.* 10	55	33
The Apparent time	2	10	40	41
Place of Moon in Ecliptick	4	24	57	27
Reduction between the true Opposition and middle of Eclipse Ad.			2	47
	D.	*H.*		
Middle of Eclipse	2	10	43	34
Continuance of Eclipse		2	55	06
Digits Eclips'd		9	55	
Beginning of Eclipse	2	9	16	01
End of Eclipse		12	11	07
End of Eclipse by the Moon's Place from Sir *Isaac Newton*'s Theory.		12	02	00
End by Observation		12	01	30
End by Calculation from *Horrox*'s Theory		12	11	08

XXVII. *Eclipse of the Moon at* Upminster, Jan. 12. 1711-12. *by Mr.* Derham, *n.* 336. *p.* 512.

6h. 15″. A darkishness on the N E. side of the Moon.
6 36 A thick Penumbra on the Moon.
6 37 The Penumbra so dense, that it may be taken for the beginning of the Eclipse.
6 39 The Eclipse undoubtedly is begun.

6h 41′ The Shadow so dark, that it nearly hid the Moon's NE Limb.
7 21 Moon's Diameter, by the Micrometer 1612 equal parts; equal to 31′. 25″.
7 25 The Shadows distance from the opposite luminous Limb of the Moon, represented by the Line, *l. n.* was 1025 parts of the Micrometer, equal to 20′.
8 31 End of the Eclipse is very near.
8 32 End of the Eclipse.
8 32 45″ Eclipse is undoubtedly ended.
8 36 A Penumbra left.

Having disorder'd my Micrometer at the beginning, I could not take with any Exactness the Inclination of the Cusps, and some other Matters; neither can I warrant the Micrometrical Measures of the Moon's Diameter, and her eclips'd parts otherwise than somewhat near the Truth. I have made a Type of the Eclipse as well as I could by guess. *Fig. 1. Plate 4.*

m. i o. r. represents the two Claspers of the Micrometer parallel to the Equator.

N. the Northern, *S.* the Southern part of the Moon's Disk running between the Claspers of the Micrometer.

L. n. The enlighten'd part of the Moon, being 1025 Micrometrical parts, or 20′.

XXVIII. *A Lunar Eclipse and Occultation of a Star, by Dr.* Fr. Blanchini, *n.* 340, *p.* 88. Nov. 21. 1713.

H. post Merid.			
12h	53′	34″	Stella *Bayero* τ Tauri proxime appellit ad limbum Lunæ, observata per Telescopium duodecim palmorum.
12	54	34	Eadem jam occultata est ab ea parte Lunaris Limbi, quæ media ferme est inter maculas Aristarchi & Galilei. Parallelus diurnus a centro Lunæ descriptus apparet Australior quam stella τ partibus Micrometri $7\frac{1}{2}$, qualium Lunæ diameter subtendit 37. Stellæ igitur τ declinatio Borealior est declinatione apparente Lunaris centri minutis circuli maximi $5'\frac{1}{2}$ circiter.
14	00	14	Sirius attingit Meridianum: unde verificata sunt tempora.
14	32	57	Stella τ, quæ aliquot minuta excesserat e limbo Lunæ in revolutione diurna præcedit limbum occidentalem Lunæ secundis horariis 0′. 33″, eademque præcedit centrum Lunæ secundis 103″ sive 1′. 43″.
14	42	50	Eadem præcedit limbum Lunæ secundis 48″, & centrum 1′. 58″.
14	50	37	Differentia Ascensionis rectæ stellæ & limbi est 1′. 03″, centri vero Lunæ & ejusdem stellæ 2′. 13″.
15	0	00	In limbo Lunæ Penumbra, quæ antea erat dilutior, sensim fit densior.
15	2	20	Penumbra fit evidentior, sed nondum apparet Umbra vera.
15	4	20	Initium incidentiæ Lunæ in Umbram veram, ea in parte limbi quæ proxima est maculæ *Schickardi*.
15	5	21	Umbra vera jam obtegit partem unam, qualium Lunæ diameter in Micrometro obtinet 37.

15 7

h	'	"	
15	7	20	Jam partes duæ obteguntur, qualium Lunæ diameter est 37.
15	16	20	Jam obteguntur Lunaris diametri $\frac{5}{37}$.
15	31	20	Latent Lunaris diametri $\frac{2}{37}$.
16	12	00	Jam latent in diametro partes $\frac{15}{37}$.
16	17	20	Partes latentes 15, ut antea.
16	50	20	Jam partes latentes $\frac{1}{7}$.
16	54	35	Incipit emergere prior limbus Tychonis.
16	56	9	Jam totus Tycho emergit.
17	13	30	Latent Lunaris diametri partes 5 è 37.
17	27	45	Umbra vera excedit e limbo Lunæ, in loco designato per diametrum dictam inter Aristarchum & Platonem situ medio.

Hæc occultatio pluris æstimanda, quod occultatio stellæ τ acciderit tam vicina Opposito Solis, ut inde locus Solis inter fixas rite examinari poterit.

XXIX.
A Lunar Eclipse at Wanstead, *by Mr.* Pound, *n.* 347. *p.* 402.

Observ.	Apparent Time. h.	'	"	*October* 30. 1715.	'	"
1	15	09	00	The Eclipse had been for sometime begun.		
2		17	00	The Moon's Diameter measured by a Micrometer.	34	04
3		22	25	The Chord connecting the Horns	30	23
4		35	45	The enlightened part of the Diameter, continued to the Chord between the Horns	19	58
5		43	24	The enlighten'd part of the Diameter	13	52
6		49	50	The same repeated	12	02
7		52	43	The same repeated	11	44
8		56	51	The inlightened part of the Diameter continued to the Chord between the Horns	15	22
9		59	27	The inlighten'd part of the Diameter	10	35
10	16	04	04	The same repeated	9	43
11		18	34	The same repeated	9	07
12		23	45	The Chord between the Horns	32	35
13		26	30	The same repeated	33	07
14	16	31	16	The same again At which time also the Shade passed thro' the middle of *Schickardus*.	33	19
15		37	15	The Chord between the Horns, agreeing with the ☽'s Diameter	33	57
16		40	45	The inlightened part of the Diameter	11	56
17		43	40	The same produced to the Chord between the Horns	16	13
18		46	55	The same repeated	17	28
19		47	57	The inlightned part of the Diameter	13	28
20		52	57	The same	15	30

	h	′	″		′	″
21	h	55′	27″	The Edge of the Shadow paſſed through the middle of *Gaſſendus* ——	′	″
22		56	12	The inlightned part produced to the Chord between the Horns ——	19	58
23	17	02	45	The Chord between the Horns ——	32	12
24		8	20	The ſame repeated	30	28
25		10	39		29	56
26		13	00		28	31
27		15	29	The ſame again —— ——	27	33
28		17	37		26	35
29		19	35		25	36
30		21	47	The ſame again —— ——	24	38
31		23	24		23	39
32		24	54		22	40
33		26	27	The ſame again —— ——	21	41
34		27	57		20	42
35		29	08		19	43
36		30	20	The ſame again —— ——	18	44
37		31	07		17	45
38		32	04		16	46
39		32	50	The ſame again —— ——	15	47
40		34	12		13	48
41	17	35	20	The ſame again repeated ——	11	42

At $17^{h}.39'$. the Eclipſe was thought to be ended; and was viſibly ſo at $17^{h}.41'$: But by comparing the laſt Obſervations of the Chords between the Horns, it follows that the true end of the Eclipſe was at $17^{h}.38'.20''$. at $17^{h}.43'$. the Moon's Diameter was $33'.40''$.

The middle cannot be ſuppoſed to be very accurately determined by theſe Obſervations, which were not ſufficiently diſtant from the time of the greateſt Obſcuration. However by comparing ſeveral of them together, the middle will be obtained, *viz.*

	h	′	″
By Obſ. 3. compared with Obſ. 24. at	16^{h}.	15′.	21″
By Obſ. 4. compared with Obſ. 22. at	16	15	58
By Obſ. 5. compared with 19. and 20. at	16	16	00
By Obſ. 6. and 7. compared with 16. at	16	15	48

By reaſon of Clouds I could not ſee the Beginning of the Eclipſe, nor make ſuch Obſervations of the Moon's immerging into the Shadow as I did of her emerging out of it.

By Obſervation 11. compared with Obſervation 15. the Digits eclips'd were $8\frac{3}{4}$.

The Angles were meaſured by a *Micrometer* in a 15 Foot Teleſcope, I have not conſidered how far they are conſiſtent with one another; they being ſet down here exactly as they were firſt taken.

This Eclipſe is the more conſiderable, as happening very near the Moon's *Perigee*, and therefore uſeful to verify her *Anomaly*; as alſo to limit the greateſt Diameter of the Shadow of the Earth, and conſequent-

ly

ly the Parallax of the Moon. This may very properly be compared with that of the 19th of *October*, 1697, whose middle was at 7h. 41′ *P. M.* at *London*, and Quantity the same as now.

The Times by the Clock were 17′. 45″. sooner than the apparent time, as was found by the following Observations of *Cor Leonis* and *Arcturus*, which through the Clouds were but just discernible.

Apparent Zenith Distance		Time by the Clock			Apparent Time by Calculat.			The Difference.		Mean Diff.	
of Cor ♌											
°	′	h.	′	″	h	′	″	′	″	′	″
70	16½	13	32	43	13	50	35	17	52		
69	38		36	50		54	44	17	54		
69	09		40	06		57	51	17	45	17	50
68	40		43	09	14	00	59	17	50		
68	08		46	37		04	26	17	59		
of *Arctu.*											
65	19	17	37	40	17	55	24	17	44		
65	06		39	12		56	48	17	36		
64	41		41	49		59	29	17	40	17	40
63	47		47	40	18	05	17	17	37		
					Clock too slow					17	45

The Latitude of *Wansted* is 51°. 34.′ Its Longitude is 8″. in time Eastward from the *Observatory* at *Greenwich*.

The Account given of this Eclipse by Mr. Derham, *who observ'd it at* Upminster, *is agreeable to this, as far as Clouds would permit him to observe.*

XxX. *Another at* Wansted, *by Mr.* Pound, *n.* 357. *p.* 845.

Observ.	*Tempus Apparens*			*Eclipsis* Lunæ *observata apud* Wansted, 29. *Augusti*, 1718.		
	h.	′	″		′	″
1	6	53	38	Chorda inter Cuspides Micrometro mensurata	22	37
2		55	8	Eadem repetita	21	14
3		56	31	Repetita	19	51
4		57	49	Iterum	18	28
5		59	38	Denuo	15	00
6	7	2	41	Immersio Totalis in Umbram		
7	8	36	13	Stella clara in Catalogis omissa occultata est a *Luna*, infra *Paludem Mareotida Hevelii*	10	2
8	8	48	18	*Luna* cœpit emergere ex Umbra		

9 51 13

Obs.	T. appar. h	'	"	*Eclipsis* Lunæ, *Aug.* 29. 1718.	'	"
9	51^h		13	Terminus Umbræ per med. *Maræotidis*; simul Chorda inter Cuspides	15	0
10		53	7	Chorda inter Cuspides	18	28
11		54	16	Eadem repetita	19	51
12		54	59	Iterum	21	14
13	8	56	18	Denuo	22	37
14	9	0	48	*Porphyrites* emersit ex umbra.		
15		8	3	Mons *Sinai* incepit emergere.		
16		9	17	Umbra per medium *Sinæ*.		
17		10	6	Jam totus *Sinai* extra Umbram.		
18		11	20	Umbra per medium *Ætnæ*.		
19		17	23	Per medium *Corsicæ*.		
20		20	0	Per medium *Lacus Nigri majoris*.		
21		27	54	Per medium *Besbici*.		
22		28	45	Emersit Stella prædicta.		
23		32	34	*Byzantium* & *Horminius* simul emergunt.		
24		33	58	Stella eandem habuit Declinationem cum Cuspide Aust. Eclipseos.		
25		43	28	Chorda inter Cuspides ——— 18′ 28″.		
26		47	2	Eadem repetita ——— 15 00		
27	9	53	0	Desiisse videbatur Defectus.		

10^h. 30′, Capta est Lunæ diameter 29′. 45″. Collatis autem inter se Observationibus, ubi Chordæ partis deficientis æquales deprehensæ sunt, provenit Eclipseos medium.

	Medium.		
Ex Observ. prima & decima tertia	7^h.	54′	58″
Ex secunda & duodecima	7	55	3
Ex tertia & undecima	7	55	24
Ex quarta & decima	7	55	28
Ex quinta & nona	7	55	25
Ex sexta & octava	7	55	29
Quorum omnium Medium fit	7	55	18

2. *At* London *by*---*ib.p.*857.

Luna per fumum Urbis & Vapores ægre visa.	6^h.	38′.	0″.
Chorda inter Cuspides utcunque, 21′ 27″	6	54	13
Immersio Totalis in umbram.	7	2	0
Stella fixa satis clara distabat in limbo Lunæ orientali 19′ 21″.	7	42	15
Eadem fixa occultata est, 10 circiter minutis centro Lunæ Australior.	8	35	18
vel, ut quibusdam visum est, uno minuto tardius Luna cœpit emergere.	8	45	50
Talus *Mareotis* primo margine emersit.	8	49	38
	8	50	14

9	7	39	Primus margo *Sinæ* emersit.
9	9	8	Mons *Sinai* totus extra umbram.
9	10	35	Umbra per medium *Ætnæ*.
9	12	0	Totus mons *Ætna* extra umbram.
9	18	51	Umbra per medium *Lacus Nigri* majoris.
9	27	35	Insula *Besbicus* tota emersit.
9	42	21	Chorda inter Cuspides 19′. 9″.
9	51	25	Finis Eclipseos ut quibusdam visum est.
9	52	45	Finis ex præcedente distantia Cuspidum conclusa.
9	56	45	Lunæ diameter 29′ 45″, iterumque 29′ 48″.

Erat autem Umbra admodum diluta, unde orta est difficultas in dijudicandis Emersionis & Finis momentis: Atque Maculæ etiam obscuriores clare conspectæ sunt, pluribus minutis antequam Umbræ marginem attingerent. Stella vero quæ durante Eclipsi occultata est, locum tunc habuit ♓ 17°. 16′$\frac{1}{2}$ cum Lat. Aust. 1° 6′ 30″ proxime.

Observationes hujus Eclipseos a Rev. Dom. *Derham*, apud *Upminster* in agro *Essexiensi* habitas; a Dom. *Wright*, apud *Crew* in agro *Cestriensi*; & a Dom. *Hawkins* apud *Wakefield* in *Eboracensi*, cum præmissis ubique fere consentiunt, si adhibeantur meridianorum differentiæ: posito scil. quod *Upminster* sit 1$\frac{1}{3}$ min. *Londino* orientalius, *Crew* vero 10 min. & *Wakefield* 5 min. occidentaliora.

XXXI. I had accidentally a Journal of an Officer of the Ship *Emperor* put into my Hands, who in his return from *India*, on the fifth of *March* 1718. observ'd the end of a Lunar Eclipse, when the visible altitude of the Moon's Center was 13°. 25′. he being then in the Latitude of 34°. 23′. *South*, and as they found afterwards, just 180 Leagues to the Eastwards of Cape *Bonne Esperance*. By Calculation I find that in that Latitude the Moon had that height at 7h. 17′$\frac{1}{2}$ *P.M.* and by comparing this Eclipse with that we observed with great exactness on *Feb.* 11°. 1682. (which agrees perfectly well with our Numbers) I conclude the middle of this to have been at *London* at 3h. 48′. *P. M.* to which adding 1h. 46′. for the Semiduration (this being very certain from the observ'd Continuance of the Eclipse of 1682.) the end will be found to have been at *London* at 5h. 34′. The Ship was therefore in a Meridian 26° to the Eastwards of *London*: But she was at that time 180 Leagues to the Eastwards of the *Cape*, which distance in that Latitude gives eleven Degrees of Longitude; this therefore being deducted from the Longitude of the Ship, leaves just fifteen *grad.* or one Hour, for the difference of Meridians between *London* and the *Cape*. So that by this account the *Cape* is yet nearer our Meridian than I had * formerly made it, and near six Degrees nearer than M. *De la Hire* places it in his *Tables*.

End of a Lunar Eclipse May 5. 1718, near the Cape of Good Hope, to determine its Longitude, by Dr. Halley, n. 361. p. 992.

* Ph. Tr. *n.* 185. *Abr. Vol.* I. *p.* 567.

This Eclipse was attended with all the Circumstances requisite to make the Conclusion as certain as the nature of the thing will admit of: For the Moon was nearly *in Perigæo*, and the Eclipse almost central; for which Reasons she emerged out of the Shadow as swiftly as possible: The

The Sea was very ſmooth, there having been little Wind for above 30 Hours before; and the Moon was not too high to be well obſerved with a Foreſtaff. Nor were they long at Sea before they made the Land, for in leſs than five Days, on the tenth of *March* at Noon, they had paſt Cape *d' Agulhas* the moſt Southerly Promontory of *Africa*, which then bore from them *North Eaſt*, about ſeven Leagues diſtant.

It may not be amiſs to inſert an Obſervation or two I procured to be made at the *Cape*, by Mr. *Alexander Brown* a *Scotch* Gentleman, who went to reſide in *India* on our Companies account. He carried with him a very good *Braſs Quadrant* of above two Foot Radius, and at the *Dutch* Settlement at *Table Bay*, having rectify'd his *Pendulum-Clock* by correſpondent Altitudes, on the fourth of *Auguſt* 1694, at $5^h. 59'$ *Mane*, the diſtance of the bright Limb of the Moon from the right Shoulder of *Orion* was obſerved to be $25°. 3'$. And the next Morning, *Aug.* 5. at $5^h. 21'. 12''$, the ſame Limb was diſtant from *Procyon* $25°. 57'$, and at $5^h. 36'. 48''$. from the *Lucida Arietis* $58°. 29'$.

For want of accurate Obſervations made thoſe Mornings at ſome place in *Europe*, whoſe Longitude from them is well known, I had recourſe to the Period of the Lunar Motions, which is performed in eighteen Years and ten or eleven Days, after which the Errors of our Lunar Computations return very nearly the ſame; and I found among my own old Obſervations, one that tallyed well with that of the fourth of *Auguſt*, *Viz.* Anno 1676. *July* 23°. $13^h. 11'. 35''$. at *Oxford*, I obſerv'd the Moon to apply to the Star *in medio Collo Tauri*, by *Bayer* markt *A*. The Star at that time was diſtant from the Southern and neareſt Cuſp of the Moon by the Micrometer $20'. 32''$. and at $13^h. 17'. 15''$. when it ſeem'd to immerge upon the bright Limb of the Moon, it was diſtant from the Northern Cuſp $23'. 20''$; but this leſs certain by reaſon of the hazey Air. The Star at that time was in ♉ $28°. 56'$. with $1°. 13'. 20''$. *North* Lat. whereby I found that our Lunar Tables, founded on Sir *Iſaac Newton*'s correct Theory of her Motion, gave her place at that time only two Minutes too ſlow; which Error being allowed on the 4th of *Auguſt* 1694. the reſult was, that $5^h. 59'$. at Cape *Bonne Eſperance* was at *London* $4^h. 53'$. whence the difference of Longitude $16\frac{1}{2}$ degrees, ſufficiently near what we had before determin'd.

Rectification of the Motion of the Circum-Saturnials, *with Obſervations of them, by Mr.* Pound, *n.* 355. *p.* 768.

XXXII. About thirty Years ago, Mr. *Chr. Huygens* made the *Royal Society* a Preſent of the Glaſſes of a Teleſcope of 125 Foot length, with the *Apparatus* for uſing them without a Tube; But thoſe here that firſt tried to make uſe of this Glaſs, finding for want of Practice, ſome Difficulties in the Management thereof, were the occaſion of its being laid aſide for ſome time. Afterwards it was deſigned for making perpendicular Obſervations of the fixt Stars paſſing by our *Zenith*, to try if the Parallax of the *Earths* annual Orb might not be made ſenſible in ſo great a Radius, according to what Dr. *Hook* had long ſince propoſed: but in this we miſcarried alſo, for want of a place of ſufficient height and

and firmneſs, whereon to fix the Object Glaſs; ſo that it lay by neglected for many Years.

In the mean time we could not but remark a great reſerve in the *French* Aſtronomers, in relation to the Satellites, of which they have given us in their Yearly *Memoirs* no Obſervations till very lately, nor have they ſeemed willing to ſhew them in their Glaſſes to ſuch as requeſted it: ſo that it might poſſibly occaſion in ſome Perſon's a Suſpicion of the reality of this Diſcovery: And Mr. *Derham*, having borrowed of the Society their long Glaſs, could not thereby aſſure himſelf that the ſmall Stars he ſometimes found about *Saturn*, were really his Satellites, their Situation not agreeing with their places derived from the Tables of their Motions exhibited in * N°. 187. of *Phil. Tranſact.* beſides that he wanted a ſufficient height to raiſe the Object Glaſs, ſo as to view *Saturn* to advantage, above the Vapours of the Horizon. But in the *Memoirs* for 1714, publiſhed but about a Year ſince, M. *Caſſini*, the worthy Succeſſor of his great Father, has given us ſome Obſervations which clear up the Point, and by ſhewing the Errors of thoſe firſt Tables, has enabled us to be aſſured, that we have ſeen the whole *Satellitium* of *Saturn* ourſelves.

* *Abr. Vol. I. p.* 376.

Anno 1714. *Maii* 6. *St.N.* about Mid-night, *Saturn* being then Stationary in ♍ 4°. 27′, the fifth and outermoſt Satellite was in its ſuperiour Conjunction with the Planet, and at the ſame time, the Earth was nearly in the Plain of this Satellit's Orbit, ſo that it appeared to paſs very near the Center of *Saturn*: From hence, and from ſome other preceeding Obſervations, Mr. *Caſſini* concludes that the Nodes of this Satellit's Orb are in four degrees of ♍ and ♓, and that its Inclination to the Ecliptick is not much more than half that of the other Satellites. Hence it ſhould follow that the Ellipſes it deſcribes by its apparent Motion about *Saturn*, when in ♊ and ♐ are much flatter and nearer to his Body, than thoſe of the other four, which he allows to move in the plain of the *Ring*, and to have their Nodes in 21 *gr.* of ♍ and ♓, with an Inclination to the Ecliptick of 31 degrees. To confirm this Diſcovery, he produces another Obſervation of his Fathers, near thirty Years before, *viz.* that *Anno* 1685. *Martii* 31. *St. N.* about Noon, the ſame Satellite was obſerved in ſuperiour Conjunction with *Saturn*, with leſs than one Diameter of the Ring North Latitude, *Saturn* being then in ♍ 11°. 18′. So that the Satellite wanted but 7°. 21′. of compleating 134 Revolutions, in the Interval of time between them. From theſe *Data* it was eaſy to ſettle the Theory of this Satellit.

As to the *Fourth* or the *Hugenian* Satellite; in the *Memoirs* for 1715, but juſt now come to Hand, we find a very curious Obſervation of it, and the firſt of its kind, *viz.* that *Mart.* 25°. *S. N.* about 11^h. *P.M.* this fourth Satellite, then in *Apogeo*, did immerge behind the Body of *Saturn.* With this Emendation, the place of this Satellit may for the future be computed with a ſufficient Exactneſs.

The Third Satellite, by an original Miſtake in the Letters in N°. 187 is all wrong; its daily Motion being there printed 2^s. 18°. 41′. 50″. inſtead

ſtead of $2^s. 19^{\circ}. 41'. 50''$; as may be perceived by the Period thereof being determined, in the aforeſaid *Memoirs* of 1714, to be 4*d*. 12^h. $25'$. $12''$. that is, that it makes 400 Revolutions in 1807 Days. This Satellite was obſerved by Mr. *Caſſini*, *April* 4 . *St. N.* $1c^h$. *P. M.* to have newly paſt its inferior Conjunction with *Saturn*, and a perpendicular from it fell on the Extremity of the weſtern *Anſe*, ſo that at about 5^h. *P. M.* it was with the Center of the Planet then in ♍ $5^{\circ}. 23'$. and conſequently in ♓ $5^{\circ}. 23'$. but *ineunte anno Gregoriano* 1686, the *Epoche* thereof was ♍ $9^{\circ}. 39'$. So that from the Noon of the laſt of *December* 1685, to *April* 4°. 6^h. $18'$. *anno* 1714, that is, in 10320 Days 6^h. $18'$. there have been made $2284\frac{1}{2}$ Revolutions of this Satellite to the Equinoctial; from which *Data*, the Tables of its Motion are readily derivable.

The Radix of the penintime or ſecond Satellite, according to the aforeſaid Letter, *ineunte anno Greg.* 1686. was in ♍ $9^{\circ}. 10'$. But by the Obſervations of Mr. *Caſſini* made the Nights before and after, this Satellite was in its ſuperior Conjunction *anno* 1714. *April* 4*d*. $21'\frac{1}{2}$. *St. N.* that is, in ♍ $5^{\circ}. 21'$, where *Saturn* then was: So that *April* 4*d*. 22^h. $12'$, an entire Number of Revolutions were performed ſince the *Epoche* of 1686, that is, in 10320 Days 22^h. 12: which number can be no other than 3771, according to the Period thereof given in this *Memoire*, *viz.* 2 *d*. 17^h. $41'$. $22''$.

Laſtly, the innermoſt or firſt Satellite, at the ſame time, *viz.* 1714, *April* 4°. 21^h. $30'$. *St. N.* was in its inferiour Conjunction *proxime*, and conſequently in ♓ 5° $21'$. But the *Epoche* thereof for 1686, is ♑ 24°. $50'$. which place the Satellite had paſt 40 *gr*. $31'$. at the time of the Obſervation. This Arch it moves in 5^h. $6'$: wherefore from the time of the *Epoche* to *April* 4 *d*. 16^h. $24'$, 1714, or in 10320 Days 16^h. $24'$. the Satellite has performed 5467 Revolutions, its Period being determined to be 1 Day, 21 Hours, $18'$. $27''$, in this *Memoire*.

Having by the help of theſe late Obſervations corrected the Motions of the Satellites which it was not poſſible for their firſt Diſcoverer to ſettle truly, in the ſhort Interval before 1687; and having fixed their *Epoches* for the preſent Year, we were enabled to know where to expect them with more Certainty, and to diſtinguiſh them one from another, and from the ſmall fixt Stars appearing with them: And the Reverend Mr. *James Pound*, (whoſe indefatigable Induſtry is no way inferiour to his incomparable Skill in Aſtronomical Matters) having, by means of his Steeple of *Wanſted*, provided a *Gnomon* high enough for the purpoſe, and having fitted a very commodious *Apparatus* for uſing the Societies aforeſaid long Teleſcope, ſoon diſcovered by it all theſe five Satellites; and lately communicated to them the following very curious Obſervations.

1718. *April* 21 *d*. 10^h. $40'$. The third and fourth Satellits of *Saturn* were in *Apogæo*, a little paſt their Conjunction with *Saturn*: A perpendicular from the fourth to the Tranſverſe *Axis* of the Ring (or Line of the

the *Anſæ*) fell a little without the Eaſtern *Anſa*; and a Line through the fourth and third touched the Eaſtern Limb of *Saturn*, *Fig.* 2. Plate 4.

The firſt was Northward of the Line of the *Anſæ* (and therefore in the *Apogæon* Semicircle alſo) diſtant from the ſaid Line about as far as the end of the Conjugate Axis of the Ring was from the Center of ♄, *viz.* nearly $\frac{3}{4}$ of *Saturn's* Semidiameter; and it was about a Semidiameter of the Ring from the Weſtern *Anſa*.

The ſecond was a very little Southward of the Line of the *Anſæ* (and therefore in the *Perigæon* Semicircle) above a Semidiameter of the Ring (or about the Semidiameter of the Ring + the Semidiameter of ♄) from the Weſtern *Anſa*. And the third, firſt and ſecond were in a ſtraight Line.

At 10h. 50′. A perpendicular from the 3d to the Line of the *Anſæ* fell almoſt on the middle of the bright part of the Eaſtern *Anſa*, but ſomewhat nearer the Center than the ſaid middle.

April 22*d.* 11h. 5′. The four innermoſt Satellits were all Eaſtward of ♄. The 2d and 4th in the *Apogæon*, and the 1ſt and 3d in the *Perigæon* Semicircle. A Line through the 2d and 4th touched the *South Eaſt* Limb of ♄. A Line paſſing through the 3d and the end of the Conjugate *Axis* of the Ring, was parallel to the Line of the *Anſæ*.

At 11h. 10′. A perpendicular from the firſt to the Line of the *Anſæ* fell on the Eaſtern Extremity of the Ring. *Fig.* 3.

Theſe Diſtances and Directions were taken only by Eſtimation, and not by any actual Meaſurement.

The fifth (or outermoſt) Satellite being at this time near its greateſt Elongation Eaſtward, among ſeveral very ſmall Teleſcopick Stars, he could not determine its Poſition. But by obſerving the Motion of this ſome other Nights before, he was now fully ſatisfied, from the Motions rectified as above, that there are five Satellits of *Saturn*, as Mr. *Caſſini* had long ſince aſſerted.

In the bright part of each *Anſa* was a darkiſh Ellipſe, nearer to the out-ſide than the inſide of the Ring, as if it was compoſed of two Rings near to one another.

On the Body of ♄, beſide the Ring on the Southſide, there appeared on the Northſide a Zone not ſo far from the Center as the Ring, and not much unlike the ſmalleſt of *Jupiter*'s Belts. Theſe Appearances were firſt taken notice of by Mr. *Caſſini*, as may be ſeen in *Phil. Tranſ.* No. 128. p. 690 *. *v. Fig.* 4.

It is not to be expected that theſe Satellites, exceedingly minute in themſelves, and ſo faintly illuminated, ſhould appear when the Air is but ordinarily ſerene, they requiring not only the *Medium* to be *ſummo modo* defecate and limpid, but withal in perfect Darkneſs. For which reaſons it may well be underſtood why the Gentlemen of the *Pariſian* Obſervatory may have ſometimes made a Difficulty to undertake to ſhew them upon Demand.

* *Abr. Vol.* I. p. 367.

Circa

Tables of the Motions of Saturn's Satellits n. 356. p. 776.

XXXIII. Circa finem Anni 1686. D. *Jo. Dom. Caſſini*, Reg. Soc. Sodalis, & in Aſtronomicis nemini ſecundus, cum Societate noſtra inventa ſua de motibus quinque Satellitum *Saturni* communicavit, Epochaſque ſingulorum ad annum ineuntem 1686, eorumque motus diurnos in Epiſtola * No 187. harum *Tranſact.* edita exhibuit: E quibus datis motuum Tabulas concinnavimus, dictæque Epiſtolæ ſubjunctas una edidimus. Cum vero deinde per triginta fere annos nullas omnino Obſervationes eorum tradiderint, qui ſoli poterant, Aſtronomi *Galli*; cumque aliunde, ob intervallum temporis nimis breve, non niſi laxe periodos Satellitum, præſertim interiorum, definire potuerit præclariſſimus Inventor; non prius dictarum Tabularum defectus corrigere datum eſt, quam in nuperis Actis Academiæ Regiæ *Pariſienſis* Phyſicis & Mathematicis, obſervata ea, quæ ſupra protulimus, prodiere.

* *Abr. Vol.* I. *p.* 376.

Horum vero ope facta, aliquali Motuum caſtigatione, tum demum Teleſcopio *Hugeniano* omne Saturni Satellitium ipſi agnovimus; adhibitiſque accuratis Rev. D. *Jac. Poundi* obſervationibus, Tabulas ſubſequentes cælo ſatis conſonas obtinuimus. Addendo ſc. motui annuo *Interioris*, 2*gr.* 9′; *Penintimi* vero 3°. 25′. retentis Epochis D. *Caſſini* ad Annum 1686. Augendo etiam motum annuum *Extimi* 9 min. ſublatis vero 16 grad. ab *Epocha*, quæ in Epiſtola dicta No. 187. perperam ſcribitur ♓ 16°. 19′. pro ♓ 0°. 16′. *Hugenianum* 6′. annuatim tardiorem invenimus. *Tertii* autem Tabellas, ob motum diurnum falſo in Epiſtola illa traditum, de integro recudere neceſſe habuimus, retenta ſaltem *Epocha*.

Tabula

Tabula Mediorum Motuum Intimi Satellitis Saturni à Caſſino *detecti Anno* 1686.

Annis Chriſti ineunt.	*Epocha.* s	°	'	*Annis*	*Mot. Med.* s	°	'	*Diebus*	*Mot. Med.* s	°	'	*H M*	*Mot. Med. Sex.* °	'	"	*Min.*	*Motus Medi.* °	'
1681	♒	8	48	1	4	4	43	1	6	10	42	1	0	7	57	31	4	6
1686	♐	13	4	2	8	9	25	2	0	21	24	2	0	15	53	32	4	14
1701	♍	6	34	3	0	14	8	3	7	2	6	3	0	23	50	33	4	22
1714	♎	9	57	4	10	29	33	4	1	12	47	4	0	31	47	34	4	30
1715	♒	14	39	5	3	4	16	5	7	23	29	5	0	39	44	35	4	38
1716	♊	19	22	6	7	8	59	6	2	4	11	6	0	47	40	36	4	46
1717	♉	4	47	7	11	13	42	7	8	14	53	7	0	55	37	37	4	54
1718	♍	9	30	8	9	29	6	8	2	25	35	8	1	3	34	38	5	12
1719	♑	14	13	9	2	3	49	9	9	6	17	9	1	11	31	39	5	10
1720	♉	18	55	10	6	8	32	10	3	16	59	10	1	19	28	40	5	18
Menſ.	*Mot. Med.*			11	10	13	15	11	9	27	41	11	1	27	24	41	5	26
Anni				12	8	28	40	12	4	8	22	12	1	35	21	42	5	34
Com.	s	°	'	13	1	3	23	13	10	19	4	13	1	43	18	43	5	42
Jan.	0	0	0	14	5	8	5	14	4	29	46	14	1	51	15	44	5	50
Febr.	5	1	38	15	9	12	48	15	11	10	28	15	1	59	11	45	5	58
Mar.	3	1	10	16	7	28	13	16	5	21	10	16	2	7	8	46	6	5
April	8	2	48	17	0	2	56	17	0	1	52	17	2	15	5	47	6	13
Maii	6	23	44	18	4	7	39	18	6	12	34	18	2	23	1	48	6	21
Junii	11	25	22	19	8	12	21	19	0	23	16	19	2	30	58	49	6	29
Julii	10	16	19	20	6	27	46	20	7	3	57	20	2	38	55	50	6	37
Aug.	3	17	57	40	1	25	32	21	1	14	39	21	2	46	52	51	6	45
Sept.	8	19	35	60	8	23	19	22	7	25	21	22	2	54	49	52	6	53
Octob.	7	10	31	80	3	21	5	23	2	6	3	23	3	2	45	53	7	1
Nov.	0	12	9	100	10	18	51	24	8	16	45	24	3	10	42	54	7	9
Dec.	11	3	5	120	5	15	37	25	2	27	27	25	3	18	39	55	7	17
								26	9	8	9	26	3	26	35	56	7	25
								27	3	18	50	27	3	34	32	57	7	33
								28	9	29	32	28	3	42	28	58	7	41
								29	4	10	14	29	3	50	25	59	7	49
								30	10	20	56	30	3	58	22	60	7	57

In Anno Biſſextili poſt Februarium *adde unius diei motum.*

Tabulæ

Tabula Mediorum Motuum Satellitis Saturni *Penintimi*, *à* Cassino *detecti Anno* 1686.

Annis Jul. ineunt	Epochæ s.	°	'	Annis.	Med. Mot. s	°	'	Diebus.	Med. Mot. s	°	'	H M	Mot. Med. Sex. °	'	"	Min.	Motus Medi. °	'
1681	♓	3	23	1	4	10	2	1	4	11	32	1	0	5	29	31	2	50
1686	♉	5	25	2	8	20	4	2	8	23	4	2	0	10	58	32	2	56
1701	♓	22	3	3	1	0	6	3	1	4	36	3	0	16	26	33	3	1
1714	♑	7	5	4	9	21	40	4	5	16	8	4	0	21	55	34	3	7
1715	♉	17	7	5	2	1	42	5	9	27	40	5	0	27	24	35	3	12
1716	♍	27	9	6	6	11	44	6	2	9	12	6	0	32	53	36	3	17
1717	♊	18	43	7	10	21	46	7	6	20	44	7	0	38	22	37	3	23
1718	♎	28	45	8	7	13	20	8	11	2	16	8	0	43	51	38	3	28
1719	♓	8	47	9	11	23	22	9	3	13	49	9	0	49	19	39	3	34
1720	♋	18	49	10	4	3	24	10	7	25	21	10	0	54	48	40	3	40
Mens.	*Med. Mot.*			11	8	13	26	11	0	6	53	11	1	0	17	41	3	45
Anni				12	5	5	0	12	4	18	25	12	1	5	46	42	3	51
Com.	s	°	'	13	9	15	2	13	8	29	57	13	1	11	15	43	3	56
Jan.	0	0	0	14	1	25	4	14	1	11	29	14	1	16	44	44	4	1
Feb.	3	27	34	15	6	5	6	15	5	23	1	15	1	22	13	45	4	7
Mar.	6	20	32	16	2	26	40	16	10	4	33	16	1	27	42	46	4	12
April	10	18	6	17	7	6	42	17	2	16	5	17	1	33	11	47	4	17
Maii	10	4	7	18	11	16	42	18	6	27	37	18	1	38	39	48	4	23
Junii	2	1	41	19	3	26	46	19	11	9	9	19	1	44	8	49	4	28
Julii	1	17	43	20	0	18	20	20	3	20	41	20	1	49	37	50	4	34
Aug.	5	15	17	40	1	6	40	21	8	2	13	21	1	55	6	51	4	39
Sept.	9	12	51	60	1	25	0	22	0	13	45	22	2	0	35	52	4	45
Octob.	8	28	53	80	2	13	20	23	4	25	17	23	2	6	4	53	4	5
Nov.	0	26	27	100	3	1	40	24	9	6	49	24	2	11	32	54	4	56
Dec.	0	12	28	120	3	20	0	25	1	18	22	25	2	17	1	55	5	1
								26	5	29	54	26	2	22	30	56	5	7
In Anno Bissextili post Februarium adde unius diei motum.								27	10	11	25	27	2	27	59	57	5	12
								28	2	22	58	28	2	33	27	58	5	18
								29	7	4	30	29	2	38	56	59	5	23
								30	11	16	2	30	2	44	25	60	5	29

Tabula

Tabula Mediorum Motuum Satellitis Saturni *Medii, à* Cassino *detecti Anno* 1671.

Annis Jul. ineunt	*Epochæ* s. ° '	*Annis.*	*Med. Mot.* s ° '	*Diebus.*	*Med. Mot.* s ° ' "	*H M*	*Mot. Med. Sex.* ° ' "	*Min.*	*Motus Medi.* ° '
1681	♍ 12 16	1	9 17 2	1	2 19 41	1	0 3 19	31	1 43
1686	♏ 27 6	2	7 4 3	2	5 9 23	2	0 6 38	32	1 46
1701	♎ 1 17	3	4 21 5	3	7 29 4	3	0 9 58	33	1 49
1714	♎ 11 43	4	4 27 48	4	10 18 46	4	0 13 17	34	1 53
1715	♋ 28 45	5	2 14 50	5	1 8 27	5	0 16 36	35	1 56
1716	♉ 15 47	6	0 1 52	6	3 28 9	6	0 19 55	36	2 0
1717	♉ 22 30	7	9 18 53	7	6 17 50	7	0 23 15	37	2 3
1718	♓ 9 32	8	9 25 36	8	9 7 31	8	0 26 34	38	2 6
1719	♐ 26 34	9	7 12 38	9	11 27 13	9	0 29 53	39	2 10
1720	♎ 13 35	10	4 29 40	10	2 16 54	10	0 33 12	40	2 13
Mens. Anni Com.	*Med. Mot.* s °	11	2 16 42	11	5 6 36	11	0 36 31	41	2 16
		12	2 23 25	12	7 26 17	12	0 39 51	42	2 19
		13	0 10 26	13	0 15 59	13	0 43 10	43	2 23
Jan.	0 0 0	14	9 27 28	14	1 5 40	14	0 46 29	44	2 26
Feb.	10 10 24	15	7 14 30	15	3 25 21	15	0 49 48	45	2 29
Mar.	0 21 44	16	7 21 13	16	6 15 3	16	0 56 8	46	2 33
April	11 2 9	17	5 8 15	17	9 4 44	17	0 56 27	47	2 36
Maii	6 22 51	18	2 25 16	18	11 24 26	18	0 59 46	48	2 39
Junii	5 3 16	19	0 12 18	19	2 14 7	19	1 3 5	49	2 43
Julii	0 23 58	20	0 19 1	20	5 3 49	20	1 6 24	50	2 46
Aug.	11 4 23	40	1 8 2	21	7 23 30	21	1 9 44	51	2 49
Sept.	9 14 47	60	1 27 4	22	10 13 11	22	1 13 3	52	2 53
Octob.	5 5 30	80	2 16 5	23	1 2 53	23	1 16 22	53	2 56
Nov.	3 15 54	100	3 5 6	24	3 22 34	24	1 19 41	54	2 59
Dec.	11 6 37	120	3 24 7	25	6 12 16	25	1 23 11	55	3 3
				26	9 1 57	26	1 26 20	56	3 6
				27	11 21 39	27	1 29 39	57	3 9
				28	2 11 20	28	1 32 58	58	3 13
				29	5 1 1	29	1 36 17	59	3 16
				30	7 20 43	30	1 39 36	60	3 19

In Anno Bissextili post Februarium adde unius diei motum.

***Tabula Mediorum Motuum Penextimi Satellitis* Saturni *ab* Hugenio *inventi Anno* 1655.**

Annis Jul. ineunt.	Epochæ. s ° '	Annis	Med. Mot. s ° '	Diebus	Med. Mot. s ° '	H. M	Mo. Me. ° ' "	Min.	Motus Medi. ° ' "
1641	♑ 2 48	1	10 20 35	1	0 22 35	1	0 56	31	29 10
1661	♓ 13 23	2	9 11 10	2	1 15 9	2	1 53	32	30 6
1681	♈ 27 58	3	8 1 45	3	2 7 44	3	2 49	33	31 3
1686	♏ 3 28	4	7 14 55	4	3 0 18	4	3 46	34	31 59
1701	♊ 12 33	5	6 5 30	5	3 22 53	5	4 42	35	32 55
1714	♓ 17 53	6	4 26 5	6	4 15 28	6	5 39	36	33 52
1715	♒ 8 28	7	3 16 40	7	5 8 2	7	6 35	37	34 48
1716	♐ 29 3	8	2 29 50	8	6 0 37	8	7 31	38	35 45
1717	♐ 12 13	9	1 20 25	9	6 23 12	9	8 28	39	36 41
1718	♏ 2 48	10	0 11 0	10	7 15 46	10	9 24	40	37 38
1719	♍ 23 23	11	11 1 35	11	8 8 21	11	10 21	41	38 34
1720	♌ 13 58	12	10 14 45	12	9 0 55	12	11 17	42	39 31
1721	♋ 27 8	13	9 5 20	13	9 23 30	13	12 14	43	40 27
Menſ.	*Mot. Med.*	14	7 25 55	14	10 16 5	14	13 10	44	41 24
An. Co.	s ° '	15	6 16 30	15	11 8 39	15	14 7	45	42 20
Jan.	0 0 0	16	5 29 40	16	0 1 14	16	15 3	46	43 17
Febr.	11 9 54	17	4 20 15	17	0 23 48	17	16 0	47	44 13
Mar.	8 12 2	18	3 10 50	18	1 16 23	18	16 56	48	45 10
April	7 21 55	19	2 1 25	19	2 8 58	19	17 52	49	46 6
Maii	6 9 14	20	1 14 35	20	3 1 32	20	18 49	50	47 3
Junii	5 19 7	40	2 29 10	21	3 24 7	21	19 45	51	47 59
Julii	4 6 26	60	4 13 45	22	4 16 42	22	20 42	52	48 56
Aug.	3 16 18	80	5 28 20	23	5 9 16	23	21 38	53	49 52
Sept.	2 26 12	100	7 12 55	24	6 1 51	24	22 35	54	50 49
Octob.	1 13 30	120	8 27 30	25	6 24 25	25	23 31	55	51 45
Nov.	0 23 24	140	10 12 5	26	7 17 0	26	24 27	56	52 42
Dec.	11 10 42	160	11 26 40	27	8 9 35	27	25 24	57	53 38
				28	9 2 9	28	26 20	58	54 35
In Anno Biſſextili poſt Februarium *adde unius diei motum.*				29	9 24 44	29	27 17	59	55 31
				30	10 17 18	30	28 13	60	56 27

Tabulæ

Tabula Mediorum Motuum Satellitis Saturni *Extimi, a* Cassino *detecti Anno* 1671.

Annis Juliæ ineunt	Epochæ s	°	'	Annis	Med. Motus s	°	'	Diebus	Med. Motus s	°	'	H	Mo. Me. °	'	M	Med. Mot. '	"
1681	♈	8	40	1	7	6	32	1	0	4	32	1	0	11	31	5	51
1686	♈	15	50	2	2	13	3	2	0	9	5	2	0	23	32	6	3
1701	♉	11	53	3	9	19	35	3	0	13	37	3	0	34	33	6	14
1714	♓	20	20	4	5	0	39	4	0	18	9	4	0	45	34	6	25
1715	♎	26	52	5	0	7	10	5	0	22	42	5	0	57	35	6	37
1716	♊	3	23	6	7	13	42	6	0	27	14	6	1	8	36	6	48
1717	♑	14	27	7	2	20	13	7	1	1	46	7	1	19	37	7	0
1718	♌	20	58	8	10	1	17	8	1	6	18	8	1	31	38	7	11
1719	♓	27	30	9	5	7	49	9	1	10	51	9	1	42	39	7	22
1720	♏	4	2	10	0	14	20	10	1	15	23	10	1	53	40	7	34
Mens. Anni Com.	*Med. Mot.* s	°	'	11	7	20	52	11	1	19	55	11	2	5	41	7	45
				12	3	1	56	12	1	24	28	12	2	16	42	7	56
				13	10	8	27	13	1	29	0	13	2	27	43	8	8
Jan.	0	0	0	14	5	14	59	14	2	3	32	14	2	39	44	8	19
Febr.	4	20	41	15	0	21	30	15	2	8	5	15	2	50	45	8	30
Mar.	8	27	46	16	8	2	34	16	2	12	37	16	3	1	46	8	42
Apr.	1	18	27	17	3	9	6	17	2	17	9	17	3	13	47	8	53
Maii	6	4	37	18	10	15	37	18	2	21	42	18	3	24	48	9	4
Jun.	10	25	18	19	5	22	9	19	2	26	14	19	3	35	49	9	16
Julii	3	11	27	20	1	3	13	20	3	0	46	20	3	47	50	9	27
Aug.	8	2	9	40	2	6	26	21	3	5	18	21	3	58	51	9	38
Sept.	0	22	50	60	3	9	38	22	3	9	51	22	4	9	52	9	50
Oct.	5	8	59	80	4	12	51	23	3	14	23	23	4	21	53	10	1
Nov.	9	29	41	100	5	16	4	24	3	18	55	24	4	32	54	10	12
Dec.	2	15	50	120	6	19	17	25	3	23	28	25	4	43	55	10	24
In Anno Bissextili post Februarium adde unius Diei motum.								26	3	28	0	26	4	55	56	10	35
								27	4	2	32	27	5	6	57	10	46
								28	4	7	5	28	5	17	58	10	58
								29	4	11	37	29	5	29	59	11	9
								30	4	16	9	30	5	40	60	11	21

Motibus mediis Satellitum ad hunc modum constitutis, proveniunt Revolutiones eorum jam veris proximæ, scilicet

Primi sive Intimi	1d.	21h.	18′.	26″ $\frac{1}{2}$
Secundi Penintimi	2.	17	41.	10 $\frac{1}{2}$
Tertii sive *Medii*	4.	12	25.	10
Quarti Hugeniani	15.	22	41.	28
Quinti sive *Extimi*	79.	7	46.	00

Posito autem, juxta regulam Naturæ (saltem in hoc nostro Systemate) universalem, quæque tam in Jovialium ac Lunæ motibus, quam Planetarum Primariorum circa Solem, obtinet, Vires centrum Saturni petentes esse in duplicata ratione distantiarum reciproce, ac proinde Cubos distantiarum a centro esse ut quadrata Temporum periodicorum; ex data distantia & periodo *Hugeniani*, fiunt reliquorum distantiæ ut sequitur.

	Semidiam. Annuli,	*Semidiam. Globi* ♄
Dist. Primi	1.9289	4,3400
Secundi	2 4708	5,5593
Tertii	3.4508	7,7643
Quarti	8.0000	18,0000
Quinti	23.3146	52,4578

Quæ quidem distantiæ cum D. *Cassini* observatis satis quadrant. Quatuor autem interiores Satellites juxta planum Annuli Saturni orbitas suas describunt proxime; in plano, *sc.* Æquatoris nostri plano quoad sensum parallelo, quicquid in contrarium proferant nonnulli. Quintum vero orbem situ paulo diversum a cæteris describere, nuper deprehendit D. *Jac. Cassinus* prioris filius, ut videre est in Actis Academiæ Scientiarum *Parisiensis* Anni 1714.

Occultation of a Fixt Star by Jupiter, *n.* 51. *p.* 546.

XXXIV. D. *Martinus Folkes Londini*, præsentibus aliis nonnullis e Societate Regia, *Jan.* 11. 1717. 8h. P. M. vidit *Jovis* centrum una diametro corporis ejus Fixam sequi, quæ dicto centro Borealior erat quasi dodrante semidiametri Jovis. Postea Nubes Jovem occuparunt, sed habita ratione motus Jovis, paulo post medium Noctis stellam Jovi conjunctam fuisse, & a Borea disci ejus parte occultatam, conclusit.

Reverendus Dominus *J. Theoph. Desaguliers*, R. S. S. & D. *Stephanus Grey*, *Westmonasterii*, viderunt Fixam, Hora Sexta vespertina, integra Jovis diametro distare a limbo ejus, Corum versus. Unde & ex sequentium dierum Observationibus, circa medium noctis incidisse conjunctionem evincitur.

Reverendus quoque D. *J. Pound*, apud *Wansted*, infrascriptas nactus est observationes, quas utique accuratissimas, Tubo scil. prælongo & Micrometro captas, huc transcribere non pigebit.

Itaque *Januarii* Quinto 5h. 6′. T. æq. Jovis centrum distabat a dicta Fixa 31′. 49″. quam 5h. 38′. sequebatur 34′. 12″. Ascensionis rectæ: simulque limbus *Jovis* Austrinus eandem habuit Declinationem cum Stella.

Die

Die autem Nono sequente 6h. 6'. Jovis centrum distabat a stella 10' 49"; & post octo minuta erat differentia Ascensionum rectarum 11'. 32": & tum centrum Planetæ, tantillo, ita ut vix perciperetur, erat Stella Australius.

Die Undecimo 5h. 30' T. æq. erat distantia centrorum 1'. 24". simulq; visa est stella quasi quadrante diametri Jovis Borealior centro ejus. Diameter autem minima *Jovis* inventa est 0' 43". Deinde Nubes.

Die vero Duodecimo 5h. 17'. erat distantia centrorum 3'. 7"; ac 5h. 50'. Jupiter stellam præcedebat 3'. 30". Ascen. Rect. Eodemque tempore limbus Jovis Boreus eandem habuit Declinationem quam Fixa accurate.

Collatis autem his Observationibus manifestum est Fixam hanc *Jovi* conjunctam, *Januarii* undecimo 13h. circiter, non nisi 17". vel 18". centro ejus Borealiorem fuisse, ac proinde occultatam.

Fixa hæc, etiamsi nulli Catalogo hactenus ascripta, Locum tunc habuit ♊ 22°. 13'. cum Lat. Aust. 0°. 13'½; Comitemque habet 17 min. eam Præcedentem & 7 min. Borealiorem, sive in ♊ 21°. 56'. cum Lat. Aust. 0°. 6'½, cui *Jupiter* conjungi visus est *Jan.* 16. 6h. 30'. vesperi.

Sic spatio minus bimestri *Jupiter* corporaliter eclipsavit duas Fixas, cujus rei ne singulare quidem exemplum ab invento Telescopio extat: proinde hæc observata inter pretiosissima Uraniæ κειμήλια, in usum Posterorum, merito reponenda sunt.

Nostra autem stellula anno 1634. *Feb.* 6. *Jovi* Stationario conjuncta, tribus ejus diametris Australior erat, observante *Gassendo*: unde constabit, calculo rite instituto, *Joves* Nodos quoad sensum immobiles hæsisse, per 83 annos ultimo elapsos, idque ad 2s 8°. 35'. a 1a * ♈.

XXXV. *Feb.* 16. 1719. At 6h¼ through a short Tube, we saw all the 4 Satellites, the 3 outermost on the East side of *Jupiter*, and the innermost near the Western Limb approaching to an Eclipse. The Fourth at that time was about half a Semidiameter of *Jupiter* from the Eastern Limb. Then it proved cloudy till about 8h, at which time (through the long Glass) we could see only the second and third Satellites, the first being behind *Jupiter* in the Shadow, and the fourth entred upon the Disque. We saw at this time a dark Spot, a little Northward of the great Northern Zone, and near the Eastern Limb, where the Satellite was to enter on the Disque; which Spot we took for the Shade of the Satellite. The Clouds then again intercepted our View, till 8h. 53'. *Æq. T.* at which time the first Satellite was lately emerged out of the Shadow; and the Spot advanced so far, that we perceived it would arrive at the middle of *Jupiter*, near two Hours sooner than the Shade ought to have done by our Computation; but not imagining that this dark Spot could be any thing else but the Shade, we concluded there had been some Error in the Calculation, which we thought to re-examine afterwards. On this Presumption we left off observing till 9h. 35'. at which time we were surprized to see a Notch in the Limb of *Jupiter*,

Transit of Jupiter's 4th Satellite over the Disk of the Planet, by Mr. Pound, n. 359. p. 900.

near the place where the former Spot entred. This last Appearance agreeing well with the time that the Shade of the Satellite ought to have entred the Disque, soon made us alter our former Opinion, and conjecture that this, and not the other Spot was the said Shade. At 9^h. $39'\frac{1}{2}$ *Æq. T.* the Notch vanishing, a round black Spot appeared within the Limb, but in contact with it. At 9^h. $45'$. we judged the first Spot, and at 11^h. $45'$. the second, to be in the middle of *Jupiter*.

At 11^h. $50'$. the first Spot touched the Limb, being within the Disque, soon after which the Limb in that place seem'd a little protuberant. At 12^h. $5'$. appeared the fourth Satellite just come out of the Disque, and touching the Limb in the place where the Protuberancy was. At 12^h. $7'$. we could perceive the Satellite separated from the Limb. At 13^h. $56'$. the second black Spot, still within the Disque, just touched the Western Limb; soon after which there appeared a Notch in this part of the Limb, as it did on the other at the coming on of this Spot. At 14^h. $6'$. the Spot was all gone off, and the Limb appeared clear and entire. The first Spot, when in the middle of *Jupiter*, was almost as black as the second when near the Limb, but somewhat less, and a little more *Northerly*.

At the time that the first Spot was in the middle of the Disqne, the three innermost Satellites appeared to the *East* of *Jupiter*; the first (as aforesaid) having lately emerged out of the Shadow; the second being almost at its greatest distance; and the third having passed the Axis of the Shade about twelve Hours before, and appearing at this time about three Diameters of *Jupiter* from his Limb. The times that these Spots arrived at the middle of the Disque are agreeable to the times found by Calculation, in which the fourth Satellite and its Shade ought to have appeared there. From all which 'tis very plain, that the first of these Spots was the fourth Satellite it self, and the second its Shadow.

We have seen the first and second Satellites appearing not as dark Spots, but as bright ones (somewhat different from the light of *Jupiter*) for some little time after they entred his Disque, but as they approached nearer the middle we lost sight of them. And we have frequently observed that the same Satellites appear brighter at some times than at others; and that when one of them hath shined with its utmost Splendour, the Light of another hath been considerably diminished. From whence 'tis very probable at least, not only that the Satellites revolve upon their proper Axes, but also that some parts of their Surfaces do very faintly (if at all) reflect the Solar Rays to us *.

* *v. Mem. Ac. Roy. for* 1707. *and* 1714.

Emersion of the innermost Circumjovial at Rome, *n.* 340. *p.* 89.

XXXV. *Sept.* $\frac{1}{2}\frac{1}{2}$ 1713. Romæ, 8^h. $38'$. $20''$. *P.M.* Intimus Jovis Satelles incipit emergere, è regione spatii inter utramque Jovis fasciam protensi. Observatio peragebatur Telescopio D. *Andreæ Chiarelli* longitudinis 40 palm. Romanorum. Deinde 8^h. $44'$. Tertius Satelles ita apparebat Quarto conjunctus, ut ambo viderentur unicus Satelles, Distabant a centro Jovis diametris

diametris Jovialibus circiter $5\frac{1}{4}$. Hora vero 9h. 4′, jam disjuncti videbantur. Quartus situ inverso apparuit paulo depressior Tertio, & paulo elongatior a Jove: quare erat Tertio Borealior.

Sept. $\frac{18}{29}$ 10h. 36′. 23″, Primus seu intimus Satelles incipit emergere ab Umbra, Tubo 25 Palm. Domini *Campani*.

November $\frac{2}{2}$ 7h. 32′. 22″, Primus Satelles incipit emergere, conspectus per Tubum Domini *Chiarelli* palm. 40. Deinde eadem nocte 7h, 46′. Primus & Secundus proximi sunt, & 7h. 53′ iidem ita sunt vicini ut vix punctulo distinguantur.

Decemb. 9. *N.S.* vel *Novemb.* 28. *V.S.* 5h. 45′. 45″, Primus Satelles incipit emergere ab umbra Jovis.

Decemb. 21. *V. S.* 5h. 50′. 22, iterum visus est primus Satelles incipiens emergere ex umbra.

Ex his observationibus accurato calculo subjectis, manifestum est æquationem secundam, quam a motu Luminis progressivo ortam supponimus, necessario locum habere. Nam post 57 *satellitis intimi revolutiones, quibus Jupiter a Terra plusquam Radio Orbis magni recessit, novem fere minutis tardius conspecta est Eclipsis ultima, quam debuit juxta tenorem Observationis primæ: quod quidem Hypothesibus D.* Cassini *consonum est.* Remarks, ib. p. 90.

Ex iisdem etiam confirmatur (quod nos quoque antea annotavimus, nempe) *quod motus Intimi Satellitis Jovis paulo celerior sit quam in Tabulis elaboratissimis D. Cassini, ante viginti annos cum publico communicatis, & calculi facilitate plurimum se commendantibus. Errorculus autem iste vix excedere videtur duo temporis minuta in singulis Jovis revolutionibus, sive duodecim annis; quibus cœlum anticipat* Cassini *calculum. Hac vero adhibita correctione, satis accuratus habebitur consensus.*

XXXVI. In *Numb.* * 214. of these *Transactions*, we exhibited an Epitome of Mr. *Cassini*'s curious Tables then newly published, for computing the Eclipses of the first Satellite of *Jupiter*, without the help of any other Numbers. The ease of this *Calculus* gave great Satisfaction to those that delight in Telescopick Observations; and has been of good use to encourage Astronomers to ascertain the Geographical Longitudes of many places, by help of these Eclipses, whose frequency seems to afford us the properest means for that purpose. *Abridg. Vol. I. p. 409. *Tables for computing of the Eclipses of the first Satellite of* Jupiter, *by Addition only,* n. 361. p. 1021.

But it being now twenty six Years since those Tables were published, length of time has discovered that this *Satellit*'s Motion is a small matter swifter than Mr. *Cassini* had supposed it; and the Reverend Mr. *Pound* being provided with all the Qualifications requisite for such a Work, has of late apply'd himself to rectify by frequent Observation what he found amiss in the aforesaid *Calculus*; and withal has put it into another Form yet much more easy and compendious, by bringing what M. *Cassini* had given us in odd Numbers, to the Millesimals of a Circle, both as to *Numb.* I. which he calls *Numb. A.* being the mean Anomalie of *Jupiter* in such parts; as also to *Numb.* II. or our *Numb. B*, which is the distance of the mean place of *Jupiter*, from

from the true place of the *Sun*, and which with the Addition of the Equation of *Numb. B.* gives the true Angle of *Commutation* in the ſame Milleſimals of a Circle. And having deducted from the *Epoches* the greateſt Equations both of *Numb. A.* and *B.* he reſtores them by adding as much to the Equations themſelves, by which means they all become Affirmative, ſo that the whole Computation is performed by Addition only.

The Reader is ſuppos'd to be acquainted with the Method of M. *Caſſini's Calculus*, which is at large explain'd in the aforeſaid *Tranſact. Numb.* 214. For which reaſon this ſhort Deſcription may ſuffice at preſent.

EPOCHÆ

Epochæ Conjunctionum Primi Satellitis cum Jove.

An. Jul. Curr.	Conjunct. D.	H.	′	″	Num. A.	Num. B.	An. Jul. Curr.	Conjunct. D.	H.	′	″	Num. A.	Num. B.
1719	1	6	11	13	872	396	1749	0	11	9	34	400	866
1720	0	20	22	40	956	310	1750	0	1	21	1	485	780
21	1	5	2	44	40	229	51	1	10	1	5	569	698
22	0	19	14	11	125	143	52	1	0	12	33	653	612
23	0	9	25	38	209	57	53	1	8	52	37	738	531
1724	1	18	5	42	293	971	1754	0	23	4	4	822	445
1725	0	8	17	10	377	889	1755	0	13	15	32	906	359
26	1	16	57	13	462	808	56	0	3	27	0	990	273
27	1	7	8	41	546	722	57	0	12	7	3	75	191
28	0	21	20	8	630	636	58	0	2	18	30	159	110
1729	1	6	0	12	715	554	1759	1	10	58	34	243	24
1730	0	20	11	39	799	468	1760	1	1	10	1	328	938
31	0	10	23	7	883	382	61	1	9	50	35	412	856
32	0	0	34	34	967	296	62	1	0	1	2	496	770
33	0	9	14	38	52	215	63	0	14	13	0	580	684
1734	1	17	54	41	136	133	1764	0	4	24	27	665	598
1735	1	8	6	9	220	47	1765	0	13	4	31	749	517
36	0	22	17	36	305	961	66	0	3	15	58	833	431
37	1	6	57	40	389	880	67	1	11	56	2	918	349
38	0	21	9	7	473	794	68	1	2	7	29	2	263
1739	0	11	20	35	557	708	1769	1	10	47	33	86	182
1740	0	1	32	2	642	622	1770	1	0	59	0	171	96
41	0	10	12	6	726	540	71	0	15	10	28	255	10
42	0	0	23	33	810	454	72	0	5	21	56	339	924
43	1	9	3	37	895	373	73	0	14	2	0	423	842
1744	0	23	15	4	979	287	1774	0	4	13	27	508	761
1745	1	7	55	8	63	205	1775	1	12	53	31	592	675
46	0	22	6	35	148	119	76		3	4	58	676	589
47	0	12	18	3	232	33	77	1	11	45	1	761	507
48	0	2	29	30	316	947	78	1	1	56	28	845	421
1749	0	11	9	34	400	866	1779	0	16	7	56	929	335

Revolutiones Primi Satellitis Jovis in mensibus:

Januarii. D. h. ′ ″	N. A.	Nu. B.
1 18 28 36	0	5
3 12 57 12	1	9
5 7 25 48	1	14
7 1 54 24	2	18
8 20 23 0	2	23
10 14 51 36	2	27
12 9 20 12	3	32
14 3 48 48	3	37
15 22 17 24	4	41
17 16 46 0	4	46
19 11 14 36	4	51
21 5 43 12	5	55
23 0 11 47	5	60
24 18 40 23	6	64
26 13 8 59	6	69
28 7 37 35	7	73
30 2 6 11	7	78
31 20 34 47	7	82
Februarii		
0 20 34 47	7	82
2 15 3 23	8	87
4 9 31 59	8	92
6 4 0 35	9	96
7 22 29 11	9	101
9 16 57 47	9	105
11 11 26 23	10	110
13 5 54 59	10	114

Februarii. D. h. ′ ″	N. A.	Nu. B.
15 0 23 35	11	118
16 18 52 11	11	123
18 13 20 47	11	128
20 7 49 23	12	132
22 2 17 59	12	137
23 20 46 35	13	141
25 15 15 11	13	146
27 9 43 47	13	150
Martii.		
1 4 12 23	14	155
2 22 40 59	14	159
4 17 9 35	15	164
6 11 38 10	15	168
8 6 6 46	16	173
10 0 35 22	16	177
11 19 3 58	16	182
13 13 32 34	17	186
15 8 1 10	17	190
17 2 29 46	18	195
18 20 58 22	18	199
20 15 26 58	18	204
22 9 55 34	19	208
24 4 24 10	19	213
25 22 52 46	20	217
27 17 21 22	20	221
29 11 49 58	20	225
31 6 18 34	21	230

Aprilis

Revolutiones Primi Satellitis Jovis in mensibus.

Aprilis. D.	h.	′	″	N. A.	Nu. B.
0	6	8	34	21	230
2	0	47	10	21	235
3	19	15	46	22	239
5	13	44	22	22	244
7	8	12	58	22	248
9	2	41	34	23	252
10	21	10	10	23	257
12	15	38	46	24	261
14	10	7	22	24	265
16	4	35	58	25	270
17	23	4	33	25	274
19	17	33		25	279
21	12	1	45	26	283
23	6	30	21	26	287
25	0	58	57	27	292
26	19	27	33	27	296
28	13	56	9	27	300
30	8	24	45	28	304
Maii					
0	8	24	45	28	304
2	2	53	21	28	309
3	21	21	57	29	313
5	15	50	33	29	317
7	10	19	9	29	322
9	4	47	45	30	326
10	23	16	21	30	330
12	17	44	57	31	335
14	12	13	33	31	339

Maii. D.	h.	′	″	N. A.	Nu. B.
16	6	42	9	31	343
18	1	10	45	32	348
19	19	39	21	32	352
21	14	7	57	33	356
23	8	36	33	33	361
25	3	5	9	33	365
26	21	33	45	34	369
28	16	2	21	34	373
30	10	30	57	35	378
Junii					
0	10	30	57	35	378
1	4	59	32	35	382
2	23	28	8	36	386
4	17	56	44	36	391
6	12	25	20	36	395
8	6	53	56	37	399
10	1	22	32	37	403
11	19	51	8	38	408
13	14	19	44	38	412
15	8	48	20	38	416
17	3	16	56	39	420
18	21	45	32	39	425
20	16	14	8	40	429
22	10	42	44	40	433
24	5	11	20	40	438
25	23	39	56	41	442
27	18	8	32	41	446
29	12	37	8	42	450

Revolutiones Primi Satellitis Jovis in mensibus.

Julii. D.	h.	′	″	N. A.	Nu. B.
1	7	5	44	42	455
3	1	34	20	42	459
4	20	2	56	43	463
6	14	31	32	43	468
8	9	0	8	44	472
10	3	28	44	44	476
11	21	57	20	45	480
13	16	25	55	45	485
15	10	54	31	45	489
17	5	23	7	46	493
18	23	51	43	46	498
20	18	20	19	47	502
22	12	48	55	47	506
24	7	17	31	47	510
26	1	46	7	48	515
27	20	14	43	48	519
29	14	43	19	49	523
31	9	11	55	49	528
Augusti.					
0	9	11	55	49	528
2	3	40	31	49	532
3	22	9	7	50	536
5	16	37	43	50	541
7	11	6	19	51	545
9	5	34	55	51	549
11	0	3	31	51	554
12	18	32	7	52	558
14	13	0	43	52	562

Augusti. D.	h.	′	″	N. A.	Nu. B.
16	7	29	19	53	567
18	1	57	55	53	571
19	20	26	31	54	575
21	14	55	7	54	580
23	9	23	43	54	584
25	3	52	18	55	588
26	22	20	54	55	593
28	16	49	30	56	597
30	11	18	6	56	602
Septembris.					
1	5	46	42	56	666
3	0	15	18	57	610
4	18	43	54	57	615
6	13	12	30	58	619
8	7	41	6	58	624
10	2	9	42	58	628
11	20	38	18	59	632
13	15	6	54	59	637
15	9	35	30	60	641
17	4	4	6	60	646
18	22	32	42	60	650
20	17	1	18	61	655
22	11	29	54	61	659
24	5	58	30	62	663
26	0	27	6	62	668
27	18	55	42	62	672
29	13	24	18	63	677

Octobris

Revolutiones Primi Satellitis Jovis in mensibus.

Octobris. D.	h.	'	"	N. A.	Nu. B.
1	7	52	54	63	681
3	2	21	30	64	686
4	20	50	6	64	690
6	15	18	41	65	695
8	9	47	17	65	699
10	4	15	53	65	704
11	22	44	29	66	708
13	17	13	5	66	713
15	11	41	41	67	717
17	6	10	17	67	721
19	0	38	53	67	726
20	19	7	29	68	730
22	13	36	5	68	735
24	8	4	41	69	739
26	2	33	17	69	744
27	21	1	53	69	749
29	15	30	29	70	753
31	9	59	5	70	758
Novembris.					
0	9	59	5	70	758
2	4	27	41	71	762
3	22	56	17	71	767
5	17	24	53	71	772
7	11	53	29	72	776
9	6	22	5	72	781
11	0	50	41	73	785
12	19	19	17	73	790
14	13	47	53	74	794

Novembris. D.	h.	'	"	N. A.	Nu. B.
16	8	16	29	74	799
18	2	45	5	74	804
19	21	13	40	75	808
21	15	42	16	75	813
23	10	10	52	76	817
25	4	39	28	76	822
26	23	8	4	76	827
28	17	36	40	77	831
30	12	5	16	77	836
Decembris.					
0	12	5	16	77	836
2	6	33	52	78	840
4	1	2	28	78	845
5	19	31	4	78	849
7	13	59	40	79	854
9	8	28	16	79	859
11	2	56	52	80	863
13	21	25	28	80	868
14	15	54	4	80	873
16	10	22	40	81	877
18	4	51	16	81	882
19	23	19	5	82	886
21	17	48	28	82	891
23	12	17	4	82	897
25	6	45	40	83	900
27	1	14	16	83	905
28	19	42	52	84	909
30	14	11	28	84	194

Primæ Æquationes Conjunctionum Primi Satellitis cum Jove.

Num A	Æquat. Conjun. Adde.		Æq Nu. B	Num. A	Æquat. Conjun. Adde.		Æq Nu. B	Num. A	Æquar. Conjun. Adde.		Æq Nu. B	Num. A.	Æquat. Conjun. Adde.		Æq Nu. B
	′	″			′	″			′	″			′	″	
0	39	8	15	128	12	7	26	256	0	1	31	384	11	52	26
4	38	12	16	132	11	27	26	260	0	0	31	388	12	37	26
8	37	16	16	136	10	47	26	264	0	1	31	392	13	23	25
12	36	21	16	140	10	9	27	268	0	3	31	396	14	11	25
16	35	26	17	144	9	31	27	272	0	7	31	400	14	59	25
20	34	30	17	148	8	45	27	276	0	12	31	404	15	48	24
24	33	35	17	152	8	19	27	280	0	19	31	408	16	38	24
28	32	40	18	156	7	44	28	284	0	28	30	412	17	30	24
32	31	45	18	160	7	10	28	288	0	38	30	416	18	22	23
36	30	50	19	164	6	38	28	292	0	50	30	420	19	15	23
40	29	56	19	168	6	7	28	296	1	3	30	424	20	9	23
44	29	3	19	172	5	37	28	300	1	17	30	428	21	4	22
48	28	10	20	176	5	8	29	304	1	33	30	432	22	59	22
52	27	16	20	180	4	41	29	308	1	50	30	436	22	55	2
56	26	23	20	184	4	15	29	312	2	8	30	440	23	53	21
60	25	30	21	188	3	49	29	316	2	28	30	444	24	51	21
64	24	38	21	192	3	24	29	320	2	51	30	448	25	49	21
68	23	47	21	196	3	1	29	324	3	15	29	452	26	48	20
72	22	56	22	200	2	40	30	328	3	40	29	456	27	48	20
76	22	5	22	204	2	20	30	332	4	6	29	460	28	48	19
80	21	15	22	208	2	1	30	336	4	34	29	464	29	49	19
84	20	26	23	212	1	42	30	340	5	3	29	468	30	50	19
88	19	37	23	216	1	25	30	344	5	34	29	472	31	51	18
92	18	48	23	220	1	10	30	348	6	5	28	476	32	53	18
96	18	0	24	224	0	58	30	352	6	38	28	480	33	55	17
100	17	14	24	228	0	47	30	356	7	13	28	484	34	57	17
104	16	28	24	232	0	36	30	360	7	50	28	488	35	59	17
108	15	42	24	236	0	26	30	364	8	27	27	492	37	1	16
112	14	57	25	240	0	18	30	368	9	6	27	496	38	5	16
116	14	13	25	244	0	12	31	372	9	46	27	500	39	8	15
120	13	30	25	248	0	7	31	376	10	27	27	504	40	11	15
124	12	48	26	252	0	4	31	380	11	9	26	508	41	15	14
128	12	7	26	256	0	1	31	384	11	52	26	512	42	17	14

Primæ

Primæ Æquationes Conjunctionum Primi Satellitis cum-Jove.

Num. A.	Æquat. Conjunct. Adde. ′	″	Æq Nu. B.	Num. A.	Æquat. Conjun. Adde. ′	″	Æq. Nu. B.	Num. A	Æquat. Conjun. Adde. ′	″	Æq Nu. B.	Num. A.	Æquat. Conjun. Adde. ′	″	Æq Nu. B.
512	42	17	14	640	70	26	3	768	77	40	0	896	61	48	6
516	43	19	14	644	71	3	3	772	77	29	0	900	61	2	7
520	44	21	13	648	71	38	3	776	77	18	0	904	60	15	7
524	45	23	13	652	72	11	2	780	77	6	0	908	59	28	7
528	46	25	13	656	72	42	2	784	76	51	1	912	58	39	8
532	47	26	12	660	73	13	2	788	76	34	1	916	57	50	8
536	48	27	12	664	73	42	2	792	76	15	1	920	57	1	8
540	49	28	11	668	74	10	2	796	75	56	1	924	56	11	9
544	50	28	11	672	74	36	1	800	75	36	1	928	55	20	9
548	51	28	11	676	75	1	1	804	75	15	1	932	54	29	9
552	52	27	10	680	75	25	1	808	74	52	1	936	53	38	10
556	53	25	10	684	75	48	1	812	74	27	1	940	52	46	10
560	54	23	9	688	76	8	1	816	74	1	2	944	51	53	10
564	55	21	9	692	76	26	1	820	73	35	2	948	51	0	11
568	56	17	9	696	76	43	0	824	73	8	2	952	50	6	11
572	57	12	8	700	76	59	0	828	72	39	2	956	49	13	11
576	58	7	8	704	77	13	0	832	72	9	2	960	48	20	12
580	59	1	8	708	77	26	0	836	71	38	3	964	47	26	12
584	59	54	7	712	77	38	0	840	71	6	3	968	46	31	12
588	60	46	7	716	77	48	0	844	70	32	3	972	45	36	13
592	61	38	6	720	77	57	0	848	69	57	3	976	44	41	13
596	62	28	6	724	78	4	0	852	69	21	3	980	43	46	13
600	63	17	6	728	78	9	0	856	68	45	4	984	42	50	14
604	64	5	5	732	78	13	6	860	68	7	4	988	41	55	14
608	64	53	5	736	78	15	0	864	67	29	4	992	41	0	14
612	65	39	5	740	78	16	0	868	66	49	4	996	40	4	15
616	66	24	5	744	78	15	0	872	66	9	5	1000	39	8	15
620	67	7	4	748	78	12	0	876	65	28	5	1004	38	12	16
624	67	49	4	752	78	9	0	880	64	46	5	1008	37	16	16
628	68	30	4	756	78	4	0	884	64	3	5	1012	36	21	16
632	69	10	4	760	77	58	0	888	63	19	6	1016	35	26	17
636	69	49	3	764	77	50	0	892	62	34	6	1020	34	30	17
640	70	26	3	768	77	40	0	896	61	48	6	1024	33	35	17

Secunda

Secundæ Æquationes Conjunctionum Primi Satellitis cum Jove.

Addendæ.

Num. B. Æqu	0 Æqu.	100 Æqu.	200 Æq.	300 Æq.	400 Æq.	500 Æq.	600 Æq.	700 Æq.	800 Æqu.	900 Æqu.
	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
0	14 0	12 52	9 45	5 30	1 37	0 0	1 37	5 30	9 45	12 52
4	14 0	12 46	9 36	5 20	1 30	0 0	1 44	5 40	9 54	12 58
8	13 59	12 40	9 26	5 9	1 23	0 1	1 52	5 51	10 3	13 2
12	13 59	12 35	9 17	4 59	1 16	0 2	1 59	6 1	10 12	13 7
16	13 58	12 29	9 7	4 48	1 9	0 3	2 7	6 11	10 21	13 11
20	13 57	12 23	8 58	4 38	1 3	0 4	2 15	6 22	10 31	13 16
24	13 56	12 17	8 48	4 28	0 57	0 5	2 24	6 33	10 40	13 20
28	13 54	12 11	8 38	4 18	0 52	0 7	2 32	6 44	10 49	1 25
32	13 53	12 4	8 28	4 8	0 46	0 10	2 41	6 55	10 57	13 29
36	13 51	11 56	8 17	3 58	0 40	0 13	2 50	7 5	11 5	13 33
40	13 49	11 49	8 7	3 58	0 35	0 16	2 59	7 16	11 11	13 36
44	13 47	11 42	7 57	3 38	0 31	0 19	3 9	7 26	11 20	13 38
48	13 44	11 34	7 47	3 29	0 27	0 23	3 19	7 36	11 27	13 41
52	13 41	11 27	7 36	3 19	0 23	0 27	3 29	7 47	11 34	13 44
56	13 38	11 20	7 26	3 9	0 19	0 31	3 38	7 57	11 42	13 47
60	13 36	11 13	7 16	2 59	0 16	0 35	3 48	8 7	11 49	13 49
64	13 33	11 5	7 5	2 50	0 13	0 40	3 58	8 17	11 56	13 51
68	13 29	10 57	6 55	2 41	0 10	0 46	4 8	8 28	12 4	13 53
72	13 25	10 49	6 44	2 32	0 7	0 52	4 18	8 38	12 11	13 54
76	13 20	10 40	6 33	2 24	0 5	0 57	4 28	8 48	12 17	13 56
80	13 16	10 31	6 22	2 15	0 4	1 3	4 38	8 58	12 23	13 57
84	13 11	10 21	6 11	2 7	0 3	1 9	4 48	9 7	12 29	13 58
88	13 7	10 12	6 1	1 59	0 2	1 16	4 59	9 17	12 35	13 59
92	13 2	10 3	5 51	1 52	0 1	1 23	5 9	9 26	12 40	13 59
96	12 58	9 54	5 40	1 44	0 0	1 30	5 20	9 36	12 46	14 0
100	12 52	9 45	5 30	1 37	0 0	1 37	5 30	9 45	12 52	14 0

Tertiæ Æquationes Addendæ.				*Semidurationes Eclipsium Primi Satellitis Jovis.*																
Nu. A.	Æquationes ′	″	Num. A.	Nu. A.	Semidurationes H.	′	″	Nu. A.	Semidurationes H.	′	″	Nu. A.	Semidurationes H.	′	″	Nu. A.	Semidurationes H.	′	″
0	3	30	1000	0	1	5	9	250	1	7	0	500	1	5	9	750	1	7	46
20	3	29	980	10	1	4	56	260	1	7	15	510	1	4	53	760	1	7	57
40	3	28	960	20	1	4	44	270	1	7	31	520	1	4	39	770	1	8	7
60	3	25	940	30	1	4	33	280	1	7	45	530	1	4	26	780	1	8	15
80	3	19	920	40	1	4	23	290	1	7	57	540	1	4	15	790	1	8	22
100	3	12	900	50	1	4	13	300	1	8	7	550	1	4	7	800	1	8	26
120	3	4	880	60	1	4	7	310	1	8	15	560	1	4	3	810	1	8	28
140	2	56	860	70	1	4	4	320	1	8	22	570	1	4	1	820	1	8	30
160	2	46	840	80	1	4	2	330	1	8	27	580	1	4	0	830	1	8	28
180	2	34	820	90	1	4	0	340	1	8	28	590	1	4	3	840	1	8	26
200	2	22	800	100	1	4	2	350	1	8	29	600	1	4	7	850	1	8	22
220	2	10	780	110	1	4	3	360	1	8	27	610	1	4	13	860	1	8	16
240	1	57	760	120	1	4	6	370	1	8	24	620	1	4	23	870	1	8	8
260	1	44	740	130	1	4	12	380	1	8	17	630	1	4	35	880	1	8	0
280	1	30	720	140	1	4	21	390	1	8	9	640	1	4	49	890	1	7	50
300	1	17	700	150	1	4	31	400	1	7	58	650	1	5	4	900	1	7	37
320	1	5	680	160	1	4	42	410	1	7	46	660	1	5	19	910	1	7	22
340	0	53	660	170	1	4	55	420	1	7	31	670	1	5	36	920	1	7	8
360	0	41	640	180	1	5	9	430	1	7	14	680	1	5	54	930	1	6	55
380	0	31	620	190	1	5	23	440	1	6	58	690	1	6	10	940	1	6	40
400	0	22	600	200	1	5	39	450	1	6	40	700	1	6	28	950	1	6	23
420	0	14	580	210	1	5	55	460	1	6	20	710	1	6	46	960	1	6	8
440	0	8	560	220	1	6	11	470	1	6	2	720	1	7	2	970	1	5	54
460	0	4	540	230	1	6	26	480	1	5	45	730	1	7	17	980	1	5	37
480	0	2	520	240	1	6	43	490	1	5	26	740	1	7	33	990	1	5	22
500	0	0	500	250	1	7	0	500	1	5	9	750	1	7	46	1000	1	5	9

The use of the foregoing Tables. The Eclipses of the first Satellite of *Jupiter*, as has been already said, afford the best means of determining the Longitude of places on the Land, where Telescopes of a convenient length may be used; thirteen of these Eclipses happening every 23 Days; but it is requisite that the Observer know near the matter when these Opportunities offer themselves, least on the one Hand he let them slip, or else grow weary by a too long Attendance on them.

Those therefore who are curious to observe them, may readily compute the times of the Immersions or Emersions of this Satellite, and that with great Exactness, by the following very short Precepts, which admit of no Exception or Caution, *viz.*

Out of the first Table take the *Epoche* for the Year, with its corresponding *Numb. A.* and *Numb. B*; and to them add, out of the Tables of Months, the Day, Hour, Minute and Second, nearest less than the time of the Eclipse you seek for, together with its *Numb. A* and *B*: the Sum of the Times is the mean time of the middle of the Eclipse. 2. With *Numb. A.* thus collected, take out the first Æquation of the Conjunctions; as also the Æquation of *Numb. B.* always to be added to *Numb. B.* before found. 3. With *Numb. B.* so equated, take out the second Æquation of the Conjunctions; and in the last Table, the third Æquation, as also the Semi-duration of the Eclipse answering to *Numb. A.* 4. To the mean time of the middle of the Eclipse, add all those three Æquations; the Sum shall be the true equated time of the middle of the Eclipse sought. 5. If *Numb. B.* equated be less than 500, substract the Semiduration, and you will have the time of the Immersion, or if it be more than 500, adding the same, it will give the time of the Emersion.

But Note, the times thus found are equal time, still to be reduced to the Apparent: and that in the *Bissextile* Year, after *February*, one Day is to be deducted from the Day of the Month.

The less skilful may perhaps be pleas'd with an Example or two, which may serve them to imitate. Let it be required to find the time of the Immersion of this Satellite into *Jupiter*'s Shadow, *November* the 9th 1719. in the Morning. The Work stands thus,

	D.	h.	′	″	Nu. A.	Nu. B.	
1719.	1.	6.	11.	13	872	396	
Novemb.	7.	11.	53.	29	72	776	
Conj. Med.	8.	18.	4.	42	944	172	
Æquat. I.			51.	53		10	Æq. B.
Æquat. II.			10.	26			
Æquat. III.			3.	26		182	B. Æquat.
	8.	19.	10.	27			
		1.	6.	33	*Semidur. Sub.*		
Novemb.	8.	18.	3.	54			

So

So that by this *Calculus*, on the ninth of *Novemb.* at 4 Minutes after 6 in the Morning, equal Time, may be ſeen the *Immerſion* of this Satellite into *Jupiter*'s Shadow.

Another Example ſhall be of the *Emerſion* on the fifth of *April* 1720, *viz.*

	D.	H.	′	″		Nu. A.	Nu. B.	
1720.	0.	20.	22.	40		956	310	
April	4.	13.	44.	22	Biſs.	22	244	
Conj. Med.	5.	10.	07.	02		978	554	
Æquat. I.			44.	13			13	Æq. B.
Æquat II.			0.	45			567	B. Æquat.
Æquat. III.			3.	29				
		1.	5.	40	*Semidur. Add.*			
April	5.	12.	01.	09				

Hence it appears that at one Minute after Midnight following the fifth of *April*, equal Time, will happen the Emerſion required. Nor do we doubt but that the Event will very nearly anſwer.

We have learnt, by the Experience of many Years Obſervation, that the ſecond inequality of this Satellite proceeds from the progreſſive Propagation of Light, and is common to all the reſt of the Satellites; Light being found to proceed in about ſeven Minutes of time as far as from the Sun to the Earth, whether with an equable Motion or otherwiſe is ſtill a Queſtion. For this reaſon we have added a *third Æquation*, whereby to account for the greater diſtance of *Jupiter* from the *Earth* in *Aphelio* than in *Perihelio*, as the *ſecond Æquation* anſwers to the greater diſtance of the Planet when near the Conjunction of the Sun, than when near his Oppoſition.

Tranſit of Mars, *near a Fixt Star, n.* 351. *p.* 548.

XXXVIII. *Februarii* Quinto Mane, vel Quarto 16h. 1717. *Mars* viſus eſt adeo vicinus Boreæ Frontis Scorpii ut ea nudis oculis non conſpiceretur; ſed per Teleſcopium inventa eſt ſupra & ad ortum, adeoque *Mars* nondum ei conjunctus. Hora 16h. 10′. T. app. *Mars* erat in recta cum Borea frontis & Teleſcopica quæ eam ſequitur ad Boream, ad diſtantiam octo circiter minutorum 16h. 35′. *Mars* intermedius erat in recta cum Borea & Media Frontis; & poſt horæ quadrantem, cum Auſtrina Frontis, ita ut 16h. 54′. T. app. æſtimabatur Conjunctio ipſa quoad Longitudinem, quo tempore *Mars* ſat accurate duobus tantum minutis auſtralior erat ſtella. Obſervavit etiam D. *Pound* Conjunctionem reſpectu Aſcenſionis Rectæ 17h. 25′. T. app. cum diſtantia centrorum 2′. 07″. Jucundum autem erat ſpectaculum, *Martem* videre ſtellam pedetentim aggredientem, motumque ſuum, lentiſſimum licet, manifeſte prodentem.

Conferatur cum hac Obſervatio *Horroxii* noſtri anno 1638. *Februarii* Septimo mane, quam vide in Epiſtolis ejus pag. 304. Tunc enim *Mars* ad eandem ſtellam appulſus, etiam multo propius ad eam acceſſit, ſed ante ortum ejus præterierat Conjunctio.

His adde *Saturni* obſervationem *Januarii* 25to. $12^h. 25'$. T. æq. a D. *Pound* habitam. Cum Planeta diſtabat a ſtella 58va. *Virginis Catal. Brit.* $13'. 16''$. verſus Auſtrum, eamque ſequebatur $2'. 30''$. Aſc. Rect. Stellæ in ♒ $19. 21'. 52''$. cum lat. Bor. $2°. 47'. 25''$.

The Cauſe of the Appearance of Venus, *for many Days together in the Day time, by* Dr. Halley, *n.*349. *p.*456.

XXXIX. The Appearance of *Venus* in the Day-time, in the Summer, 1716. generally taken notice of about *London* and elſewhere, was by ſome reckoned to be prodigious. This put me upon the Enquiry, how it came to paſs that at that time the *Planet* ſhould be ſo plainly ſeen by Day, whereas ſhe rarely ſhews her ſelf ſo, unleſs to thoſe who know exactly where to look for her. To reſolve this, the following Problem aroſe, *viz.* To find the Situation of the *Planet* in reſpect of the *Earth*, when the *Area* of the illuminated part of her Disk is a *Maximum.*

To inveſtigate this *Maximum*, I found it requiſite to aſſume the following *Lemmata*. I. That the viſible *Areas* of the Disk of the ſame *Planet*, at differing Diſtances, are always reciprocally as the Squares of thoſe Diſtances; which is evident from the firſt Principles of *Opticks*. II. That the *Area* of the whole Disk of the *Planet* is to the *Area* of the illuminated part thereof, as the Diameter of a Circle to the Verſed-Sine of the exteriour Angle at the *Planet*, in the Triangle at whoſe Angles are the *Sun*, *Earth*, and *Planet*. III. That in all plain Triangles, four times the Rectangle of the Sides containing any Angle, is to the exceſs of the Square of the Sum of the Sides above the Square of the Baſe, as the Diameter is to the Verſed Sine of the Complement of the contained Angle to a Semicircle, which I call the exterior Angle: This is a new *Theorem* of good uſe in *Trigonometry*, and eaſily proved from the 12th and 13th of the II. *Elem. Euclid.*

This premiſed, putting m for the Diſtance of the *Sun* and *Earth*, and n for that of the *Sun* and *Venus*, and x for the diſtance of the *Earth* and *Venus*, or the third ſide of the Triangle which we ſeek; by the third *Lemma*, $4nx$, will be to the exceſs of the Square of $n+x$, above the Square of m, as the *Area* of the whole Disk of *Venus* to the *Area* of the part illuminated; and by the firſt *Lemma*, the *Area*'s of her whole Disk are at all times as the Squares of x reciprocally; whence the Quantity $\frac{nn+2nx+xx-mm}{4nx^3}$ will in all Caſes be proportional to the *Area* of the illuminated part.

Now that this ſhould be a *Maximum*, it is required that the Fluxion thereof be equal to 0, or that the Negative parts thereof be equal to the Affirmative, that is, that $\overline{2n\dot{x}+2x\dot{x}}\times 4nx^3 = 12nx^2\dot{x}\times\overline{nn+2nx+xx-mm}$; and dividing all by $4nx^2x$, the Equation becomes

comes $2nx+2xx=3nn+6nx+3xx-3mm$. Conſequently $3nn+4nx+xx=3mm$, and therefore $x=\sqrt{3mm+nn}-2n$.

From hence a ready and not inelegant Geometrical Conſtruction (if I may be allowed to ſay ſo) becomes obvious; for with the Center *S* and Radius *S T*$=m$, deſcribe the Semicircle *T D A*; and with the ſame Center and Radins *S E*$=n$, the Semicircle *E V B*; which two Semicircles ſhall repreſent the Orbs of the *Earth* and *Venus*. Make the Chord *A D* equal to the Radius *S T*, and from *D* towards *A*, lay off *D F*=*S E*; draw *T F*, and thereon place *F G*=*B E*$=2n$; and with the Center *T*, and *Radius T G* deſcribe the Arch *G V*, cutting the Semicircle *B V E* in *V*; and draw the Lines *S V*, *T V*. I ſay the Triangle *S T V* is Similar to that at whoſe Angles are the *Sun*, *Earth* and *Venus*, at the time when the *Area* of the enlightned part of that Planet's Disk, as ſeen from the *Earth*, is greateſt. How this Geometrical Effection follows from the Equation is too evident to need Repetition. *Fig.* 5. *Plate* 4.

In conſequence of this Solution, I find this *Maximum* always to happen, when the Planet is about forty Degrees diſtant from the *Sun*; and the times thereof, about the middle between her greateſt Elongations on both ſides from him, and her retrograde Conjunctions with him; when little more than a quarter of her viſible Disk is luminous, and reſembling the Moon of about five days old; and notwithſtanding that her Diameter is at that time but 50 Seconds, yet ſhe ſhines with ſo ſtrong a Beam, as to ſurpaſs the united Light of all the Fixt Stars that appear with her, and caſts a very ſtrong Shade on the Horizontal plain whereon they all ſhine: an irrefragable Argument to prove that the Disks of the fixt Stars are unconceivably ſmall, and next to nothing; ſince ſhining with a native Light, ſo many of them do not equal the reflex Light of one quarter of a Disk of leſs than a Minute diameter.

In this Situation *Venus* was found in *July* laſt, on the tenth day, about which time, when the *Sun* grew low, ſhe was very plainly ſeen in the day-time, for many days together; as ſhe might have been in the Mornings, about the latter end of *September*. But this ariſing from the Cauſes we have now ſhewn, is nothing uncommon; for every eighth Year it returns again, ſo that the Planet may be ſeen on the ſame day of the Month and Hour, very nearly in the ſame place; as all acquainted with the Heavenly Motions muſt know.

Laſtly, it may not be amiſs to note that the Equation $x=\sqrt{3mm+nn}-2n$ has a Limit; for if n be equal to $\frac{1}{4}m$, the point *V* will fall on *B*; and the whole Disk of a Planet at that diſtance from the Sun would be the *Maximum*, *viz.* when in its ſuperior Conjunction with the *Sun*. And the like if n were leſs than $\frac{1}{4}m$; the Arch *G V* in ſuch Caſe not interſecting the Semicircle *B E*.

XL. Veteres *Ægyptii* & *Chaldæi*, ſiqua fides *Diodoro Siculo*, longa obſervationum ſerie inſtructi, Cometarum ἐπιτολὰς prænuntiare valuerunt. Cum autem iiſdem artibus etiam Terræ motus ac tempeſtates prævidiſſe dican- *The Aſtronomy of Comets, by Dr.* Halley, *n.* 297. *p.* 1882.

dicantur, extra dubium eſt Aſtrologiæ potius calculo fatidico, quam Aſtronomicis motuum Theoriis eorum de his rebus ſcientiam referendam eſſe. Ac vix alia a Græcis utriuſque populi victoribus reperta eſt apud eos doctrina; adeo ut eam, quam nunc eouſque proveximus Aſtronomiam, Græcis ipſis, præſertim magno *Hipparcho*, uti inventoribus, acceptam debeamus. Apud hos vero *Ariſtotelis* ſententia, qui Cometas nihil aliud eſſe voluit quam Vapores ſublunares vel etiam Meteora aërea, tantum effecit, ut hæc Aſtronomicæ ſcientiæ pars longe ſubtiliſſima omnino neglecta manſerit, cum nemini operæ pretium viſum fuerit, vagas & incertas fluitantium in æthere vaporum ſemitas adnotare ſcriptiſque mandare; unde factum ut ab illis nihil certi de motu Cometarum ad nos tranſmiſſum reperiatur.

Seneca autem Philoſophus, perpenſis duorum inſignium ſui temporis Cometarum Phænomenis, non dubitavit iis loca inter corpora cœleſtia aſſignare, Sydera eſſe cum mundo duratura exiſtimans, quanquam Motus eorum legibus nondum compertis regi fateatur. Tandemque Vaticinio non irrito promittit aliquando futura ſecula, quibus hæc tam occulta *dies extraheret ac longioris ævi diligentia*: quibuſque admirationi foret hæc *Veteres* neſcire potuiſſe; poſtquam *Demonſtraverit aliquis* Naturæ Interpres *in quibus Cœli partibus Cometæ errent, quanti, qualeſque ſint*. Ab hac autem *Senecæ* ſententia in diverſas partes abiit pene omnis Aſtronomorum Cohors; ac ipſe *Seneca*, neque Phænomena Motus quibus opinionem hanc tueretur, neque tempora adſcribere dignatus eſt, quæ poſteris ad hæc definienda uſui forent. Ac evolutis plurimis Cometarum Hiſtoriis, nihil omnino invenio quod huic negotio inſervire poſſit, ante annum a *Chriſto* nato 1337. quo *Nicephorus Gregoras* Hiſtoricus & Aſtronomus *Conſtantinopolitanus* nobis Cometæ ſemitam inter fixas ſatis accurate deſcripſit: tempora autem nimis laxe conſignavit, ita ut non niſi quod abhinc quadringentis pene Annis apparuerit, lubricus & incertus hic Cometa Catalogo quem damus inſeri mereatur. Dein Cometa anni 1472 omnium velociſſimus ac terris proximus *Regiomontanum* habuit obſervatorem. Hic magnitudine ac Coma terribilis, unius diei ſpatio 40 gradus ſub circulo Cœli maximo emenſus eſt, ac omnium primus eſt de quo obſervata idonea ad nos pervenere. Quotquot autem Cometas conſiderarunt, uſque ad tempora *Tychonis Brahe* magni illius Aſtronomiæ reſtauratoris, eos ſublunares eſſe autumarunt, adeoque parvi penderunt, utpote pro Vaporibus habitos.

Anno autem 1577, (*Tychone* jam ſtudio aſtrorum ſerio incumbente, comparatiſque Machinis ingentibus pro dimetiendis cœli arcubus, majori cum cura & certitudine quam Veteribus ſperare fas erat) Emerſit Cometa ſatis conſpicuus, cui obſervando ſtrenue ſeſe accinxit *Tycho*: multiſque & fidis experimentis deprehendit, nulli quæ ſentiretur Parallaxi diurnæ obnoxium fuiſſe, adeoque non tantum non fuiſſe Vaporem aereum, ſed & etiam multo ſuperiorem extitiſſe Luna: immo nihil obſtabat quin inter ipſos Planetas collocaretur; fruſtra interim contra obſtrepentibus Scholaſticorum nonnullis.

Tychonis

Tychonis vero eximiam in obſervando induſtriam excepit *Kepleri* ſagaciſſimum & pene divinum ingenium. Hic *Tychonis* laboribus fretus Syſtema Mundi verum & Phyſicum adinvenit, ac ſcientiam Aſtronomicam in immenſum auxit; Monſtrato ſc. Planetas omnes in Planis per Solis centrum tranſeuntibus revolvi, Curvaſque Ellipticas deſcribere, ea lege, ut Areæ Sectorum Ellipticorum, ad centrum Solis in Ellipſeos foco conſtituti, temporibus, quibus deſcribantur arcus, ſemper proportionales ſint. Invenit etiam Diſtantias Planetarum a Sole eſſe in ſeſquialtera ratione temporum periodicorum, ſive Cubos diſtantiarum eſſe ut Quadrata Temporum. Tanto autem Artifici affulſere duo Cometæ, quorum alter maxime illuſtris. Ex horum obſervatis concluſit *Keplerus* non uno parallaxis annuæ indicio, Cometas inter Orbes planetarum liberrime quaquaverſum ferri, motu quidem non multum a rectilineo diverſo, ſed quem nondum definire licuit. Ac *Hevelius*, *Tychonis* æmulus, *Kepleri* veſtigiis inſiſtens, eandem Hypotheſim Motus rectilinei amplexus eſt, ipſe plurium Cometarum Obſervator perquam ſubtilis. Cœlo tamen Calculum ſuum non penitus conſentire queſtus eſt, Viamque Cometicam verſus Solem incurvari ſuboluit.

Tandem de ſummo Cœlo lapſus eſt prodigioſus ille Cometa Anni 1680. quaſi Caſu perpendiculari Solem petens, & exinde pari velocitate aſſurgens: Hic per quatuor Menſes continuos viſus, inſigni ac peculiari Curvitate Orbitæ ad inveſtigationem Motus Theoriæ præ cæteris idoneus erat: Inſtructis autem jampridem Regiis Obſervatoriis, *Pariſienſi* & *Grenovicenſi*, ac Aſtronomorum Clariſſimorum curæ commiſſis, accidit ut hujus Cometæ Motus apparens, quantum forſan mortalibus fas eſt, accuratiſſime a *Caſſino* & *Flamſtedio* obſervaretur.

Non multo poſt, dum Geometrarum Princeps illuſtriſſimus *Newtonus* operam dabat *Principiis Philoſophiæ Mathematicis*; non ſolum Inventa *Kepleri* in Syſtemate Planetario neceſſario locum habere demonſtravit, verum etiam Cometarum Phænomena omnia ex iiſdem Principiis evidenter conſequi. Id quod exemplo prædicti Cometæ anni 1680 abunde illuſtravit, modumque docuit Geometrice conſtruendi Orbitas Cometarum; Problemaque arduum ac tanto Oedipo dignum ſumma cum omnium admiratione reſolvit. Cometam autem hunc in orbe parabolico Solem circumiſſe probat, ita ut Areæ ad Centrum Solis æſtimatæ Temporibus proportionales fuerint.

Tanti viri veſtigia inſecutus eandem methodum calculo Arithmetico accommodare aggreſſus ſum, nec irrito Conamine. Undique enim conquiſitis Cometarum Obſervationibus, Tabellam hanc, immenſi pene Calculi fructum, obtinui, exiguum quidem ſed non ingratum Aſtronomis munus. Hi etenim numeri vim habent omnia quæ de motu Cometarum hactenus obſervata ſunt accuratiſſime repræſentandi, ope ſolius Tabulæ Generalis inſequentis, cui adornandæ nullis ſane peperci laboribus, ut perfecta prodiret, utpote Poſteritati conſecrata, ac cum Scientia Aſtronomica duratura.

Come-

Cometarum Omnium hactenus rite Observatorum, Motuum in Orbe Parabolico Elementa Astronomica.

Cometæ Anni.	Nodus Ascend.	Inclin. Orbitæ.	Perihelion in Orbe.	Perihelion in Ecliptica.	Latitudo Perihelii.	Distantia Perihelia a Sole.	Log. dist Periheliæ à Sole.	Temp. æquat. Perihelii Londini.
	gr. ′ ″	gr. ′ ″	gr. ′ ″	gr. ′ ″	gr. ′ ″			die. h. ′
1337	♊ 24. 21. 0	32. 11. 0	♉ 7. 59. 0	♉ 12. 45. 15	22. 40. 30 B	40666	9. 609236	*Junii* 2. 6. 25
1472	♑ 11. 46. 20	5. 20. 0	♉ 15. 33. 30	♉ 15. 40. 20	4. 25. 50 A	54273	9. 734584	*Feb.* 28. 22. 23
1531	♉ 19. 25. 0	17. 56. 0	♒ 1. 39. 0	♒ 0. 48. 15	17. 3. 05 B	56700	9. 753583	*Aug.* 24. 21. 18½
1532	♊ 20. 27. 0	32. 36. 0	♋ 21. 7. 0	♋ 16. 59. 40	15. 57. 00 B	50910	9. 706803	*Oct.* 19. 22. 12
1556	♍ 25. 42. 0	32. 6. 30	♑ 8. 50. 0	♑ 11. 6. 00	31. 10. 20 B	46390	9. 666424	*Apr.* 21. 20. 3
1577	♈ 25. 52. 0	74. 32. 45	♌ 9. 22. 0	♍ 7. 53. 00	69. 35. 20 A	18342	9. 263447	*Octo.* 26. 18. 45
1580	♈ 18. 57. 20	64. 40. 0	♋ 19. 5. 50	♋ 19. 17. 10	64. 40. 0 B	59628	9. 775450	*Nov.* 28. 15. 00
1585	♉ 7. 42. 30	6. 4. 0	♈ 8. 51. 0	♈ 8. 59. 10	2. 55. 25 A	109358	0. 038850	*Sept.* 27. 19. 20
1590	♍ 15. 30. 40	29. 40. 40	♏ 6. 54. 30	♏ 2. 55. 50	22. 45. 50 A	57661	9. 760882	*Jan.* 29. 3. 45
1596	♒ 12. 12. 30	55. 12. 0	♏ 18. 16. 0	♏ 22. 44. 35	54. 44. 30 B	51293	9. 710058	*Julii* 31. 19. 55
1607	♉ 20. 21. 0	17. 2. 0	♒ 2. 16. 0	♒ 1. 29. 40	16. 10. 5 B	58680	9. 768490	*Oct.* 16. 3. 50
1618	♊ 16. 1. 0	37. 34. 0	♈ 2. 14. 0	♈ 6. 10. 00	35. 50. 0 A	37975	9. 579498	*Oct.* 29. 12. 23
1652	♊ 28. 10. 0	79. 28. 0	♈ 28. 18. 40	♊ 10. 41. 35	58. 14. 0 A	84750	9. 928140	*Nov.* 2. 15. 40
1661	♊ 22. 30. 30	32. 35. 50	♋ 25. 58. 40	♋ 21. 37. 30	17. 17. 0 B	44851	9. 651772	*Jan.* 16. 23. 41
1664	♊ 21. 14. 0	21. 18. 30	♌ 10. 41. 25	♌ 8. 40. 35	16. 1. 50 A	102575½	0. 011044	*Nov.* 24. 11. 52
1665	♏ 18. 02. 0	76. 05. 0	♊ 11. 54. 30	♉ 24. 6. 35	23 8. 0 B	10649	9. 027309	*Apr.* 14. 5. 15½
1672	♑ 27. 30. 30	83. 22. 10	♉ 16. 59. 30	♋ 9. 26. 00	69. 27. 40 B	69739	9. 843476	*Feb.* 20. 8. 37
1677	♏ 26. 49. 10	79. 03. 15	♌ 17. 37. 5	♋ 16. 21. 05	75. 44. 10 B	28059	9. 448072	*Apr.* 26. 00. 37½
1680	♑ 2. 2. 0	60. 56. 0	♐ 22. 39. 30	♐ 27. 26. 50	8. 11. 10 A	00612½	7. 787106	*Dec.* 8. 00. 6
1682	♉ 21. 16. 30	17. 56. 0	♒ 2. 52. 45	♒ 2. 0. 30	16. 59. 20 B	58328	9. 765877	*Sept.* 4. 07. 39
1683	♍ 23. 23. 0	83. 11. 0	♊ 25. 29. 30	♋ 10. 36. 55	82. 52. 00 B	56020	9. 748343	*Julii* 3. 2. 50
1684	♐ 28. 15. 0	65. 48. 40	♏ 28. 52. 0	♐ 15. 15. 25	26. 35. 20 A	96015	9. 982339	*Maii* 29. 10. 16
1686	♓ 20. 34. 40	31. 21. 40	♊ 17. 00. 30	♊ 16. 24. 00	31. 17. 35 B	32500	9. 511883	*Sept.* 6. 14. 33
1698	♐ 27. 44. 15	11. 46. 0	♑ 00. 51. 15	♑ 10. 47. 20	0. 38. 10 A	69129	9. 839660	*Oct.* 8. 16. 57

Hæc Tabula vix indiget explicatione, cum ex titulis satis pateat quid sibi velint Numeri. Distantiæ autem periheliæ æstimantur in ejusmodi partibus quales media distantia Terræ à Sole habet centies millenas.

Tabula

Tabula Generalis pro Supputando Motu Cometarum in Orbe Parabolico.

Medius motus.	*Angulus à perihelio.*	*Logarith. pro distan. à Sole*	*Medius motus.*	*Angulus à perihelio.*	*Logarith. pro distan. à Sole.*
o	gr. ′ ″		o	gr. ′ ″	
1	1. 31. 40	0.000077	31	42. 55. 07	0.062400
2	3. 3. 15	0.000309	32	44. 3. 16	0.065835
3	4. 34. 43	0.000694	33	45. 10. 26	0.069316
4	6. 6. 0	0.001231	34	46. 16. 35	0.072839
5	7. 37. 1	0.001921	35	47. 21. 36	0.076396
6	9. 7. 44	0.002759	36	48. 25. 33	0.079984
7	10. 38. 2	0.003745	37	49. 28. 29	0.083604
8	12. 7. 53	0.004876	38	50. 30. 23	0.087249
9	13. 37. 17	0.006151	39	51. 31. 11	0.090912
10	15. 6. 6	0.007564	40	52. 30. 54	0.094594
11	16. 34. 20	0.009115	41	53. 29. 42	0.098298
12	18. 1. 54	0.010798	42	54. 27. 32	0.102019
13	19. 28. 47	0.012609	43	55. 24. 22	0.105752
14	20. 54. 53	0.014550	44	56. 20. 11	0.109490
15	22. 20. 14	0.016607	45	57. 15. 5	0.113240
16	23. 44. 43	0.018783	46	58. 9. 2	0.116995
17	25. 8. 22	0.021072	47	59. 2. 5	0.120756
18	26. 31. 7	0.023470	48	59. 54. 13	0.124518
19	27. 52. 55	0.025969	49	60. 45. 26	0.128278
20	29. 13. 52	0.028551	50	61. 35. 45	0.132035
21	30. 33. 39	0.031263	51	62. 25. 14	0.135792
22	31. 52. 31	0.034045	52	63. 13. 50	0.139541
23	33. 10. 23	0.036916	53	64. 1. 38	0.143288
24	34. 27. 12	0.039864	54	64. 48. 38	0.147029
25	35. 42. 59	0.042892	55	65. 34. 50	0.150762
	36. 57. 41	0.045989	56	66. 20. 14	0.154482
27	38. 11. 20	0.049154	57	67. 04. 51	0.158192
28	39. 23. 56	0.052383	58	67. 48. 22	0.161890
29	40. 35. 26	0.055668	59	68. 31. 51	0.165578
30	41. 45. 50	0.059010	60	69. 14. 16	0.169254

Tabula Generalis pro Supputando

Medius motus.	Angulus à perihelio.	Logarith. pro distan. à Sole.	Medius motus.	Angulus à perihelio.	Logarith. pro distant. à Sole.
o	gr. ′ ″		o	gr. ′ ″	
61	69. 55. 58	0.172914	91	86. 20. 34	0.274176
62	70. 36. 56	0.176557	92	86. 46. 20	0.277239
63	71. 17. 16	0.180188	93	87. 11. 43	0.280284
64	71. 56. 56	0.183803	94	87. 36. 45	0.283306
65	72. 35. 57	0.187404	95	88. 01. 27	0.286308
66	73. 14. 15	0.190978	96	88. 25. 49	0.289293
67	73. 51. 59	0.194540	97	88. 49. 48	0 292252
68	74. 29. 6	0.198085	98	89. 13. 32	0.295201
69	75. 05. 38	0.201614	99	89. 36. 54	0.298122
70	75. 41. 35	0.205122	100	90. 00. 00	0.301030
71	76. 16. 56	0.208612	102	90. 45. 14	0.306782
72	76. 51. 43	0.212080	104	91. 29. 18	0.312469
73	77. 25. 57	0.215529	106	92. 12. 14	0.318060
74	77. 59. 41	0.218963	108	92. 54. 4	0.323587
75	78. 32. 54	0.222378	110	93. 34. 52	0.329042
76	79. 5. 35	0.225769	112	94. 14. 40	0.334424
77	79. 37. 45	0.229142	114	94. 53. 30	0.339736
78	80. 9. 23	0.232488	116	95. 31. 22	0.344979
79	80. 40. 34	0.235809	118	96. 8. 22	0.350153
80	81. 11. 16	0.239127	120	96. 44. 30	0.355262
81	81. 41. 31	0.242416	122	97. 19. 48	0.360306
82	82. 11. 19	0.245684	124	97. 54. 17	0.365284
83	82. 40. 40	0.248933	126	98. 28. 00	0.370200
84	83. 9. 34	0.252159	128	99. 00. 57	0.375052
85	83. 38. 4	0.255366	130	99. 33 11	0.379842
86	84. 6. 8	0.258552	132	100. 4. 43	0.384576
87	84. 33. 49	0.261720	134	100. 35. 45	0.389252
88	85. 1. 5	0.264865	136	101. 5. 48	0.393868
89	85. 27. 58	0.267989	138	101. 35. 22	0.398428
90	85. 54. 27	0.271092	140	102. 4. 19	0.402930

Motu

Motu Cometarum in Orbe Parabolico.

Medius motus.	*Angulus à perihelio.*	*Logarith. pr. distan. à Sole.*	*Medius motus.*	*Angulus à perihelio.*	*Logarith. pr. distan. à Sole.*
o	gr. ′ ″		o	gr. ′ ″	
142	102. 32. 41	0.407380	204	113. 37. 25	0.523406
144	103. 00. 31	0.411784	208	114. 9. 52	0.529705
146	103. 27. 47	0.416132	212	114. 41. 23	0.535886
148	103. 54. 31	0.420430	216	115. 12. 02	0.541958
150	104. 20. 43	0.424676	220	115. 41. 51	0.547922
152	104. 46. 22	0.428866	224	116. 10. 52	0.553782
154	105. 11. 33	0.433012	228	116. 39. 7	0.559538
156	105. 36. 16	0.437110	232	117. 6. 38	0.565199
158	106. 00. 32	0.441164	236	117. 33. 27	0.570762
160	106. 24. 23	0.445178	240	117. 59. 35	0.576233
162	106. 47. 47	0.449144	244	118. 25. 5	0.581616
164	107. 10. 44	0.453060	248	118. 49. 57	0.586912
166	107. 33. 17	0.456936	252	119. 14. 14	0.592122
168	107. 55. 27	0.460772	256	119. 37. 56	0.597252
170	108. 17. 14	0.464208	260	120. 1. 6	0.602301
172	108. 38. 37	0.468318	264	120. 23. 44	0.607274
174	108. 59. 39	0.472030	268	120. 45. 52	0.612174
176	109. 20. 20	0.475705	272	121. 7. 30	0.616998
178	109. 40. 40	0.479340	276	121. 28. 39	0.621750
180	110. 00. 40	0.482937	280	121. 49. 22	0.626438
182	110. 20. 20	0.486498	284	122. 9. 38	0.631056
184	110. 39. 41	0.490022	288	122. 29. 28	0.635608
186	110. 58. 44	0.493512	292	122. 48. 54	0.640098
188	111. 17. 28	0.496965	296	123. 7. 57	0.644525
190	111. 35. 55	0.500384	300	123. 26. 36	0.648893
192	111. 54. 05	0.503769	310	124. 11. 40	0.659559
194	112. 11. 58	0.507121	320	124. 54. 36	0.669880
196	112. 29. 34	0.510441	330	125. 35. 34	0.679876
198	112. 46. 55	0.513729	340	126. 14. 44	0.689568
200	113. 4. 00	0.516984	350	126. 52. 12	0.698970

Tabula Generalis pro Supputando Motu Cometarum in Orbe Parabolico.

Medi. motus. °	*Angulus a peribelio* gr. ′ ″	*Logarith. prodistan. a Sole.*	*Medius motus.* °	*Angulus a peribelio.* gr. ′ ″	*Logarith. prodistan. a Sole.*
360	127. 28. 6	0.708104	820	141. 49. 24	0.970836
370	128. 2. 33	0.716976	840	142. 10. 00	0.978397
380	128. 35. 38	0.725606	860	142. 29. 56	0.985771
390	129. 7. 27	0.734006	880	142. 49. 10	0.992970
400	129. 38. 4	0.742186	900	143. 7. 48	1.000000
410	130. 7. 34	0.750160	920	143. 25. 51	1.006871
420	130. 36. 2	0.757930	940	143. 43. 21	1.013586
430	131. 3. 30	0.765516	960	144. 00. 18	1.020155
440	131. 30. 2	0.772918	980	144. 16. 46	1.026583
450	131. 55. 41	0.780148	1000	144. 32 46	1.032876
460	132. 20. 30	0.787216	1500	149. 26. 8	1.158188
470	132. 44. 32	0.794122	2000	152. 26. 15	1.246058
480	133. 7. 50	0.800882	2500	154. 32. 20	1.313703
490	133. 30. 25	0.807494	3000	156. 7. 27	1.368678
500	133. 52. 20	0.813969	3500	157. 22. 49	1.414974
520	134. 34. 18	0.826522	4000	158. 24. 36	1.454950
540	135. 14. 0	0.838600	4500	159. 16. 36	1.490125
560	135. 51. 28	0.850187	5000	160. 1. 12	1.521521
580	136. 27. 6	0.861369	5500	160. 40. 5	1.549874
600	137. 00. 57	0.872155	6000	161. 14. 24	1.575718
620	137. 33. 13	0.882575	6500	161. 45. 00	1.599460
640	138. 3. 58	0.892649	7000	162. 12. 34	1.621417
660	138. 33. 21	0.902401	7500	162. 37. 34	1.641838
680	139. 1. 29	0.911866	8000	163. 00. 23	1.660922
700	139. 28. 25	0.921012	8500	163. 21. 20	1.678834
720	139. 54. 16	0.929907	9000	163. 40. 42	1.695708
740	140. 19. 5	0.938549	9500	163. 58. 38	1.711662
760	140. 42. 56	0.946951	10000	164. 15. 20	1.726784
780	141. 05. 55	0.955124	50000	170. 52. 0	2.197960
800	141. 28. 3	0.963082	100000	172. 45. 44	2.399655

Tabulæ Generalis Conſtructio & Uſus.

Ut Planetæ in Orbibus Ellipticis, ita Cometæ in Parabolicis Solem in Foco communi ſitum ambiunt; ea lege ut Areæ æquales æqualibus temporibus deſcribantur. Quoniam vero Parabolæ omnes inter ſe ſimiles ſunt, ſi determinata aliqua pars Areæ datæ Parabolæ dividatur in partes quotlibet; in omnibus Parabolis fiet ſimilis diviſio ſub iiſdem angulis, atque diſtantiæ erunt proportionales: ideoque una noſtra Tabula pro Cometis *Fig. 6.* omnibus ſufficiet. Calculi autem hujus Tabulæ hæc eſt ratio. In Sche- *Plate 4.* mate ſit *S* Sol, *POC* orbita Cometæ, *P* Perihelion, *O* Locus ubi Cometa quadrante diſtat a Perihelio, *C* Locus quivis alius. Junge *CP*, *CS*; ac fiant *ST*, *SR* æquales ipſi *CS*; ac ductis rectis *CR*, *CT* (quarum hæc Curvæ Tangens eſt, illa perpendicularis) in axem *PSR* demitte normalem *CQ*. Jam data quavis Area *COPS*, oportet angulum *CSP*, & diſtantiam *CS* inquirere. Quoniam ob naturam Parabolæ recta *RQ* ubique æqualis eſt ſemilateri recto, ponatur latus rectum$=2$, adeoque $RQ=1$: ac ſit recta $CQ=z$: erit itaque $PQ=\frac{1}{2}zz$, ac Segmentum parabolicum $COP=\frac{1}{12}zzz$. Triangulum autem CSP erit $=\frac{1}{4}z$: adeoque Area mixtilinea $COPS$ erit $\frac{1}{12}z^3+\frac{1}{4}z=a$, ac $z^3+3z=12a$. Quare reſoluta hac æquatione Cubica, z ſive ordinatim applicata *CQ* innoteſcet. Proponatur jam Area *OPS* in partes centenas dividenda: hæc Area duodecima pars eſt quadrati lateris recti, adeoque $12a$ æquantur Quadrato illo$=4$: Si itaque ſucceſſive extrahantur radices æquationum $z^3+3z=$ 0,04: 0,08: 0,12: 0,16, &c. habebuntur totidem z ſive ordinatim applicatæ *CQ* reſpective; ac diviſa erit area *SOP* in partes centenas. Eodemque modo ultra locum *O* continuandus eſt calculus. Radix autem hujus æquationis, cum RQ ſit$=1$, Tangens eſt Tabularis anguli *CRQ*, ſive dimidii anguli *CSP*, adeoque angulus *CSP* datur. Ejuſdemque anguli *CRQ* ſecans *RC*, Media proportionalis eſt inter *RQ* ſive Unitatem & *RT*, quæ dupla eſt ipſius *SC*, ut ex Conicis notiſſimum eſt. Quod ſi *SP* ponatur 1, adeoque latus rectum$=4$ (ut in Tabula noſtra), ipſa *RT* erit diſtantia quæſita; duplum ſcilicet ipſius *SC* in priore Parabola. Ad hunc modum itaque præcedentem Tabulam elaboravi repræſentandis omnium Cometarum motibus inſervientem: hactenus enim nullus ex obſervatis Parabolæ leges reſpuit.

Reſtat jam præcepta Calculi tradere, modumque ſupputandi locum Cometæ viſum ex his Numeris exhibere. Cometæ autem in Parabola moventis Velocitas ubique eſt ad velocitatem Planetæ gyrantis in Circulo circa Solem, ad eandem a Sole diſtantiam, ut $\surd 2$ ad 1: ut conſtat ex *Principiis Phil. Nat. Math. Lib.* I. *Prop.* 16. *Corol.* 7. Si itaque Cometa in perihelio ad diſtantiam æqualem diſtantiæ Terræ a Sole ſupponatur, erit area diurna, quam deſcriberet Cometa, ad aream quam deſcribit Terra, ut $\surd 2$ ad 1. ac proinde tempus annuum, ad tempus quo Cometa talis deſcriberet Quadrantem Orbitæ ſuæ a Perihelio ut 3. 14159, &c. (hoc eſt ut area circuli) ad $\surd\frac{32}{9}$. Cometa igitur deſcriberet Quadrantem illum diebus 109. 14^h. 46$'$. adeoque area illa Parabolica areæ *POS* analogia in centum particulas diſtributa, ſingulis diebus competunt particulæ 0,912280.

Cujus

Cujus Logarithmus nempe 9,960128 in perpetuum usum servandus est. Tempora autem, quibus Cometa in distantia majore vel minore Quadrantes similes describeret, sunt ut Revolutiones in Circulis; hoc est in sesquiplicata ratione distantiarum: adeoque areæ diurnæ in partibus centesimis Quadrantis æstimatæ (quas medii Motus mensuras, instar Graduum, ponimus) sunt in singulis in subsesquialtera ratione distantiæ Periheliæ a Sole.

His necessario præmissis proponatur alicujus e Cometis nostris Locum visum ad datum tempus supputare. Primum itaque Solis Locus ab Æquinoctio in promptu sit, ejusdemque distantiæ a Terra Logarithmus. 2°. Capiatur intervallum Temporis inter Tempus Perihelii & Tempus datum, in diebus partibusque diei decimalibus. Hujus numeri Logarithmo addatur Logarithmus constans 9,960128 ac complementum Arithmeticum sesquialterius Logarithmi distantiæ Periheliæ a Sole: summa Logarithmus erit Motus medii in prima Columna Tabulæ Generalis quærendi. 3°. Cum motu medio capiatur in Tabula correspondens angulus a Perihelio, & Logarithmus pro distantia a Sole: ac in Cometis Directis adde, in Retrogradis subduc, si fuerit Tempus post Perihelium; vel in Directis subduc & in Retrogradis adde, si fuerit ante Perihelium, angulum sic inventum a loco vel ad locum Perihelii. Et habebitur Locus Cometæ in orbita propria: Et ad Logarithmum pro Distantia ibidem inventum addatur logarithmus distantiæ Periheliæ, summa erit Logarithmus distantiæ veræ Cometæ a Sole. 4°. Cum Loco Cometæ in Orbita, dato Loco Nodi, capiatur distantia Cometæ a Nodo: ac data Inclinatione plani, dabuntur, Notissimis Trigonometriæ præceptis, Locus Cometæ ad Eclipticam reductus, cum Inclinatione sive Latitudine Heliocentrica, ac Distantiæ Curtatæ Logarithmus. 5°. Ex his datis, iisdem omnino regulis quibus loca Planetarum, ex dato Loco & Distantia Solis; obtinebitur Locus Visus seu Geocentricus cum Latitudine visa. Id quod exemplo uno vel altero operæ pretium erit illustrare.

Exemp. I. Quæritur Locus Cometæ Anni 166 4/5 *Martii* 1°. 7h. 00. P. M. *Londini*. Hoc est 96d. 19h. 8'. post Perihelion ejus *Novemb.* 24°. 11h. 52'. Celebratum.

Log. dist. Perihel.	0. 011044	*Perihel.* ♌	10. 41. 25	*Log. pro dist.*	0. 255369
Log. Sesquialt.	0. 016566	*Ang. Corresp.*	83. 38. 05	*Log. Perihel.*	0. 011044
Comp. Arith.	9. 983434	*Comet. in Orb.* ♉	17. 3. 20	*Co. sin. Incl.*	9. 990754
	9. 960128	☊ ♊	21. 14. 00	*Log. dist. Curt.*	0. 257167
Log. Temp.	1. 985862	*Com. a Nodo*	34. 10. 40	*Log. dist.* ☉	9. 997918
Log. Med. Mot.	1. 929424	*Red. ad Eclip.*	32. 19. 05	☉ ♓	21. 44. 45
Medius Motus	85. 001	*Com. Helioc.* ♉	18. 54. 55	*Com. Visus* ♈	29. 18. 30
		Incl. Bor.	11. 46. 50	*Lat. Visa*	8. 36. 15. *Bor.*

Exemp. II. Quæritur Locus Cometæ Anni 1683 *Julii* 23° 13^h. 35'. P. M. *Londini*. Vel 13^h. 40'. T. æquat. hoc eſt 2rd. 10^h, 50'. poſt Perihelion.

Log. diſt. Perihel.	9. 748343	*Perihel.* ♊	25. 29. 30	*Log. pro diſt.*	0. 111336
Log. Seſquialt.	9. 622514	*Ang. Correſp.*	56. 47. 20	*Log. Perihel.*	9. 748343
Comp. Arith.	0. 377486	*Comet. in Orb.* ♈	28. 42. 10	*Co. ſin. Incl.*	9. 913187
	9. 960128	☊ ♓	23. 23. 00	*Log. diſt. Curt.*	9. 772866
Log. Temp.	1. 310723	*Com. à* ☋	35. 19. 10	*Log. diſt.* ☉	0. 005104
Log. Med. Mot.	1. 648337	*Red. ad Eclip.*	4. 48. 30	☉ *Locus* ♌	10. 41. 25
Medius Motus.	44. 498	*Com. Helioc.* ♓	28. 11. 30	*Com. Viſus* ♋	5. 11. 50
		Incl. Bor.	35. 2. 00	*Lat. Bor.*	28. 52. 00

Momento autem primi Exempli, *Londini* obſervatum eſt Cometam applicari ad Stellam ſecundam *Arietis*; ita ut novem minutis illa borealior repertus ſit, ac tribus minutis orientalior: Obſervante Dno. *Roberto Hookio*. In ſecundo autem Exemplo ipſe, in vicinia *Londini*, inſtrumentis quibus olim Stellas Auſtrales obſervaveram, Cometæ locum deprehendi ♋ 5°. 11'. $\frac{1}{2}$, cum Latitudine Boreali, 28°. 52', conſentiente ad amuſſim obſervatione *Grenovicenſi* eodem pene momento facta.

Cometa autem Anni 1680, qui pene Solem attigit, (non enim triente ſemidiametri corporis Solaris a ſuperficie ejus diſtabat in Perihelio) cum Latus rectum exiguum admodum ſit, Tabula Generali haud coerceri potuit, ob immanem Motus medii velocitatem: præſtat itaque in hoc, poſtquam inventus fuerit Motus medius, ex eodem, ope præcedentis æquationis $z\,z\,z + 3\,z = 1\frac{4}{7}$ *Mot. Med.* Tangentem dimidii anguli a Perihelio elicere, una cum Logarithmo pro diſtantia a Sole. Quibus datis iiſdem omnino regulis ac in cæteris procedendum eſt.

Ad hunc itaque modum Aſtronomico Lectori examinare licet numeros a me poſitos, quos ſumma cura ex obſervationibus quæ ſuppetebant exantlavi; neque enim, antequam probe ad incudem redacti fuerint, ac multorum annorum ſtudio quantum fieri poſſit politi, in publicum prodeunt. Hoc autem ſpecimen Aſtronomiæ Cometicæ, futuri operis Prodromum, editum eſſe volui; ne forte ſuperveniente fato perirent lucubrationes noſtræ, ob Calculi difficultatem non cuivis homini denuo ſuſcipiendæ. Monendus autem eſt Lector, quinque priores ordine Cometas, quorum tertius & quartus eſt a *Petro Apiano* obſervatus, quintus vero a *Paulo Fabricio*, uti & decimus a *Mæſtlino* (ni fallor) anno 1596 conſpectus, non eundem certitudinis gradum cum reliquis præ ſe ferre. Neque enim debitis organis nec cura ad hoc requiſita obſervationes ipſæ peractæ ſunt; adeoque inter ſe diſſidentes nullo modo cum computo regulari conciliari poſſunt. Cometam Anni 1684 unus vidit *Blanchinus* obſervator *Romanus*: ultimum vero Anni ſc. 1698 *Pariſienſes* ſoli conſpexerunt, ejuſque curſum inſolito modo deſignarunt. Obſcurus hic admodum, etiamſi velox ac terris ſatis vicinus, noſtros ſane oculos alioquin non incurioſos effugit. Inſignes autem duos hac noſtra ætate Cometas, alterum Anno 1689 Menſe *Novembri*

ortum,

ortum, alterum Mense *Februario* Anni 1702, Catalogo subjungere non licuit, propter defectum observationum. Etenim versus mundi plagas Australes cursum dirigentes, ac in *Europa* vix conspicui, contemplatores non habuere negotio pares. Quod si forsan ex partibus *Indicis* advectæ fuerint accuratæ observationum series ad hoc necessariæ; lubens calculum repetere, horumque Orbitas, reliquorum ad modum, Numeris designandi laborem suscipere non gravabor.

Angustia autem paginæ 328, factum est, ut omissa sit necessaria illa Columella quæ ostendat an directe vel retrograde moti fuerint Cometæ. Sciat itaque Astronomus undecim e nostris Cometis directo cursu secundum seriem signorum processisse, nempe illos annorum 1532, 1556, 1580, 1585, 1618, 1652, 1661, 1672, 1680, 1684 & 1686. Reliquos vero tredecim motu retrogrado contra seriem signorum cursum tenuisse. Quibus perpensis, ac collatis inter se cæteris horum Cometarum motuum Elementis, videre est, nullo ordine dispositos esse Orbitas; neque ipsos, Planetarum more, *Zodiaco* comprehendi posse, quaquaversum tam Retrograde quam directe indifferenter latos; unde manifestum est eos motu vorticali nullo modo circumagi. Quinetiam distantiæ Periheliæ nunc majores nunc minores reperiuntur; unde pronum est suspicari etiam multo plures esse Cometas, qui in partibus a Sole remotioribus, obscuri caudaque destituti, adeoque nobis inconspicui, præterlabi possunt.

Hactenus Cometarum Orbes consideravimus ut perfecte Parabolicos; quo supposito consequeretur Cometas, vi Centripeta versus Solem impulsos, a spatiis infinite distantibus descendere, casuque suo velocitatem tantam acquirere, ut iterum in spatia Mundi remotissima sese abdere possint, perpetuo nisu sursum tendentes, ac ad Solem nunquam reversuri. Cum autem satis frequentes sint Cometarum adventus; ac eorum nullus reperiatur motu ferri Hyperbolico, seu velociore quam cadendo ad Solem acquirere debeat, credibile est potius in Orbibus valde Excentricis revolvi eos circa Solem, ac post longissimas periodos reverti. Sic enim Numerus eorum præfinitus esset, ac fortasse non usque adeo magnus. Spatia autem inter Solem Fixasque tanta sunt, ut Cometæ revolventi cum Periodo quantumvis longa satis loci sit. Latus autem rectum Ellipsis est ad Latus rectum Parabolæ eandem Periheliam distantiam habentis, ut distantia Aphelia in Ellipsi est ad Axem totum Ellipsis; Velocitates autem sunt in dimidiata ratione eorundem: quapropter in Orbibus valde Excentricis ratio hæc accedit proxime ad rationem æqualitatis. Tantilla autem differentia, quæ intercedit ratione majoris in Parabola velocitatis, facillime in situ Orbis determinando compensatur. Hujus itaque Tabulæ Elementorum Motuum usus præcipuus est, atque etiam propter quem illam construere operæ pretium duxi, ut, si quando novus Cometa emerserit, possimus collatis elementis dignoscere an poterit esse aliquis ex antiquis, necne; ac proinde Periodum Orbitæque Axem determinare, reditumque prædicere. Ac sane multa me suadent ut credam Cometam anni 1531 ab *Apiano* observatum, eundem fuisse cum illo qui anno 1607 descriptus est a *Keplero* & *Longomontano*, quemque ipse iterum reversum

vidi

vidi ac observavi anno 1682. Quadrant Elementa omnia, ac sola inæqualitas periodorum adversari videtur: hæc autem tanta non est ut causis Physicis non possit attribui. *Saturni* enim motus a cæteris, præsertim *Jove*, ita interturbatur, ut per aliquot dies integros incertum sit hujus Planetæ tempus Periodicum. Quanto magis talibus erroribus obnoxius erit Cometa, qui quatuor pene vicibus altius excurrit *Saturno*, cujusque velocitas, vel tantillum aucta, Orbem ab Elliptico in Parabolicum possit immutare? Confirmatur etiam eundem esse potuisse ex eo, quod anni 1456 æstate, conspectus fuerit Cometa eodem pene modo inter Solem & Terram transiens retrograde: quem, licet a nemine observatus fuerit Astronomice, ex periodo modoque transitus non diversum a prædictis extitisse conjicio. Unde ausim ejusdem reditum fidenter prædicere, anno scil. 1758. Quod si hoc evenerit, nulla amplius erit dubitandi causa, quin redire debeant cæteri. Habebunt ergo Astronomi in hac arena quo se exerceant per multa sæcula, priusquam tot tantorumque Corporum circa commune centrum Solis revolventium numerus cognoscatur, ac motuum symptomata certis regulis coerceantur. Crediderim equidem Cometam etiam anni 1532, eundem fuisse cum illo, qui ab *Hevelio* observabatur ineunte anno 1661: sed observationes *Apiani*, quas solas de primo habemus, nimis rudes sunt, nec quicquam certi in re tam subtili ex iisdem elici potest. Justo volumine hæc omnia exequi mihi animus est, nec Astronomiæ promovendæ hac in re deero, si Deo O. M. visum fuerit vitam facultatesque prorogare. Interim quicunque modum Construendi Cometarum Orbes per tres observationes accurate habitas addiscere cupit, sub finem libri de Systemate Mundi, sive tertii *Philosophiæ Nat. princip. Math.* magni ipsius Inventoris methodum inveniet: Quam postea Dignissimus Collega meus *D. Gregorius*, Lib. V. pereruditæ Astronomiæ suæ Physicæ & Geometricæ plene & luculenter illustravit.

Unicum autem non abs re erit nec injucundum, hic loci Lectorem monere Astronomum; nempe quod nonnulli ex his Cometis Nodos suos habeant adeo Orbi Terræ annuo vicinos, ut si forte acciderit, tempore reditus Cometæ, *Terram* occupare Loca in orbe suo Nodo proxima, dum Cometa incredibili cum Velocitate præterierit, Parallaxin etiam habiturus sit valde observabilem, quæque fuerit ad Solis parallaxin in ratione data. Unde occasione talium transituum oblata erit ansa, rara quidem sed optima, determinandi Solis a *Terra* distantiam; quam hactenus non nisi mediante parallaxi *Martis* Acronychii, vel *Veneris* perigææ, triplo quidem solari majore, sed quæ vix ullis instrumentis sentiatur, laxe admodum concludere licuit. Quem Cometarum usum suggessit Clarissimus Geometra Ds. *Nic. Facio.* Cometa etenim anni 1472 parallaxin habuit plusquam vigesies Solari majorem. Ac si Cometa anni 1618 appulisset, juxta medium Mensis *Martii*, ad Nodum ejus Descendentem; vel si Cometa anni 1686 paulo citius ad Nodum Ascendentem pervenisset, profecto Terris admodum propinqui etiam adhuc magis notabiles habuissent parallaxes. Inter omnes vero nullus propiore appulsu Terris minatus est quam ille anni 1680: Hic inito Calculo non amplius ad Boream distabat

ab Orbe nostro annuo, quam Semidiametro Solari (sive Radio Lunaris Orbitæ, ut existimo) idque Nov. 11. 1h. 6'. *P. M.* Quo tempore, si Terræ quoad Longitudinem conjunctus fuisset, parallaxis sane Lunari æqualis in Cometæ motu observari potuisset. Hæc Astronomis dicta sunto. Quæ vero ab hujusmodi allapsu vel contactu, vel denique collisione corporum cœlestium (quæ quidem omnino non impossibilis est,) consequi debeant, rerum Physicarum studiosis discutienda relinquo.

Observations of the Comet at Rome *in* 1664 *by* Mr Ray, *n.* 309. *p.* 2350.

XLI. *December* the 20th, 1664. *S. N.* About three a Clock this Morning, I observed the *Comet*; it was in the Constellation of *Hydra*, not far from the Foot of *Crater*. It appeared about the bigness of a Star of the first Magnitude, but nothing so lucid and bright. It had a very long Tail, which pointed almost directly towards the Heart of *Hydra*: The Tail shew'd somewhat like Rays of a Candle burning in a Mist: The Figure of it was Conical; the length of it 5 or 6 Degrees; the breadth at the Base not above a Degree and half. The Body of this *Comet* was about three Degrees to the South-East of the most Southerly Star in the Foot of *Crater*; it stood very near in a Right Line with the two lowermost Stars in the Foot of *Crater*, which are common to it and *Hydra*. See the Figures.

Plate 6.

December 21. In the Morning, about the same Hour, it was removed about a Degree and half from the place where it stood, Westward, and a little to the South. The Tail pointed still towards the Heart of *Hydra*, and appeared 10 Degrees long at the least.

December 22. At the same time it was removed from the place where it stood the Day before, to the same Point, and about the same distance as the Night before. The Tail of it still pointed to *Cor Hydræ*, or a little thought above it, as the two former Days, and was rather longer than shorter: It also to my thinking, appeared brighter and larger; the Body of it being bigger than any Fixt Star, except *Sirius*.

December 23. It was removed to the same Point, and about the same Distance as the Day before; the Tail of it was as long as ever, and the *Comet* brighter. The Tail pointed almost directly to *Cor Hydræ*.

December 24, 25, 26. All these Three Nights were cloudy.

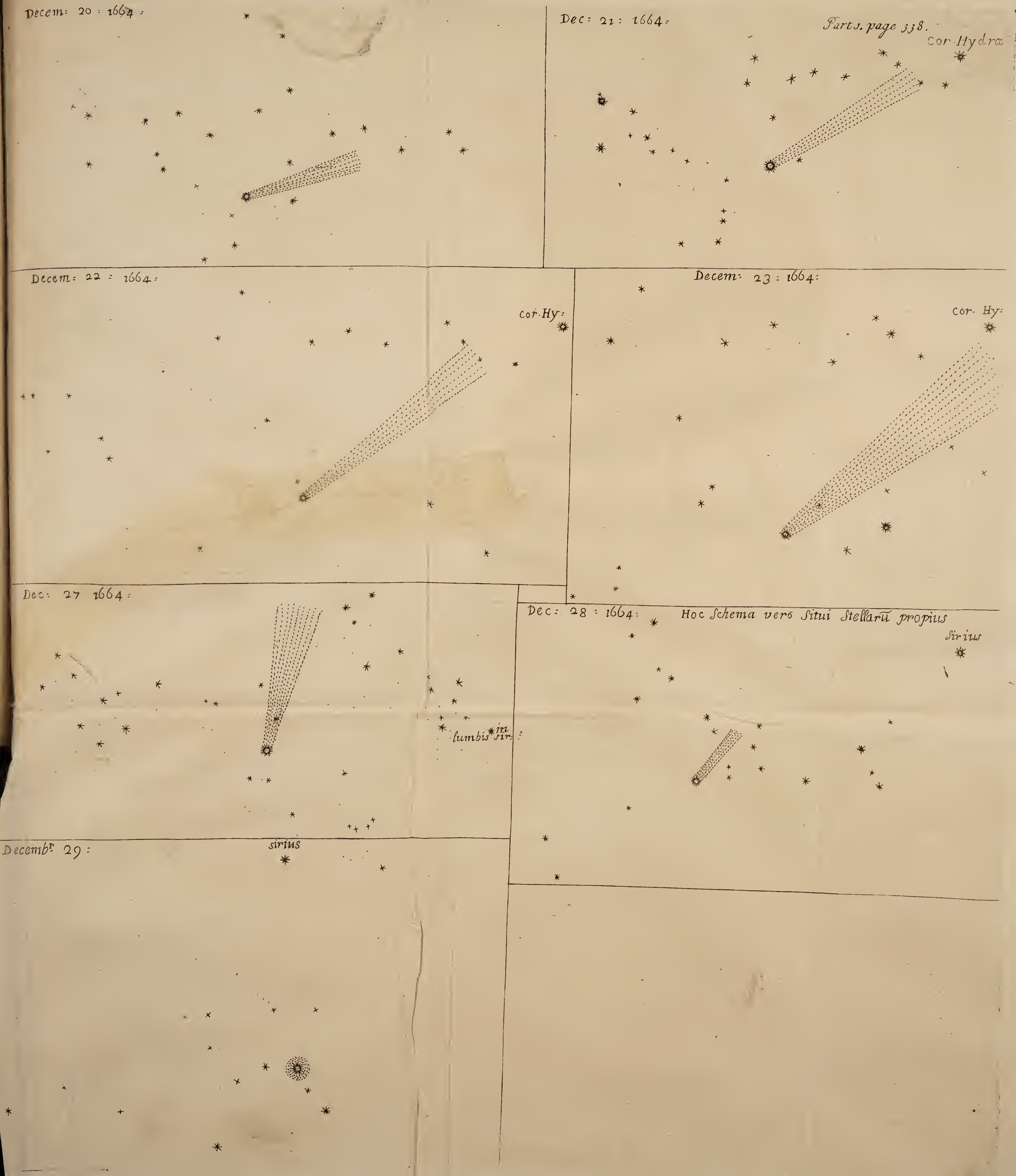
December 27. We found it strangely removed from the Place where it was: It was still Westward, and a little to the South, as before. The Body of the Star was still brighter, and the *Cauda* about it greater, and more bushy; and yet as long as before; it pointed almost directly against *Canis major*. The Body of it was among the Stars of *Argo*.

December 28. The same time it was removed above 2 Degrees towards the same Point, and come within 4 or 5 Degrees of the most Eastern Stars in the bright Triangle in the Buttocks of *Canis major*. The *Moon* shining, we could not so well judge either of the Bigness of the Body, or the length and bushiness of the Tail.

December 29. It was strangely removed, and got before, not the Eastern Star only of the mentioned bright Triangle, but also the most Northern. I think, at least, in this last 24 Hours, it had moved 4 Degrees.

The

Part 5. page 338.



The Moon shining bright, the Tail could not well be observed, yet still it seemed to point directly to *Canis minor.*

XLII. Cometa ille qui anno exeunte 1680 visus est, plurimis de causis præcipuus merito habendus est; tam ob Cursum ejus quadrimestrem, quo novem integra Signa percurrit; quam ob immensam Caudæ magnitudinem & claritatem: maxime vero ob insignem Orbitæ Curvitatem, cujus ope tandem patefacta est Cometarum Theoria. Nam dum Astronomi omni adhibita diligentia motum ejus observationibus definiendo desudarunt, conatus eorum secundavit * D. *Newtoni* in Geometricis vis pene divina, qui primus mortalium Cometas Orbes Parabolicis maxime affines describere probavit, ac datis tribus locis accurate observatis eosdem construere docuit, remque hujusce Cometæ exemplo illustravit. Accidit autem, nescio quo fato, ut Cometa hic (quem vespertinum tantopere prosecuti sunt Cœlispices) antequam Solem attigerat matutinus, nec *Parisiis* neque *Grenovici* ne semel quidem observatus sit: quique eum viderunt & observarunt, incongrua & inter se pugnantia, ac pro rei subtilitate parum idonea prodidere: neque ante *Novemb.* 17. mane a quopiam Observatorum visus est. Unde factum ut Orbitæ pars illa, qua ad Solem descendit Cometa, paulo incertius definiri potuit. Bona autem fortuna nuper incidimus in Librum meritissimi Astronomi Domini *Gottfried Kirch, Germani*, anno 1681 *Norimbergæ* impressi, cui titulus Neue Himmels Zeitung, hoc est *Novus Nuncius cœlestis*; ubi auctor diligentissimus nobis exponit, quo casu ductus Cometam hunc nondum adnata Cauda obscurum, ac vix nudis oculis conspicuum detexerit; dum scilicet Lunam & Martem ei vicinum observaturiens circumlustraret, die *Novemb.* 4. *S. V.* mane, idque *Coburgi Saxoniæ*, quod oppidum XI. Grad. *Londino* orientalius est, sub altitudine poli 50° 20′ circiter. Excitatus autem, ut ait, rumoribus Cometæ in *Germania* visi, vultu in Orientem verso pernoctavit, ut cœlo tum forte perquam sereno, si quid novi oriretur, situm ejus notaret. Luna vero jam ad stellam aliquam *Tychoni* incognitam applicata, (sed quæ in Catalogo *Flamsteedii Britannico* habetur, estque numero 44ta *Leonis*) voluit dictæ stellæ locum ex circumvicinis Fixis determinare; dumque Tubum trium graduum capacem hinc inde circumrotat, incidit in Luculam nebulosam, speciem insolitam præ se ferentem, quamque vel novum Cometam esse, vel Stellam nebulosam ad instar ejus quæ in *Cingulo Andromedæ* est, statim conclusit.

Observations on the Comet in 1680. by Mr. Gottf. Kirch, with Remarks by —— n. 342. p. 170.

** Princip. Philos. L. III. sub. fin.*

Primo autem Cometam vidit, Hora $4\frac{1}{2}$ matutina, paulo altiorem duabus stellulis Telescopicis, *a* & *e* cum quibus tamen Hora Sexta visus est in linea accurate recta; unde constabat eum moveri, idque motu directo. Inter horas vero 5. & 6. Tubo decempedali Phænomenon hoc contemplatus est, viditque duas alias stellulas contiguas prioribus minores, literisque *e* & *d* notatas, & supra has tertiam *g*. Erat autem distantia Cometæ ab *e* paulo minor quam ab *a*, major vero distantia *d e*. 6h. 38′. distantia Cometæ ab *e* dupla erat intervalli inter ipsas *d e*, ac linea *d e* producta reliquit Cometam * infra se, sic tamen ut marginem ejus superiorem

Plate 4. Fig. 7.

** Tubo scilicet objecta invertente.*

riorem attingeret. 6h. 45′. Cometa jam ſenſibiliter remotior erat ab *e* quam ab *a*, diſtabatque ab *a* paulo plus quam dimidio diſtantiæ ſtellularum *a* & *g*. Notandum vero eſt Horologium totis 14 minutis cœlum anticipaſſe, uti ex altitudinibus *Cordis Leonis* tum captis patuit.

Nobilis ſane eſt hæc obſervatio, adeoque Stellularum Cometæ tunc adjacentium loca non una methodo perquiſivimus, exercitatiſſimam manum ſuam & inſtrumenta perelegantia præbente Rev.D.*Jac. Pound*,R.S.S. Unde conſtabat ſtellulas illas tum temporis infraſcriptos habuiſſe ſitus, nempe

	Long.	Lat.
a	♌ 29°. 54′ 20″	1. 29. 20 Bor.
d	29. 27. 20	1. 8. 00
e	29. 34. 30	1. 10. 45

Circulus autem maximus per *a* & *c* ductus, deprehenſus eſt per *Ultimam Caudæ Urſæ majoris* tranſire, adeoque angulum cum circulo Longitudinis ad *a* eſſe 15° 36′½. Cumque diſtantia Cometæ ab *a* verſus *c* paulo major fuerat dimidio diſtantiæ *ag*, (quam Tubo ſedecim pedum & Micrometro deprehendimus 21 ½ min.) ponamus eam fuiſſe duodecim minutorum; & ex datis proveniet Cometæ locus ♌ 29°. 51′ cum Lat. Borea 1°. 17′¼. Hora ſcilicet Horologii 6 ½, ſed *Londini* 5h 2′. Temp. Appar.

Deinde *Novemb.* 6. mane, 4h. 42′, Tubo bipedali deprehendit Cometam omnino in linea recta inter *Martem* & Stellulam *N*; quæ quidem in *Catalogo Britannico* 45ta Leonis eſt, & tunc erat in ♍ 2° 42′ cum Lat. Auſt. 0° 16′⅙. *Mars* autem tum temporis habuit (ex collatis obſervationibus paulo antea & poſt factis) ♍ 3° 46′½ cum Lat. Bor. 1° 56′. Unde, ob datam ejus viam, Cometa occupavit ♍ 3°. 23′ cum Lat. Bor. 1° 6′. *Londini* Temp. App. 3h 58′ mane.

Novembris quoque 11. 5h 15′ mane, Cometa æqualiter diſtabat a Stellis Leonis σ & τ *Bayero*, nondum vero attigit rectam eaſdem jungentem, ſed parum abfuit ab ea. In *Catalogo Britannico* σ tunc habuit ♍ 14° 15′. cum Lat. Bor. 0 41′ fere, τ vero ♍ 17° 3′½ cum Lat. Auſt. 0° 34′. Proinde Cometæ Latitudo paulo minor erat medio inter illas, nempe quam 0° 33′½ Bor. ac Longitudo quam ♍ 15°. 39′. Sed huic non utique fidendum, cum pendeat ab æſtimata æqualitate diſtantiarum, quæ res lubrica eſt. Cauda autem jam cœpta non niſi dimidio gradu longa Tubo decempedali viſa eſt.

A ſmall Teleſcopical Comet ſeen at London, *on the* 10*th of* June, 1717. *by Dr.* Halley, *n.* 354. *p.* 721.

XLIII. That the Number of Comets traverſing our Solar Syſtem is much greater than ſome, on account of the late rareneſs of their Appearance, have ſuppoſed it, may be collected from ſeveral ſmall ones which have within few Years been deſcribed in the Memoirs of the *French* Royal Academy of Sciences; thoſe diligent Obſervers aſſuring us that they diſcover'd one in *Sept.* 1698. another in *Febr.* 1699 a third in *April* 1702. and again a fourth in *Novemb.* 1707. none of which, as far as I can learn, were ever ſeen in *England*; all of them having been

very

very obſcure and without Tails, by means whereof Comets uſually firſt ſhew themſelves. And beſides theſe, two other Comets with remarkably long Tails, the one in *Novemb.* 1689. the other in *Feb.* 1702. paſt by unobſervable in theſe our Northern Climats, they having great South Latitude, and their Motions directed toward that Pole. Hence we may juſtly conclude, that the Returns of Comets are much more frequent than is vulgarly reckoned, and that it is only contingent that for theſe 35 Years no one of them has been ſeen and obſerved by our Aſtronomers.

But there may be ſtill a much greater Number of theſe Bodies, which by reaſon of their Smallneſs and Diſtance are wholly inviſible to the naked Eye, ſo that unleſs Chance do direct the Teleſcope of a proper Obſerver, almoſt to the very Points where they are (againſt which there are immenſe Odds) it will not be poſſible for them to be diſcovered, whereof take the following Inſtance. On *Monday*, *June* 10. in the Evening, the Sky being very ſerene and calm, I was deſirous to take a View of the Disk of *Mars* (then very near the *Earth*, and appearing very glorious) to ſee if I could diſtinguiſh in my twenty four Foot Teleſcope, the Spots ſaid to be ſeen on him. Directing my Tube for that purpoſe, I accidentally fell upon a ſmall whitiſh Appearance near the Planet, reſembling in all reſpects ſuch a *Nebula* as I lately deſcribed * in *Philoſ. Tranſact.* N°. 347. but ſmaller. It ſeem'd to emit from its upper part a very ſhort kind of Radiation directed towards the Eaſt, but Northerly withal; which, conſidering its Situation, was nearly towards the Point oppoſite to the Sun. The great Light of the Moon, then very near it, and alſo very near the Full, hinder'd this *Phænomenon* from being more diſtinctly ſeen; but its Place in the Heavens was ſufficiently aſcertained from the Neighbourhood of *Mars*, from whom it was but about half a Degree diſtant towards the Southweſt, the difference of Latitude being ſomewhat more than that of Longitude; and *Mars* being at that time in ♐ 17°. 30′ with 3°. 48′. South Latitude, I concluded the place thereof in ♐ 17°. 12′ with 4°. 12′. Lat. South, or thereabouts; the which may yet be more ſecurely determined by help of two ſmall fixt Stars I found near it, the more northerly of which I judged to have the ſame Latitude with it, and to follow it at about the diſtance of ſix Minutes; the other Star was about one Minute in conſequence thereof; the Angle at the Northern Star was a little obtuſe, as of about 100 Degrees, and the diſtance of our *Nebula* from it *ſeſquialter* to the diſtance of the two Stars, or rather a little more. The Reverend Mr. *Moſes Williams*, Mr. *Alban Thomas*, and my ſelf, contemplated this Appearance for above an Hour, *viz.* from $10\frac{1}{2}^{h}$ to near twelve, and we could not be deceiv'd as to its Reality; but the Slowneſs of its Motion made us at that time conclude that it had none, and that it was rather a *Nebula* than a *Comet*.

* v. ſupr. p. 235.

However, ſuſpecting that it might have ſome Motion, I attended the next Night, *June* 11th, at the ſame Hours, and in the ſame Company, when with ſome Difficulty by reaſon of the Thickneſs of the Air, we found

found the two little Stars, but the *Nebula* could not at that time be ſeen, which we then imputed to the want of a clearer Sky. But on *Saturday*, *June* 15. the Moon being abſent, and the Air perfectly clear, we had again a diſtinct View of the two Stars, with an entire Evidence that there remained no Footſtep or Sign of it, in the place where we had firſt ſeen this *Phænomenon*, which we therefore now found to be a Comet, and that being far without the Orb of the Earth, and in it ſelf a very ſmall Body, it appeared only like a little Speck of a Cloud, ſuch as would ſcarce have been diſcerned in an ordinary Teleſcope, much leſs by the naked Eye.

A Comet at Berlin, *n.* 357. *p.* 820.

XLIV. 1. Cl. *Chriſtfridus Kirchius*, motus Corporum cœleſtium, ut munere ſibi a Societate Scientiarum Regia (*Berolini*) demandato recte fungeretur, ſedulo obſervans, a. d. XV. Kal. Febr. (*Jan.* 18. *ſt. n.*) 1717. veſperi dimidia Septima, verſus Septentiones fortuito Cometam conſpexit. Vicinus erat ad dextram (ſtellarum) γ & β *Bayeri* in *Urſa Minore*, nudoque oculo longe diſtinctius apparebat, quam β *Urſæ minoris*, licet ea inſignis ſit Stella ſecundæ magnitudinis, cum longe pallidior quidem majore tamen diametro, atque ſatis clara luce maxime circa centrum, conſpiceretur. Per Tubum viſus lucidam rotundamque referebat nubeculam: Caudæ autem nullum obſervari potuit veſtigium, neque Nucleus dignoſci. Motu celerrimo ab hora VII. ad XI. proceſſit, graduſque quatuor cum dimidio abſolvit, ut ex obſervationibus colligitur.

Die 19no & 20mo *Januarii* cœlum nubibus fuit obductum. Die vero 21mo Cometa longe receſſerat a loco ſuo nupero, atque in *Caſſiopea* deprehendebatur, ubi cum ſtellis ε & δ triangulum conficiebat (*an æquicrure?*) ſcil. Hora 5^h 45′ in 17° 34′ ♉, ſub latitudine Boreali 49°. 54′ hærebat: Deinde 9^h 15, in 16° 38′, ♉ ſub Lat. Bor. 49°. 2′ conſpiciebatur. Cæterum multum decreverat, atque a celeritate ſua remiſerat; præterquam enim quod pallidior quam ante apparebat, ſtellas etiam quartæ dignitatis magnitudine haud ſuperare nudo oculo conſpectus videbatur, inque orbita ſua, quatuor cum dimidia horis, non ultra ſeſquigradum proceſſerat: Tubi autem beneficio diameter ejus 7 min. inveniebatur.

Jan. 23. hora IV. mat. Cometa cum δ & φ *Caſſiopeæ* triangulum æquicrurum efficiebat, cum ab utraque 2° 41½ abeſſet. Hoc mane duarum horarum ſpatio vix dimidium gradum abſolvit; hora decima veſpertina cum δ *Caſſiopeæ* & φ *Perſei* in linea recta cernebatur, atque a priori 3° 38′, a poſteriori 3° 9′ diſtabat. Diameter ejus erat 5 min. nudoque oculo conſpectus ſtellam quintæ magnitudinis referebat.

Die 24to *Jan.* hora VI. mat. nondum attigerat φ *Perſei*, ſed cum *v* & *g* ejuſdem Aſteriſmi triangulum æquicrurum ſiſtebat, & ab utraque non plane 3½ grad. aberat.

A Remark by Dr. Halley, *ibid.*

2. *Deſiderantur Obſervationes Diei* 18væ, *cum Cometa velociſſime motus terræ proximus erat, unde certius de Via ejus tam vera quam apparente judicium ferre poſſemus. Manifeſtum autem eſt eum Polo Æquatoris Boreo viciniſſimum die* Januarii 19no *tranſiiſſe. Quod ſi cui libeat has Obſervationes ad*

ad examen revocare, calculoque accurato subjicere; in illius gratiam, loca Stellarum fixarum, quarum hic fit mentio, ex Catalogo Britannico *excerpta, subnectuntur: Unde etiam patebit nonnulla in hac motus Cometæ descriptione haud rite se habere; quæ tamen, a Cl.* Kirchio *corrigi, in pleniori ejus quam promisit historia, spes est.*

Stellarum fixarum Loca ineunte Anno 1718.

BAYERO		Long. ° ′ ″	Lat. Bor. ° ′ ″
Ursæ minoris	β	♌ 9 18 0	72 58 10
	γ	♌ 17 35 15	75 13 15
Cassiopeæ	δ	♉ 14 00 35	46 23 25
	ε	♉ 20 50 8	47 31 50
	φ	♉ 11 36 35	45 4 5
Persei	υ	♉ 8 32 0	35 23 45
	φ	♉ 10 41 35	36 49 15
	g	♉ 12 15 20	36 18 37

XLV. 1. *Astronomical Observations at* Greenwich, *by Mr* Flamsteed, *n.* 337. *p.* 65.

Observationes SATURNI.

Temp. per Horolog.	Tempora correcta.	Die Veneris Januarii 5. 1711.	Distantiæ a Vertice.
10. 14. 26	10. 14. 41	Calx Castoris μ ♊ transiit	28. 50. 10
11. 11. 38	11. 11. 53	In Inguine Pollucis, δ transf.	28. 59. 00
12. 4. 45	12. 5. 00	Saturnus transit ——	30. 23. 20
		Ascensio recta ♄ 119. 01. 00	
		Dist. a Polo Bor. 68. 55. 20	
		Die Solis Januarii 7.	
11. 2. 53	11. 0. 31	Geminorum δ transiit ——	28. 59. 20
11. 50. 36	11. 48. 14	In pede Bor. ♋ μ transiit ——	29. 5. 20
11. 55. 22	11. 53. 00	Saturnus transiit ——	30. 21. 50
12. 15. 41	12. 13. 19	Cancri η *Bayero* transit ——	30. 4. 30
12. 26. 18	12. 23. 56	Cancri γ, Asellus Bor. transf. ——	28. 59. 50
		Ascensio Rect. ♄ 118. 52. 00	
		Dist. a Polo Bor. 68. 53. 50	
		Die Lunæ, Januarii 8.	
10. 58. 35	10. 57. 49	Geminorum δ transiit ——	28. 59. 15
11. 50. 46	11. 50. 00	Saturnus transit ——	30. 20. 35
12. 11. 25	12. 10. 30	Cancri η transiit ——	30. 4. 25
		Ascensio Rect. ♄ 118. 46. 30	
		Dist. a Polo Bor. 68. 52. 35	

Temp.

Temp. per Horolog.	Tempora correcta.	Die Jovis, Januarii 25. 1711.	Distantia a Vertice.
9. 45. 53	9. 46. 26	Geminorum ♌ transiit	28. 59. 10
10. 32. 27	10. 33. 00	Saturnus transiit	30. 4. 00
11. 10. 54	11. 11. 27	Cancri ♌, Asellus Aust. trans.	32. 17. 50
		Ascensio Rect. ♄ 117. 23. 00	
		Dist. a Polo Bor. 68. 36. 00	
		Die Saturni, Januarii 27.	
9. 37. 25	9. 38. 2	Geminorum ♌ transiit	28. 59. 20
10. 13. 11	10. 13. 48	Sub latere Pollucis *l* transiit	30. 51. 50
10. 23. 23	10. 24. 00	Saturnus transiit	30. 2. 30
10. 40. 59	10. 41. 36	Post Caudam ♋ *d* transiit	32. 14. 10
10. 50. 16	10. 50. 53	Cancri *n* transiit	30. 4. 35
		Ascensio Rect. ♄ 117. 14. 00	
		Dist. a Polo Bor. 68. 34 30	
		Die Martis, Januarii 30.	
9. 29. 45	9. 26. 55	Geminorum ♌ transiit	28. 59. 40
10. 14. 50	10. 12. 00	Saturnus transiit	30. 0. 00
10. 33. 20	10. 30. 30	Cancri *d* transiit	32. 14. 10
10. 42. 35	10. 39. 45	Cancri *n* transiit	30. 4. 45
		Ascensio Rect. ♄ 117. 00. 30	
		Dist. a Polo Bor. 68. 32. 00	
		Die Mercurii, Feb. 28.	
8. 18. 32	8. 15. 00	Saturnus transiit	29. 43. 00
8. 27. 11	8. 23. 39	In Boreo pede Cancri *μ* trans.	29. 5. 40
9. 2. 50	8. 59. 18	Asellus Boreus Cancri *γ* trans.	29. 0. 2
10. 39. 59	10. 36. 27	Lucida Colli Leonis *γ* trans.	30. 12. 05
		Ascensio Rect. ♄ 115. 31. 30	
		Dist. a Polo Bor. 68. 15. 00	
		Die Jovis, Mart. 1. 1711.	
8. 14. 5	8. 11. 00	Saturnus transiit	29. 42. 50
8. 22. 47	8. 19. 42	Cancri *μ* transiit	29. 5. 35
8. 58. 28	8. 55. 23	Asellus Boreus transiit	29. 0. 15
10. 35. 36	10. 32. 31	Lucida Colli Leonis transiit	30. 12. 00
		Ascensio Rect. ♄ 115. 30. 30	
		Dist. a Polo Bor. 68. 14. 50	
		Die Veneris, Nov. 9.	
17. 21. 12	17. 21. 23	Sequens ad *π* Cancri transiit	35. 21. 10
17. 29. 49	17. 30. 00	Saturnus transiit	34. 9. 50
18. 13. 25	18. 13. 36	Austrina Colli Leonis *n* trans.	33. 18. 50
		Ascensio Rect. ♄ 136. 58. 00	
		Dist. a Polo Bor. 72. 42. 00	

Temp.

Temp. per Horolog.	Tempora correcta.	Die Lunæ, Nov. 19.	Distantia a Vertice.
16. 46. 20	16. 41. 00	Saturnus transit	34. 8. 00
16. 59. 31	16. 54. 11	Telescopica *a* transit	33. 45. 30
17. 29. 59	17. 24. 33	Austrina Colli Leonis *n* transf.	33. 18. 30
		Ascensio Rect. ♄ 136. 58. 30	
		Dist. a Polo Bor. 72. 40. 05	
		Die Jovis, Nov. 22.	
16. 24. 36	16. 19. 30	Cancri sequens ad *π* transiit	35. 21. 30
33. 06	16. 28. 00	Saturnus transit	34. 6. 50
46. 27	16. 41. 21	Telescopica dicta transiit	33. 45. 10
17. 16. 51	17. 11. 45	Austrina Colli Leonis transiit	33. 18. 30
		Ascensio Rect. ♄ 136. 55. 45	
		Dist. a Polo Bor. 72. 39. 00	
		Die Solis, Decembris 30. 1711.	
11. 9. 43	10. 52. 42	Lucidus pes ♊ *γ* transit	34. 51. 30
13. 29. 25	13. 12. 24	Cancri præced. ad *ο* transiit	35. 4. 00
13. 49. 01	13. 32. 00	Saturnus transiit	33. 30. 00
14. 39. 50	14. 22. 49	Lucida Colli Leonis transiit	33. 18. 40
		Ascensio Rect. ♄ 135. 10. 00	
		Dist. a Polo Bor. 72. 2. 5	
		ANNO MDCCXII.	
		Die Saturni, Januarii 12.	
11. 39. 52	11. 33. 06	Ad caudam Cancri *ζ* transiit	32. 59. 10
12. 40. 46	12. 43. 00	Saturnus transit	33. 12. 00
12. 43. 16	12. 36. 30	Cancri sequens ad *π* transiit	35. 22. 20
13. 36. 44	13. 29. 58	Cor Leonis transiit	38. 6. 55
		Ascensio Rect. ♄ 134. 12. 00	
		Dist. a Polo Bor. 71. 44. 00	
		Die Saturni, Jan. 19.	
11. 45. 31	11. 36. 55	Asellus Austrinus transiit	32. 17. 10
11. 58. 14	49. 38	Austrina ad *ο* Cancri transiit	35. 3. 50
12. 11. 36	12. 3. 00	Saturnus transiit	33. 2. 5
12. 20. 00	11. 24	Telescopica *b* transiit	32. 33. 20
		Ascensio Rect ♄ 133. 37. 00	
		Dist. a Polo Bor. 71 34. 10	
		Die Solis, Jan. 27.	
10. 57. 50	11. 4. 31	Asellus Austrinus transiit	32. 17. 10
11. 10. 54	11. 17. 35	Borea ad *ο* Cancri transiit	34. 48. 10
11. 21. 19	12. 28. 00	Saturnus transiit	32. 50. 20
11. 32. 20	39. 01	Telescopica *b* transiit	32. 33. 25
		Ascensio Rect. ♄ 132. 58. 00	
		Dist. a Polo Bor. 71. 22. 20	

Temp. per Horolog.	Tempora correcta.	Die Lunæ, Martii 31. 1712.	Distantia a Vertice.
7. 8. 46	7. 7. 41	Asellus Austrinus transiit ——	32. 16. 50
7. 20. 35	7. 19. 30	Saturnus transiit ——	32. 03. 30
7. 36. 50	7. 35. 45	Cancri præced. ad π transiit	35. 19. 10
7. 39. 36	7. 38. 31	Sequens ad π transiit ——	35. 21. 20
8. 31. 51	8. 30. 46	Austrina colli Leonis transf.	33. 18. 40
		Ascensio recta ♄ 130. 02. 00	
		Dist. a Polo Bor. 70. 35. 30	
		Die Veneris Novemb. 7.	
18. 17. 13	18. 22. 00	Saturnus transiit ——	37. 57. 30
18. 26. 13	18. 31. 00	Leonis 40 *Catal. Brit.* transf.	41. 14. 10
18. 28. 35	18. 33. 22	Ejusdem 41 transiit ——	40. 15. 20
		Ascensio Rect. ♄ 150. 15. 30	
		Dist. a Polo Bor. 76. 29. 40	
		Die Lunæ, Nov. 17.	
17. 33. 19	17. 34. 59	Cor Leonis transiit ——	38. 7. 20
17. 42. 20	17. 44. 00	Saturnus transiit ——	38. 00. 50
17. 57. 48	17. 59. 28	In Axilla Leonis ρ transiit	40. 41. 45
18. 14. 19	18. 15. 59	In ventre Leonis *l* transiit ——	39. 24. 50
		Ascensio Rect. ♄ 150. 31. 00	
		Dist. a Polo Bor. 76. 33. 00	
		Observationes J O V I S.	
		Anno MDCCXI.	
		Die Saturni, Maii 26. 1711.	
12. 44. 28	12. 39. 44	Serpentarii 58 *Cat. Br.* transf.	76. 12. 00
52. 40	47. 56	Sagittarii Nebulosæ *a* transiit	75. 39. 20
13. 8. 44	13. 4. 00	Jupiter transiit	74. 37. 00
15. 43	10. 59	Sagittarii 11 transiit	72. 5. 20
		Ascensio Rect. ♃ 270. 19. 00	
		Dist. a Polo Bor. 113. 11. 50	
		Die Solis, Maii. 27.	
10. 47. 24	10. 43. 2	Media frontis ♏. δ transf.	73. 12. 00
12. 57. 46	53. 24	Telescopica *c* Jovem præced.	75. 7. 50
13. 4. 22	13. 00. 00	Jupiter transiit	74. 36. 50
11. 53	7. 31	Sagittarii 11 transiit	72. 5. 20
		Ascensio Rect. ♃ 270. 11. 00	
		Dist. a Polo Bor. 113. 11. 40	
		Die Solis Junii 3.	
10. 15. 41	10. 10. 45	Media frontis Scorpii transiit	73. 12. 5
12. 14. 11	12. 9. 15	Nebulosæ Sagittarii *b* transiit	75. 10. 20
12. 17. 09	12. 12. 13	Ejusdem Nebulosæ *a* transiit	75. 39. 30

Temp. per Horolog.	Tempora correcta.	Die Solis Junii 3.	Distantia a Vertice.
12. 28. 56	12. 24. 00	Jupiter transiit	74. 38. 00
13. 8. 36	13. 03. 40	Sagittarii in Oculo præced. transf.	74. 29. 15
		Ascensio Rect. ♃ 269. 15. 00	
		Dist. a Polo Bor. 113. 12. 50	
		Die Lunæ, Junii 4. 1711.	
10. 11. 51	10. 7. 19	Media frontis Scorpii transiit	73. 12. 00
12. 10. 18	12. 5. 46	Nebulosæ Sagittarii *b* transf.	75. 10. 20
12. 24. 32	12. 20. 00	Jupiter transiit	74. 38. 10
13. 4. 45	13. 0. 13	Præced. ad *v* Sagittarii transf.	74. 29. 10
13. 5. 41	13. 1. 09	Sequens ad *v* transiit	74. 24. 50
		Ascensio Rect. ♃ 269. 07. 00	
		Dist. a Polo Bor. 113. 13. 00	
		Die Saturni, Junii 9.	
9. 52. 16	9. 49. 5	Media frontis Scorpii transf.	73. 11. 55
11. 50. 44	11. 47. 33	Nebulosæ *b* transf.	75. 10. 15
12. 2. 11	11. 59. 00	Jupiter transiit	74. 38. 25
12. 45. 8	12. 41. 57	Præced. ad *v* Sagittarii transf.	74. 29. 10
		Ascensio Rect. ♃ 268. 25. 00	
		Dist. a Polo Bor. 113. 13. 15	
		Die Solis, Junii 10.	
9. 48. 22	9. 44. 38	Media frontis Scorpii transiit	73. 12. 00
11. 46. 51	11. 43. 07	Nebulosæ *b* transiit	75. 10. 15
11. 57. 44	11. 54. 0	Jupiter transiit	74. 38. 35
12. 41. 14	12. 37. 30	Præcedens ad *v* Sagitt. transf.	74. 29. 15
		Ascensio Rect. ♃ 268. 16. 45	
		Dist. a Polo Bor. 113. 13. 25	
		Die Saturni, Julii 14.	
8. 58. 10	8. 50. 4	Serpentarii 48, *C* transiit	75. 7. 20
9. 10. 32	9. 2. 26	Ejusdem 54 sive *D* transiit	72. 56. 00
9. 21. 6	9. 13. 00	Jupiter transiit	74. 37. 50
9. 40. 49	9. 32. 43	Sagittarii in arcu *μ* transiit	72. 31. 30
		Ascensio Rect. ♃ 264. 12. 30	
		Dist. a Polo Bor. 113. 12. 40	
		Die Solis, Julii 15.	
8. 54. 8	8. 46. 24	Serpentarii *C* transiit	75. 7. 20
9. 6. 28	8. 58. 44	Serpentarii *D* transiit	72. 56. 00
9. 16. 44	9. 9. 00	Jupiter transiit	74. 37. 50
9. 36. 46	9. 29. 2	Sagittarii *μ* transiit	72. 31. 35
		Ascensio Rect. ♃ 264. 07. 45	
		Dist. a Polo Bor. 113. 12. 40	

Temp. per Horolog.	Tempora correcta.	ANNO MDCCXII. Die Jovis, Julii 3.	Diſtantia a Vertice.
12. 36. 3	12. 25. 14	Capric. ſub Oculo σ tranſiit	71. 24. 40
12. 44. 6	12. 33. 17	Capric. in roſtro π tranſiit	70. 33. 00
12. 45. 42	12. 34. 53	Capricorni ρ tranſiit	70. 09 50
12. 57. 49	12. 47. 00	Jupiter tranſiit	71. 26 20
13. 16. 24	13. 5. 35	Capric. 20 *Catal. Brit.* tranſ.	71. 33. 10
		Aſcenſio Rect. ♃ 306. 09. 20	
		Diſt. a Polo Bor. 110. 0. 30	
		Die Martis, Julii 15.	
11. 39. 34	11. 39. 33	Capricorni σ tranſiit	71. 24. 45
55. 1	11. 55. 00	Jupiter tranſiit	71. 49. 20
12. 19. 53	12. 19. 52	Capricorni 20 tranſiit	71. 33. 10
24. 38	12. 24. 37	♑ni in medio corpore ν tranſ.	72. 23. 25
		Aſcenſio Rect. ♃ 304. 34. 35	
		Diſt. a Polo Bor. 110. 23. 30	
		Die Mercurii, Sept. 17.	
7. 32. 37	7. 31. 56	Teleſc. Jovem præced. tranſ.	72. 49. 40
7. 38. 41	7. 38. 00	Jupiter tranſiit	72. 51. 10
7. 42. 28	7. 41. 37	Teleſc. Jovem ſequens tranſ.	73. 14. 40
		Aſcenſio Rect. ♃ 299. 43. 00	
		Diſt. a Polo Bor. 111. 25. 30	
		Die Veneris, Septembris 19.	
7. 30. 55	7. 32. 00	Jupiter tranſiit	72. 50. 45
8. 19. 47	8. 20. 52	Capricorni ν tranſiit	72. 23. 40
		Aſcenſio Rect. ♃ 299. 45. 00	
		Diſt. a Polo Bor. 111. 25. 05	
		Die Lunæ, Octobris 6.	
6. 25. 35	6. 31. 30	Jupiter tranſiit	72. 39. 50
6. 36. 22	6. 42. 17	Capricorni in roſtro ο tranſ.	70. 56. 25
6. 46. 36	6. 52. 31	Capricorni in Cervice υ tranſ.	70. 33. 30
		Aſcenſio Rect. ♃ 300. 39. 00	
		Diſt. a Polo Bor. 111. 14. 10	

Obſervationes MARTIS.

Anno MDCCXI.

		Die Solis Jan. 7.	
12. 40. 35	12. 38. 56	Cancri Auſt. ad ο tranſiit,	35. 04. 00
12. 40. 56	12. 39. 17	Cancri Bor. ad ο tranſiit	34. 48. 00
12. 58. 45	12. 57. 6	Cancri ſequens ad π tranſiit	35. 21. 20

Temp.

Temp. per Horolog.	Tempora correcta.	Die Solis Jan. 7.	Distantia a Vertice.
13. 27. 15	13. 25. 36	Leonis ι transiit	36. 8. 50
13. 50. 52	13. 49. 13	Austrina Colli Leonis η transf.	33. 19. 00
14. 00. 39	13. 59. 00	Mars transiit	34. 51. 30
14. 30. 14	14. 28. 35	Borea Ventris ♌, *k* transiit	35. 45. 40
		Ascensio Rect. ♂ 150. 20. 00	
		Dist. a Polo Bor. 73. 23. 35	
		Die Saturni, Januarii 27. 1711.	
10. 40. 59	10. 41. 30	Cancri 20, prima ad *d* transiit	32. 14. 10
10. 50. 16	10. 50. 47	Cancri η transiit	30. 4. 35
12. 8. 29	12. 9. 00	Mars transiit	32. 14. 00
12. 25. 27	12. 25. 58	Austrina Colli Leonis η transf.	33. 19. 00
12. 37. 59	12. 38. 30	Lucida Colli Leonis γ transiit	30. 11. 50
		Ascensio recta ♂ 143. 37. 00	
		Dist. a Polo Bor. 70. 36. 00	
		Die Lunæ, Januarii 29.	
12. 1. 50	11. 59. 00	Mars transiit	31. 58. 50
12. 21. 58	12. 19. 08	Leonis η transiit	33. 19. 05
12. 34. 31	12. 31. 41	Lucida Colli Leonis γ transiit	30. 11. 50
		Ascensio Rect. ♂ 142. 49. 30	
		Dist. a Polo Bor. 70. 30. 50	
		Die Martis, Jan. 30.	
10. 33. 30	10. 30. 25	Cancri 20, prima ad *d* transf.	32. 14. 10
10. 42. 35	10. 39. 30	Cancri η transiit	30. 4. 45
11. 56. 5	11. 53. 00	Mars transiit	31. 51. 35
12. 17. 49	12. 14. 44	Leonis η transiit	33. 19. 5
12. 30. 20	12. 27. 15	Leonis γ transiit	30. 11. 50
		Ascensio Rect. ♂ 142. 25. 00	
		Dist. a Poloa Bor. 70. 23. 35	
		Die Mercurii, Feb. 28.	
8. 27. 11	8. 23. 58	In pede Bor. Cancri μ transf.	29. 5. 40
9. 2. 50	8. 59. 37	Asellus Boreus γ transf.	29. 00. 20
9. 31. 13	9. 28. 00	Mars transf.	30. 5. 10
10. 27. 28	10. 24. 15	Leonis η transf.	33. 19. 55
10. 39. 39	10. 36. 26	Leonis γ transf.	30. 12. 5
		Ascensio Rect. ♂. 133. 45. 00	
		Dist. a Polo Bor. 68. 37. 10	
		Die Jovis, Martii 1. 1711.	
8. 22. 47	8. 19. 27	Cancri μ transf.	29. 5. 35
8. 58. 28	8. 55. 8	Asellus Boreus transf.	29. 00. 15
9. 26. 20	9. 23. 00	Mars transf.	30. 6. 20

Temp.

Temp. per Horolog.	Tempora correct.	Die Jovis Martii 1. 1711.	Distantia a Vertice.
10. 23. 04	10. 19. 44	Leonis η transf.	33. 19. 50
10. 35. 36	10. 32. 16	Lucida Colli Leonis transf.	30. 12. 00
		Ascensio Rect. ♂ 133. 37. 30	
		Dist. a Polo Bor. 68. 38. 20	
		ANNO MDCCXII.	
		Die Lunæ, Novemb. 17.	
18. 21. 7	18. 19. 52	Leonis 55 *Catal. Brit.* transf.	43. 45. 40
18. 25. 53	18. 24. 38	Sub Ventre Leonis *c* transf.	43. 49. 40
18. 30. 14	18. 28. 59	Leonis χ transiit	42. 35. 10
18. 43. 15	18. 42. 00	Mars transiit	43. 7. 25
		Ascensio Rect. ♂ 165. 48. 00	
		Dist. a Polo Bor. 81. 39. 45	
		Die Jovis, Novemb. 20.	
18. 13. 8	18. 6. 40	Leonis 55 transf.	43. 45. 40
18. 17. 53	18. 11. 25	Leonis 58, *c* transf.	43. 49. 50
18. 22. 14	18. 15. 46	Leonis χ transf.	42. 35. 15
18. 38. 18	18. 31. 50	In Poplite Leonis σ transf.	43. 52. 10
18. 40. 28	18. 34. 00	Mars transf.	43. 36. 55
		Ascensio Rect. ♂ 167. 6. 30	
		Dist. a Polo Bor. 82. 9. 15	

Observationes SOLIS.

Anno MDCCXI.

Die Saturni, Januarii 6.	
Solis centro per planum Arcus Meridionalis transeunte, limbus ejus remotus & Austrinus distabat a Vertice.	72. 31. 10
Die Veneris, Januarii 26.	
Solis Meridiani limbus proximus a Vertice	66. 35. 10
Die Martis, Januarii 30.	
Solis limbus proximus a Vertice	65. 38. 40
Die Lunæ, Junii 4.	
Solis limbus remotus, &c.	28. 24. 18
Die Saturni, Julii 14.	
Solis limbus remotus	37. 55. 20
Die Martis, Novembris 20.	
Solis limbus remotus	73. 30. 40
Die Martis, Decembris 4.	
Solis limbus remotus a Vertice	74. 59. 30

Anno

Anno MDCCXII.	Distantia a Vertice.
Die Mercurii, Januarii 2.	
Solis limbus remotus a Vertice	73. 17. 30
Die Saturni, Januarii 12.	
Solis limbus remotus	71. 18. 00
Die Veneris, Martii 7.	
Solis limbus remotus	52. 31. 00
Die Veneris, Maii 9.	
Solis limbus remotus	31. 40. 50
Die Martis, Octobris 7.	
Solis limbus remotus a Vertice	61. 29. 50

Observationes LUNÆ.

Anno MDCCXI.

Temp. per Horolog.	Tempora correcta.	Die Saturni, Maii 19. 1711.	Distantia a Vertice.
10. 22. 12	10. 18. 32	Scorpii γ sive Libræ 15 transf.	75. 32. 50
10. 50. 55	10. 47. 15	☽ Limbus transf. centro a Vert.	75. 38. 40
10. 52. 10	10. 48. 30	☽ Centrum transiit proximo	75. 22. 00
11. 8. 30	11. 4. 50	Scorpii *b* transiit	76. 15. 20
11. 11. 10	11. 7. 30	Scorpii *A* transiit	75. 50. 55
		Ascensio Rect. ☽ 229. 18. 30.	
		Dist. a Polo Bor. 114. 13. 40	
		Die Lunæ, Novemb. 19.	
16. 41. 28	16. 36. 8	Lunæ centrum transf. remoto	37. 6. 30
17. 42. 16	16. 36. 56	Lunæ limbus transiit, centro	36. 52. 30
16. 46. 20	16. 41. 00	Saturnus transiit	34. 8. 00
16. 59. 31	16. 54. 11	Stella Telescopica *a* transiit	33. 45. 30
17. 29. 53	17. 24. 33	Austr. Colli Leonis *n* transf.	33. 18. 30
		Ascensio Rect. ☽ 135. 41. 40	
		Dist. a Polo Bor. 75. 23. 20	

Anno MDCCXII.

		Die Saturni, Januarii 12. 1712.	
7. 40. 50.	7. 34. 00	Medium Eclipsis Lunaris, quo tempore Chorda partis in Luna deficientis erat 24. 30; maximus autem defectus 8. 30 a parte Borea. Lunæ Diam. 30. 48.	
11. 39. 52	11. 33. 6	Cancri ζ *Bayero* transiit	32. 59. 10
12. 13. 46	12. 7. 00	☽ centrum transiit, remoto	34. 14. 10

Temp.

Temp. per Horolog.	Tempora correcta.	Die Saturni, Januarii 12.	Diſtantia a Vertice.
12 40. 46	12. 34. 90	Saturnus tranſiit	33. 12. 00
43. 16	12. 36. 30	Cancri ſequens ad π tranſiit	35. 22. 20
13. 36. 44	13. 29. 58	Cor Leonis tranſiit	38. 6. 55
		Aſcenſio Rect. ☽ 127. 25. 30	
		Diſt. a Polo Bor. 72. 31. 00	
		Die Jovis, Martii 6	
7. 39. 8	7. 38. 8	♊ 80 *Catal. Brit.* tranſiit	30. 29. 30
7. 49. 42	7. 48. 42	Geminorum 86 *l* tranſiit	30. 51. 45
7. 54. 45	7. 53. 45	Lunæ limbus tranſiit, centro	30. 50. 40
7. 55. 48	7. 54. 48	☽ centrum tranſiit, proximo	30. 35. 30
8. 1. 41	8. 0. 41	Nona Cancri μ tranſiit	29. 4. 45
		Aſcenſio Rect. ☽ 116. 14. 40	
		Diſt. a Polo Bor. 69. 22. 50	
		Die Mercurii, Maii 7.	
9. 47. 38	9. 46. 2	Virginis 70 *Catal. Brit.* tranſiit	66. 9. 50
9. 48. 48	9. 47. 12	Virginis 71 tranſiit	65. 45. 30
9. 52. 51	9. 51. 15	Ejuſdem 75 tranſiit	68. 7. 40
10. 9. 36	10. 8. 00	Lunæ limbus tranſ. centro	69. 12. 30
10. 10. 43	10. 9. 7	Lunæ centrum tranſ. remoto	69. 28. 35
		Aſcenſio Rect. ☽ 208. 4. 30	
		Diſt. a Polo Bor. 107. 45. 50	
		Die Jovis, Maii 8. 1712.	
10. 52. 2	10. 50. 00	Libræ 8 *Catal. Brit.* tranſiit	74. 51. 50
11. 1. 35	10. 59. 33	Scorpii γ ſive Libræ 15 tranſ.	75. 32. 30
11. 8. 2	11. 6. 00	Lunæ limbus tranſiit, centro	73. 42. 30
11. 09. 16	11. 7. 14	Lunæ centrum tranſ. remoto	73. 58. 20
11. 50. 33	11. 48. 31	Scorpii *A* tranſit	75. 51. 10
11. 57. 34	11. 55. 32	Media frontis ♏ *A* tranſit	73. 12. 10
		Aſcenſio recta ☽ 223. 45. 00	
		Diſt. a Polo Bor. 112. 16. 10	
		Die Saturni, Maii 10.	
12. 18. 05	12. 16. 30	Cor Scorpii *Antares* tranſiit	77. 10. 00
13. 3. 48	13. 2. 13	Serpentarii A tranſit	77. 32. 50
13. 18. 13	13. 16. 38	Lunæ centrum tranſ. remoto	78. 1. 30
13. 19. 30	13. 17. 55	Lunæ limbus tranſiit, centro	77. 46. 05
13. 35. 25	13. 33. 50	Prima Sagittarii *p* tranſiit	79. 4. 20
14. 16. 8	14. 14. 33	Sagittarii λ tranſiit	76. 57. 00
		Aſcenſio Rect. ☽ 258. 03. 20	
		Diſt. a Polo Bor. 116. 20. 15	

n. 344. p. 285.

Observationes SATURNI.

Temp. per Horolog.	Tempora correcta.	Die Solis, Januarii 25. 1713.	Distantia a Vertice.
8. 30. 15	8. 28. 5	Geminorum τερτιος transiit	28. 13. 20
8. 41. 7	38 57	Pes Castoris η transiit	28. 54. 50
8. 49. 8	8. 46. 58	Calx ejusdem sive μ transiit	28. 50. 40
12. 10. 49	12. 8. 39	Leonis ♃ *Bayero* transiit	36. 9. 30
12. 25. 24	23. 14	Leonis ν transiit	37. 40. 30
12. 34. 21	32. 11	Saturni centrum transiit	36. 51. 45
12. 35. 44	12. 33. 34	Cor Leonis transiit	38. 7. 00
		Ascensio Rect. ♄ 147 55 10	
		Dist. a Polo Bor. 75 23 55	
		Longitudo ♌ 25 8 15	
		Latitudo Bor. 1 31 27	
		Die Jovis Februarii 5.	
11. 20. 52	11. 14. 10	Leonis ξ *Bayero* transiit	38. 55. 20
30. 9	23. 27	Leonis ο transiit	40. 17. 30
35. 18	28. 36	Leonis 16*ta Cat. Brit.* transf.	38. 21 20
47. 8	40. 26	Leonis ν transiit	37. 40. 30
52. 42	46. 00	Saturni centrum transiit	36. 32. 50
11. 57. 25	50. 43	Cor Leonis transiit	38. 7. 5
12. 0. 35	53. 53	Leonis 31*ma* transiit	36. 42. 50
5. 40	11. 58. 58	Leonis 34*ta* transiit	36. 19. 30
12. 10. 50	12. 4. 8	Leonis 38*va* transiit	35. 3. 50
		Ascensio recta ♄ 147. 4. 45	
		Distant. a Polo 75. 5 00	
		Longitudo ♌ 24. 14. 8	
		Latitudo Bor. 1. 32 16	
		Die Veneris Februarii 6.	
8. 22. 53	8. 14. 43	Lucidus pes Pollucis, ♊ γ tr.	34. 52. 00
11. 29. 22	11. 21. 12	Leonis ♃ transiit	36. 9. 15
11. 43. 58	11. 35. 48	Leonis ν transit	37. 40. 30
11. 49. 10	11. 41. 0	Saturni centrum transiit	36. 31. 00
		Ascensio recta ♄ 146. 59. 0.	
		Distantia a Polo 75. 3. 10	
		Longitudo ♌ 24. 8. 17	
		Latitudo Bor. 1°. 32. 8	

Temp. per Horolog.	Tempora correcta.	Die Mercurii Febr. 18.	Distantia a Vertice.
11. 0. 15	10. 53. 0	Saturni centrum transiit	36. 12. 00
11. 8. 53	11. 1. 38	Cor Leonis transiit	38. 7. 5
11. 17. 7	11. 9. 52	Leonis 34*ta Cat. Brit.* transf.	36. 19. 35
11. 22. 16	11. 15. 1	Ejusdem 38*va* transiit	35. 3. 55
		Ascensio Rect. ♄ 146. 5. 00	
		Distantia a Polo 74. 44. 10.	
		Longitudo ♌ 23. 12. 47	
		Latitudo Bor. 1. 32 41	
		Die Lunæ Martii 2. 1713.	
10. 6. 30	9. 52. 3	Leonis ♃ transiit	36. 9. 30
19. 27	10. 5. 00	Saturni centrum transiit	35 55. 20
31. 21	16. 54	Cor Leonis transiit	38. 7. 5
34. 31	20. 4	Leonis 31*ma* transiit	36. 42. 50
10. 39. 35	10. 25. 8	Leonis 34*ta* transiit	36. 19. 40
		Ascensio Recta ♄ 145. 16. 00	
		Distant. a Polo 74. 27. 30	
		Longitudo ♌ 22. 22. 40	
		Latitudo Bor. 1. 32. 52	
		Die Martis Aprilis 7.	
7. 43. 1	7. 41. 15	Leonis ♃ transiit	36. 9. 25
50. 46	7. 49 00	Saturni centrum transiit	35. 31. 10
57. 36	55. 50	Leonis ν *Bayero* transiit	37. 40. 20
8. 6. 39	8. 4. 53	Leonis in collo ν transiit	33. 19. 25
16. 7	14. 21	Ejusdem 34*ta Cat. Br.* transf.	36. 19. 30
8. 21. 17	8. 19. 31	Ejusdem 38*va* transiit	35. 3. 50
		Ascensio recta ♄ 143. 57. 45	
		Distantia a Polo 74. 3. 15	
		Longitudo ♌ 21. 3. 32	
		Latitudo 1. 31. 20	
		Die Mercurii Aprilis 8.	
7. 47. 43	7. 46. 00	Saturni centrum transiit	35. 31. 15
54. 34	52. 51	Leonis ν transiit	37. 40. 20
8. 3. 37	8. 1. 54	Leonis ν transiit	33. 19. 20
13. 4	11. 21	Leonis 34*ta* transiit	36. 19. 35
8. 18. 14	8. 16. 31	Leonis 38*va* transiit	35. 3. 45
		Ascensio Recta ♄ 143. 57. 30	
		Distantia a Polo 74. 3. 20	
		Longitudo ♌ 21. 3. 20	
		Latitudo 1. 31. 10	
		Saturno pene stationario.	

Temp.

Temp. per Horolog.	Tempora correcta.	Die Jovis Novemb. 5. 1713.	Distantia a Vertice.
18. 22. 00	18. 15. 37	Leonis ϖ in genu seq. transf.	42. 4. 00
30. 8	23. 45	Cor Leonis transiit	38. 7. 5
54. 35	48. 12	Leonis in Axilla ρ transiit	40. 42. 0
19. 11. 5	19. 4. 42	Leonis in ventre *l* transiit	39. 25. 15
19. 26. 23	19. 20. 00	Saturni centrum transiit	42. 10. 40
		Ascensio Rect. ♄ 162. 23. 20	
		Dist. a Polo 80. 43. 00	
		Longitudo ♍ 10. 13. 40	
		Latitudo Bor. 1. 39. 37	

Observationes JOVIS.

Anno MDCCXIII.

		Die Solis Augusti 9.	
12. 40. 4	12. 37. 27	Aquarii λ in effusione Aquæ transiit	60. 32. 50
12. 48. 37	12. 46. 0	Jovis centrum transiit	60. 48. 35
12. 52. 36	12. 49. 59	Aquarii 73*ta Cat. Brit.* prima ad *h* transiit	60. 46. 20
13. 4. 21	13. 1. 44	Aquarii in aqua χ transiit	60. 49. 10
		Ascens. Rect. ♃ 341. 33. 5	
		Dist. a Polo Bor. 99. 21. 40	
		Longitudo ♓ 9. 26. 00	
		Latitudo Aust. 1. 25. 8	
		Die Lunæ Augusti 10.	
12. 36. 21	12. 33. 55	Aquarii λ transiit	60. 32. 50
12. 44. 26	12. 42. 00	Jovis centrum transiit	60. 52. 00
12. 48. 53	12. 46. 27	Aquarii 73*ia* transiit	
		Ascens. Rect. ♃ 341. 26. 5	
		Distant. a Polo 99. 25. 5	
		Longitud. Jov. ♓ 9. 18. 17	
		Latitudo Aust. 1. 25. 40	
		Die Lunæ Octobris 26.	
7. 29. 16	7. 28. 42	Aquarii in Clune σ transiit	63. 34. 40
7. 36. 34	7. 36. 0	Jovis centrum transiit	63. 00. 5
8. 14. 34	8. 14. 0	Aquarii 80*ma* prima ad ψ transf.	62. 5. 20
8. 17. 45	8. 17. 11	Aquarii 84*ta* seq. ad ψ transf.	62. 37. 5
		Ascensio Rect. ♃ 335. 41. 30	
		Distantia a Polo 101. 33. 20	
		Longitudo Jovis ♓ 3. 16. 00.	
		Latitudo Aust. 1. 19. 8	

Temp. per Horolog.	Tempora correcta.	Die Martis Octob. 27.	Distantia a Vertice.
7. 25. 40	7. 23. 34	Aquarii Clunis σ transiit	63. 34. 35
7. 33. 6	7. 31. 00	Jovis centrum transiit	62. 59. 15
8. 11. 00	8. 8. 54	Aquarii prima ad ♃ transiit	62. 5. 15
8. 14. 10	8. 12. 4	Sequens ad ♃ transiit	62. 37. 10
		Ascensio Rect. ♃ 335. 43. 20	
		Dist. a Polo 101. 32. 30	
		Longitudo ♓ 3. 17. 58	
		Latitudo Aust. 1. 19. 00	
		Die Jovis Octob. 29.	
7. 18. 29	7. 15. 19	Aquarii σ transf.	63. 34. 40
26. 10	7. 23. 00	Jovis centrum transiit	62. 57. 20
8. 3. 47	8. 0. 37	Prima ad ♃ transiit	62. 5. 20
6. 55	8. 3. 45	Sequens ad ♃ transiit	62. 37. 10
		Ascensio Rect. ♃ 335. 47. 45	
		Dist. a Polo 101. 30. 35	
		Longitudo Jov. ♓ 3. 22. 41	
		Latitudo Aust. 1. 18. 49	

Observationes MARTIS.
Anno MDCCXIII.

		Die Mercurii Feb. 18. 1713.	
12. 28. 38	12. 21. 20	Leonis in poplite τ transiit	47. 2. 15
12. 51. 6	12. 43. 48	In ancone Alæ ♍ β transf.	48. 5. 40
13. 10. 21	13. 3. 3	Virginis 10*ma Catal. Brit.* r transf.	47. 57. 40
13. 13. 18	13. 6. 00	Martis centrum transiit	47. 2. 5
13. 21. 8	13. 13. 50	In cervice Viginis *c* transiit	46. 33. 30
		Ascensio Rect. *Martis* 179. 29. 20	
		Dist. a Polo 85. 34. 35	
		Longitudo ♍ 27. 46. 00	
		Latitudo Bor. 3. 51. 10	
		Die Martis Martii 3.	
11. 55. 52	11. 53. 2	In vertice Virginis ν transf.	43. 20. 00
12. 4. 50	12. 2. 00	Martis centrum transf.	45. 14. 20
10. 54	8. 4	In vultu Virginis π transf.	43. 15. 25
20. 8	17. 18	Undecima Virginis s transf.	44. 3. 30
12. 30. 25	27. 35	Virginis 16 in Cervice *c* transf.	46. 33. 25
12. 32. 37	12. 29. 47	Virginis 17*ma Cat. Br.* transiit	44. 33. 45
		Ascensio recta *Martis* 175. 1. 15	
		Dist. a Polo Bor. 83. 46. 45	
		Longitudo ♍ 22. 57. 33	
		Latitudo Bor. 3. 43. 37	

Temp.

Temp. per Horolog.	Tempora correcta.	Die Martis Aprilis 7. 1713.	Distantia a Vertice.
9. 4 47	9. 3. 10	Sub Ventre Leonis χ transf.	42. 35. 25
9. 17. 37	9. 16. 00	*Martis* centrum transf.	42. 42. 50
9. 38. 10	9. 36. 33	Prima Virg. *Cat. Br.* ω transf.	41. 44. 50
9. 45. 00	9. 43. 23	Borea in Vertice ♍ ξ transf.	41. 37. 15
		Ascensio Rect. *Martis* 165. 45. 40	
		Dist. a Polo 81. 15. 10	
		Longit. *Martis* ♍ 13. 30. 40	
		Latitudo Bor. 2. 26. 31.	
		Die Mercurii Aprilis 8.	
9. 1. 44	9. 0. 29	Leonis χ transiit	42. 35. 30
9. 14. 15	9. 13. 00	*Martis* centrum transf.	42. 43. 40
9. 35. 7	9. 33. 52	Virginis ω transiit	41. 44. 55
9. 41. 58	9. 40. 43	Virginis ξ transiit	41. 37. 15
		Ascensio Rect. *Martis* 165. 41. 00	
		Distantia a Polo 81. 16. 00	
		Longitud. *Martis* ♍ 13. 26. 45	
		Latitudo Bor. 2. 23. 58	
		Die Veneris Maii 1.	
7. 55. 9	7. 50. 00	*Martis* centrum transf.	44. 17. 30
8. 18. 12	8. 13. 3	In Vertice Virginis ν transf.	43. 20. 00
8. 33. 14	8. 28. 5	In Vultu Virginis π transf.	43. 15. 30
		Asc. Rect. *Martis* 166. 59. 40	
		Dist. a Polo 82. 49. 50	
		Longitudo ♍ 15. 15. 00	
		Latitudo Bor. 1. 27. 40	
		Die Saturni Maii 2.	
7. 52. 45	7. 47. 00	*Martis* centrum transf.	44. 24. 20
8. 15. 7	8. 9. 22	Virginis ν transiit	43. 20. 5
8. 30. 8	8. 24. 23	Virginis π transiit	43. 15. 25
		Asc. Rect. *Martis* 167. 10. 00	
		Distantia a Polo 82. 56. 40	
		Longitudo ♍ 15. 27. 5	
		Latitudo Bor. 1. 25. 20	

Observationes LUNÆ.

Anno MDCCXIII.

		Die Solis Januarii 25.	
8. 9. 33	8. 7. 23	Telescopica *a* transf.	28. 26. 20
8. 15. 5	8. 12. 55	Tauri 123*ia Catal. Brit.* transiit	27. 2. 30
8. 20. 20	8. 18. 10	Lunæ limbus præced. transiit, centro a Vertice	27. 32. 40

Temp.

Temp. per Horolog.	Tempora correcta.	Die Solis Januarii 25. 1713.	Distantia a Vertice.
8. 21. 23	8. 19. 13	Lunæ centrum transit limbo remoto a Vertice	27. 47. 40
8. 22. 52	8. 20. 42	Lunæ cuspis Bor. a Vertice	27. 17. 40
8. 30. 15	8. 28. 5	Geminorum προπὺς transit	28. 13. 20
8. 41. 7	8. 38. 57	Pes Castoris η transit	28. 54. 50
8. 49. 8	8. 46. 58	Calx ejusdem μ transit	28. 50. 40
		Asc. Rect. Cent. ☽ 84. 26. 55	
		Dist. a Polo visa 66. 4. 40	
		Sed adhibit. Paral. 65. 39. 50	
		Longit. Lunæ ♊ 24. 56. 30	
		Latitudo Bor. 0. 57. 00	
		Die Lunæ Januarii 26.	
8. 26. 41	8. 24. 36	Propus transit	28 13. 30
8. 37. 31	8. 35. 26	Pes Castoris η transit	28. 54. 50
8. 45. 31	8. 43. 26	Calx Castoris μ transit	28. 50. 40
9. 9. 43	9. 7. 38	Lunæ limbus præcedens transit, centro a Vertice	28. 45. 00
9. 10. 50	9. 8. 45	Lunæ centrum trans. limbo remoto a Vertice	29. 0. 5
9. 12. 30	9. 16. 25	Lunæ cuspis Bor. a Vertice	28. 30. 10
9. 27. 50	9. 25. 45	♊orum 46*ta Bat Brit* trans.	28. 26. 10
9. 42. 44	9. 40. 39	♊orum in Inguine δ trans.	28 59. 30
		Asc. Rect. cent ☽ 97. 43. 50	
		Dist. a Polo visa 67. 17. 5	
		Adhibita Parallaxi 66. 51. 15	
		Longit. Lunæ ♋ 7. 6. 18	
		Latitudo Aust. 0. 8. 48.	

Observationes SATELLITUM JOVIS.

		Die Veneris Octob. 30.
6. 56. 30	6. 52. 35	Quartus Satelles visus est emergens ab umbra, diametro Jovis distans a *tertio* ei proximo ad dextram, Tubo scil. octo pedum.
7. 4. 00	7. 00. 00	Clare explenduit, & linea ducta a proximo illo per centrum Jovis emergentem reliquit ad Austrum; *situ scilicet inverso*.
7. 36. 31	7. 32. 30	Pegasi μ tran. per planum Arcus meridionalis
		Die Saturni Novemb. 7.
7. 13. 2	7. 5. 00	Secund. Satelles emergebat, vel potius emergere incipiebat Tubo octo pedum.
9. 5. 11	8. 57. 00	Piscium ϑ in Lino australi transit.

N. B.

N. B. Stella illa Teleſcopica *a* quæ die Januarii 25° Lunam præceſſit, Aſcenſionem rectam tunc habuit 81° 28′½, & diſtabat a Polo 66° 58′ 20″, unde fit Longitudo ejus ♊ 22° 9′½ cum Latitudine Auſtrali 0° 13′½. Hæc autem eſt ea ipſa ſtella ad quam applicabatur Jupiter in Statione ſecunda, anno 1634 Februarii 6, eamque non niſi tribus ſui corporis diametris ad Auſtrum reliquit, obſervante *Gaſſendo* · ut habetur inter Obſervata ejus p 174. Et ad eandem *Mars* obſervatus eſt Septembris 6to anno 1644. mane, ut videre eſt in Prolegomenis Selenographiæ *Hevelianæ* p 65. & Fig. 1

A Remark by Dr. Halley, ibid. p. 291.

2. Having after Midnight carefully corrected the Clock by no leſs than ten Obſervations of the Altitude of the *Lucida Arietis*, the Error thereof was found 5′. 13″. too faſt, the extreams not differing above 6″: And in the Morning about 7h, by as many Altitudes of the *Sun*, with a like Agreement, the ſame Error was found 5′. 14″, to be deducted from the Times ſhewn by the Clock.

An Occultation of Jupiter, by the Moon at Wanſtead, *by Mr.* Pound, *n.* 347. *p* 401.

Julii 13°. 1715. P. M. N.	Time by the Clock.	Time correct.
	h. ′ ″	h. ′ ″
The third Satellite of *Jupiter* was hid by the Moon	13 27 33	13 22 20
The firſt Satellite was hid	13 32 35	13 27 22
The ſecond Satellite was hid	13 34 25	13 29 11
The firſt Contact of the Limbs of ♃ and ☾	13 34 54	13 29 41
Jupiter wholly hid	13 36 23	13 31 10
The third Satellite came out from behind the dark ſide of the Moon	14 7 25	14 2 12
The firſt Satellite	14 12 25	14 7 12
The ſecond Satellite	14 14 38	14 9 25
The firſt Limb of *Jupiter* came out	14 14 45	14 9 32
The following Limb of *Jupiter*, or laſt Contact	14 16 15	14 11 2
The fourth Satellite emerged	14 18 49	14 13 36

Jupiter and the Satellites were to the Northward of the viſible way of the Moon's Center.

This Occultation was obſerved through a Teleſcope, in which the focal length of the Object Glaſs was 4½ Feet, and of the Eye Glaſs 2½ Inches. And the Aperture of the Object was 1½ Inch.

I could perceive no Colours on *Jupiter's* Limb, either at his Immerſion or Emerſion, when the Axis of the Tube was directed to him.

Anno

Observations by the same, n. 350. p. 506. 3. Anno 1715. *Augusti* 21°. 8h. 25′ ½ Temp.æq. *Mars* præcedebat scil. Ascensione recta, Mediam frontis Scorpii (*Bayero* δ) 6′. 54″. Borealior Fixa 9′. 47″.

Sept. 18°. 7h. 30′. *Mars* præcedebat Claram in pede Serpentarii (*Bayero* θ) 17′. 48″. & eandem habuit Declinationem accurate.

Novemb. 30. 18h. 8′. *Saturnus* præcedebat γ, sive secundam Alæ Virginis 23′. 19″. & erat Fixa Australior 25′. 3″. *Decembris* autem 4to 17h. 25′. præcedebat eam 10′ 50″. & Australior erat 29′. 00″.

Anno autem 1716. *Feb.* 22°. 7h. 23′. T. æq. *Mars* præcedebat ζ Piscium, sive Sequentem trium clariorum in Lino Aust. Piscium 3′. 35″. eademque Australior erat 1′. 23″. quam proinde obtegere debuit ante bihorium, forsan corporaliter.

Junii 22°. 8h. 52′. T. æq. *Venus* sequebatur Cor Leonis 34′. 50″. & fixa Australior erat 7′. 23″.

Aug. 14°. 15h. 00′. *Jupiter* præcedebat *Propoda* uno tantum minuto cum Declinatione Bor. minore 14′. 26″.

Aug. 19°. 13h. 2′. *Jupiter* præcedebat fixam Telescopicam, quæ vocetur *b*, 50′, 08″. eandem habens Declinationem accurate.

Aug. 24°. 12h. 19′. *Jupiter* Micrometro distabat a prædicta *b*, 5′. 54″. simulque ab alia Fixa clariore *a* 7′. 17″. Distantia fixarum 12′. 31″. Tunc minor *Jovis* diameter 0′. 38″.

Sept. 12°. 17h. 00. *Venus* recens a Statione secunda sequebatur Telescopicam 17′. 40″, eaque Australior erat 5′. 30″. Hæc autem Fixa tunc occupavit ♌ 27°. 44′ ½ cum Lat. Aust. 5°. 39′.

Octob. 15°. 17h. 12′. ½ *Venus* distabat Microm. a Fixa τ in Crure Leonis 27′. 55″.

Novemb. 20°. 6h. 18′ ½ *Jupiter* regressus est ad stellas *a* & *b*, ad quas observatus est *Aug.* 24°. & distabat a *b* 6′. 21″. ab *a* vero 11′. 36″.

Novemb. 21. 7h. 38′. *Jupiter* distabat a *b* 9′. 19″, & ab *a* 3′. 48″. Fixæ inter se 12′. 30″. *Jovis* diameter minor sive Axis 0′. 44″. Deinde hora 18h. 50′. visa est stella *a* limbo *Jovis* quasi adhærere, eratque quasi ⅓ semidiametri vel 0′. 15″. centro *Jovis* Borealior. Juxta has autem observationes constat medium Occultationis Fixæ, interposito *Jovis* corpore, contigisse *Nov.* 21°. 19h. 55′. vel proxime. Deinde

Nov.	30.	5h. 41′.	♃. præced. *Propoda*	12′. 36″	*Australior*	7′. 36″
Dec.	4.	6. 0.	♃. Sequeb. eam	22. 49		7. 47
Dec.	5.	6. 0	Repet.	31. 35		7. 50
Dec.	6.	6. 0	Repet.	40. 30		7. 52
Dec.	7.	6. 0	Iterum	49. 15		7. 54

Ex his ultimis Observationibus liquet *Jovem* & *Propoda* eandem habuisse Longitudinem *Dec.* 1°. 15h. 29′. quo tempore *Jupiter* Australior erat stella 7′. 40″. Ex iisdem etiam constabit *Jovem* in opposito Solis fuisse, quoad Longitudinem, *Dec.* 6°. 12h. 46′.

N. B. Stellas illas Telescopicas *a* & *b* vocatas, haberi in Catalogo Fixarum Britannico D. *Flamsteedii*, ubi ipsi *a* Locus datur, ad annum scil. 1690 ineuntem, ♊ 27°. 54′. 29″ cum Lat. Aust. 0 21′. 55″; alteri vero

b ♊

b ♊ 28°. 5'. 24" cum Lat. Aust. 28'. 5". Neque aliam novimus Fixam a corpore *Jovis* occultatam & ab invento Telescopio observatam, præter jam dictam Stellam *a*; ad quam olim arctissime applicabatur *Jupiter*, ante annos 83. *Decembris* nono St. nov. Anni 1633. Vesp. cum *Gassendus Diniæ* vidit *Jovem* huic Fixæ conjunctum, nec nisi quinque semidiametris corporis sui superiorem. Unde calculo debite inito, constabit Nodos hujus Planetæ Planumque Orbis ejus situm in Sphæra Fixarum servare immobilem, vel saltem lentissimo motu * cieri.

* *Gassend. Obser. Tom.* IV. p. 162.

4. Anno 1717, *Aprilis* 15°. 9h. 49'. T. æq. observavit D. *Pound* apud *Wansted*, *Jovem* jam reversum ad stellam illam, quam *Novemb.* 22°. 1716. mane corpore suo texerat. *Jovis* autem centrum tum temporis distabat ab ea Stella (quæ tertia est Geminorum in *Catalogo Britannico*) 23'. 22". boream versus; simulque ab alia vicina, quæ quarta est Geminorum in dicto Catalogo, 27' 11". atque huic fere conjunctus erat planeta.

Astronomical Observations, com. to the R. S. n. 357. p. 847.

Aprilis 25to sequente, eodem observatore ac loco, 10h. 3'. T. æq. *Jupiter* apud quatuor Fixas exiguas visus est, eas omnes præcedens, & in ipso quasi principio Cancri. Centrum autem planetæ distabat ab *e* 13' 00". ab *h* 11'. 32". ab *f* 19'. 53". & a *g* 9'. 27".

Postridie vero *Apr.* 26°. 9h. 7'. *Jovis* centrum distabat ab *e* 8'. 35", ab *f* 9'. 00", a *g* 4' 5", & ab *h* 13'. 50". Jamque prætererat omnes præter *f* ad quam tendebat, quamque parum admodum die crastino infra se relinquere debuit.

Eodem fere momento, hora scil. nona, *Londini* visa est stella *g* in vertice Trianguli Isoscelis ac fere Isopleuri cum *Jovis* centro ac tertio Satellite, tum sex *Jovis* diametris ad occasum distante, nisi quod parum admodum base longiora erant crura; ac intra quadrantem horæ, angulus ad *Jovis* centrum, qui prius major erat angulo ad Satellitem, factus est eo sensibiliter minor.

Tres autem Stellæ *h*, *g*, *e*, sunt 10ma, 11ma, & 12na, Geminorum in *Catal. Britan.* juxta quem tum temporis situm habuere, *h* in ♋ 0° 22'. 55". cum Latit. Borea 0° 11'. 25". Et *g* in ♋ 0°. 28'. 25". Lat. Bor. 0°. 3'. 40". *e* vero in ♋ 0°. 29'. 20". cum Lat. Aust. 0°. 8'. 05". Distat autem quarta *f* a Stella *g* 11'. 40", ab *e* 12'. 50". ac denique ab *h* 20'. 36", unde constabit locus ejus. Ex his manifestum est *Jovem* Latitudinem habuisse parvam admodum Borealem, nec semiminuto majorem, saltem si dictis stellarum locis habenda fides. Hæc posteris usui esse possunt definiendo Nodorum *Jovis* motu, si quem habeant.

Ejusdem anni *Junii* 18vo 10h. *Londini*, in ædibus Societatis Regiæ, visus est *Saturnus* Stellæ fixæ Telescopicæ admodum propinquus, a qua vix distabat ad Austrum una Annuli diametro, & normalis in lineam Ansarum a Stella demissa incidebat in medium Ansæ orientalis. Fixa hæc parvula nullique Catalogo adscripta tunc habuit ♎ 12° 58'$\frac{1}{2}$, cum Lat. Bor. 2°. 33'. proxime; comitemque habet sibi adjunctam ac luce æqualem, quatuor minutis ad ortum distantem, ac paulo australiorem, unde facile dignosci poterit, locusque ejus si cui libeat verificari.

Eadem nocte 10h. 30′ *Mars* visus est prope Stellam quæ præcedit 35. *Scorpii*, a qua distabat Tubo 24 pedum mensurata 7′. 16″; idque in recta per claram in pede Ophiuchi θ & dictam Stellam producta. Hæc autem Stella præcedit 35. *Scorpii* 30′. 27″. Asc. Rect. eaque Australior est 2′. 28″, unde fit locus ejus tum temporis *Sagitt.* 15°. 24′. 20″. Lat. Aust. 3°. 59′. 25″. Sed θ Ophiuchi tunc habuit *Sagitt.* 17°. 28′, & Lat. Aust. 1°. 47′. 38″. *Mars* itaque Stellam præcedebat Longitudine 4′. 58″, australior ea 5′. 30″

Deinde *Sept.* 13°. 8h. 5′. T. æq. *Mars* visus est a Dom. *Pound* præcedere claram in humero *Sagittarii* σ 11′. 54″. Asc. Rect. simulque borealior erat Stella 22′ 56″. Hora 8h. 25′. erat distantia Planetæ a Stella 25′. 00″. accurate.

Decemb. 5. 18h. 30′. T. æq. consensu sæpius repetitarum observationum invenit D. *Pound Saturnum* præcedere Telescopicam claram sibi vicinam 27′. 19″. Asc. Rect. Stellaque australiorem esse 1′. 59″. Simul *Saturnus* præcedebat κ in Syrmate *Virginis* 1°. 25′. 21″, eaque australior erat 4′. 05″. Hinc *Saturni* locus *Libra* 29°. 16′. 21″. Lat. Bor. 2°. 22′. 21″. Telescopica autem tunc habuit *Libr.* 29°. 40′. 56″. Lat. Bor. 2°. 33′. 43″.

Anno 1718. *Jan.* 7. 5h. 30′. T. æq. *Venus* apud duas Stellas in *Catal. Britan.* omissas observata est. Erat autem Planeta utraque Fixa Borealior, distans a præcedente 32′. 30″, a sequente 17′. 30″. Stella præcedens tunc habuit *Pisc.* 14°. 42′. 20″, cum Lat. Aust. 0°. 40′. 10″; altera vero sequens *Pisc.* 15°. 21′. 55″. Lat. Austral. 0°. 27′. 15″. prout ex observationibus D. *Flamstedii* colligere licet.

Jan. 15. 8h. 00′, T. æq. *Jupiter* præcedebat η in pectore *Cancri* 3°. 30′. 50″. Asc. Rect. fixaque Australior erat 14′. 15″. Hinc provenit *Jovis* locus *Canc.* 28°. 20′. cum Latitudine Borea 0°. 36′. 45″.

Martii 11. 10h. 36′ T æq. *Saturnus* præcedebat κ in Syrmate *Virginis* 18′. 51″, eaque Fixa australior erat 5′. 23″. Hinc fit Locus *Saturni Scorp.* 0. 18′. 34″. cum Lat. Bor. 2. 44′. 8″. Posito scilicet, juxta *Catal. Britan.* κ *Virginis* occupare ♏ 0. 34′. 10″, cum Lat. 2. 55′. 40″. Eadem nocte 17h. 00′. *Westmonasterii* observarunt DD. *Desaguliers* & *Gray Saturnum* præcedere Stellam 19′. 00″ cum declinatione majore in Austrum 4′. 45″.

April 8. 11h. 30′. *Londini* visus est *Saturnus* nuper Acronychus parum admodum occidentalior Telescopica clara, eademque 5 minutis borealior. Unde Fixæ locus *Libræ* 28. 18′. 30″. Lat. Bor. 2. 41′. Circulus autem magnus per hanc Stellam & Saturnum ductus dirigi videbatur ad Stellam 5tæ magnitudinis in *Catal. Brit.* omissam, sed quæ *Hevelio* est *in cuspide Alæ Boreæ Virginis*, cuique locum assignat *Libr.* 26. 10′, cum Lat. 14. 43′ Bor.

Eadem nocte 13h. 20′, apud *Wansted*, perpendiculum a dicta Stella Telescopica in lineam Ansarum *Saturni* demissum præcedebat centrum planetæ quasi sesquialtera diametro annuli; aberat autem Stella ad Austrum ab Ansarum axe 4′. 30″. Simul Ansæ orientalis extremitas deprehensa est in linea recta inter hanc Stellam & aliam eidem quasi longitudine conjunctam, quæ tunc a *Saturno* distabat 24′. 48″. versus Boream. Locus autem prioris Stellæ tunc fuit *Libr.* 28. 18′. 30″. cum Lat. Bor. 2. 41′. proxime.

Sept.

Sept. 7. circa meridiem incidit conjunctio *Jovis* & *Veneris* arctiſſima, cujus quidem ſpectaculum Aſtronomis noſtris inviderunt Nubes. Die autem ſexto præcedente mane, vel 5d 22h. 57'. 30''. T. æq. apud *Wanſted*, *Venus* occidentalior diſtabat a *Jove* 1. 3'. 28''. Die autem 7 17h. 21', *Venus* jam facta orientalior a *Jove* aberat 43'. 8''; ac 17h. 34', *Venus* auſtralior erat *Jove* differentia declinationum 14' 23''. Et 17h. 39'. capta eſt diſtantia Planetarum 44'. 4''. Hinc calculo accuratiſſimi Obſervatoris conjuncti ſunt *Sept.* 7. 0h. 9'. T. æq. *Veneris* centro tum *Jovis* auſtraliore non niſi 1'. 42''.

Denique *Sept.* 18. mane, apud *Wanſted*, *Jupiter* viſus eſt prope *Cor Leonis*, quocum die præcedente conjunctus fuerat. *Sept.* 17. 16h. 51'. T. æq. *Jovis* centrum aberat a *Corde Leonis* 24'. 22''; & 17h. 6'. 20''. erat diff. Declin 12'. 43''. Dein poſt Horam, nempe 17h. 54', facta eſt diſtantia 24'. 44''; ac 18h. 7'. differentia Declinationum inventa eſt 12'. 35''. Hinc ſupputante Dom. *Pound*, fit *Sept.* 17. 18h. 00'. T. æq. *Jovis* locus ♌ 26. 11'. 7''. cum Lat. Bor. 45'. 39''.

1718. *Octob.* 10°. mane, applicabatur *Jupiter* ad Fixas Teleſcopicas, n.363.p.1109. quarum loca, occaſione primæ apparitionis Cometæ anni 1680. (de qua vide *Phil. Tranſ.* * N°. 342) ſedulo inquiſivit Rev. D. *Pound*, ac nuper veri- * v. ſupr. p. ficata nobiſcum communicavit, una cum accurata obſervatione tranſitus 340. Jovis juxta eas hac vice, ac deinde altera *Febr.* 11°. ſtatim ab oppoſitione Solis & Jovis. Ineunte autem *Januario* 1719. loca ſtellarum ſic ſe habuere.

	Long.	Lat. Bor.		Long.	Lat. Bor.
d ♌	29°. 59'. 43''	1. 7. 50	*a* ♍	00. 25'. 41''	1°. 28'. 54''
e ♍	0. 6. 13	1. 10. 18	*x* ♍	0. 5. 43	0. 51 56
c ♍	0. 3. 13	0. 32. 50			

Ubi notandum ſtellas *d* & *e* eandem præciſe hoc ſeculo ſortiri declinationem, *x* vero exiguam eſſe ſtellulam in priore deſcriptione ob parvitatem omiſſam.

Jam *Octob.* 9°. 17h. 50'. T. æq. Jovis limbus orientalis attigit lineam ſtellas *e* & *c* jungentem, ſimul centrum ejus diſtabat ab *e* 21'. 20''. & a *c* 16'. 25''. ſtatimque aberat a *d* 19'. 35''. Parvula *x* Jovi proxima latuit, luce ejus obumbrata.

Decemb. 11°. 18h. 30. T. æq. *Saturni* centrum diſtabat a *μ Libræ Bayero*, 28'. 32'', & fixa Borealius erat 4'. 31''. Hinc concluſit D. *Pound* Obſervator *Saturni* locum ♏ 10°. 41'. 10'', cum Lat. Boreal. 2°. 16'. 43.

1719 *Feb.* 11°. 6h. 56'½ T. æq. *Jovis* retrogradi centrum diſtabat a ſtella *d* ſuperius deſcripta —— —— 10'. 42''
6. 58¼ Idem centrum diſtabat ab *e* —— —— 6 7
9. 37⅓ Iterum diſtantia capta a *d* —— —— 10. 9
9. 43½ Iterum ab *e* —— —— 6 11
9. 49½ Jovis centrum diſtabat ab *a* —— —— 25. 21
9. 58½ Idem centrum a parvula *x* —— —— 24 38

Circa Horam ſeptimam *Jovis* limbus orientalis attigit lineam per *x* & *e*

 pro-

productam; *Jupiter* itaque tunc habuit ♏ 0° 6 cum Latitudine Boreal. 1° 16′ 30″ Deinde,

Feb. 13° 8ʰ 0′ T. æq. Declinatio centri Jovis, Micrometro mensurata, Borealior erat ea stellæ utriusque *d* & *e* 11′ 37″, & 8ʰ 20′ eadem differentia inventa est 11′ 36″. Hora vero 8ʰ 48′ centrum Jovis distabat ab *e* 17′ 40′.

Apr. 22° 10ʰ 45′ T. æq. *Saturni* centrum sequebatur *μ Libræ* 4″½ Temp. sive 1′ 8″ Asc. Rectæ. Micrometro autem Borealior inventus est fixa 35′ 25″ Stella autem in *Catalogo Britannico* tunc habuit ♏ 10° 16′ 8″ Lat. Bor. 2° 3′ 54″

Maii 16° 8ʰ 00′ T. æq. ♃ sequebatur *Cor Leonis* 1° 34′½ Ascensionis rectæ; Borealior autem erat stella illa 0′. 41″½ Temporis, hoc est, 10′ 7″ Arcus cœlestis.

Eadem nocte, 15ʰ 18′ T. app. Observavit D. *Stephanus Grey Martem*, ratione Ascensionis rectæ, sequi stellam *in Cauda Capricorni* orientalem 16′ 15″; simul non nisi 0′ 11″ australior erat quam Fixa.

Junii 7° 10ʰ 15′. T. app. *Jupiter* directus iterum reversus est ad stellas Telescopicas prædictas, & tum sequebatur stellam *d* 0′ 35″ Ascensionis rectæ, & 10ʰ 30′ distabat fixa a limbo *Jovis* proximo 4′ 18″.

Postridie *Junii* 8°. 10ʰ 20′, *Jupiter* sequebatur stellam alteram *e* 1′ 30″ Ascensionis rectæ, ac statim distantia limbi *Jovis* proximi a stella capta est Micrometro 7′ 30″.

Julii 5. 8ʰ 26′. T. app. Conjungebantur arcte *Jupiter* & *Venus*, quæ tum Borealior præcedebat *Jovem* secundum Ascensionem rectam 1′. 20″: Centrorum autem distantia ex decies repetitis media, capta est 13′ 36″. Hæc tria *Londini* observata communicavit harum Scientiarum eximius Cultor D. *Martinus Folkes*, R. S. Soc.

Aug. 3. 12ʰ 20′ T. æq. *Mars* pene Acronychus sequebatur stellam *τ Aquarii Bayero* 10′ 58″ Temporis, sive 2° 44′ 57″ Ascensionis Rectæ. Erat autem fixa *Mars* Borealior 0′ 36″ tantum; unde concesso loco stellæ *Britannico* fit locus *Martis* observatus ♓ 7° 10′ 10″ cum latitudine Australi 6° 38′ 10″.

Aug. 10° 11ʰ 50 T. æq. *Mars* sequebatur fixam minorem quæ præcedit *τ Aquarii* 1° 39′ 30″ ratione Ascensionis rectæ; Australior vero quam fixa 10′ 42″.

Aug. 16° 7ʰ 18′ T. æq. *Spica Virginis* præcedebat *Veneris* centrum 5″¼ secundis temporis, sive 1′ 20″ Ascensionis rectæ, australior Planeta 18″½ temp. sive 4′, 35″.

Aug. 17° *Mars* pridie Acronychus ac Terris proximus observatus est ad duas stellulas contiguas, Parallaxis ejus investigandi gratia, juxta methodum a D. *Cassino*, in libro de Cometa anni 1680. exhibitam. Harum stellularum borea tum temporis locum habuit ♓ 3° 5′. 50″ cum Latitudine australi 6° 6′¼: altera vero Australior habuit ♓ 3° 5′ 30″, cum Lat. Aust. 6° 10′¼ proxime. Hora vero 10ʰ 40′ T. æq. Australem sequebatur *Mars* 41 min. 40″ Ascensionis rectæ, eaque adhuc Australior erat 7′ 50″.

Sept.

Sept. 18 9h 20′ T. æq. *Mars* visus est præcedere stellam in *Catalogo Britannico* Aquarii 53 iam. 3′ 45″ temporis, sive 56′ 24″ Ascen. Rectæ; simulque Stella Borealior erat limbo Martis boreo, non nisi una Planetæ diametro. Locus stellæ ♒ 29′ 57″½ Lat. Aust. 4°. 48½.

Octob. 30. Vesperi 5h 45″ T. app. *Mars* proximus stellis duabus contiguis ad *h* ♒ *Bayero*, quæ sunt ♒ 73 ia & 74 ta *Catal. Brit.* Præterierat rectam per easdem ductam, eratque angulus ad *Martis* centrum ad sensum rectus: Borea vero stellarum eandem habuit declinationem cum limbo Planetæ austrino. 5h 53′ distantia stellæ a centro *Martis* 2′ 30″ 5h 56′ centrum *Martis* distabat a tertia & Australiore ad *h*, sive 75ta aquarii, 17′.04″. 6h 18′ distantia centri a Borea sive 73ta erat 3′ 5″. Hinc concludere licet *Martem*, hora 3h 30′ proxime, stellæ Boreæ conjunctum fuisse, eamque uno tantum minuto ad Boream reliquisse. Fixæ autem locus e *Catalogo Britannico* tunc erat ♓ 10° 29′ 00″ cum Lat. Aust. 1° 40′¼. 74 ta vero habuit ♓ 10°. 29′. 50″ cum Lat. Aust. 1° 44′¼.

Novemb. 16° 19h 18′ T. æq. *Venus* præcedebat *Lancem Libræ* Austrinam 3′ 13″ Temp. sive 48′ 23″ Ascen. Rect. simulque fixa borealius erat centrum *Veneris* 7′ 45″. *Venus* quasi Stationaria apud Nodum ejus ascendentem.

Dec. 3. 19h T. æq. Saturnus præcedebat tertiam ad ζ Libræ, sive Libræ 29. *Cat. Brit.* 0′ 46″ Temp. sive 11′ 32″ Asc. Rect. Erat autem fixa Australior 15′ 29″ differentia per Micrometrum capta. Unde Saturni locus ♍ 20° 25′ ¼ cum Lat. Bor. 2° 5′¼.

5. Anno 1717. *Jan.* 12. *Westmonasterii* observavit Dom. *Stephanus Gray* Lunæ appulsum ad quatuor Stellas contiguas sub cornu Austrino *Tauri*, apud quas observata est Luna Anno 1683. *Mart.* 23. st. v. ab *Hevelio* & *Flamstedio*. Itaque 9h 45′ T. app. Luna gibba visa est quasi conjuncta cum Stella e quatuor præcedente, quæ est *Tauri* 107. *Catal. Brit.* quæque tunc Australior erat Lunæ limbo Aust. sesquialtero minuto. 11h 29′ altera, quæ minor est, & ideo in Catalogo omissa, occultabatur paulo infra medium obscuri limbi. Ad 12h 24′ Tertia & clarior (110 *Tauri*) in ipsa fere conjunctione sex minutis distabat a limbo boreo. Denique 12h 54′ sequens e quatuor (111 *Tauri*) limbo Boreo superior erat 3′ 30″. Locus autem præcedentis, sive 107 *Tauri*, ex dicto Catalogo tunc erat *Gemini* 18. 12. Lat. Aust. 5h. 18′; *Tauri* autem 110 habuit *Gem.* 19. 26′¼ cum Lat. Aust. 4° 44′: Sequens vero sive 111 *Tauri*, erat in *Gem.* 19 45′. Lat. Aust. 4. 48′⅙. Secunda parvula, ut ex aliis observationibus constat, Locum tunc habuit *Gem.* 19 17′ Lat. 5. 5′ fere. *Lunar Observations and Eclipses. n. 357. p. 852.*

Eodem anno *Mart* 16. mane, erat Eclipsis Lunæ partialis, apud nos ob cœlum nubilum inconspicua. At apud *Cambridg Nov-Anglorum*, Dom. *Robie* Astronomiæ peritissimus vidit Eclipseos initium circa horam nonam. Finem vero, juxta *Paludem Mæotida*, ad 11h 42′ 30″ sat accurate. Est autem *Cambridg* sub altitudine Poli 42h 25′, *Londino* 71 grad. sive 4h 44′ occidentalior, ut ex pluribus antea observatis constat. Apud *Limam Peruviæ*, observante D. *Petro Peralta*, Mathematico Regio, Initium hujus *n. 363. p. 113.*

hujus Eclipsis fuit 8h 41' 8". Finis autem 11h 19' 55". Et ad insulam quam *Virgine Gorda* vocant, observante D. *Candler* Navarcha Regio, desiit Eclipsis 12h 13'. *P. M.* fine per cœlum sudum distincte viso. Denique, *Parisiis* observabant etiam D. *Cassini*, & D. *De la Hire*. Hic Initium æstimavit 13h 54'. Finem vero certius 16h 38' 10". At ille initium 13h 55'. Finem 16h 38' 25'. Maxima obscuratio huic $7\frac{1}{3}$ Dig. illi $7\frac{1}{2}$ Dig.

Hinc ex Fine, in singulis locis ut videtur accuratius sumpto, proveniunt Longitudinum differentiæ inter *Parisios* & *Limam* 5h 18' 20", Inter *Parisios* & *Cambridg* 4h 55' 50" Inter *Parisios* & Insulam *Virgine Gorda* 4h 25' 20". E quibus si 9' 40" subduxeris, provenient Longitudines ad occasum *Londini*, nempe *Limæ* 77° 10'. *Cambridg Nov-Anglorum* $71°\frac{1}{2}$, ac denique insulæ *Virgine Gorda* 63° 55'. unde Insularum adjacentium situs Geographici certo corrigi poterint.

n. 357. p. 852. *Sept.* 9. vesperi, in ædibus Societatis Regiæ *Londini*, observarunt nonnulli e Sociis finem Eclipseos Lunaris 7h 26'. Luna autem orta est juxta medium Eclipseos, nec nisi paulo ante finem e nubibus horizontem ob-
n. 363. p. 1113. sidentibus sese extricaverat. *Parisiis* vero D. *Cassino* Finis 7h 34' 50", D. *Maraldo* 7h 35' 30", & Dno *De la Hire* 7h 34' 15". Simul D. *Wurtzelbaur Noribergæ* eundem Finem vidit 8h 10' 45' Hinc confirmantur Meridianorum differentiæ *Londinum* inter & *Parisios*, præsertim ex observatione D. *Maraldi*, nempe 9' 30"; uti & inter *Londinum* & *Noribergam* 44' 45", quantam sæpius olim experti sumus.

n. 357. p. 853. *Sept.* 14. Vesperi, hac prima vice post longum intervallum rediit Luna ad occultandum *Palilicium*. Favit autem admodum cœlum *Londini* præter solitum purum, ita ut Luna & Stellæ exorientes in ipso quasi Horizonte simul conspicerentur. Incidit Immersio Stellæ 9h 6' 20", Luna nondum 3° alta, in ipso quasi medio Limbi orientalis, e regione scilicet Boreæ partis maculæ illius exiguæ quam *Hevelius* Stagnum *Mœridis* vocat, quamque *Ricciolus* sui ipsius nomine insignivit. Emersit autem paulo infra medium limbi obscuri ad 9h 58' 20", in ictu oculi tota sua claritate effulgens; unde etiam in tam illustri Stella quasi nullitas diametri de-
n. 363. p. 1113. monstratur. *Parisiis*, observantibus sigillatim DD. *Maraldo* & *Delisle* Juniore, Evanuit stella e regione Maculæ *Grimaldi* sive *Paludis Marcotidis*, Hora 9h 11' 35". Emersit autem e limbo Lunæ obscuro 10h 3' 55".

n. 357. p. 853. *Septembris* 23. vesperi, incidit Eclipsis Solis vix ullibi in *Europa* conspicua. Ex *America* autem nostra duplicem obtinuimus ejus observationem; alteram ex literis illustris Viri D. *Keith* Provinciæ *Pensylvaniæ* Præfecti dignissimi, qui *Philadelphiæ*, sub altitudine Poli 40° 00' fere, vidit Eclipsin jam cœptam (sed quæ ante minutum temporis nondum inceperat) ad 11h 55'. Circa medium Digiti erant quasi decem. Finis autem visus est accurate ad 2h 46' 35".

Altera autem hujus observatio habita est ad *Cambridg Novæ Angliæ* Academiam, a Dom. *Robie*, de quo supra: Initium Eclipseos ibi observatum est 0h 23' 00" post meridiem. Ad 1h 47' defecere IX Digit. Ad 3h 5' 10" desiit Eclipsis, Sole integro per Tubum 24 pedum conspecto. Hæc ex literis accurati Observatoris communicavit D. *Guil. Derham*.

Dec.

Dec. 5. Luna paulo supra *Palilicium* invecta est: Transitum autem satis arctum observavit D. *Jac. Bradley*, A. M. eruditus Juvenis, ingenio simul & industria pollens. Hic, cum Luna jam propemodum plena esset, Stellam contulit cum insigni illa Macula quam *Ricciolus Tychonem*, *Hevelius Sinam* appellat, & ex pluribus æqualibus distantiis Micrometro ante & post captis, Stellam dictæ maculæ centro proximam apparuisse conclusit ad $11^h\ 15'\ 8''$ T. æq. apud *Wansted*. Ad $11^h\ 15'\ 42''$ distabat Palilicium a limbo Lunæ proximo & Austrino $5'\ 55''$. Macula autem *Tycho* ab eodem limbo aberat $4'\ 16''$. Ad $11^h\ 18'\ 42''$ Stella erat in linea recta cum maculis *Tychonis* & *Copernici*, sive *Sinæ* & *Ætnæ*; & $11^h\ 25'\ 27''$ T. æq. erat in recta cum *Tychone* & *Keplero*. Inter hæc observata est Lunæ diameter $32'\ 45''$.

Anno 1718. *Jan.* 29. vesperi, DD. *Desaguliers* & *Gray*, *Westmonasterii* alteram *Palilicii* Occultationem præstolabant; sed nubium interventu impediti, viderunt saltem quod $5^h\ 52'$ nondum immerserat Stella; attenuatis autem postea nubibus conclusa est Emersio ad $7^h\ 20'$, e regione Promontorii *Sarmatiæ Asiaticæ Hevelii*.

Feb. 19. mane. Iidem observatores ibidem varie cum nubibus colluctati Eclipsin Solis ægre conspexerunt: Hora tamen 6. 59′ visi sunt deficere duo Digiti, & post unum temporis minutum chorda inter Cuspides visa est æqualis semidiametro Solis.

Apud *Wansted* autem D. *Pound* notavit ad $6^h\ 54'\ 7''$ T. app. chordam inter Cuspides $18'\ 30''$. Ad $7^h\ 17'\ 00''$ erat $10'\ 18''$. Ad $7^h\ 19'\ 30''$ eadem inventa est $8'\ 05''$ Desiit autem Eclipsis ad $7^h\ 23'\ 20''$.

Feb. 25. vesperi $6^h\ 44'$ T. app. *Westmonasterii*, Stella prima *Hyadum* in Naribus *Tauri* (γ *Bayero*) visa est in recta per cuspides Lunæ, adeoque propemodum conjuncta; distabat autem a limbo Lunæ Austr. $5'\ 51''$. Diameter Lunæ $31'\ 45''$ mensurata Micrometro.

Feb. 28. $8^h\ 36'$ T. app. etiam *Westmonasterii*, visa est immersio Stellæ in poplite Pollucis (γ Geminorum *Bayero*) sub limbi Lunæ obscuri ea parte, quæ paulo Borealior erat, ea macula quam Hevelius Cretam vocat. Emersio ipsa ob cœlum minus purum non conspecta est: sed ad $9^h\ 51'$ egressa erat Stella e limbo lucido, a quo distabat $3'$ circiter, e regione Boreæ partis Insulæ Majoris Caspii.

Aug. 8. Luna orta est paulo infra Palilicium, cum quo tamen ob nubes conferri non potuit. Apud *Wansted* autem $13^h\ 2'\ 00''$. T. app. visa est præcedens contiguarum, ad σ Tauri *Bayero*, in linea recta per cuspides Lunæ, distans ab Austrino $4'\ 36''$ Ad $13^h\ 7'\ 25''$ Stella *p* ejusdem Catalogi emersit paulo intra medium obscuri limbi. Ad $13^h\ 9'\ 4''$ emersit sequens contiguarum dictarum, tantum distans a Cornu Austrino quantum contiguæ illæ inter se, *h. e.* 7 Min.

Sept. 5. Mane, Sole jam fere 30 gr. alto, vidi apud *Wansted* arctissimum Lunæ infra Palilicium transitum, cujus distantiam, a limbo proximo, ad $7^h\ 59'\ 00''$ T. æq. Micrometro inveni $5'\ 38''$. Ad $8^h\ 17'\ 5''$ distabat a limbo $1'\ 25''$. Stella autem ad $8^h\ 33'\ 15''$ erat in linea recta, per Lunæ cuspides tum obtusiusculas, nec nisi $0'\ 13''$ distabat a

By Mr. James Bradley, *ibi* p. 858.

Borea

Borea. 8^h 41′ 00″ jam Cuſpidem illam reliquerat 3′ 42″. Et 8^h 45′ 37″ ab eadem diſtabat 5′ 36″. Lunæ diameter ad 8^h 58′ capta eſt 31′ 7″.

XLVI. *Papers Omitted.*

*n.*291. *p.*1594. 1. Obſervatio Eclipſis Lunæ *Dec.* 12. 1703. a *J. Hodgſon, Londini.* The ſubſtance of this Tract is repeated in *Engliſh*, p. 285.

*n.*354. *p* 692. 2. An Advertiſement to Aſtronomers of the Advantages that may accrue from the Obſervations of the Moon's frequent Appulſe to the *Hyades* during the Years 1718, 1719 and 1720.

XLVII. *Accounts of Books.*

*n.*283. *p.*1312. Aſtronomiæ Phyſicæ & Geometricæ Elementa. Auctore Davide Gregorio, *M. D.* Aſtr. Prof. Savil. & R. S. S. *Oxon.* 1702.

CHAP. IV.

Mechanicks, Acouſticks.

I. *Account of Dr.* Hook's *Marine Barometer, by Dr.* Halley, *n.* 269. *p.*791.

THe Mercurial Barometer requiring a perpendicular poſture, and the Quick-ſilver vibrating therein with great Violence upon any Agitation, is therefore uncapable of being uſed at Sea, (though it hath lately been contrived to be made portable) ſo it remained to find out ſome other Principle, wherein the Poſition of the Inſtrument was not ſo indiſpenſably neceſſary. It is about forty Years ſince, that the *Thermometers* of *Robt. de Fluctibus*, depending on the Dilatation and Contraction of included Air by Heat and Cold, have been diſuſed, upon diſcovery that the Airs preſſure is unequal; that inequality mixing it ſelf with the Effects of the warmth of the Air in that Inſtrument. And inſtead thereof was ſubſtituted the ſeal'd Thermometer, including Spirit of Wine (firſt brought into *England*, out of *Italy*, by Sir *Robert Southwell*) as a proper Standard of the Temper of the Air in relation to Heat and Cold; that ætherial Spirit being of all the known Liquors the moſt ſuſceptible of Dilatation and Contraction, eſpecially with a moderate degree of either Heat or Cold. Now this being allowed as a Standard, and the other Thermometer that includes Air being graduated with the ſame Diviſions, ſo as at the time when the Air was included, to agree with the Spirit-Thermometer in all the degrees of Heat and Cold, noting at the ſame time the preciſe height of the Mercury in the common Barometers: It will readily be underſtood that whenſoever theſe two Thermometers ſhall agree, the preſſure of the Air is the ſame it was, when the Air was in-

included and the Inftrument graduated: That if in the Air-Thermometer the Liquor ftand higher than the Divifion marked thereon, correfponding with that on the Spirit-Glafs, it is an Indication that there is a greater preffure of the Air at that time, than when the Inftrument was graduated. And the contrary is to be concluded when the Air-glafs ftands lower than the Spirit, *viz.* that then the Air is fo much lighter, and the Quick-filver in the ordinary Barometers lower than at the faid time of Graduation.

And the fpaces anfwering to an Inch of Mercury will be more or lefs, according to the quantity of Air fo included, and the fmallnefs of the Glafs Cane, in which the Liquor rifes and falls, and may be augmented almoft in any proportion, under that of the fpecifick gravity of the Liquor of the Thermometer to Mercury. So as to have a Foot or more for an Inch of Mercury, which is another great Convenience.

It has been obferved by fome, that in long keeping this Inftrument, the Air included either finds a means to efcape, or depofits fome Vapours mixt with it, or elfe for fome other Caufe becomes lefs Elaftick, whereby in procefs of Time it gives the height of the Mercury fomewhat greater than it ought; but this, if it fhould happen in fome of them, hinders not the ufefulnefs thereof, for that it may at any time very eafily be corrected by Experiment, and the rifing and falling thereof are the things chiefly remarkable in it, the juft height being barely a Curiofity.

In thefe parts of the World, long Experience has told us, that the rifing of the Mercury forebodes fair Weather after foul, and an Eafterly or Northerly Wind; and that the falling thereof, on the contrary, fignifies Southerly or Wefterly Winds, with Rain, or ftormy Winds, or both; which latter it is of much more confequence to provide againft at Sea than at Land; and in a Storm, the Mercury beginning to rife is a fure Sign that it begins to abate, as has been experienced in high Latitudes both to the Northwards and Southwards of the Equator.

The Form of this Inftrument is fhown in the Cut: wherein, *Plate* 4.

A B reprefents the Spirit-Thermometer, graduated from 0, or *Fig.* 9. the freezing Point, through all the poffible degrees of the heat or cold of the Air, at leaft in thefe Climates.

C D is the Air-Thermometer, graduated after the fame manner, with the like degrees.

E F is a Plate applyed to the fide of the Thermometer *CD*, graduated into Spaces anfwering to Inches and Parts of an Inch of Mercury, in the common Barometers.

G. A Hand ftanding on the Plate at the height of the Mercury thereon, as it was when the Inftrument was graduated, as fuppofed here at $29\frac{1}{2}$ Inches.

L M a Wire on which the Plate *EF* flips up and down, parallel to the Cane of the Thermometer *C D*.

K, Any point at which the Spirit ſtands at the time of Obſervation; ſuppoſe at thirty eight on the Spirit-Thermometer; ſlide the Plate *E F* till the Hand *G* ſtand at 38 on the Air-Thermometer, and in caſe the Liquor therein ſtand at 38 likewiſe, then is the preſſure of the Air the ſame as at the time of Graduation; *viz.* 29, 5; but if it ſtand higher, as at 30 at *I*, then is the preſſure of the Air greater; and the diviſion on the ſliding Plate againſt the Liquor ſhews the preſent height of the Mercury to be 29 Inches 7 Tenths.

I had one of theſe Barometers with me in my late Southern Voyage, and it never failed to prognoſticate and give early notice of all the bad Weather we had, ſo that I depended thereon, and made Proviſion accordingly; and from my own Experience I conclude that a more uſeful Contrivance hath not for this long time been offered for the benefit of Navigation.

A new Baroſcope, by Mr. Caſwell, n. 290 *p.* 1597. *Fig.* 10. *Plate* 4.

II. Suppoſe *ABCD* is a Bucket of Water, in it the Baroſcope *xrezyosm* which conſiſts of a Body *xrsm*, and a Tube *ezyo*, the Body and Tube are both concave Cylinders communicating with each other, and made of Tin (for want of Glaſs:) the bottom of the Tube *zy* has a Lead-weight to ſink it, ſo that the Top of the Body may juſt ſwim even with the Surface of the Water by the Addition of ſome Grain-weights on the Top. The Water when the Inſtrument is forc'd with its Mouth downwards gets up into the Tube to the height *yu*. There is added on the top a ſmall concave Cylinder, which I call the Pipe, to diſtinguiſh it from the bottom ſmall Cylinder, which I call the Tube: This Pipe is to ſuſtain the Inſtrument from ſinking to the Bottom; *md* is a Wire; *ms*, *de* are two Threads oblique to the Surface of the Water, which Threads perform the Office of Diagonals: For that while the Inſtrument ſinks more or leſs by the Alteration of the Gravity of the Air, there where the Surface of the Water cuts the Thread, is form'd a ſmall Bubble, which Bubble aſcends up the Thread while the Mercury of the common Baroſcope aſcends.

The Circumference of the Body is 21 Inches, therefore its Area$=35$: the Altitude $ms=4$, therefore the Bodies Solidity$=140$, each Baſe xm, rs, has a Convexity whoſe Altitude is $.65$, therefore the Conoid on each Baſe is nearly$=11\frac{1}{2}$, therefore d the whole Body is $=(140+11\frac{1}{2}+11\frac{1}{2}=)$ 163, and b the entire Altitude of the Body $=(4+.65+.65=)5.3$. The Inner Circumference of the Tube is 5.014, therefore its Area $n=2$ the length of the Tube$=4.5$, therefore the Tube's Capacity$=9$, therefore c, the Content of the Body and Tube$=163+9=172$ Cubic Inches, that is almoſt $2\frac{1}{2}$ Quarts.

Suppoſe the Air's Preſſure when greateſt$=30.5$ Inches of ☿$=(30.5\times14=)$ 427 of Water, and $f=427$, therefore $fc=73444$. Put a for the Depth ou, of the Air in the Tube when the Body is juſt all immers'd, the Air in the Inſtrument on Immerſion contracts ſomewhat by the cold of the Water; this Contraction I find is nearly as much as would be produc'd

duc'd by an Addition of 1 Inch to the Atmosphere's Altitude 427, this in cold Weather, but in warm Weather 'tis probably twice as much: but we will now suppose it $=1$, therefore the Depth of the Surface of the Water in the Tube below the Surface of the outer Water is $=b+a$; therefore the Pressure on that inner Surface is as the Altitude of the Atmosphere above it $=f,+b+1+a=F+a$ (putting $F=f+b+1$.) Then for that the Spaces into which the Air is contracted, are reciprocal to their respective Pressures, and for that while the Instrument is out of the Water the Pressure f answer'd to the Space c, therefore $F+a:f::c:\frac{fc}{F+a}=$ Space which the Air takes up in the Instrument under Water; therefore, $\frac{fc}{F+a}-d=$ that part of the Tube which is possess'd by Air $=an$ (supposing the Tube's Area $2=n$.) Therefore $fc-Fd-ad=Fan+aan$. Therefore $aa+F+\frac{d}{n}\times a=\frac{fc-Fd}{n}$. Put $F+\frac{d}{n}=2g$, therefore $aa+2g$ $a=\frac{fc-Fd}{n}$ therefore $a=\sqrt{:\frac{fc-Fd}{n}+gg}\,-g$

Then suppose the Atmosphere's Gravity less so much as to sink the ☿ $\frac{1}{10}$ Inch $=1.4$ of Water, and therefore putting $\varphi=F-1.4$, and in the last Equation α instead of a, and γ instead of g, you have $\alpha=\sqrt{\frac{fc-\varphi d}{n}+\gamma\gamma}:-\gamma$. Thus I find $\left.\begin{matrix}a=2.72\\ \alpha=2.94\end{matrix}\right\}$ and therefore $\alpha-a=.22$, which $.22\times n$ gives .44 Cubic Inches, and (supposing a Cube-Inch $=253$ Grains) $.44\times 253=111$ Grains weight of Water that was gotten up into the Tube in the 1st Case more than in the 2d, and therefore the Baroscope requires an Addition of 111 Grains on its Top to sink it with the Level of the Water in the 2d Case more than in the 1st, and this upon the sinking of the ☿ in the common Baroscope only $\frac{1}{10}$ Inch: Now 1 Grain in this new Baroscope is nearly as discernable as $\frac{1}{10}$ Inch in the common, and therefore this new Baroscope is more exact than the common 111 Times.

Put $f=427$. $c=172$. $d=163$. $n=2$ as above, only change F, put F 437.3, that is, suppose the Body sunk in Water 4 Inches lower; in this Case $\alpha=2.08$, therefore $a-\alpha=.64$ which multiplyed into $\varphi n=1,28$ Cubic-Inches, which $\times$ 253 gives 324 Grains, and so much the Body's top $x\,m$ being sunk 4 Inches under Water, the Body becomes heavier, than while $x\,m$ was at the Surface of the Water. Therefore this 1.28 divided by the aforesaid Depth 4 gives .32 the Area of the top Pipe such as would ballance or buoy up the Body at any Depth. Strictly speaking, the Pipe should be gradually bigger upward in order to sustain the Instrument at any Depth, but as to Sense 'tis Cylindrical, and its Circumference $=2.005$. But for that the least Alteration of the Air would make the Body's top $x\,m$ in that Case pass through the 4 Inches (which 4 Inches I suppose all the Variety of Depth that the Instrument has room given it in the Bucket to

aſcend or deſcend) therefore the Pipe is made a ſmall matter bigger, (*viz.*) its Circumference is 2.14; whereby the Pipe, according as the Body ſinks more, gives more reſiſtance to the deſcending Body. The Pipes Area is .3643: Therefore the Capacity of the Pipe in 4 Inches Altitude is =1.457. But as aboveſaid to give juſtly no Reſiſtance, its Capacity ſhould be 1.28. Therefore this 1.28 taken from 1.457, leaves .177 the actual Reſiſtance in 4 Inches depth, *viz.* (.177 × 253=) 44 Grains.

But this Reſiſtance will not be the ſame in all Weathers, in order therefore to calculate what it will be when the ☿ of the common Baroſcope is very low, for example, but 28 Inches high=392 of Water; f muſt be ſuppos'd=392, therefore $F=f+b+1=398.3$, and the reſt as before; *viz.* $d=163$, $fc=67424$. $Fd=649229$. Thence by the aforeſaid Equation $a=2.59$, $\alpha=2.84$ Therefore $\alpha-a=.25$, which × n gives .50 Cubic Inches, which ×253=126 Grains. So that this Baroſcope when the ☿ is loweſt, is more exact than the common 126 Times, ſuppoſing the Body immers'd afreſh when the ☿ is ſo low.

Next while the ☿ is ſo very low, ſuppoſe the top of the Body depreſs'd four Inches under Water; therefore $\varphi=F+4=402.3$, the reſt are as before, *viz.* $fc=67424$, then α will be 1.9: but before, while the top of the Body was at the Surface, α was 2.59. Therefore the Difference .69 × Tube's Area 2, gives 1.38 Cubic-Inches, which × 253 gives 349 Grains, and ſo much the Baroſcope is heavier when the top xm is 4 Inches under Water, or which comes to the ſame, ſuppoſing that ☿ at 28, and xm at the Surface; this Baroſcope by the ☿'s aſcending $\frac{4}{14}$ Inch will become heavier 349 Grains. The Pipe's Capacity in 4 Inches Altitude was 1.457, from which take the aboveſaid 1.38, the reſidue =.077, which × 253 gives 19 Grains in 4 Inches; ſo that the Pipe will ſuſtain the Baroſcope, and alſo 44 when the ☿ is $30\frac{1}{2}$ high, and but 19 Grains when the ☿ is 28 high. The fewer Grains difference there are in its ſinking, through 4 Inches, the more nice the Baroſcope will be.

There where the Thread cuts the Surface of the Water, is form'd a Bubble; therefore this Bubble while the Inſtrument ſinks in Water 4 Inches, which is all the room that I give it, moves on the 2 Diagonal Threads twenty Inches; it follows therefore that 120 Grains difference would make the Bubble walk over 120 Inches, if the Threads were ſo long; but, as it has been above calculated, about 120 Grains difference of Weight of the Inſtrument is produc'd by ſo much of the Alteration of the Air, as would make the ☿ of the common Baroſcope $\frac{1}{10}$ Inch: therefore when the ☿ aſcends $\frac{1}{10}$ Inch, the Bubble of this new Baroſcope aſcends 120 Inches; therefore this new Baroſcope is more exact than the common Baroſcope by about 1200 times.

Obſervations with this Baroſcope.

1. While the ☿ of the common Baroſcope is often known to be ſtationary 24 Hours together, the Bubble of the new Baroſcope is rarely found to ſtand ſtill one Minute.

2. Suppoſe the Air's Gravity encreaſing, and accordingly the Bubble aſcending, during the Time that it aſcends 20 Inches, it will have many ſhort

ſhort deſcents, of the quantity of ½ Inch, 1, 2, 3, or more Inches, each of which being over it will aſcend again. Theſe retroceſſions are frequent, and of all Varieties in quantity and duration, ſo that there is no judging of the general courſe of the Bubble by bare Inſpection though you ſee it moving, but by waiting a little time.

3. A ſmall Blaſt of Wind will make the Bubble deſcend; a Blaſt that can't be heard in a Chamber of the Town, will ſenſibly force the Bubble downward. The Blaſts of Wind ſenſible abroad cauſe many of the abovesaid Retroceſſions, or Accelerations in the general Courſe; as I found by carrying my Baroſcope to a place where the Wind was perceptible.

4. Clouds make the Bubble deſcend. A ſmall Cloud approaching to the Zenith works more than a great Cloud near the Horizon. In cloudy Weather the Bubble deſcending, a break of the Clouds (or clear place) approaching to the Zenith, has made the Bubble to aſcend; and after that break had paſs'd beyond the Zenith a conſiderable Space, the Bubble again deſcended.

5. All Clouds (except one) hitherto by me obſerv'd, have made the Bubble to deſcend. But the other Day the Wind being *North*, and the courſe of the Bubble deſcending, I ſaw to the Windward a large thick Cloud near the Horizon, and the Bubble ſtill deſcended; but as this Cloud drew near the Zenith it turn'd the way of the Bubble making it to aſcend, and the Bubble continued aſcending till the Cloud was all paſs'd, after which it reſumed its former deſcent. It was a Cloud that yielded a cold Shower of ſmall Hail.

An Improvement of the Heſſian Bellows, by Mr. Papin, n. 300. p. 1990. Plate 4.

III. The ſhape of the *Tympanum* was Cylindric, as may be ſeen Fig. 11. where *D A F C* is the Circumference: *CP*, *DP*, *AP*, are the *Radii* which bear the Wings *Cm*, *Dn*, *Ao*: *Be* is the aperture through which the Wind muſt be driven in the Direction of the Tangent *CB*: And it may be obſerv'd, that when the Engine is working, every Wing from the end of the aperture *e*, till it comes to the beginning of the ſame aperture *C*, drive always the ſame Air, with the ſame Swiftneſs, and at the ſame diſtance from the Center: So that in peruſing all that Circumference, the Air doth find reſiſtance by Friction, and gets nothing at all. I do therefore now make the Circumference of the *Tympanum* in a ſpiral ſhape, which is to be ſeen Fig. 12. where the ſpiral Circumference is *A F G B*, the *Radii* are *AP*, *CP*, *DP*, &c. The Wings are *AM*, *CN*, *DO*, &c. The aperture is *AB*. And it is to be obſerved, that every Wing in going round drives new Air, becauſe the Air which is firſt in Motion finds place to recede from the Center towards the ſpiral Circumference; and ſo it gives room to new Air to come to the Wing: and when the Wings come near to the aperture, they drive their new Air into the aperture without any Friction; and the Air which hath been firſt driven and removed from the Wing, cannot loſe its Swiftneſs, becauſe the Wings which continually follow, do continually drive new Air, which keeps that which is before always in the ſame Swiftneſs. This new ſhape of the *Heſſian* Bellows affords alſo another advantage; becauſe the Air in going

going round follows the ſpiral Line, which is nearer to the ſtraight Line than a circular Circumference; and when the Air comes to the Aperture, it gets into it without any loſs of ſubſtance; but in the Cylindrical Machine, Fig. 12. the air doth always go round in a circular Circumference; and when it comes to the aperture, the Wind is driven directly in the direction of the Tangent but juſt in the beginning at *C*; and afterwards the Impulſion is oblique: and this obliquity is always increaſing until the Wing comes to the *Punctum A*: Now it is known how much Diminution ſuch an obliquity can make to the ſtrength. I believe therefore that this ſpiral Figure is a good Improvement to this Engine. And indeed I have made ſuch Bellows where the *Radius A P* is but 10½ Inches, the Wing *A m* 2 Inches broad and 9 Inches high; becauſe the *Tympanum* is alſo ſo high, or a little more; the aperture *A B* is alſo 9 Inches, or a little more, ſo that it makes a ſquare hole. When I work this Engine with my Foot, it makes ſuch a Wind, that it may raiſe up 2 Pounds weight; and without doubt a ſtronger Man could do much more: But this is more than ſufficient for our purpoſe, ſince we muſt but drive air enough for the Reſpiration of ſuch Men that can work in the Mine; and we may eaſily with Boards make wooden Pipes, to carry the Wind to the very bottom: So that the Air within will be continually renewed as well as without.

Of the Motion of Sound, by Mr. Derham, *n.* 313. *p.* 2.

IV. Diſſenſus inter authores celeberrimos de Soni Velocitate facili intuitu in ſequente Tabella conſpici poteſt: in qua (Pedibus Anglicanis) Spatium exhibetur quod Sonorum Progreſſui in uno Minuto Secundo Temporis aſcribunt.

	Pedes,	
D. Iſ. Newton, Eq. Aur.	968	Prin. Ph. Nat. Math. L. 2. Prop. 50.
Nobilis D. Roberts	1300	Philoſ. Tranſact. N. 209.
Nobilis D. Boyle	1200	Eſſay of Languid Motion p. 24.
D. Walker	1338	Philoſ. Tranſact. N. 247.
Merſennus	1474	Baliſtic. Prop. 39.
D. Flamſteed & Halley	1142	
Florentini celebres	1148	Exp. per Acad. del Cimen. p. 141.
Galli celebres	1172	Du Hamel Hiſt. Acad. Reg.

Inter ultimum & penultimum diſſenſus non eſt magnus, & *Gallorum* non multo major; cæterorum vero magnus eſt. Et Ratio manifeſte hæc eſt; vel ſcilicet ab Inſtrumenti defectu; vel a Diſtantia; vel a Ventis.

1 *Inſtrumentum*, quo nonnulli virorum horum inclytorum dimenſi ſunt, non fuit *Automaton*; ſed *Bolis funipendula*, quæ Minuta Secunda vibrat. Sed omnibus, in hiſce rebus exercitis, manifeſtum eſt, Bolidem multo minus commodam eſſe, nec tam accuratam ac Automaton; quoniam neceſſarium eſt, Oculum primo occupatum eſſe in obſervando Coruſcationem, deinde ad Bolidem, ſive Pendulum reſpicere: quod tempus conterit, & confuſionem creat. Hoc autem, una cum Senſuum, & Captus ſive attentionis

tionis nostræ tarditate, magnum errorem efficere potest; uti bene notum est iis qui Experimenta de his fecerunt. Præsertim si

2. *Intervallum* inter rem sonantem, & Observatorem parvum fuerit. At vero manifestum est, quod plerique istorum laudatorum Virorum Experimenta sua fecerunt ad intervallum tantum paucorum Pedum, & per Soni reditum, sive Echo dimensi sunt. Horum enim nonnulli vix ultra 6 vel 700 pedes mensurationem extendebant, aliique non ultra Milliare unum. Sed semper observavi ambiguitatem oriri in tam parva distantia, quamvis optimum adhiberetur Instrumentum. Errorque levissimus in tantillis distantiis, magnus est habendus. Nam Pendulum forsan dimidium sui diadromi, sive arcus, præteriit ab ultima pulsatione, cum Sonus primo fuerit emissus: Sed nos istum Pulsum numeramus, ac si Vibratio fuisset tota & completa; vel forsan Vibrationem anticipamus. Et postquam Sonus nos pertigit, forsan plus vel minus quam par est numeramus.

Vel si Distantia sat fuerit longa, tamen error exinde potest oriri, si

3. *Ventorum* ratio non sit habita. De quo in sequentibus.

Hæc sunt certa, inevitabilia, & perpetua incommoda, quæ Mensurationem progressus Sonorum comitantur: quæ in parvis intervallis (ut dixi) præsertim si Instrumenta mala sint, magnos errores producere possunt: & sine dubio maxima fuere causa tanti inter tantos authores dissensus.

Sed observari potest, quod Spatia a tribus ultimis in Tabella Observatoribus assignata, quam proxime conveniunt. Quod proculdubio hinc provenit, quia nempe bonis Automatis instructi fuerunt. In quorum usu, *Auris* sola occupatur in Vibrationibus Penduli excipiendis, dum *Oculus* attendit Coruscationem, sive aliquam aliam Soni emissionem. Hæ quoque Observationes intervallis longinquis factæ fuere, in quibus error pusillus non magni erit. Dominorum enim celeberrimorum *Flamsteedii* & *Halleii* Observationes factæ sunt ad intervallum trium fere milliarium (paucis Perticis plus vel minus exceptis) ab Observatorio Regio, super Collem *Shooterianum*: & Sonus advenit in $13\frac{1}{2}$ Secundis Temporis. Nobiles isti *Florentini* & celeberrimi ex *Acad. del Cimento* ad idem fere intervallum Expe imenta sua fecerunt; & quædam ad intervallum unius tantum Milliaris. Et denique celeberrimi D. D *Cassini*, *Picard*, & *Roemer* ad intervallum 1280 Hexapedarum Gallicarum, quod est plus quam $1\frac{1}{2}$ Milliare Anglicanum.

Ut veritas inter prædicta dissidia innotescat, experimenta plurima ad varia intervalla feci; Scil. ab uno, ad 12 milliaria, & plura. Et ad Tempus dimetiendum, habeo accuratissimum *Automaton portabile*, cum Pendulo Semisecunda vibrante.

Tutius autem ut procederem, sequentes Quæstiones a meipso discutiendas proposui.

1. Quantum Spatium Sonus percurrit in Secundo Minuto Temporis, vel alio Temporis intervallo?

2. An Sclopus versus Observatorem displosus, in eodem temporis intervallo Sonum mittit, ac cum in contrariam partem displodatur?

3. An

3. An in quolibet Atmofphæræ ftatu, cum Mercurius in Barometro afcendit vel defcendit, Soni percurrunt idem Spatium in eodem Temporis intervallo?

4. An Soni velocius Die quam Nocte moventur?

5. An Ventus favens Sonum accelerat, & adverfus retardat? Sive an, & quomodo Venti Sonum afficiunt?

6. An tranquillo Cœlo Sonus velocius movetur, quam Vento flante?

7. An vehemens Ventus in tranfverfum flans accelerat, an retardat motum Soni?

8. An Soni eundem habent motum Æftate ac Hyeme, Die ac Nocte?

9. An etiam in Nivofo, ac Sudo Cœlo?

10. An Sonus magnus & exiguus eundem habent motum?

11. An in omnibus Sclopeti elevationibus, viz. Horizontali, 10 gr. 20 gr. ad 90 gr. Sonus in eodem temporis intervallo Obfervatoris aurem pertingit?

12. An omnimodi Soni, Sclopetorum, Campanarum, Malleorum, &c. eundem habent motum?

13. An variæ Pulveris Pyrii vires motum Soni variant?

14. An in Culminibus Montium altorum, & Vallibus; five in fummis Atmofphæræ partibus, & imis, Soni idem percurrunt Spatium in eodem Temporis intervallo?

15. An Sonus acclivis & declivis eundem habet motum? Sive an a Jugo Montis defcendit in Radicem eodem paffu, ac a Radice afcendit in Jugum?

16. An Sonus principio velocius, & in fine tardius movetur, ut in plurimis aliis motibus violentis accidit?

17. Annon potius fit æquabilis? Nempe Annon in dimidio Temporis, dimidium Spatii; in quarta parte Temporis, quartam partem Spatii, &c. movetur?

18. An in omnibus Regionibus, Septentrionalibus, & Auftralibus, in *Anglia*, *Gallia*, *Italia*, *Germania*, &c. eundem habent motum?

19. An Sonus recta, five breviffimo itinere, a loco in locum tranfit; an fecundum fuperficiem interjacentis telluris?

Ad hæc determinanda Amicos Generofos mihi vicinos petii (quorum beneficia hic gratiffime agnofco) ut Sclopos ex Turribus, aliifque locis eminentibus difploderent, ad intervallum 1, 2, 3, ufque ad 8 millia paffuum (quod maximum effe intervallum reperi, ex quo Sclopi Sonum audire potui in his partibus, arboribus, *&c.* obfitis.) Hæc Sclopeta magno mihi fuerunt ufui. Sed Tormenta, quæ maxime propofito inferviebant, bellica illa fuerunt apud *Blackheath* [*Sakers* vocata] quæ exercentur in educandis Tyronibus Tormentariis Inclytiffimæ noftræ Reginæ miniftraturis. Horum Tormentorum micantes flammulas ex Ecclefiæ meæ turre videre, & fragorem audire potui in omnibus fere cœli tempeftatibus; etiam interdiu, ope Telefcopii. Ideoque cum omni cura & diligentia me ad horum Tormentorum obfervationem accinxi, ufque a Februario 170¾.

Poft paucas obfervationes inter eorum difplofiones factas, fpeciale quoddam experimentum faciendum comparavi, benignitate nuperi D. Baronis *Granville*

Granville tunc Præfecti, & cæterorum Virorum clarorum qui in *Turre Londinenſi* Rei Tormentariæ Regiæ miniſtrant (quorum beneficia hic gratiſſime agnoſco.) Duo Tormenta bellica (*Sakers* vocata) juxta ſe ſita ſunt, adverſo unius ore, averſo alterius. Hæc duo Tormenta Feb. 13. 170$\frac{4}{5}$. diploſa fuere, unaquaque Semihora ab Hora ſexta pomeridiana ad mediam noctem, leni aura directe adverſus Sonum ſpirante. Temporis intervallum inter Coruſcationem ſinguli Tormenti (quam nudo oculo videre potui) & Soni adventum, ſemper fuit circiter 120 vel 122 Semiſecunda Temporis. Dixi 120 vel 122, quoniam Sonus duplicatus advenit; ſcil. prior Sonus intra 120 Semiſecunda (qui languidior) ſecundus intra 122 (qui intentior.) Et eodem modo, per totum obſervationis tempus, ſingulorum Tormentorum fragor advenit, nempe duplicatus.

Hæc Reduplicatio Soni mihi videtur Echo, repercuſſa, ut opinor, a *Molendino Blackheathenſi*, vel Domibus juxta ſitis. De quo nullam habeo Dubitandi rationem, præter ſententiam contrariam Amici cujuſdam docti, & ſagacis Philoſophi; qui credit nullam Echo audiri, niſi quæ facta eſt per Objecta Phonocamptica non procul ab Obſervatore, non per ea prope Vocale, ſive Sonorum, vel alia longinqua Objecta.

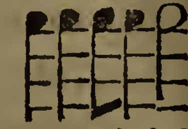
§. 2. *De Sonis longe repercuſſis, ſive Echo longinqua.*

Pro Digreſſione forſan habebitur hæc Diſquiſitio: ſed quoniam ad Soni ſubjectum attinet, ideo paucas de hac re Obſervationes ingenioſis non fore ingratas ſpero.

Et primo hoc Legibus Echus non contrarium credo. Deinde notandum eſt, hunc duplicem Sonum directe a *Blackheath* veniſſe: neque enim prior Sonus inde veniebat, & alter (Echus more) aliunde; nempe ultra me, vel a dextra, vel ſiniſtra, vel ab ulla alia parte. Idemque ſæpius obſervavi, cum Tormenta magna e Navibus diſploderentur in *Fluvio Thameſi* (præcipue ſi aer fuerit ſerenus & tranquillus veſperi & mane, cum Tormenta Vigilaria (Anglice *Watch-guns*) exonerarentur. Poſtquam Fragor Tormenti aurem pertigit, audivi eum longe percurrentem ſecus Fluvium, & a Ripa, Montibus, & Scopulis (juxta littus *Cantianum* confertim ſitis) per plura Milliaria reboantem.

Hæc omnia, inquit Amicus, a Repercuſſione Domuum, &c. prope Te proveniunt. Sed nequid de Debilitate Soni dicam, poſtquam plurima Milliaria percurrit, & de ejuſdem incapacitate, ſi tam procul veniſſet, ut repelleretur per objecta Phonocamptica juxta Obſervatorem, potius quam per Objecta Phonocamptica juxta Sonorum ſita; (ut nihil de his dicam) exemplum unum vel alterum dabo, unde plane conſtabit, quod Echo facta per Objecta Phonocamptica prope Rem vocalem ſive ſonantem poteſt per plura milliaria audiri, æque ac primarius Sonus, aliquando etiam eodem intentior.

Sæpe obſervavi Tormenta magna bellica e Navibus in *Thameſi Fluvio* veſpere diſploſa circa loca vocata *Deptford* & *Cuckolds-Point*, plerumque fragorem edere duplicatum, triplicatum, quadruplicatum, vel adhuc amplius multiplicatum; & quod Fragores poſteriores ſunt magis Sonori. Et

cum hinc & illinc Stadium, etiam quadrantem vel dimidium Milliaris in tranſverſum ivi, Sonus tamen idem fuit. Memini quod octavo Martii noviſſime elapſi plurimæ Bombardæ magnæ diſploſæ fuere alicubi inter *Deptford* & *Cuckolds-Point* prædictas, e Nave quam in *Thameſi* ex mea Eccleſia proſpexi. Harum fragor quinquies vel ſexies hoc modo repetitus  eſt. Inter Coruſcationem & Sonum 122 Semiſecunda numeravi Vento in tranſverſum flante. Tunc temporis ideo Tormenta a me diſtabant pluſquam 13 millia paſſuum. Duo primi crepitus languidiores erant quam tertii; ſed crepitus ultimi omnium maxime Sonori. Et cum a dextra quadrantem milliaris tranſiviſſem, idem fuit multiplicatus Sonus: & cum a ſiniſtra, idem. Et præterea in aliquibus meis Stationibus, præter multiplicatum Sonum, plane audivi languidam Echo ab Eccleſia mea, ſive Domibus adjacentibus repercuſſam: quod ſæpenumero tunc obſervavi, quoties Tormenta diſploſa fuere.

Alia hujus generis Obſervatio fuit quodam die Dominico, circa biennium vel triennium abhinc, ex Sono Tormenti magni militaris diſploſi alicubi in *Thameſi* fluvio cis vel trans Oppidum *Graveſend* vocatum. Hujus Tormenti fragor fuit multiplicatus ad minimum octies, novies, vel decies, ſecundum hanc temporis menſuram  Hunc multiplicatum Sonum plurimi (ad Dei cultum tunc temporis accedentes) putabant eſſe Fragores multorum Tormentorum e Nave dimicanti: (Sed ut opinor, nil aliud fuit niſi Echo polyphona, ex unius vel alterius & alterius Tormenti diſploſi Sono, a Navibus pluribus vel Littore juxta ſitis repercuſſo) Quod autem pro me facit, hoc eſt, quod non ipſe ſolum (dum in meo Horto deambularem) audivi, ſed etiam multi alii qui procul diſtabant. Item D. *Barret* R. S. S. domi ſuæ eundem repetitum Sonum audivit, ad intervallum 4 fere milliarium ab *Upminſter*, ubi ipſe audivi.

Ex quibus omnibus luculenter conſtat ſententiam prædicti Amici (plurimis nominibus colendi) eſſe falſam.

§. 3. *De Echo, ſive Repercuſſione Sonorum in Aere.*

His de *Echo* dictis, ſpero non ingratum fore exemplum adjicere de *Repercuſſione Soni ab aereis particulis*; quod poteſt confirmare quæ dicta ſunt.

Cum audivi Fragores magnorum Tormentorum bellicorum, præcipue in tranquillo & ſereno Cœlo, ſæpenumero obſervavi *Murmur*, excelſe in aere, præceſſiſſe Crepitum. Et in *Nebula tenui*, ſæpe Bombardarum Sonum audivi in Sublime, ſupra caput, in aere, per plurima Milliaria percurrentem; adeo ut Murmur iſtud per 15″ temporis perduraverit. Diutinum hoc Murmur, meo judicio, provenit a Particulis Vaporoſis in atmoſphæra ſuſpenſis, quæ curſui Undulationum Soni oppugnant, eaſque Obſervatoris auribus reverberant, indefinitarum Ἠχῶν more: quas *Murmur in aere* vocamus.

His rite perpenſis, manifeſtum erit Echo longinque factam poſſe audiri; & Reduplicationem iſtam prædictam fragoris Tormentorum *Blackheathenſium* proculdubio veniſſe ab ipſo *Blackheath*, prout modo aſſerui.

§. 4.

§. 4. *De Sonis Sclopetorum omnifariam displosorum*, &c.

Ut autem redeam a Digressione de Repercussione Sonorum, pergam ad Observationes meas de eorum *Progressu*, quas ex plurimis Experimentis feci. Et quod jamjam de Sono Bombardarum *Blackheathensium* suggessi, in omnibus aliis reperi, viz. Motum Soni nec velociorem nec tardiorem esse, sive Tormentum versus Observatorem, sive a contrario displodatur. In omnibus item *Sclopi positionibus*, Horizontali, Erecta; & in omnibus ejusdem elevationibus, 10 gr. 20 gr. &c. nulla est variatio Soni. Adeo vera est illa de hac re observatio Nobilium & Inclytorum istorum Virorum ex *Academia del Cimento Florentiæ*.

Pulveris Pyrii quoque Vis, sive sit fortis, sive debilis, ejusque major vel minor Quantitas, licet augeat, vel minuat Sonum, non tamen accelerat, aut retardat ejusdem Motum.

§. 5. *De Motu Soni in qualibet Cæli, & Anni tempestate.*

Kircherus dicit *se semper diversam Soni celeritatem invenisse, diversis temporibus, mane, meridie, vesperi, noctu.* Sed ego (meliore Chronometro fretus, & commodiore distantia) nunquam reperi celeritatem Soni esse diversam his temporibus In omni autem tempestate, sive Cœlum sit *Sudum* & *Serenum*, sive *Nubilosum* & *Turbidum*; sive *Nix* decidit, sive *Nebula* (quæ ambæ fortiter retundunt Soni Audibilitatem) seu *Tonat*, aut *Fulgurat*; sive *Æstus* vel *Frigus* adurit; sive *Dies*, vel *Nox* sit, *Æstas* vel *Hyems*; sive *Mercurius* in *Barometro* ascendit vel descendit: verbo dicam, in omnibus quibuscunque Atmosphæræ mutationibus (Ventis tantum exceptis) Motus Soni nec velocior nec tardior est; tantum magis vel minus clarus est ex illa Medii variatione. Quod forsan *Kircherum* sagacem decepit.

Phonurg. l. 1. c. 1. *Prælus.* 3. *Sect.* 2.

Hinc sequitur Conclusiones *D. Walker* a Doctoris *Plot*, *Kircheri*, & suis ingeniosis Observationibus deductas erroneas fuisse.

Philos. Transj. N. 247. *Abr. Vol.* I. *p.* 506.

§. 6. *De Motu Soni intensi & languidi, & diversorum corporum Sonantium.*

Licet *Kircherus* contra sentiat, attamen non dubito quin omnium corporum Strepitus, Sclopetorum, Campanarum, Malleorum, *&c.* eandem Velocitatem habeant. Anno 1704. Pulsationes Mallei & Fragorem Sclopeti comparavi, ad Milliaris intervallum (maximum spatium ad quod Mallei sonum audire potui) & reperi utrorumq; Sonum in eodem tempore advenisse: & quod $\frac{3}{4}$, $\frac{1}{2}$, & $\frac{1}{4}$ ejusdem Spatii pertransiverunt in $\frac{3}{4}$, $\frac{1}{2}$ & $\frac{1}{4}$ ejusdem temporis. Ibid.

Quod ad Sonos *Intensos* & *Languidos* attinet, non dubito quin idem Spatium in eodem Temporis intervallo percurrant. Ut ex his Experimentis aliqua ex parte manifestum erit.

Jan. 13. 170$\frac{4}{5}$. Archibombardarius Arcis *Tilburiensis* meo hortatu displosit unum & alterum Sclopetum, & Tormentum magnum Militare, in

quod injectum pulverem pyrium bene fiſtulaverat. Horum omnium Strepitus in eodem tempore ad me, tria circiter milliaria diſtantem, pervenit.

Archibombardarius quoque *Angliæ* Sept. 11. 1705. poſt Solis occaſum, in mei gratiam, diſploſit ſuper *Blackheath* nonnullos Sclopos (Anglice *Muskets*,) Tormenta magna bellicoſa (*Sakers* vocata,) & Pyrobola (Anglice *Mortars*). Sclopos exaudire non potui, propter magnam diſtantiam, vel quia aer non ſat ſerenus fuit. Sed Tormentorum & Pyroborum Sonos in eodem Temporis intervallo exaudivi, licet Fragor Pyroboli fuit multo torpidior & remiſſior, quam Tormentorum.

§. 7. *De Æquabilitate Motus Soni.*

Hanc talem eſſe deprehendi, qualem *Academia del Cimento* illuſtris dudum præfinivit. Soni quippe progrediuntur Dimidium Spatium in Dimidio Temporis intervallo; Quartam Spatii partem in Quarto Temporis intervallo; & ſic deinceps. Quod ex exemplis in ſequenti Tabella conſtabit.

Locus quo diſploſio facta fuit.	Penduli Vibrat. numerus.	Diſtantia Locorum. Trigonometrice.	Diſtantia Locorum. Per Sonum	Ventorum Tendentia.
		Milliaria.	Milliaria.	
Hornchurch Eccleſia	9	0, 9875		tranſverſo.
Okendon Bor. Eccleſ.	$18\frac{1}{2}$	2, 004	2, 0	tranſverſo.
Mola Upminſterienſ.	{ $22\frac{1}{2}$	} 2, 4	{ 2, 4	favent.
	23		2, 48	nive: tranſv.
Warley parvæ Eccleſ.	$27\frac{1}{2}$	3, 0	2, 97	forte faven.
Rainham Eccleſ.	$33\frac{1}{4}$	3, 58	3, 59	tranſverſo.
Mola Alveleientis	33	3, 58	3, 57	tranſverſo.
Dagenham Eccleſ.	35	3, 85	3, 78	faventi.
Weal Auſtrin. Eccleſ.	45	4, 59	4, 86	tranſverſo.
ThorndonOrient.Ecc.	$46\frac{1}{2}$	5, 09	5, 03	paulo fav.
Barking Eccleſia	$70\frac{1}{2}$	7, 7	7, 62	favente
TormentaBlackheath	116	12, 5	12, 55	tranſverſo.

Intervalla Locorum ab *Upminſter* (ubi obſervavi) in hac Tabella notata, quanta potui accuratione dimenſus ſum vel Virga Menſuratoria, vel Arte Trigonometrica. Et ex magna conſonantia inter intervalla hoc modo, eademque per Soni motum dimenſa, cum Inſtrumentorum meorum præſtantia, tum Obſervationum & Calculorum veritas patet. Differentia enim inter Intervalla dimenſa, & eadem Sono capta, aut prorſus nulla eſt, aut tantum paucularum centeſimarum partium, niſi cum Ventus fuerit ſecundus (Eccleſia *Weal Auſtrinæ* excepta, de quo poſthac) Ita nempe in Obſervationibus ex Eccleſiis *Dagenhamenſi*, *Warleienſi*, *Thorndonienſi*, & *Barkingenſi* factis, diſtantiæ per Sonum notatæ paulo breviores viſæ ſunt; quia Ventus Sonum acceleravit. At in conficienda hac Diſtantiarum per Sonum Columna, nihil propter Ventorum accelerationem conceſſi; ſed numerum Vibrationum, ſive Semi-ſecundorum tantum diviſi

divisi per $9\frac{1}{4}$, vel 9,25 (numerum Semi-secundorum in quibus Sonus Mille passus transiit.)

Æquabilitas quoque motus Soni ex hac Tabella manifesta est; prout patebit ex collatione Vibrationum & Distantiarum: sive ex sola Columna Distantiarum per Sonum.

Ut autem nihil deesset in harum rerum confirmationem, iter feci ad *Arenas Foulnesianas* in littore nostro *Essexiano.* Hæ Arenæ (Maris quotidiano Æstu allutæ, & obtectæ) faciunt magnam & exactam Planitiem multorum milliarium. Super hanc Planitiem tantum sex milliaria dimensus sum, quia neque Maris æstus, neque mora mea, ut majus intervallum dimetirer, permitterent. Ad cujusque fere Milliaris finem experimenta feci per Sclopetorum Explosionem, non sine magno Vitæ periculo, ex Maris Fluxu, & Noctis tenebris. Ex quibus Experimentis comperi Observationes meas priores omnes justissimas, & veras fuisse, scilicet Sonum unum Milliare pertransire in $9\frac{1}{4}$ Semi-secundis: duo Milliaria in $18\frac{1}{2}$ Semi-secundis: tria Milliaria, in $27\frac{1}{4}$ Semi-secundis, & sic deinceps.

§. 8. *De Acclivi & Declivi Sonorum Motu: sive De eorundem Ascensu & Descensu.* Item an *recta, vel secundum interjacentis telluris superficiem a loco in locum transeunt?*

Quod ad 15 & 19 Quæsita attinet; ingenue fateor me nunquam ullis quæ feci hactenus Experimentis mihimet ipsi super his rebus satisfecisse.

De progressu Soni per brevissimam viam, ratio dubitandi fuit discrepantia inter Spatium *Weal* Villæ & *Upminster* Trigonometrice, & per Sonum dimensum; prout in Tabella præcedenti exhibetur. Mensuratio Trigonometrica tot modis, & tam bonis Angulis capta est, ut de ea nullus dubitem. Sed quoniam per Soni motum distantia major videtur, & Superficies interjacentis Soli hujusmodi formam induit, qualis in figura exhibetur; ideo subdubitavi annon paululum tortuose Sonus vagatur? *Fig.* 13. sive annon Acclivitas illa interjacens in (*A*) Soni Undulationibus oppugnando retundit, easque tardat?

Ut nodum hunc quodammodo solverem, Experimentum fieri curavi, Sono Sclopi a Cacumine *Collis Langdoniensis* in Vallem subjacentem, ad intervallum 3,79 millia passuum. Intervallum Trigonometrice, ex Angulis & Basi fat magnis, bene dimensum est; & Experimentum factum, cum lenis aura paululum Sono opposuerit. Inter Coruscationem & Crepitum. $35\frac{1}{2}$ Semi-secunda numeravi. Qui numerus ad intervallum adeo quadrat, & cum cæteris experimentis tam proxime convenit, ut non dubitandum sit quin Sonus a Cacumine in vallem recta (per aerem) descenderit, & non juxta Superficiem curvatam interjacentis Soli.

Errorem igitur aliqualem fuisse credo in Observationibus *Wealiensibus* prædictis, quoniam nec in Experimento novissimo *Langdoniensi*, neque in ullis aliis tale aliquid observavi.

§. 9. *De Sonorum translatione sive Motu in Italia.*

Ingeniosus *Richardus Townleius* Armiger (Literis ad me datis Anno 1704.) significaverat, 'Sonos raro exaudiri *Romæ* tam longe ac in *Anglia*,
' nostris-

'nostrisque *Borealibus Regionibus*. Speciatim vero aiebat, Se, cum *Romæ* 'commoratus est, dum Bombardæ quædam *Castelli S. Angelo vocati*, ob læta 'nuncia disploderentur, atque ipse super Montem *Trinitade* dictum staret, 'observasse Sonum multo languidiorem eo loci fuisse, quam in ullo alio 'ad eandem distantiam sito'. Et post ejus mortem Frater ejusdem mihi Scripto retulit, quod Anno 1688, 'Cum relicta *Roma* ad *Castellum Gondolfo* (eminentiorem quendam locum prope *Lacum Albanum*, duodecim 'circiter Italica Milliaria a *Roma*) ut bonas horas contereret, se contulerit, animadvertisse sonum Bombardarum magnarum a Castello (prædicto) *St Angelo* obstrepentium, sibi tamen imminutum & debilem videri. Alio quoque tempore, cum Curru circa prædicti Castelli mænia 'veheretur, Bombardæque ingentes exinde boarent, nec talem tantumve 'Sonum ibi loci ac alibi emittere videbantur.

Cl. D. Doctor *Newton*, *Florentiæ* Ablegatus *Britannicus*, super hac re rogatus, narrat quod in itinere a *Bononia*, *Florentiam* versus, Bombardarum exoneratarum strepitum ad *S. Michaelis in Bosco* urbem (in *Bononiæ* vicinia) exaudiverit, quæ tamen Bombardæ *Mirandulæ* displosæ ad 40 millia passuum distabant; quem locum *Gallorum* acies obsidione tum cinxit. Ac nocte insequenti eundem Sonum cum in *Appenninis* pernoctaret (20 millia passuum longius remotus) exaudiverit.

Observationes vero & Experimenta, quæ idem Vir insignissimus ab aliis fieri curavit, specialem locum merito sibi vendicant.

'In inferiori *Florentiæ* arce, Bombarda *Colubrina* inter horas primam & tertiam Noctis crebro displodebatur; Virique quidam *Ligurni* 'asservabantur, qui diligenter observare jussi sunt, an ejusdem crepitum 'exaudire possent. Quorum nonnulli qui ad *Lanternam*, & *Marzocco* positi erant, nullum audiebant, (forte quia Maris fremitus Sonum infuscabat:) alii vero qui stabant super *Veteris Arcis* munimenta (quæ *Donjon* 'appellant) quique ad *Montem Rotonda* dictum (qui quinque circiter milliaria a *Ligurno* abest versus *Montem Nero*) missi, auribus exceperunt. 'Et quotiescunque exonerabatur, toties ejus fragor iisdem in locis clare 'exauditus est. Hujus autem *Arcis Florentiæ* distantia a *Monte Rotondo* 'recta linea vix minor 55 milliaribus censetur. Et notatu dignum est, 'quod interjacentia rura plerisque collibus obsita sunt, qui paulo impeditiorem Soni viam reddant necesse est. His accedit, quod eodem vespere 'ventus quidam Occidentalis leniter spirabat, qui (cum *Ligurnus* situs sit 'ad Libonotum respectu *Florentiæ*) liberiorem Soni expansionem aliquatenus præpedire merito credatur'.

'Quo autem locus apertus & undique patens haberetur, seligebatur tractus ille Maris qui *Ligurnum* & *Portum*, *Ferraio* dictum, interjacet, cujus 'distantia secundum peritissimorum Nautarum calculum 60 milliarium esse 'deprehenditur. Tormentorum autem militarium fragor a *Ligurno* ad 'prædictum *Portum Ferraio*, locaque vicina haud raro pervenit. Nec aspirantium Ventorum auxilio opus est, ad promovendum huncce Soni progressum, quo nempe exaudiatur. Imo vero ventus quilibet, sive sit secundus, sive adversus, eidem impedimento est, ipsumque Sonum minus

'sono-

'sonorum reddit: fortean, quia Maris hinc agitati fremitus magis obest, 'quam Aeris eodem confluentis cursus prodest. Proinde tunc solum exau'ditur Sonus, cum Ventus prorsus silet, vel tantum lenissime susurrat, cum 'Aer serenus est, & Mare tranquillum. Neque tum quidem ab omnibus 'locis indiscriminatim exauditur, sed ab iis solis quæ paulo eminentius 'sita sunt; cujusmodi sunt duo ista Propugnacula, quæ *Stella* & *Falcon* 'nominantur & Locus *Mulini* dictus. Præterea requiritur ut ipse Obser'vator quam attentissimum se præbeat, nec ulla obstrepentium voce aut 'clamore impediatur, & infestetur. Tum vero interdiu æque ac noctu '(modo Atmosphæra sit serena & tranquilla) exaudiat; nisi quod nocturno 'tempore fortior & acutior aliquanto videatur Sonus, cum nulli occurrunt 'Strepitus, qui luci sæpius aures offendere solent.'

'Porro nobis nunciatum fuit a Testibus fide dignissimis, quod pluribus 'abhinc annis, cum grassaretur *Seditio Messanensis*, ipsaque Urbs obsidione 'premeretur, Tormentorum bellicorum fragor *Augustæ* & *Syracusanorum* 'Incolarum aures percelleret.'

'Item cum *Galli* Tormentis muralibus *Genuam* concuterent, constat 'quod eorundem crepitus ad *Montem* usque *Nigrum*, qui *Ligurno* supere'minet, pertigerit.'

'Ex hisce Observationibus proclives sumus ad credendum nullum esse 'super hac re discrimen inter *Italiam* & *Plagas Boreales*.'

'Quod autem ad alterum Quæsitum attinet; utrum Ventus directe vel 'adverse spirans, Sonum accelerat vel retardat? Eidem hactenus certo re'sponderi haud potest. Neque enim ipsa quæ adhibuimus Experimenta, 'quibus veritatem indagatam fore speravimus, Quæstioni dirimendæ suf'ficiunt. Quippe æstivo tempore (quo plerumque interdiu venti spirant 'a Mari, & Occidente; cum autem advesperascit, silere solent) defuere 'nobis commodissimæ occasiones hanc rem sæpius & certius experiundi. 'Speramus tamen, inclinante Anno, postquam alia successerit Tempestas, 'opportuniora nancisci tempora, quibus hujusmodi Experimenta felicius '& iterato, & cum majori ἀκριβείᾳ institui & comprobari possint. Impræ'sentiarum autem referre sufficiat quid nobis evenerit 10 Augusti postremo 'elapsi, cum quæ sequuntur experimenta capere licuerit.'

'*Coleurina* quædam (60) super Cortinam inferioris Propugnaculi *Flo'rentiæ* adducebatur, ibique sic posita ut Os ejusdem versus *Artemino* 'spectaret (quod est Rusticanum *Magni Hetruriæ Ducis* Palatium, Colle 'quodam altiore situm, prædictique Propugnaculi latus Occidentale re'spiciens, a quo etiam circiter 12 millia passuum distat.) Diem quendam 'selegimus cum Ventus occidentalis aliquanto fortius spiraret, ut Soni 'motus contrario vento repelleretur. Hoc autem parum juvabat: quippe 'sub vesperam, cœlum omnino tranquillum erat, vel saltem adeo tenui 'aura agitabatur, ut candelæ flammam haud disjiceret.

'Hic Lo i relictis quibusdam harum rerum peritis, quibus antea quæ 'potissimum cu.arent in mandatis dedimus, ad prædictum *Palatium* '*Artemino* concessimus. Prout jussimus, inter Horas primam & ter'tiam Noctis, *Coleurina* sæpius exonerata est; & 49 Secunda Mi-

'nuta

'Minuta inter ejusdem Coruscationem & Fragorem jugiter numeravimus. 'Nos etiam in *Artemino* Bombardas quasdam accendimus; atque inter 'harum coruscationem & fragorem prædicti Spectatores (quos in Arce re-'liquimus) tantum 48 Minuta Secunda numeraverunt. Unde constabat 'Sonum unius tantum Minuti Secundi intervallo velocius ab *Artemino* ad '*Florentiam*, quam retrorsum ferri.'

'Haud adeo nostræ observationi confidimus, ut minutulum hoc Veloci-'tatis discrimen ad Venti conspirantis aut renitentis vim referre audea-'mus. Quippe ipsius Observatoris error, qui Penduli vibrationes nume-'rabat, huic fortean occasionem dederit. Quod sane facile fieri possit. 'Sæpius enim eveniat necesse est, ut emicantem flammam non nisi post 'inceptam Penduli Vibrationem videat, Sonitusque fragorem nondum ter-'minata Vibratione exaudiat: adeo ut ipse Calculum una Vibratione 'auctiorem, quam par est, hoc pacto faciat, dum interea Temporis spa-'tium sit utrinque par & idem.'

'Sperabamus autem proximo mane Ventum forte contrarium exori-'turum, (Sæpius enim hic loci, primo saltem diluculo, Ventus ab Oriente 'spirare solet) qui inceptis Experimentis magis inserviret. *Colubrinam* 'igitur rursus, cum illuxerit Dies, exonerari jusseramus: Ventus autem 'nec Votis nec Operi favebat; quippe qui paululum tantum ad Borealem 'plagam se convertisset. Adeo ut variatio Temporis, & Velocitatis Soni, 'in tantilla Venti mutatione, vix perciperetur. Solitas proinde 49 Pen-'duli Vibrationes, ut prius, numerabamus. Interea temporis hæc eadem 'Experimenta expendere speramus, quamprimum tempestas magis idonea 'occurret, Ventorumque crebriores mutationes, commodiores occasiones 'dabunt eadem melius experiendi, unde tandem plenissime nobis satisfiat.'

Quod ad Spatium attinet, quod Soni quovis assignato tempore percurrunt, de eodem nondum inter se constat; sed ab experimentis quibusdam conjiciebant rem ita se habere, prout Experimenta *Academiæ del Cimento* testabantur.

Ex his Observationibus, aliisque nobiscum communicatis abunde patet, Multo longius exaudiri posse Sonos in *Italia*, quam prædictus Amicus ingeniosus nos docuit. Ipse enim eximius *Ablegatus* Bombardarum ingentium strepitum ad 60 Milliarium distantiam auribus percepit. Quæ etiam ejusdem suasu *Florentiæ* exonerabantur, eædem 55 Millia passuum audiebantur. Tormenta militaria *Ligurni* displosa ad 60 Milliarium intervallum aures feriebant. Quæ *Messanæ* exonerabantur (ut ex Tabulis Geographicis patet) eorem aures, qui centum fere Italica Milliaria semoti sunt, percellebant. Quæ denique in concutienda *Genua* displosa sunt, eorundem fragor plus quam 90 Millia passuum Italicorum (ut ex Mappis) pervenit.

Quibus omnibus in mentem revocatis, & serio perpensis, vix possum quin credam non minus late propagari Sonos in omnibus *Meridionalibus*, quam in hisce *Borealibus* Terrarum plagis. Quamvis haud desunt exempla longioris Sonorum progressus in quibusdam Septentrionalibus Terræ partibus.

bus. Generosus quidam *Danus* (insignissimi nostri *Danici Principis* Famulus) mihi inter confabulandum narravit se, cum in *Dania* vitam ageret, Bombardarum *Carolscroomiæ* displosarum crepitum, 80 Milliaria Anglicana (ni fallit memoria) remotum clare exaudivisse. *Doctor Hearn* (Regis *Sueciæ* Medicus) narrationem quandam ad *Regalem* nostram *Societatem* misit, de Bombardis *Holmiæ* explosis, cum exequiæ unius ex regiis Principibus celebrarentur, A. D. 1685; quarum fragor 30 Suevorum Milliarium intercapedinem percurrit, quæ 180 Milliaria Anglicana fere exæquant. Navali etiam illa Pugna quæ gesta est *Angliam* inter & *Hollandiam* A. D. 1672. Tormentorum bellicorum Strepitus plus quam ducentis Milliaribus interjacentium attonitas aures percussit; quippe qui trans *Insulam nostram* ad *Salopiam* usque & *Walliam* pertingebat.

Quod proinde ambo Fratres *Townleii* observarunt, idem prædicto *Castello S. Angelo*, vel *Romæ* saltem, proprium omnino est, & peculiare. Neque enim perspicax eorundem ingenium, fidamve curam male suspicari licet. Ista igitur Soni diminutio, quam iidem animadvertebant (nisi male auguror) vel ad prædicti Castelli Situm, vel ad interjacentes Domos (passim & ubique in ista confertissima Urbe surgentes) vel ad Strepitus ejusdem undique personantes, vel ad Ventos adversos, vel demum aliam consimilem causam referendum est: Vel forsan hi Viri prædictas suas Observationes fecerunt eo Aeris statu, quo Soni, quamvis maxime secundos habeant Ventos, multo tamen languidiores sunt, quam aliis temporibus, cum prorsus adversi sunt.

§. 10. *De varia Sonorum Remissione & Intensione (sive Audibilitate) pro diverso Atmosphæræ statu.*

Sæpius *Æstate*, cum jam incaluerit aer, observavi Sonos supra modum languidiores videri, debilesque admodum ad aures ferri; cum alia tempestate, præsertim *Hyeme*, si forte gelascit, multo magis argutos & stridulos eosdem fuisse, fortiusque aures perculisse. Spirante etiam *Borea* vel *Euro* (quantumvis adverse) Sonos clariores, magisque stridulos esse sensi, quam si ex contrariis plagis Venti spirarent: ut *Kircherus* quoque *Romæ* observavit. Hoc autem non constans & perpetuum est.

Neque quid magis certum ex *Mercurii in Barometro* ascendentis vel descendentis inspectione colligerem, quod tamen credulus autumabam. Sonos enim aliquando maxime claros & argutos, aliquando maxime debiles & languidos cum ad summum ascenderet; e contra aliquando maxime stridulos, interdum maxime deficientes, cum Mercurius ad imum descenderet, comperi.

Pariter etiam incerte se res habet quoad Serenum & Nebulosum aerem. Tempore pluvioso & humido sæpe observavi Sonos obtundi, & "Post "imbres vehementiores plurimum virium acquirere, ut *Kircherus Romæ*. Sed contrarium quoque sæpe evenit. Maii 31. A. D. 1705. Aer hic loci magis serenus, Vaporumque expers fuit, quam unquam antea me vidisse memini. Tam purum etenim liquidoque serenum erat Cœlum, ut objecta longissime remota clare facileque prospicerem. Sed tamen Bombardas in

agro *Blackheath* tunc temporis explosas exaudire non potui (si unam excipias, cujus fragorem jam prorsus languentem auribus perceperim) quamvis omnium eminus micantem flammulam clare cernerem. Eodemque tempore Nubium & Venti motus cum sono conspirabat; Aura etiam lenissima tunc spirabat, quæ compositos crines vix moveret; & omnia denique ad Soni vim motumque promovendum necessaria concurrere videbantur. E contra vero, cum prorsus mutatus fuerit Aeris & Cœli status, cum omnia turbida viderentur, & Atmosphæra vaporibus plena, sæpe stridulos Sonos, nec minus crebro eosdem hebetes & remissos exaudivi.

Nebulas spissas certum est Sonos quam maxime hebetare. Soni enim tunc admodum languidi & obtusi plerumque videntur. Quod ab interpositis vaporibus, & spissis particulis, quæ Nebulam constituunt, certissime provenit, Idem etiam de *Nivoso Cœlo* observavi. Cum enim Nix recens in terram decidit, protinus hebescunt Soni. Cum vero glaciata fuerit ejus superficies, Soni repente acutiores fiunt, Campanasque & Bombardas tinnientes & reboantes eo usque exaudivi, ac si Nix humum non consperserat. *Townleius* prædictus haud ita pridem se observasse aiebat (cui non absimile egomet expertus sum) dum per oppidum quoddam equo veheretur, Campanarum (quæ tum haud ita procul pulsabantur) Sonum ad aures vix posse pervenire, si quando Domus Nive tecta occurreret interjacens. Adeo ut ipse, oppidulum ingressus, plurimum miratus sit, Campanas tam subito silere, dum primas interjectas ædes prætergrederetur; deinde repente resonare, cum proximum vacuum intervallum præteriret. Quod quidem per totum viæ cursum in eodem oppido observavit, Campanarum nempe Sonum ad aures pertingere, vel non; prout ædificia nive obsita occurrerent interposita, vel non.

§ 11. *De Ventorum vi, sive influentia in Soni Motum.*

Illustrissima *Academia del Cimento* ab experimentis invenit Sonorum motum nec ab adversis Ventis retardari, nec a secundis accelerari: Sed utcunque spirarent Venti, semper idem Spatium in eodem tempore percurrere. In ista sententia fuit *Gassendus*, cæterique fere omnes qui antea vel postea Philosophati sunt. Quoniam vero contrarium hujus patet ab ipsa Experientia, erroris coarguendi sunt. In quem ideo incidisse videntur, quod ad nimis breve intervallum Experimenta sua instituerentur. Omnino enim verisimile est hosce Philosophos ad unius tantum, vel ad summum, duorum, triumve Milliarium distantiam observationes suas fecisse. Quas proinde vitiosas esse haud miror. Sin autem ad 10 aut 12 Millia passuum, accuratis Instrumentis adhibitis, rem tentassent (quod ipse sæpius feci) errorem facile agnoscerent.

Quem communem errorem Egomet (horum Virorum authoritate fretus) diu admisi; donec tandem Bombardarum in agro *Blackheath* observatione triennali, & amplius, eundem feliciter detexi. Cum autem primum Sonos aliquando citius, aliquando tardius ad aures pertigisse sensi, erroris cujusdam a me facti suspicio animum subiit, vel quod Automati vibrationes minus recte numeravi, vel coruscantem Bombardæ flammulam

mulam male obſervavi; vel in alium conſimilem errorem haud attentus inciderim. Poſtquam vero Bombardæ de induſtria in mei gratiam exonerarentur ſingulis ſemihoris, ab hora ſexta veſpertina uſque ad mediam noctem, Sonumque perpetuo ſine ulla notabili varietate, 120 vel 122 ſemiſecundorum ſpatio pervenire ſenſi, quamvis Ventus directe adverſus fuiſſet; aliis autem temporibus, cum Ventus ſecundus ſpiraret, ſive e directo, ſive ex tranſverſo, aut obliquo, earundem Bombardarum Sonum 111, 112, 113, 114, 115, 116 vel ad ſummum 117 Semiſecundorum ſpatio advenire deprehendi; tum demum me certiſſime perſuaſum habui, reale aliquod diſcrimen fuiſſe, quod iſtam in Obſervationibus varietatem perperiſſet.

Neque ſolum *Secundi* aut *Adverſi Venti Sonorum motum accelerant aut tardant*, ſed etiam *pro graduum varietate, quo vehementius aut lenius ſpirant, eo magis minuſve eundem promovent aut impediunt.* De quibus omnibus in majorem certitudinem, ſpeciales quaſdam obſervationes in ſequenti Tabella ſubnectam; poſtquam prænotavi Bombardas in agro *Blackheath* circiter 60 gradus a Meridie diſtare, hoc eſt ad Plagam a S W b W aliquanto remotiorem vergere.

Tabella Sono.um Bombardarum in Agro *Blackheath*, pro Ventorum, Viriumque quibus agitantur, varietate.

Dies Menſis & Anni.	Hora Diei.	Numerus Vibrationum.	Ventorum Plaga.	Nubium Plaga.	Altitudo ☿.	
1704.						
Febr. 13 {	6 h. ad med. noc. }	120 122	NEbE 1	NEbE	29	99
11	11 ½ mane	119	E 2	E	30	22
1705.						
Mar. 30	10 mane	113	SW 7	SW	29	30
Apr. 2	8 ½ p. M.	114½	SbW 1			
3						
3	10 mane	116½	S 4 {	Inferior S Sup. Wb. N }	29	80
5	1 p. M.	111	SWbW 7	SWbW	29	70
13	8 ½ mane	120	NbE 2		29	26
24	5 p. M.	116	SWbW 0	NW	29	59
Sept. 11 {	6 ½ p. M. 7 p. M.	115 115½	W 2 WbN 2 }	WbN {	Saker. Mortar.	
29	10 ½ mane	112	SSW 6	SSW	29	38
Octob. 6	10 mane	117	ESE 1 & 2	SE	29	34
Nov. 30	meridie	115	SSW 4	SSW	29	10
Febr. 15	11 mane	116	SbW 1	SW	29	60
1706.						
Nov. 29 {	11 ½ mane meridie	116 118	SW 0 SWbS 1 }	SWbW	30	06
Febr. 7	meridie	113	SWbW 4	W	29	83

Ab Experimentis Ap. 5. & Sept. 29. factis, patet Ventos vehementiores urgere & maturare Sonorum motum. Quinto enim Aprilis, cum prope conſpiraret Venti Soniquemotus, fortior etiam aliquanto idem Ventus fuerit (prout figura [7] annexa denotat, pariter ac Cifra [0] tranquillum Cœlum; & Figuræ 1, 2, 3, 4, &c. varias Ventorum vires ſignificant) tunc temporis, inquam, Sonus 111 Semiſecundorum ſpatio iter confecit. Aprilis autem 24, cum Ventus ab eadem Plaga ſpiraret, & Aer tranquillus eſſet, idem itineris ſpatium non niſi 116 Semi-ſecundorum intervallo Sonus peragraſſet. Ita etiam Feb. 7. 1706, cum ab eodem Cardine ſpiraret Ventus, & ſecum deferret Sonum, viribus autem jam dimidio minoribus, 113 Semi-ſecunda elapſa ſunt priuſquam Sonus aſſuetum confecit iter. Ita demum Sept. 29. 1705. Vento vehementiore ſpirante, & minus ſecundo, Sonus intra 112 Semi-ſecunda progreſſum abſolvit. Ex quibus,

bus, aliisque in Tabella exemplis liquidò conſtat, *Ventos Fortiores Soni propagationem adjuvare, Leniores autem eandem minus promovere.*

Idem etiam conſtat de iis Ventis, ſive Aeris torrentibus, qui Soni progreſſui e directo favent, vel obſtant; eos nimirum *ejuſdem motum celeriorem vel tardiorem reddere.* Quique *Intermedii* volvuntur Atmoſphæræ fluxus, eos *Intermedium pariter Soni progreſſum,* ſive Penduli Vibrationum numerum efficere.

Maximum diſcrimen, quod in Soni progreſſu per 13 fere milliarium ſpatium hactenus animadverti, novem circiter aut decem Semiſecunda exæquat, cum nempe Venti fortes promovent, & lenes tantum impediunt Sonum. Cum vero tenues ſolum aut prorſus nulli obſtant, vel adjuvant eundem, tum quidem differentia duo vel tria Semi-ſecunda haud ſuperat.

§. 12. *De Ventorum Velocitate.*

Ut quantum ſpatium quovis aſſignato tempore perflent Venti pro certo ſcirem, leviuſcula quædam corpora in parandis experimentis adhibui. Cujuſmodi ſunt Pappus, Plumæ leves, &c. quæ propoſito melius inſervire videbantur, quam Inſtrumentum illud quod in * *Actis Philoſophicis*, No. 24. deſcriptum habemus; vel etiam illud alterum magis commodum Molæ alatæ figuram referens, ab acutiſſimo D. *Doctore Hook*, ni fallor, excogitatum.

* Abr. Vol. II. p. 42.

Ex plurimis quæ feci experimentis, leviuſculorum corporum ope, cum variæ Ventorum vires fuerint, deprehendi, Ventum vehementiſſimum vix 60 Millia paſſuum horæ ſpatio percurrere. Exempli gratia; Aug. 11. 1705. tantam Procellam excitavit Venti vehementia, ut ipſam Molam Pneumaticam, juxta locum quo obſervationes meas feci, pene ſubverteret. [Ventorum Vires (uti modo diximus) hiſce Characteribus plerumque notavi; 0, 1, 2, 3, 4, 5, 6, uſque ad 10, 15, aut plures gradus.] Prædicti autem Venti vires 12 circiter aut 14 graduum horum reſpondere æſtimavi: & a quamplurimis iteratis experimentis animadverti, Turbinem iſtum circiter 33 pedes, ſpatio Semi-ſecundi Minuti percurrere, ſive 45 Milliaria in Hora. Unde colligo Ventum concitatiſſimum & maxime nimboſum (illo vehementiſſimo, qui Menſe Novembris 1703 furebat, haud excepto) non plus quam 50 aut 60 millia paſſuum horæ ſpatio prætervolare.

Poſtquam rapidorum Ventorum Velocitatem dimenſi ſumus, quæ ſit minus rapidorum celeritas conjicere haud difficile eſt. Horum enim curſus pariter notavi, variiſque ab Experimentis edoctus ſum, horum nonnullos 15, quoſdam 13, alios multo plura, alioſque multo pauciora milliaria horæ ſpatio conficere: quoſdam autem tam lento motu ferri, ut vix unum milliare in hora peragrent. Alii porro Venti ſunt adeo tardigradi, ut eoſdem aliquis, equo vel pedibus iter faciens, facile prævertat. Quod Senſibus patet; quoties gradum ſiſtimus, lenem auram nos placide ventilantem percipimus: ſin autem cum eodem pergimus, nullam prorſus ſentimus: ſi vero celerius pedem movemus, comitantis & conſpirantis Auræ loco, adverſantem, & in ora vultuſque ſpirantem

aerem

aerem perſentimus. Ita, quieſcente prorſus Atmoſphæra, & ſtagnante, ſi forte ambulamus, aut equitamus, lenem Auram nos tunc prementem ſentimus, tantarum nempe virium, quantæ motui quo ferimur reſpondent. Eodemque Motus gradu, ſive Velocitate, fertur Venti aura, ſive Aeris fluxus, cum pari impetu nos morantes aut ceſſantes premat.

Hoc unum inſuper quoad Sonos obſervabo: nempe, cum eorum Motus Vento celerior ſit, patet quod iſtæ Atmoſphæræ partes quibus imprimuntur, aut deferuntur Soni, non ſunt eædem ac illæ ex quibus conflantur Venti, ſed quædam aliæ magis æthereæ & volatiles, quantum divinare licet. *Venti* enim celerrimi haud plus quam 60 milliaria Horæ ſpatio prætervolant: *Soni* vero plus quam 700 millia paſſuum eodem tempore percurrunt.

Sin autem objiciatur quod Venti Sonos celeriores aut tardiores reddunt: Reſponderetur; Hoc non a ſolo proprioque ventoſarum particularum fluxu, ſeu tendentia proficiſci, ſed potius ab omnium Atmoſphæræ particularum, cum craſſiorum, tum ætherearum conjuncto & conſpirante motu. Quæ Curſus, ſive Motus directio, ſi Sonorum Undulationibus faveat, Sonorum appulſum exinde accelerari; ſin adverſetur, retardari, omnino veriſimile eſt.

§. 13. *De Sonorum Velocitate.*

Ex quamplurimis quæ prænotavimus, firmiſſime concludo, Sonos hoc Velocitatis gradu propagari, Nempe, *ad Milliaris intervallum (ſive* 5280 *Pedum Anglicanorum) ſpatio* 9 $\frac{1}{4}$ *Semi ſecundorum percurrere:* Vel (quod eodem redit) Pedes 571 unius Semi-ſecundi, vel 1142 Pedes unius Secundi Minuti Temporis ſpatio.

Hoc autem prædictum ſpatium pertranſeunt Soni, ſi tranſverſus Atmoſphæræ fluxus intercurreret, & eſt *Medius* eorum *Progreſſus* ſive *Motus.* Sin autem Ventus Soni rapiditatem augeat, poſſibile eſt ut pluſquam 600 Pedes Semi-ſecundi ſpatio prætereat. Vel e contra, ſi moram ei innectat, haud plus quam 560 Pedes eodem temporis intervallo progrediatur.

Prædictæ Obſervationes, & Experimenta non parum conducere videntur,

1. *Philoſopho*; qui vel hinc aliqua ex parte inſtructior ſit ad arcanam Sonorum Naturam inveſtigandam; & eorundem plurima Phænomena abſtruſa explicanda.

2. *Nautæ*; qui hinc diſcat quanto intervallo abſunt Naves, quas procul fluctuantes, vel ad anchoras ſtantes cernit: quam longe item Tellus, aut optata Arena, eminus conſpecta, diſtat: Quæ ex Sclopetis de induſtria exploſis, ſigno quodam dato, facile certoque innoteſcant.

3. *Militi*: ad inveniendum quam procul Hoſtis Caſtra locavit; ad quam diſtantiam ſita eſt Urbs obſeſſa, Arx, Armamentarium, &c. ad Tormenta muralia libranda, & dirigendos Pyrobolos, glandeſque ignivomas.

4. *Geographo*: ad Locorum diſtantias facilius & certius menſurandas. Quivis enim intra horam unam aut alteram, parva pulveris pyrii copia inſtructus

inſtructus, totam fere Regionem Tabula accuratiſſime deſcriptam hoc pacto exhibeat. Sclopeta enim diſploſa Diſtantias (prout diximus) oſtendent: & quodvis Inſtrumentum Mathematicum quo metiuntur Angulos, vel Inſtrumentum illud vulgare quo Decempedatores utuntur (*the Plain Table* vocatum,) vel ſola Regula Pinacidiis inſtructa, variorum locorum Situs indicabit; quæ deinde delineare haud difficile eſt.

Hac etiam ratione in Mapparum rectitudinem, & veritatem quis facile inquirat; & ſiquos habeant errores, corrigat.

Hæc demum (Soni) obſervandi ratio, Locorum inacceſſorum, præſertim vero latiſſimorum Fluviorum, & ejuſmodi locorum haud aliter menſurabilium, diſtantiis dimetiendis magnopere inſerviat.

5. *Echometra.* De hoc ludicro & jucundo Soni Phænomeno (ſcil. *Echo*) licet plurimi docti viri olim & poſtea ſollicite quæſiverint, de plurimis tamen ad idem ſpectantibus non bene inter ſe convenit: ſpeciatim de ſpatio Loci ad repetitionem 1, 2, 3 vel plurium Syllabarum neceſſario; vel (quod eidem redit) de ſpatio ab *Echo* peragrato in certo quodam temporis intervallo. *Merſennus* —— —— paſſus ad Vocem Monoſyllabam repetendam concedit; *Blancanus* 24 paſſus (cui aſtipulatur noſtras celeberrimus Dr. *Plott*,) ſed *Ath. Kircherus* aſſerit nihil omnino certi de eo definiri poſſe, quod nempe Ventorum variatio, viriumque Soni intenſio & remiſſio, & multa alia immenſam variationem pariunt.

Rationem autem hujus diſſenſus inter laudatos hoſce viros reddere haud difficile eſt. Ex plurimis enim cauſis oriri poteſt; ex tarditate nimirum & diverſa noſtrorum Senſuum diſpoſitione; vel ex varia Sonorum audibilitate; ex Syllabarum ipſarum gravi vel acuto ſono ſive earundem contracta vel producta pronunciatione; vel ex qualibet alia cauſa temporis intervallum protrahente. Nullus enim dubito (Exempli gratia) quin ſi Objectum aliquod Phonocampticum repercutere potuerit omnes Syllabas hujuſce ſequentis carminis, viz.

Vocalis Nymphe, quæ nec reticere loquenti,

Quod haud valeret repercutere omnes Syllabas ſequentis carminis, quoniam paulo productior eſt ejus pronunciatio,

Corpus adhuc Echo, non vox erat, & tamen uſum:

Et multo minus repetere valeret aſperas omnes, & productas Syllabas ſequentis carminis licet numero pauciores,

Arx, tridens, roſtris, ſphinx, præſter, torrida, ſeps, ſtrix.

Verum a præcedentibus de Soni motu Obſervationibus concludere licet, Quod uti Soni, ita Ἠχῶ certa & determinata ſpatia in certo quodam præſcriptoque tempore percurrunt. Quod ipſum ab experientia ſæpius edoctus

ſum

ſum, ſcil. Echo redire in duplo temporis intervallo quo Vox primaria Objectum Phonocampticum pertingebat, ex. gr. Echus regreſſus in eodem temporis intervallo fuerit, in quo Primarius Sonus duo Stadia percurriſſet ſi non repercuſſus fuiſſet.

Et hoc in dimetiendis Locorum diſtantiis magno mihi uſui ſæpe fuit. Exempli gratia: Cum in *Thameſis* fluvii ripa ſtarem, Villæ *Woolwich* oppoſita, monoſyllabæ vocis Echo a Domibus adverſis repercuſſa fuit in ſex Semi-ſecundis Minutis temporis. Unde colligo Latitudinem fluvii *Thameſis*, eo loci a margine unius Ripæ ad marginem alterius, 1712 pedes Anglicanos eſſe, ſive ſupra quadrantem milliaris. Nam ut 9,25 (Semi-ſecunda): ad 5280 (pedes in milliari uno): Ita 6 (Semi-ſecunda): ad 3424, 8 pedes. Cujus dimidium eſt 1712,4 pedes.

Denique hoc pacto *Intonantium Nubium Altitudo*, & ipſius *Tonitru*, *Fulgurisque* Diſtantia facile innoteſcant.

Of the Nature and Property of Sounds, by Dr. Grandi n. 319. p. 270.

**Vid. Phi. Tran. n 156. Abrid. p. 508.*

V. Elegantiſſimum Præſulis * Armachani commentariolum de Sono ſumma animi voluptate perlegi, at circa Semiplani figuram in calæ ejus ſcripti commemoratam, vereor, ut ſatis aſſequi mentem Auctoris valuerim.

Comparat doctiſſimus Præſul ſcientiam auditus cum Theoria viſionis, atque ut hæc in directam, reflexam, & refractam dividitur, ita illam pari ratione trifariam diſtribuit, ut non modo ſonos directos, & reflexos (quod dudum in uſu fuit) ſed & refractos conſideret; quem-admodum autem eximiis inventis opticis, catoptricis, & dioptricis viſionem a majoribus noſtris magna jam ex parte perfectam fuiſſe animadvertit, ita compluribus inſtrumentis Acuſticis, Catacuſticis, & Diacuſticis, ſive Phonicis, Cataphonicis, & Diaphonicis (utrovis enim modo denominat) auditum, tam ex objecti, quam medii, vel organi parte perfici poſſe non dubitat, eoque ſpectantia problemata proponit, quæ tamen in hoc ſcripto, nedum abſque demonſtratione, ſed & abſque determinatione, aut conſtructione ulla exhibentur, unde non major ad ipſorum ſolutionem lux nobis affulget, quam quæ, ante inventa a M. Galilæo ſcientiæ motus principia, haberi potuiſſet ad enodationem problematum circa determinandam projectorum ſemitam, vel aquarum ex data altitudine deſcendentium velocitatem, propoſitorum. Neque enim affirmare verebor, perinde ignota nunc eſſe Acuſticæ doctrinæ fundamenta, certe nondum paſſim vulgata, aut inter eruditos recepta; licet fortaſſe laudatiſſimo huic Præſuli innotuiſſe videantur, ſi quæ ab illo indicata, & promiſſa ſunt, attendamus, quorum quidem uberiori expoſitione, atque aperta demonſtratione totam ſibi literariam Rempublicam demeruiſſet; cum vix credendum ſit, omnes ſimplici illa Opticorum, & Acuſticorum comparatione fore contentos, quæ vix ultra ſatis latam analogiam extenditur, ob tot diſcrimina, quibus propagatio Lucis a diffuſione Soni ſecernitur: inter quæ illud palmarium eſt, quod Lux per lineam rectam ſemper exporrigitur, dum ſonus etiam per curvas, & inflexas utcunque ſemitas

ſemitas quaquaverſum ſpargitur, atque, intercepto cujuſvis opaci corporis obice, ſenſibilis redditur.

Et vero hæc ipſa, quæ de Soni diffuſione doctiſſimus Auctor noſter ediſſerit, ejus differentiam a lucis propagatione manifeſtant: docet ſiquidem, ſonum ſecus parietes, aut fornices lævigatiſſimos, elliptica, vel cycloidali, potius quam circulari flexura donatos, blando quodam, & expeditiſſimo lapſu feliciter excurrere, necnon per mollem aquæ ſuperficiem, ſonoris tremoribus, quibus aer criſpatur, obſequentem validius promoveri: quæ vereor, ut in luminis propagatione adeo generatim obſerventur; nam de Ellipſi quidem hoc tantum habemus ex catoptrica demonſtratum, quod radii lucis ex altero ejus foco *D* emanantes, & in ellipticam curvam *A B C* impingentes, inde reflexi in altero foco *E* colligentur; at ſi ex alio quovis puncto *G*, præter focos, exeant radii, non omnes amplius in unum punctum coibunt, ſed ita reflectentur, ut curvam cauſticam *f F f* contactu ſuo efforment, ſupra cujus convexitatem exiſtentes, uno aut altero reflexo radio, non pluribus gaudere poterunt, in ipſa vero curva poſiti aliquot ex maxime vicinis participabunt; atqui intra cavitatem ejuſdem verſabuntur, ab omni reflexorum radiorum illapſu immunes erunt, adeo nullum inde ſibi emolumentum obventurum ſperabunt. *Fig.* 14.

Cycloidem quod attinet, oſtendit quidem Cl. V. Joannes Bernoullius in Actis Lipſiæ 1697. Lucis radium, ſi per media tranſiret, quorum raritates in quolibet puncto juxta rationem ſubduplicatam altitudinum variarent, ita continuo flexu refrangendum, ut in curvam cycloidis ſinuaretur: at ſeu reflexione, ſeu directa per idem medium propulſione, quid figura cycloidis ad feliciorem luminis diffuſionem conferret, plane non video; hæc enim curva focis omnino caret, adeo ut in nullo puncto radios recolligere poſſit, ſed in curvas irregulares abeunt radii ab ipſa reflexi, niſi quod ubi ad axem *K L* paralleli radii *P M*, *Q N* in Cycloidem *E M K N H* inciderent, tunc linea cauſtica per contactum reflexorum radiorum *M R*, *N S*, efformata, ex binis cycloidibus *E R L*, *H S L*, circulo ſubduplæ diametri generatis, componeretur, radioſque reflexos confertiſſimos circa utriuſque confinium *L*, ad medium baſis reflectentis cycloidis exhiberet: cæterum tam in his quam in aliis cauſticis ex qualibet luminoſi puncti radiorumque poſitione reſultantibus, eædem obſervationes locum haberent, quas ſub finem præcedentis paragraphi cauſticis per ellipſim efformatis competere diximus. *Fig.* 15.

De plana aquarum ſuperficie nihil eſt quod addam, cum pateat, lucis radios per illam aut omnino refractos tranſire, aut ordinata reflexione in adverſam partem remitti, perinde ac e chryſtalli ſolidioris ſuperficie; imo ab hac aliquanto fortius, quam ab illa, tantum abeſt, ut per illam facillime repentes in directum expeditius promoveri poſſint, illumque blandum progreſſum obtinere, quem tremoribus harmonicis, per mollem aquarum ſuperficiem, criſpatione ſua ipſorum flexui ſe accommodantem, ſerpentibus Auctor tribuit: immo & dubitare licet, an lævigatiſſimæ ſpeculorum ſuperficies, perinde ac luminis, ſic ſoni reflexionibus valde conducerent, cum Echo ipſa ſpeluncarum receſſus aſperrimos, magis quam po-

 litos,

litos, ac tenui gypſo incruſtatos parietes habitare videatur, ab incultis vallibus, ab anfractuoſis antris, atque e veterum ædificiorum ruderibus frequentius reſpondens.

Jam Acuſticum ipſum, ſeu ſpheram Phonicam ab eodem propoſitam, divinando magis, quam interpretando, exponere utcunque aggrediar, verbis illius primum adductis, ut cum meis conjecturis mox ſubjiciendis conferri queant, & quam exacte iiſdem reſpondeant cujuſlibet poſſit arbitrio judicari.

Addam hoc loco, inquit Auctor, *Semiplani Acuſtici, ſeu Spheræ Phonicæ figuram, quaſi tentamen ad magnum ſcientiæ hujus principium explicandum, quod in ſonorum progreſſione conſiſtit. Conſideretis oportet rude hoc ſemiplanum velut horizonti parallelum, nam ſi eidem perpendiculare fuerit, ſuppono extremitatem illius ſuperiorem non amplius circularem futuram, ſed hyperbolicam, partem vero inferiorem æqualem fore uni ex maximis terræ circulis: adeo ut univerſa ſphera Phonica, ſi ita appellare liceat, ſit quædam hyperbola ſolida, ſuper concava baſis ſphericæ ſuperficie erecta.* Porro Diagramma Londino tranſmiſſum hujuſmodi erat, nullis præterea notis ad
Fig. 16.
ejus illuſtrationem facientibus inſtructum.

Fig. 17. Ipſe igitur, hoc alio Schemate ſubſtituto, mentem Auctoris aperire ſatagam. Sit terræ globus $CGFE$, atque in puncto C ejus ſuperficiei ſonus aliquis excitetur. Hic per terram ipſam nec non per aerem circumquaque propagabitur, adeo ut quo tempore ad maximum terræ circulum polo C deſcriptum, nempe ad Peripheriam GBE aut reipſa (licet fortaſſe inſenſibiliter) pervenit, aut ſaltem (ſi validior eſſet) perveniret, per aerem diffuſus, quoddam ſpatium repleret, pro varia tranſitus facilitate, non prorſus ſpherice, ſed inæqualiter exporrectum, & a perimetro hyperbolæ $GLAKE$, circa axem CAO ſonoro corpori C perpendicularem poſitæ, circumſcriptum: imo vero a ſuperficie conoidis hyberbolicæ, quam hyperbola ALG circa ſuum axem rotata generat, definitum. Itaque univerſa ſphera phonica, per quam dato tempore ſonus extenditur, erit ſolidum ſpatium comprehenſum ab hyperbolica conoide $GAEB$, quæ maximo terræ circulo GBE inſiſtit, & concava ſuperficie hemiſpherica $GCEB$ inferius terminatur: quod quidem ſpatium plano ad horizontem parallelo ubilibet ſectum, exhibebit ſemicirculum LIK, qualem oſtendit Auctoris figura, quem & ſemiplanum appellat, eo quod ipſius diagrammatis proſpectus alteram ſolum medietatem ejus exhibeat, dimidia reliqua trans hyperbolam verticalem (quæ & ipſa phonicam ſpheram per axem bifariam ſecat) inconſpicua manente. Verum quæ ſit hujuſmodi hyperbolæ ſpecies, aut quibus principiis doctrina hæc fulciatur, nec Auctor indicat, nec mihi ſuppetit unde hac de re quidpiam certi conjiciam.

Quod unum ſupereſt, adnitar, ut inverſo veſtigandi ordine procedens, detegam primo per quod linearum genus tremores ſonoros diffundi oporteret, ut in ejuſmodi hyperbolam dato tempore expanderentur; ſecundo, quæ raritatis variatio foret in variis aeris altitudinibus ſupponenda, ut (ſtante refractionis communi lege, qualem radii lucis obſervant) ſonorum directiones juxta inventam linearum ſpeciem flectere poſſet; ac tertio, quæ viciſſim dicenda ſit lex refractionis, quam ſonori tremores in ejuſ-

modi

modi curvarum genus abeuntes sequuntur, supposita raritatis aeris variatione tali, qualem plerique Philosophorum, & Mathematicorum in illo agnoscunt juxta reciprocam rationem ponderis atmosphæræ incumbentis, & inferiores partes gravantis, quam experimentis congruere testantur.

Pro quo consideremus, corpus sonorum *C* tremores suos per directiones *C n*, *C m*, *C h*, quaquaversus communicare, aut certe juxta eas lineas, per quas impulsum fuerat, se restituendo repellere aerem, eumdemque frequentissimis oscillationibus protrudere, quibus crispatur, atque ad motum tremulum juxta easdem directiones diffusum sollicitatur; hi ergo tremores quodam minimo tempore pervenisse concipiantur ad puncta *n*, *m*, *h*, unde iter suum prosequentes, successive post aliud datum tempus simul propagabuntur, primus ad punctum *N*, secundus ad *M*, tertius ad *H*; iterumque post aliud datum tempus simul progredientur, prior ad *G*, alter ad *L*, postremus ad *A*. Nunc igitur lineas quidem *C n N G*, *C m M L*, *C h H A*, per quas quilibet tremor succesive diffunditur, voco *Radios sonoros*; lineas vero *n*, *m*, *h*, *N M H*, *G L A*, quas prædicti sonori radii, omnesque alii synchroni iis intermedii dato quolibet tempore simul attingunt, *Undas sonoras* appello. *Fig.* 18.

Et quidem in medio prorsus ubilibet uniformi, cessante causa, quæ tremores sonoros a sua directione in hanc, vel illam partem deflectere cogat, patet sonoros radios semper rectos procedere, seu per viam brevissimam ab uno ad alium terminum directe progredi, atque undas penitus circulares sonoro corpori concentricas efficere, quia cum non majorem hic, quam alibi transitus difficultatem inveniant, utique ad pares distantias singuli dato quovis tempore elongabuntur: Secabit autem radius quilibet undam suam perpendiculariter, atque undæ quælibet concentricæ & similes erunt, ut constat ex elementis.

At in medio difformis densitatis, velut in aere terræ circumfuso, qui diversam pro varia altitudine (nam caloris, frigoris, humiditatis, & siccitatis vices, quæ ad certam legem revocari non possunt, pro nunc seponimus) raritatem obtinet, solus radius *C H A* perpendiculariter trajiciens omnes aereas lamellas, sive superficies terræ concentricas, irrefractus transibit, ac rectus manebit; cæteri vero iisdem superficiebus oblique impingentes continuo quodam flexu in quolibet puncto refringentur, & in curvas *C m M L*, *Cn N G* sinuabuntur; nec non pro varia transitus facilitate non ad eandem ubivis distantiam dato tempore progredientur, quare puncta *A*, *L*, *G*, aut *H*, *M*, *N*, quæ eodem momento sonus per quoslibet radios emissus attingit, inæqualiter a sonoro *C* remota erunt, adeoque undæ *A L G*, *H M N*, *h m n*, omnino circuli non erunt corpori sonoro concentrici, sed alterius generis curvæ, quas tamen oportet invicem similes esse, ac similiter positas: quare in hypothesi nostri Auctoris, qui extremam illam undam *A L G*, ultimos globi terraquei fines lambentem hyperbolicam voluit, necesse est quasvis alias undas intermedias *H M N*, *h m n*, esse hyperbolas similes, ac similiter positas, diversis quidem verticibus, *A*, *H*, *h*, sed eodem centro, ad eumdem axem, & sub similibus laterum figuris descriptas; nam quæcumque sit ratio, quæ ostendat, ob simultaneum appulsum soni ad puncta *A*, *L*, *G*, per vias synchronas *C H A*, *C M L*,

$C\,N\,G$, unde $A\,L\,G$ facessere in curvam talis speciei (puta hyperbolicam:) eadem prorsus iisdem fundamentis evincet, ob simultaneum appulsum soni etiam ad puncta H, M, N, per synchronas lineas $C\,h\,H$, $C\,m\,M$, $C\,n\,N$, undam $H\,M\,N$, in curvam ejusdem speciei (nempe hoc casu in hyperbolam similem, ac similiter positam) pariter abire, ut de se constat. Nec dubium insuper, sonoros radios $C\,H\,A$, $C\,M\,L$, $C\,N\,G$, semper undas illas similes $A\,L\,G$, $H\,M\,N$, $h\,m\,n$ debere perpendiculariter, sive ad rectos angulos secare, ut in circularibus undis contingit; quod cum in simili proposito de lucidis undis ostenderit jam Vir Cl. Christ. * Hugenius, non est cur in hac observatione, pluribus momentis confirmanda, tempus teratur.

* In Tractatu Gallico, de Lumine, p. 44.

Itaque investigatio viæ, per quam radii sonori, juxta hypothesim Auctoris nostri, propagantur, ad hoc pure geometricum Problema reducitur, ut inquiratur natura curvarum, quaslibet hyperbolas similes, & circa eumdem axem, eodem centro similiter descriptas, perpendiculariter secantium. Sint hyperbolæ similes $A\,L\,G$, $H\,M\,N$, $h\,m\,n$, aliæque innumeræ intermediæ, aut supra, vel infra ipsas similiter positæ, idem commune centrum O habentes, eodemque axe $O\,A\,H$, cui alter $O\,S$ conjugatur, descriptæ: ducenda est per punctum C curva $C\,m\,M\,L$, aut $C\,n\,N\,G$, propositas omnes hyperbolas perpendiculariter secans. Describatur per datum punctum C, inter asymptotos $O\,A$, $O\,S$, hyperbola $C\,m\,M\,L$ talis naturæ, ut posita ratione transversi lateris priorum hyperbolarum $A\,L$, $H\,M$, &c. ad latus rectum earumdem æquali rationi t ad r, potestates ordinatarum $L\,Q$ denominatæ ab exponente r sint reciproce proportionales potestatibus abscissarum a centro $O\,Q$ denominatis ab exponente t, nempe facta $O\,Q = x$, & $Q\,L = y$, ita ut $y^r = \frac{1}{x^t}$; sive, ducta qualibet alia ordinata $m\,i$, $M\,I$, ita ut ratio distantiarum a centro $O\,Q$, $O\,I$ sit reciproce tam multiplicata rationis applicatarum $I\,M$, $Q\,L$, quam multiplex est fractio $\frac{r}{t}$ unitatis. Dico hanc satisfacere quæsito; ducta enim cujusvis hyperbolæ $A\,L$ tangente $L\,P$ in puncto, ubi a curva $C\,M\,L$ secatur, necnon $S\,L\,R$ tangente ipsius hyperbolæ $C\,M\,L$ in eodem puncto, patet ex his quæ in Theorematum Hugenianorum demonstratione, cap. 7. n. 9. ostendimus, fore $O\,Q$ ad $Q\,R$, ut exponens potestatis distantiarum $O\,Q$ ad exponentem potestatis ordinatarum $Q\,L$, nempe ut t ad r; sed ut t ad r, nempe ut transversum latus ad rectum, ita per 37. l. Conic. est rectangulum $O\,Q\,P$ ad quadratum $Q\,L$; igitur ut $O\,Q$ ad $Q\,R$, sive sumpta communi altitudine $Q\,P$, ut rectangulum $O\,Q\,P$ ad rectangulum $P\,Q\,R$, ita rectangulum $O\,Q\,P$ ad quadratum $Q\,L$, quod ideo æquabitur rectangulo $P\,Q\,R$; quare angulus $P\,L\,R$ rectus erit: unde curva $C\,M\,L$ perpendiculariter occurret in puncto L hyperbolæ $A\,L\,G$, eodemque modo aliis hyperbolis $H\,M\,N$, $h\,m\,n$, in punctis M, m, in quibus illas secat, perpendicularis esse ostendetur; quod erat, &c.

Fig. 19.

Hinc primo colligitur, quod si hyperbola determinans sphæram Phonicam Auctoris nostri, nempe $A\,L\,G$, aliæque similes concentricæ $H\,M\,N$, $h\,m\,n$, fuerint æquilateræ, tunc propter æqualitatem laterum t, & r, hyperbola

perbola CML erit & ipsa hyperbola Apolloniana, & quidem pariter æquilatera, ejus enim æquatio superius allata transformabitur in hanc $y=\frac{1}{x}$ ubi ratio ordinatarum simpliciter reciproca erit rationis distantiarum a centro, itaque radii pariter sonori æque ac sonoræ undæ, juxta hanc hypothesin forent hyperbolæ ejusdem speciei, diversa duntaxat positione collocatæ: Memini porro Illustrissimum Equitem Isaacum Newtonum Opticæ suæ, lib. 3. p. 287. Observ. 10. ostendere, quod & radii lucis trans duorum cultrorum acies in obscurum cubiculum admissos; in hyperbolicas fimbrias, qualis esset CML, pariter sinuari, cujus phænomeni si ratio physica afferri posset, eadem hyperbolicos pariter soni radios, quales Armachani Præsulis systema invehere videtur, fortasse persuaderet.

Secundo observandum est, quod si plures ejusmodi curvæ, seu radii hyperbolici mML, nNG, &c. secantes undas hyperbolicas ALG, HMN, &c. perpendiculariter describantur, non in unum exacte punctum C poterunt convenire; tametsi propius & propius coeant ad partes C, atque ad intervallum pervenire possint, minus quolibet dato intervallo; quare concipiendi erunt radii illi hyperbolici a corpusculo C alicujus extensionis procedere, non ab aliquo mathematico puncto, quod ipsum convenientissimum est; sonus enim ex collisione corporum nascitur, non ex unius rigorosi puncti, seu termini extensionis tremore produci potest.

Imo cum omnes undæ a sonoro corpore propagatæ esse debeant, ut supra vidimus, hyperbolæ similes; congruum est, ut concipiamus, corpus sonorum C quasi fibrillam minutissimam frequentissime oscillantem, cujus minima, & veluti initialis unda infinite propemodum exigua 2 3 4, & ipsa revera hyperbolica sit, seu potius apex physicus alicujus hyperbolæ; ita ut nimirum fibrilla oscillatoria corporis sonori C, dum pulsatur, ex situ directo 2 C 4 detrusa in situm concavum 2 5 4, vi percussionis adigatur, tum vehementissimi elateris sui, nec non propriæ tensionis vi, restituta in convexam hyperbolam 2 3 4 intumescat, ac rursus reducta alternis vibrationibus, fluctuans hinc inde, suos tremores in hyperbolicas undas, ipsimet initialibus 2 3 4, 2 5 4 perpetuo similes, sursum, ac deorsum suapte natura, & in medio utrinque libero expandat, sed obice terrestris globi CE (cujus centrum T) impedita fortasse, hyperbolicas undas suas dumtaxat sursum propaget, & Phonicam Sphæram ab Auctore nostro excogitatam describat, hemisphaerio terrestri ab inferiori parte interruptam, ac definitam. Quod si vera esset *P. Pardies* doctrina Artic. 81. suæ staticæ proposita, quod chordæ extensæ reipsa hyperbolicam figuram, qualis esset 2 5 4, cujus centrum sit idem quod centrum terræ, vi proprii ponderis assumant, nemo non videt eam ipsam confirmando Auctoris nostri systemati fore congruentissimam, hinc enim ratio haberetur, cur fibrilla quævis sonori corporis, C, dum ad vibrationes harmonicas sollicitatur, in hyperbolam 2 5 4 excurreret centrum habens in centro terræ T, similiterque in aliam æqualem 2 3 4 assurgeret, indeque per alias ampliores hyperbolas tremorem diffunderet, quarum omnium centrum esset O æque distans a sonoro corpore C, ac sonorum corpus illud remotum sit ab ipsomet centro terræ;

quare

quare diſtantia CO æqualis ſemidiametro Globi Terraquei limitem definiret, ultra quam nulla ſonora unda propagaretur, nulluſque poſſet ſonus audiri, & linea OS, utpote aſymptotos quorumvis hyperbolicorum radiorum, per quos defertur ſonus, confinium beatæ illius regionis conſtitueret, in qua ab omni terrenarum rerum ſtrepitu ſecuris in ſumma tranquillitate philoſophari liceret.

Porro ne quis ſpeculationem hanc eo nomine contemnendam putet, quod fibrilla quævis ſonori corporis, cum breviſſima ſit, ac valide diſtenta, ſemper in ſitu recto $2C4$, manere videatur, nec poſſe in concavas, aut convexas hyperbolas 2 5 4, 2 3 4 ſinuari, conſiderandum inſuper eſt, hyperbolas quo majoribus axibus præditæ fuerint, eo magis ampliari, & ad lineam rectam accedere; itaque ob ingentem diſtantiam centrorum T, vel O, ſicut lineæ quas gravia cadentia deſcribunt, licet in centrum T collimantes pro parallelis habentur, & arcus circuli horizontalis cum recta ejus tangente confunditur, ita & initiales illas hyperbolas 2 5 4, 2 3 4 ferme coincidere dicendæ ſunt cum recta $2C4$, unde ſenſibilis non eſt incurvatio fibrillarum oſcillantium in ſonoro corpore, nec ſe prodit undarum hyperbolicarum ſpecies, niſi ubi in amplius ſpatium $GLALG$ dilatatæ fuerint centro ſuo propius accedentes.

Animadvertendum adhuc tamen, his principiis poſitis, conſequens fore, ut ſonus hinc inde ad latera non excurreret ultra ſpatium ab hyperbolicis extremis radiis $298g$, $476g$ comprehenſum, quas tangerent rectæ $T2$, $T4$, a centro terræ per terminos fibræ oſcillantis adductæ; ac revera fibræ illius tremores juxta aliam directionem non procederent, quam per $T2$, $T3$, $T4$, aliaſque intermedias angulo $2T4$ comprehenſas, ſingulis particulis fibræ ejuſdem correſpondentes, itaque ſpatium extra dictas hyperbolas $298g$, $476g$ poſitum ab omni tremore harmonico vacaret, nec poſſet juxta ſenſum Auctoris phonica ſphæra ad integrum terræ hemiſpherium exporrigi; itaque oportet, nunquam reipſa unicam aliquam ſonori corporis fibrillam tremere, quin terminos aliarum fibrarum, quibus connectitur, & inter quos diſtenditur, eo ipſo trahat, & ad harmonicum tremorem pariter ſollicitet, quæ rurſus alias, quibus implicantur, abducunt, & ad tremorem exſtimulant, quemadmodum tenſa chorda muſica ligneo inſtrumento, cui alligatur, tremores ſuos evidenter communicat; itaque harmonicæ oſcillationes in alia corpora, quibus mediate, vel immediate connectitur, percuſſa fibra ſonori corporis, ſubinde transfunduntur, licet magis magiſque ſemper debilitatæ, ac demum inſenſibiles redditæ per hemiſpherii terreſtris ſuperficiem ſparguntur, & longius ac longius ſerpentes obrepunt (quod auris ipſa terræ applicata, & magnos ſaltem fragores in maxima diſtantia excitatos diſcernens teſtari poteſt) itaque ex aliis etiam locis emergunt alii ſonori radii hyperbolici per totum terræ hemiſpherium, a quibus Phonica ſphera Præſulis Armachani ſatis impleri poſſit.

Secundam quæſtionem generalius ſolvere conabor, ut major inde fructus elici queat; intelligatur quivis radius ſeu lucidus, ſeu ſonorus NnG, in cujuſvis naturæ curvam, continua ſui refractione, mutatus: quæritur qua lege variari ſupponenda ſit denſitas, aut raritas medii in ejus diverſis altitudinibus,

tudinibus, ut ſtante refractionis theoria, quæ ſinum refractionis ſemper raritati medii refringentis proportionalem ſupponit, radius ille in talis naturæ curvam obire potuerit? Sit axis curvæ $Nn\,G$, quam radius refractus efficit, recta $C\,O$, in qua ſumpto quolibet puncto C, radio quovis $C\,L$ deſcribatur quadrans circularis $L\,P\,p$, ductaque ubilibet refracti radii tangente $N\,R$, $n\,r$, agatur ex C radius dictæ tangenti parallelus, occurrens circulo in P; ductaque $P\,F$ axi parallela, occurrat ordinatæ $N\,Q$ ad axim perpendiculari in puncto F; dico inde ortam curvam $Ff\,F$ exprimere ſuis ordinatis $F\,Q$, $f\,q$ raritates medii in variis ejus altitudinibus; nam quia $C\,P$ eſt parallela ipſi $R\,N$, erit angulus $P\,C\,B$ æqualis angulo, quem radius refractus $N\,n$ in puncto N efficit cum perpendiculo; & ideo $B\,P$, ſive $F\,Q$ erit ſemper ſinus refractionis, poſito CP ſinu toto; quare cum ſuppoſita ſit lex ea refractionis, ut ſinus ejuſdem proportionalis ſit raritati medii; utique eadem $F\,Q$ exprimit medii raritatem ad altitudinem Q, ſive ad æque altum punctum N, per quod radius tranſit. Quod erat, &c.

In noſtro autem propoſito, ubi $Q\,N = \frac{1}{X^{-\frac{t}{r}}}$ propter $Y^r = \frac{1}{X^t}$, ſi $F\,Q$ exponens raritatem aeris vocetur Z, erit $Z = \sqrt{\frac{t}{X^{\frac{2r+2t}{r}} + t\,t}}$; aut ſumpta etiam r, & CP pro unitate, fiet $Z = \sqrt{\frac{t}{X^{2+2t} + t\,t}}$, atque in caſu quod unda hyperbolica fuerit æquilatera, adeoque & radius hyperbola ſimilis æquilatera, $y = \frac{1}{X}$ propter $t = 1$, fiet $Z = \frac{1}{\sqrt{X^4 + 1}}$.

Quoniam vero tum † Jacobus Hermannus tum David * Gregorius oſtendunt Curvam, quæ determinat gradus raritatum aeris eſſe logarithmicam, adeo ut altitudines $O\,Q$, $o\,q$, ſive X ſint logarithmi numerorum exponentium aeris raritates in punctis Q, q: patet radii continue refracti $N\,n$, $N\,G$ curvaturam ea lege procedere, ut ſinus complementi incidentiæ, & refractionis ad poteſtatem $\frac{r}{r+t}$ elevati rationem habeant compoſitam ex ratione ſinuum rectorum ad ſimilem poteſtatem evectorum, & ex ratione quam habent logarithmi raritatum.

† *Act. Lipſ.* 1706.
* *Aſtron. Lib.* 5.

Cæterum etſi conſenſerim, ordinariam legem refractionis lucis dare ſinus incidentiæ & refractionis proportionales raritatibus mediorum, non diſſimulo tamen id fortaſſe non adeo exactum eſſe, cum ratio ſinuum in refractione ex aere in vitrum ſit circiter ſeſquialtera, aer vero pluſquam millies vitro ſit rarior. Sed cum viderent Geometræ majorem fieri ſinum refractionis in tranſitu ad aliud medium pro majori facilitate qua illud lux penetrat in communi hypotheſi, vel pro majori difficultate juxta Carteſium, qui ſupponit e contrario lucem magis refringi ob majorem difficultatem in rariori medio quam in denſiori (ut gravia corpora ob majorem difficultatem penetrandi denſiora corpora, in his magis refringuntur, reſiliendo a perpendiculari) & utramque legem in eo convenire, quod pro majori medii

medii raritate, major fieret refractio: hinc invaluit, ut ſinus proportionales dicerentur, non quidem facilitati, aut difficultati tranſitus, quarum alterutra ab aliis in dubium vocatur, ſed raritati medii, in qua omnes conveniunt, licet vera proportio illi non prorſus reſpondeat in eadem geometrica ratione; itaque ubicunque raritatis mentio facta eſt, ſubroganda eſt fortaſſe facilitas tranſitus in communi, & difficultas in Cartheſiana hypotheſi, præterquam ubi diximus, raritatem ex pondere aeris incumbentis variatam reſpondere altitudinibus ut numeri logarithmis ſuis reſpondent; hoc enim exacte veriſſimum eſt.

Experiments concerning Sound, by Mr. Hawksbee. *Actual Sound not to be tranſmitted through a Vacuum, n.* 321. *p.* 367.

V. 1. I took a ſtrong Receiver, arm'd with a Braſs-hoop at bottom, in which I included a Bell as large as well it could contain. This Receiver I ſcrew'd ſtrongly down to a Braſs-Plate, with a wet Leather between, and was full of common Air, which could no ways make its eſcape. Thus ſecur'd, it was ſet on the Pump, where it was cover'd with another large Receiver. In this manner, the Air contain'd between the outward and inward Receivers was exhauſted.

Now here I was ſure, when the Clapper ſhould be made to ſtrike the Bell, there would be actual Sound produc'd in the inward Receiver, the Air in which was of the ſame denſity with common Air; and could ſuffer no Alteration by the *Vacuum* on its outſide, ſo ſtrongly was it ſecur'd on all ſides. And as I ſaid before, that if the ſonorous Body ſhould ſuffer in any meaſure, by being in a very rare *Medium*, ſo as to contribute to the loſs of its Sound, that this Method ſeem'd probable to diſcover it.

Thus all being ready for Trial, the Clapper was made to ſtrike the Bell; but I found that there was no Tranſmiſſion of it through the *Vacuum*, tho' I was ſure there was actual Sound produc'd in the Receiver.

This plainly ſhews, and ſeems poſitively to confirm, That Air is the only *Medium* for the Propagation of Sound.

The Propagation of Sound, paſſing from the ſonorous Body into the common Air, in one Direction only, ib. p. 369.

2. To try whether that Sound, which ſhould be propagated in a Receiver, having a Communication with the open Air at one ſmall Aperture only, but otherwiſe intirely ſurrounded by a *Vacuum*; whether I ſay, that Sound would be increas'd, or continu'd Sounding longer, at each Stroke that ſhould be given the Bell, than it would do, were not its Body encompaſs'd by ſuch a *Medium*: The Bell was included as in the pre-mention'd Experiment; only, to the upper part of its Receiver, was ſcrew'd a Box with Collars of Leather; and on the Top of the outward Receiver, was laid a Braſs-Plate with a wet Leather between: In the middle of which Plate, was likewiſe ſcrew'd another Braſs Box with Collars, as before. Theſe Receivers when plac'd on the Pump, had their Boxes ſtanding directly one over the other. Through both of them in that Poſition, I paſt a hollow Braſs Tube, which exactly fitted their Perforations: thus the inward Receiver had a Communication with the outward Air, and the outward Receiver thereby was ſecured from the Ingreſs of the Circumambient *Medium*. Now when the Air contained between the Receivers was pretty well exhauſted, and the Bell ſtruck, the Sound was ſenſibly very vigo-

vigorous, and (I think) very nearly as great as before any Air was taken away at all; yet if ones Finger was apply'd to the Apperture of the hollow Brafs Tube, the Sound would be fo much diminifh'd, as but juft to be diftinguifh'd. By this we fee, that fince the Sound in that ftate cannot be tranfmitted through the Receiver that includes it, by means of the furrounding *Vacuum*, yet the Receiver is certainly ftruck with it; but finding no conveyance that way reverberates and makes its Paffage where it finds leaft Refiftance. Nor did I obferve, that although the Sound had but one Paffage from its Receiver, and that but a fmall one, that it continued any longer from the Stroke, than if it had been made in the open Air.

3. The Receiver which contain'd the Bell was fcrew'd down to a Brafs-plate, with a Leather between, as in the former Experiment; This Receiver with its Bell, was fufpended in a large Glafs-Veffel, by four Twine-threads to the top, and as many to the bottom: whereby it remain'd in the middle between both, concluding that thefe Threads would fo abforbe the Water when it fhould come to be put in, that there could be no Apprehenfion, that any Sound fhou'd be convey'd by them from the founding Body, any more than if they were intirely Water. Thus provided, the Clapper was made to ftrike the Bell, whofe Sound was fomething lefs by the Interpofition of the Glafs, than it would be, had it been made in the open Air; however it was very audible, and might be heard at a confiderable diftance; It appear'd to the Ear to be very harfh, in refpect to the Tone it afforded us. But now, when the Water came to be pour'd in, and the inward Receiver furrounded by it, at leaft an Inch and an half from the neareft part of the outward Glafs, the Clapper again was made to give the Sound; which it did, feemingly, very little lefs, in refpect to its Audibility; but much more mellow, fweet, and grave at leaft two or three Notes deeper than it was before. *Of the Propagation of Sound through Water, ib. p. 371.*

VI. *A Paper of lefs general ufe Omitted*, viz.

A Letter from Mr. *Jofeph Williamfon* Watchmaker to the Publifher; wherein he afferts his Right to the curious and ufeful Invention of making Clocks to keep Time with the Sun's apparent Motion, againft the Pretenfions of a *French* Watchmaker, who claim'd the faid Invention. *n. 363. p. 1080.*

CHAP. V.

Hydrostaticks, Hydraulicks.

I. *The weight of common Water under different Circumstances, by Mr.* Hawksbee, *n.* 318. *p.* 221.

FIrst, I took a Glass of common Water, and having weigh'd nicely a Glass-Bottle in it, whose Bulk was equal to the Bulk of 575 Grains of the same Fluid; then I caus'd some of the same Water to be boyl'd over the Fire, and after that, it was included in *Vacuo*, and there remained till it became of the same Temperature (as to Coolness) with common Water. Thus to the utmost of my Power, I endeavour'd to extricate all the Air out of the Water, thinking in that State it would become more dense than when I weigh'd my Bottle first in't; but contrary to my Expectation, I found that the Bottle had just the same weight in it, as before, which seems to confirm the Impossibility to compress Water by force into a lesser space than it naturally possesses; for if upon the removal of such a quantity of Air from out of its Body, the Parts do not slide any closer together, how should a Weight laid upon its Surface, when its Interstices seem to be replete with Air, make any Impression on it. The Body which was forc'd out of the Water by the prementioned means, I call Air, since, for any thing to the contrary that I can discover, it is subject to all the same Laws with it; but that the Water upon its Absence should not unite more closely than before, seems very surprizing to me; for I cannot conceive what Matter must supply the Vacancies, since the Particles of Water themselves remain at the same Distances as if the Air was not withdrawn, otherwise the Water of necessity must become more dense. But to proceed, I caus'd some Water to be heated about Blood warm, when weighing my Bottle in it, I found the Bulk of Water equal to the Bulk of the Bottle, which was about three Grains less than when cold; which shews, that the component parts of the Water are easily separated by Heat, and the Matter lodg'd in its Interstices, capable of Dilation: Then I took that Water that I had purg'd of all its Air (as near as I could,) and gave it a degree of Heat, not exceeding luke-warm; upon weighing the premention'd Bottle in it, I found, that although the Heat it had receiv'd was very inconsiderable, yet the Bulk of the Water in that State, equal to that of the Bottle, was now diminish'd two Grains: which plainly shews, That notwithstanding the Water contain'd no Air that I could discover, yet there seems a Matter latent in it capable of Intumescence.

Of the weight of Bodies in Water, by the same, n. 320. *p.* 306.

II. I took a piece of Sheet Brass (which I take to be more close and solid than that which is cast) of an exact square Inch, weighing just 482 Grains; I then cut as many Inches of Brass Tinsel, as were equal to the same

ſame Weight. The Number of theſe ſquare Inches were 255. Now theſe being of an equal weight with the other ſingle piece in common Air, I concluded from the inequality of their Surfaces that a diſproportion conſiderable in their ſpecifick Gravities would enſue, by weighing them in Water; the Water in one touching ſo many more parts of the Superficies than the other; and 'twas from what is generally aſſerted, that the ſmaller Bodies are, ſo the diſproportions of their Bulks to their Superficies encreaſe; and that ſuppoſing them infinitely ſmall or as Gold diſſolv'd in *Aqua Regis*, or Silver in *Aqua Fortis* muſt be, then their Superficies being touch'd by ſo many parts by the including *Menſtruum*, which is in ſuch a diſproportion to their Diameters or Bulks of Matter, as diſpoſes them to remain ſuſpended in it. This I take to be the general Solution of that Phænomenon, and theſe Conſiderations gave birth to that Experiment, yet when I came to bring it to the Teſt, I found to my great ſurprize, (being prepoſſeſt to the contrary) but two Grains difference; the ſingle piece weigh'd in the Water about 422 Grains; all the other Bodies together, hadly two Grains leſs: And this upon two or three Tryals ſucceeded much the ſame, notwithſtanding they were made with all the Caution imaginable. Now ſince ſo ſmall an Inequality is the Matter of Fact, between Bodies of the ſame Species weigh'd in Water, whoſe diſproportions of Surfaces are, as 1 to 255, (for I reckon the Sides of all the Tinſel Bodies to be equal to the ſides of the ſingle Braſs piece,) I muſt conclude, That thoſe Bodies muſt be infinitely ſmall, whoſe inequality of their Surfaces to their Bulks does exceed thoſe in this Experiment: For ſuppoſing one of theſe thin Squares ſhould be wrought into the form of a Globe, I am very apt to think, That the Diſproportion then of its Surface to its Bulk of Matter, would not be ſo great as its preſent form renders it.

Moreover, That although the Diſproportions of the Surfaces of Bodies to their bulk of Matter be very great; yet, that that is the only Reaſon why a Metallick Body ſhould be ſuſpended in a *Menſtruum* ſpecifically lighter than it ſelf, is very doubtful: For certainly if it was ſo, we might reaſonably have expected to have met with a much greater difference in the Bodies made uſe of in the newly recited Experiment: For there it ſhould ſeem neceſſary, that where we had ſo great a difference in point of Superficies, there we ſhould alſo have had a difference ſomething proportional in point of weight; which did not happen. I think therefore that there muſt be ſome other Agent or Quality, not only to aſſiſt, but to govern in the Caſe. And what we call a corroding *Menſtruum*, I take to be a Fluid adapted to attract ſuch, or ſuch a Body, (as we find no one of them to operate alike on all;) but, as I ſaid before, *Aqua Regis* for ſeparating the parts of Gold, *Aqua Fortis* for Silver: Now this Separation of their Parts by Attraction, ſeems to proceed from the *Menſtruum*'s Affection to the Body immers'd, and the Body reciprocally to the *Menſtruum*, and both to act on one another with greater Vigour, than either of their own Particles do upon their contiguous Fellows; by which

means a Separation of Parts must (I think) consequently follow. Thus being at liberty, they with the *Menstruum* become as one Body, and remain suspended in any part of it by their mutual Attraction. And that one *Menstruum* in this Case should affect one Body more than another, is no more than why the Magnet should affect Iron only.

Different Densities of common Water, by the same, n. 319. p. 267.

III. I caus'd a Quart of Water to be heated near scalding hot, and then put it into a convenient Glass with my Thermometer, the Spirit in which soon arose into the Ball a-top, where it remain'd till the Water cooling caus'd it to descend: by this time the Spirit in the Thermometer and the Water were become of an equal Temperature; and when it had descended to 130 Degrees above the Freezing Point, I began my Observations; which take as follows. I weighed a small Bottle in't, and found the Bulk of Water equal to it in that State was 574 Grains. When the Spirit had descended to 80 Degrees above the Freezing Point, the Bulk of Water equal to the Bottle then weigh'd three quarters of a Grain more than before. At thirty degrees above the Freezing Point, the quantity of Water equal to the Bulk of the Bottle was again increased about three quarters of a Grain. At the Freezing Point, it weigh'd still something more; in all about two Grains from 130 Degrees above the Freezing Point, to that very Point. Which to me seems considerable, and ought to be taken notice of by such Gentlemen, who judge of a Mineral or any other Water by its weight, when they have not an opportunity of making the Experiment at the Fountain-head; for there I suppose the Water is at the same Degree of Temperature at all Seasons.

Now according to this Experiment, I find, that Water is condensable by Cold one 28th part of the whole, from the greatest Degree of Heat in this Climate. Supposing then, that the Water in the Sea should suffer the same Alterations by the change of the different Seasons, (as I see no reason but very nearly it must) it would be easy to compute, that a Ship which should draw two Fathoms, or 12 Feet Water, in such Weather as is understood by the greatest Degree of Heat, would draw about half an Inch less from the greater Density of the Fluid, when reduc'd to the pre-mention'd Degree of Cold; and consequently would Sail better at that time.

Of the seeming spontaneous Ascent of Water by the same, n. 319. p. 258.

IV. 1. *Exp.* I. The several Experiments, which already have been made in relation to the spontaneous Ascent of Water in small Tubes, not only by my self, but several others, with much the same Success, gave me the occasion of thinking, whether the Figure of the Vessel might, or might not, contribute to the Oddness of the Appearance. And to give my self that Satisfaction, (an Account of which I thought would not be unacceptable to the Society) I proceeded as follows.

I procured a Couple of Glass Planes, about seven Inches long, and one and a half broad; these Planes were part of a broken Looking Glass; and notwithstanding when clapt together, they seem'd to touch one another

ther in ſo many parts, yet when they came to be immerſed in a Liquid, the Liquid would aſcend between them; but ſo thin and colourleſs it was, that it could not without difficulty be diſcerned, but upon the Separation of them, when they would be found wet on all their parts: therefore to make it more obvious, I put a ſmall piece of thin Paper on each corner; by which means, when laid one on the other, they became ſeparated by ſuch a diſtance, as is equal to the thickneſs of the Paper. In this manner I plung'd one end under ſome ſtrongly ting'd Liquor; where it no ſooner arriv'd, but the Water run (but not with that Velocity as in a ſmall Tube) gradually, ſometimes higher in one part than in another, ſhooting it ſelf very pleaſingly into Branches divers ways, and ſo would continue till it had arriv'd to its greateſt height; but that would be according to the diſtance the Planes were plac'd aſunder: for if, inſtead of one, two pieces of Paper were laid on each corner of the Planes, the Water then would not aſcend ſo high between them, as when they were ſeparated only by a ſingle one. And then, if the Planes were any ways declin'd, the Water would ſtill ſpread it ſelf farther and farther, agreeable to the degree of Declination: and this on ſeveral Tryals ſucceeded much the ſame.

Exp. II. Having ſeen the Succeſs of the former Experiment in the open Air, I was willing to try what appearance it would afford in *Vacuo*; accordingly I fixt the two Planes ſo to a Braſs Wire, (which paſt through the Cover of a Receiver) that I could make them deſcend at Pleaſure. In this manner, with a Diſh of ting'd Liquor, I convey'd them within the Receiver; which having plac'd on my Pump, I proceeded to exhauſt its contain'd Air, which the Gage, in a little time, diſcover'd to be pretty nicely done. Then I plung'd the Planes (ſeparated by Pieces of thin Paper as before) into the Water, where, as in the open Air, it aroſe between them; only with this difference, that there appear'd more Intervals, or Spaces, between the Branches of the aſcending Liquid, than in the former Experiment: but when I came to let in the Air, thoſe Intervals vaniſh'd, and an intire Body of the Liquid ſucceeded; yet the exact form of the upper parts of it remain'd unalter'd.

Exp. III. By the foregoing Experiments I found, that neither the Figure of the Veſſel, nor the Preſence of the Air, did any ways aſſiſt in the Production of the forementioned Appearance. To try therefore whether a quantity of Matter would help to unriddle the Miſtery; I produc'd two Tubes of an equal Bore, as near as I could, but of very unequal Subſtances, one of them being at leaſt ten times the thickneſs of the other; yet when I came to plunge them into the premention'd Liquid, the Aſcent of it ſeem'd to be alike in both. Now ſince the form of the Veſſel, the preſence of the Air, or the quantity of Matter that compoſes the Veſſel, do not any thing contribute to the Production of the Phænomenon, it may not be amiſs, to inquire a little into the Nature and Property of ſome other Body, that operates with equal Vigour, under the pre-mentioned Circumſtances; and by a Compariſon of one with the other, we may at length arrive nearer to account for the ſame.

What

What I ſhall now uſe to compare with theſe Experiments, is the Magnet.

Firſt, A Magnet of any form will attract Iron.

So by the firſt Experiment, the Figure of the Veſſel ſeems no ways to contribute to the Aſcent of the Water.

Secondly, The Magnet is no ways leſſen'd in its vigour of Attraction, even in ſo thin a Medium as a *Vacuum*.

So by the ſecond Experiment we find the preſence of the Air to be no ways neceſſary to aſſiſt in the Aſcent of the Water, in ſmall Tubes, or between the Planes.

Thirdly, The Magnet, as ſuppoſe one of a Pound weight, that will take up or ſuſpend a piece of Iron of the like weight, and no more, (ſuppoſing it to be in every part of equal vertue) when ſeparated and broke into a number of ſmall parts, (imagining them not to weigh above half a Grain each) and theſe dreſt, and arm'd according to Art, will then be capable to ſuſpend fifty, nay perhaps a hundred times more the weight of Iron amongſt them now ſeparate, than they could when all of one Maſs; which appears to me, that the Attractive Quality of the Stone ſeems to be increas'd in Proportion as its Superficies is to its Bulk of Matter.

So by the third Experiment, I found that the Quantity of Matter, that was us'd to compoſe one Veſſel more than the other, ſignify'd nothing to the Aſcent of the Water, which ſeem'd wholly to depend on the largeneſs, or the ſmallneſs of their Cavities, as to the height it would ariſe in them; and as their Cavities are leſſen'd, ſo the Diſproportions of their inward Surfaces to their Cavities are increas'd.

And as the Magnet, when ſeparated into the prementioned number of ſmall Parts, will attract more than when united in one, and is no more than ſeparating or working the promention'd thick Body of Glaſs into a number of ſmall Tubes, that is multiplying the Surfaces; the Water then would ariſe in each of them ſingly, as it would when all in one Body, its Cavity being the ſame with the others; by which means, the quantity of Water aſcending in them is augmented from the ſame Quantity of Matter.

To conclude: There ſeems to be ſuch an agreeableneſs of the Qualities or Diſpoſitions of one with the other, that I ſee no reaſon why the Facts proceed not from one and the ſame Cauſe; for as the inward Surfaces of the Tubes are made ſmaller and ſmaller, ſo the Power of their Attraction (as is viſible by the higher Aſcent of the Water in them) is greater and greater, and is moſt demonſtrable by the Experiments of the Planes; for their inward *Area* being always the ſame, ſo that as they are placed nearer and nearer to each other, the Cavity or Space between them becomes leſs and leſs, and conſequently the Diſproportions are increaſed, whereby the Power of their Attraction is augmented.

Exp. VI. This Experiment I take to be very Analogous to thoſe lately made on the ſeeming ſpontaneous Aſcent of Water between Glaſs, Marble, and Braſs Planes, as alſo with thoſe made in Capillary Tubes; ſince

fince it feems to proceed from the fame Principle, and fubject to the fame Laws, as appears by matter of Fact; which take as follows. I took a Glafs Tube about 32 Inches long, the Diameter of its Cavity near three quarters of an Inch: This, when I had ty'd a piece of Linnen Cloth at one end, (to prevent the Afhes from falling out) I proceeded to fill with Afhes at the other: the Afhes were fifted through a pretty fine Searfe. At every fmall Portion I put in, I ramm'd them ftrongly down with a Rammer, whofe Bafis was very little lefs than the Bore of the Tube; by which means, I laid, or rather crouded them as clofe together as poffible. When the Tube was become full, I ty'd over that end of it by the Neck a fmall and limber Bladder, having firft expreft all the Air out of its Body, in order to receive that Air, which I expected would be forc'd through the Afhes upon the Afcent of the Water. In this manner I plung'd the end of the Tube, to which I had ty'd the Linnen, (as it was,) under the furface of Water in a Glafs, and found the Water prefently begin to Afcend in it: It arofe a pretty pace at firft; for in 16 Minutes time it had afcended near an Inch and three quarters: but as it arofe higher, fo its Progrefs became flower; for at the end of 24 Hours, the Water had attain'd but to 16 Inches; the Bladder at the top being near half fill'd with that Air which had deferted the Afhes as the Water afcended in them. At the fame time I found the upper part of the Tube, to which the Bladder was ty'd, to be crack'd round, and foon after drop'd off. However, I had the Satisfaction defir'd. And fo continuing the Experiment, I found at 24 Hours diftance from the laft Obfervation, the Water had afcended in the Afhes 6 Inches higher, which was very difcernible by the change of Colour it gave them, diftinct from thofe that were dry.

Again, at the like diftance of time from the laft notice, the Water had arifen 4 Inches and a half, and fomething better. On the 4th Day, at the ufual time of Obfervation, it had afcended 3 Inches higher: and when the following 24 Hours were finifh'd, the Water reach'd within half an Inch of the top, by its afcent of 2 Inches. About 10 Hours after, it had compleatly reach'd the Extremity of the Tube. Then defiring to know what Quantity of Water the Afhes had Abforb'd, I weigh'd a Glafs of Water nicely, part of which I pour'd into the Glafs, in which the Tube had all along been kept, till it reach'd the Mark the Surface of the Water ftood at, when the Tube was firft plung'd into it; and found the quantity to be equal to the weight of 1792 Grains, which is nearly the Bulk of 7 Cubical Inches; the Capacity of the whole Tube, in which it arofe, was equal but to about 13 Inches of the fame Denomination. Now this Experiment to me feems furprizing enough from the following Obfervations.

Firft, That the Water not only afcended in the Afhes, as between the prementíon'd Planes, and in the fmall Tubes, contrary to its natural Gravitation; but with fuch a Power too, as to force, and put to flight pretty ftrongly imprifon'd Air, which was contain'd in the Interftices of the ramm'd Afhes.

Secondly,

Secondly, That the removal of this imprison'd Air could not be done without a Power surmounting its Resistance, which must be great, since upon endeavouring to force Air through the Body of Ashes by the strength of my Breath, when the Tube was not above half filled, it prov'd unsuccessful. Not but that I believe, if the same force had been continu'd for some time, it would have found its way through.

Thirdly, That the Water ascends fastest at first, when there is a larger quantity of Interstitial Air to remove, (if I may call it so,) than when the Column of the dry Ashes grows shorter, by the higher Ascent of the Water in them.

Fourthly, That notwithstanding the Tube was rammed as full as it could with Ashes, yet their Interstices were so many, as to receive, or imbibe another Body, equal in bulk to above half the Content of the whole.

Fifthly, That the Water arose, not only in the Ashes adjoyning to the inward Surface of the Tube, but equally in the whole Body of it, as I found upon Examination.

Sixthly, That the Air lodg'd in the Interstices of the Ashes, was protruded through them as the Water ascended, was manifest by the Intumescence of the Bladder: And notwithstanding the Accident of the Bladders falling off, I cannot but conclude, that the Quantity of it must be equal to the like bulk of Water which supplyed its place.

I repeated the same Experiment in *Vacuo*, in a Tube much about the same Diameter of the other, but not above 10 Inches in length: This Tube, being fill'd with Ashes as before, was plac'd in *Vacuo*, where it remain'd some time, to give liberty for the Air contain'd in them to get away. Then plunging the lower end of the Tube under some Water, I found (as I expected,) that the Water arose faster in the Ashes in that Medium, than in common Air; for in about 4 Hours time, it had reach'd the Extream of its height; which plainly shews, that the Presence of the Air is so far from being necessary in the Production of this odd Phænomenon, that it is a manifest Impediment to it.

2. The Ascent of Water in Capillary Tubes has been taken notice of some *ib. p.* 265. Years ago, but that it should arise between two Glass Planes, whose Sides lie open to the Air, I had not so much as received a hint of before I first discover'd it. And I find that this *Phænomenon* is not to be ty'd up to Glass Bodies alone; for Stone, or Brass, and for ought I know, most other Bodies that have smooth Surfaces, or that their Surfaces may become nearly contiguous to one another, may give the like Appearance; as is plain by the following Experiments. I procur'd a pair of Marble Planes, that were ground as true as the Workman could make them: These when I had joyn'd together dry, without any thing between, I plung'd the edge of them about a quarter of an Inch under the Surface of the Water, and continu'd them so for some Minutes of time: then taking them out, I found I could not easily part them without sliding them one from off the other; which when I had done, 'twas easily discoverable how far the Water

Water had made its way between them, which, upon divers Tryals, I have found different; but at all times, when I had newly rubbed over the Planes with Wood Aſhes, the Water would aſcend higheſt. Now whether the ſmall Duſt of the Aſhes adhering to the Planes may contribute any thing towards it; or that they better clear them from an oily or viſcous Matter, that may be communicated to them from our Hands, I cannot yet determine: However, whatever the occaſion is, the Matter of Fact is true. Then I took a pair of round Braſs Planes, and ordered them as before; the Succeſs of which was very agreeable with the former.

There is one thing I forgot to take notice of in a former Experiment; which is the Aſcent of Spirit of Wine, or Oyl of Turpentine between two Glaſs Planes, without any thing to ſeparate them. It cannot be imagined but that theſe Planes muſt touch each other in a multitude of Parts; yet for all that, and notwithſtanding they are held forcibly together, the Spirit of Wine will inſinuate, and aſcend ſeemingly in an entire Body, between all the contiguous parts of them, as before and after their Separation nothing appears to the contrary.

To the prementioned Experiments give me leave to add what I have ſince obſerved, in plunging the Planes in Spirit of Wine, Oil of Turpentine, and common Oil: That all theſe different Fluids aroſe between as the tinged Water; only with this difference, the common Oil very ſluggiſhly; it was near an Hour ariſing ſo high between them, as the other Liquids would in leſs than half a Minute. They all aroſe in an intire Body from ſide to ſide of the Planes, without thoſe Intervals or Spaces, which generally happen on the Aſcent of the Water. I likewiſe took a couple of round Glaſs Planes, and having laid them one on another, without Paper, or any thing elſe between to keep them ſeparate; in this manner I plung'd one edge juſt under the Surface of the tinged Liquor, and found the Water almoſt inſtantly had reach'd the Extreams of them in all parts: By which we find, that the Water not only aſcends directly upwards, but runs ſideways, obliquely, or in any direction.

Conf. Part III. *p.* 181. & *ſeqq. infra.*

Of the Motion of running Waters, by Dr. Jurin. *n.* 355. *p.* 748.

V. Aquæ Motum ex imi vaſis foramine defluentis ſæpe videmus, tum in ipſa re Hydraulica, tum in ejus Principiis ad Oeconomiam Animalem applicandis, aliis cum Potentiis comparari. Cujus Motus quantitatem cum hactenus nemo, quod ſciam, recte determinaverit, uſurpare ſolent ejus loco ſcriptores Hydraulici Columnæ aqueæ pondus foramini incumbentis. Quod qui faciunt, id ſane neutiquam animum advertunt fieri omnino non poſſe, ut Motus aliquis cum pondere quieſcente conferatur. Poterit autem Aquæ defluentis Motus facili opera definiri hunc in modum

Plate 4. *Fig.* 21.

Sit *S H A H S* Aquæ ſuperficies infinita, *C C* foramen circulare in fundo factum, *A B* recta perpendicularis per foraminis centrum ducta, *S G C C G S* Columna ſive Cataracta Aquæ per foramen *C C* decurrentis, *S G C*

Curva, cujus rotatione circa Axem AB generatur Solidum, sive Cataracta, $SGCCGS$. Aqua enim cum libere, & motu accelerato descendat ad normam corporum omnium gravium, necessario in minorem amplitudinem contrahitur, prout majorem velocitatem acquirit inter cadendum, & profluit ex foramine CC ea cum velocitate, quæ cadendo ab altitudine AB comparatur.

Velocitas autem corporis gravis cadendo genita, ex *Galilæi* demonstratis, rationem obtinet subduplicatam altitudinis, unde cecidit. Quare, si ducatur ad Curvam SGC Ordinata quævis DE, atque ipsa DE vocetur y, & $AD\,x$, exponetur velocitas Aquæ in sectione EE per $\sqrt{x}$, & Factum ex ea velocitate ducta in ipsam sectionem per $\sqrt{x} \times y^2$.

Quod Factum est ut moles Aquæ dato temporis spatio per eam sectionem transeuntis; cumque eadem Aquæ moles dato tempore per singulas Cataractæ sectiones transeat, proinde Factum istud perpetuo sibi constabit, eritque $\sqrt{x} \times y^2 = 1$, & $xy^4 = 1$.

Quæ est Æquatio Curvæ SGC, cujus partem, intra datum vas comprehensam, delineavit, ejusdemque Æquationem non obscure indicavit Magnus *Newtonus*, *Prop.* 36. *Libr.* 2. *Princip.* qui primus omnium veram Aquæ effluentis velocitatem, ex genuinis Principiis deductam, Orbi Literato exposuit.

Est autem ipsa Curva Hyperboloeides quarti Ordinis, cujus altera Asymptotos est recta AS ad Horizontem parallela, altera AB eidem perpendicularis.

Hujus Potestas est Quadrato-Cubus Ordinatæ FG, ductæ ad punctum G, ubi recta AG, bisecans angulum ab Asymptotis comprehensum, Curvæ occurrit.

Spatium $SADES$, inter Curvam SGE, Ordinatam DE & Asymptotos AD, AS inclusum, æquale est quatuor partibus tertiis Rectanguli HD sub Abscissa AD & Ordinata DE contenti. Estque proinde Spatium SHE pars tertia ejusdem Rectanguli.

Solidum $SGEEGS$ convolutione spatii $SADES$, circa Axem AD, generatum, duplum est Cylindri incumbentis sectioni EE. Unde Solidum cavum, quod gignit conversio spatii $SHEGS$, circa eundem Axem, Cylindro incumbenti æquale est. Quæ omnia facili calculo inveniuntur per Methodum Fluxionum inversam.

Theorema I. Aqua ex vase amplitudinis infinitæ, per foramen circulare in fundo factum, decurrente, Motus totius Cataractæ aqueæ Horizontem versus æqualis est Motui Cylindri aquei, sub ipso foramine & altitudine Aquæ, cujus velocitas æquet velocitatem Aquæ per foramen effluentis; vel æqualis est Motui molis Aquæ, quæ dato quovis tempore effluit, cujus ea sit velocitas, qua percurratur eodem dato tempore spatium æquale altitudini Aquæ.

Demonstratio primæ partis. Ducatur ad Curvam SGC alia Ordinata de, priori DE quamproxima.

Curva circa Axem AB conversa, generabunt Ordinatæ DE, de, Circulos duos, quibus intercipitur Solidum nascens $EEee$. Id solidum æquale

est

eſt Facto ex altitudine $D\,d$ ducta in ſectionem $E\,E$, & Motus ejus æquatur Facto ex ipſo ſolido ducto in velocitatem ejuſdem, ſive Facto ex altitudine $D\,d$, ſectione $E\,E$, & velocitate Aquæ in ea Sectione. Cumque ſupra oſtenſum ſit, Factum ex quavis Sectione Cataractæ & velocitate Aquæ in ea Sectione, quantitatem eſſe conſtantem, erit proinde Motus totius Cataractæ æqualis Facto ex quantitate illa conſtante ducta in Summam omnium altitudinum $D\,d$, ſive in ipſam $A\,B$, hoc eſt, Motui Cylindri ſub ipſo foramine & altitudine Aquæ, cujus velocitas æquet velocitatem Aquæ per foramen effluentis. *Q. E. D.*

Corol. 1. Data altitudine Aquæ, erit Motus Cataractæ in ratione foraminis.

2. Dato foramine, erit Motus Cataractæ in ratione ſeſcuplicata altitudinis, ſive in ratione triplicata velocitatis, qua Aqua per foramen exit.

3. Dato Motu Cataractæ, erit foramen reciproce in ratione ſeſcuplicata altitudinis, vel reciproce in ratione velocitatis triplicata.

Demonſtratio ſecundæ partis. Moles Aquæ dato tempore effluentis eſt ad Cylindrum ſub ipſo foramine & altitudine Aquæ, ut longitudo quam Aqua effluens æquabili velocitate dato iſto tempore percurſura ſit, ad altitudinem Aquæ. Cumque velocitas, quæ tribuitur moli Aquæ effluentis, ſit ad velocitatem Cylindri reciproce in eadem ratione, erunt Motuum quantitates utrinque æquales. *Q. E. D.*

Corol. 1. Data altitudine Aquæ & mole effluente, Motus Cataractæ eſt in ratione inverſa temporis quo iſta moles effluit.

2. Data altitudine & tempore, Motus Cataractæ eſt ut moles Aquæ tempore iſto effluentis.

3. Dato tempore & mole Aquæ effluentis, erit Motus Cataractæ in ratione altitudinis.

4. Dato Motu Cataractæ & altitudine, moles effluens eſt in ratione temporis.

5. Dato Cataractæ Motu & mole Aquæ effluentis, altitudo eſt ut tempus.

6. Dato tempore & Motu Cataractæ, erit Aquæ effluentis moles reciproce ut altitudo.

Theorema II. *Fig.* 22. Si capiatur $B\,A$, quæ ſit ad $B\,D$, ut $D\,G^{4}$ ad $D\,G^{4} - B\,C^{4}$; Aqua decurrente ex dato vaſe Cylindrico ſemper pleno $G\,G\,E\,E$, per foramen circulare $C\,C$ in fundo medio factum, Motus Cataractæ aquæ Horizontem verſus æqualis erit Motui Cylindri ſub foramine & altitudine $A\,B$, cujus velocitas æquet velocitatem Aquæ per foramen exeuntis; vel erit æqualis Motui molis Aquæ quæ dato quovis tempore effluit, cujuſque ea ſit velocitas, qua percurratur eodem dato tempore ſpatium æquale altitudini $A\,B$.

Demonſtratio primæ partis. Ducatur $A\,S$ ipſi $D\,G$ parallela, & Aſymptotis $A\,S$, $A\,B$, per puncta G, C deſcripta concipiatur Curva *Newtoniana* $S\,G\,C$.

Ut conſtet Aquæ altitudo, ſupplendus eſt exeuntis locus Cylindro aqueo $g\,g\,G\,G$, deſcendente cum ea velocitate uniformi, quæ acquiritur cadendo ab A ad D, quemadmodum docet Vir incomparabilis Propoſitione prædicta.

Motui hujus Cylindri æquatur, per Theorema superius, Motus Cataractæ *S S G G*. Ergo Motus Aquæ descendentis, cum sit compositus ex Motu Cylindri aquei *g g G G*, & Motu Cataractæ *G G C C*, æqualis est Motui Cataractæ integræ *S G C C G S*, *h. e.* per Theorema primum, Motui Cylindri aquei sub foramine & altitudine *A B*, cujus velocitas æqualis sit velocitati Aquæ per foramen decurrentis. *Q. E. D.*

Pars secunda sequitur ex priore.

Corol. 1. Oriuntur hinc omnia Propositionis præcedentis Corollaria, substituendo altitudinem *A B*, pro Aquæ altitudine.

2. Si vas alia figura fuerit, atque Cylindrica; aut foraminis figura pro circulari fuerit quadrata, triangularis, vel qualiscunque; aut ipsum foramen non sit in medio fundo situm, vel etiam in latere vasis factum; idem erit Motus Cataractæ, scilicet æqualis Motui Prismatis aquei sub foramine & altitudine *A B*, cujus velocitas par sit velocitati Aquæ effluentis. Nam eadem Aquæ moles, cum eadem velocitate atque in priori Hypothesi, tum per ipsum foramen, tum per singulas Cataractæ sectiones transibit.

3. Si vasis Diameter permagnam rationem obtineat ad Diametrum foraminis, negligi poterit altitudo *A D*, & vasis ipsius altitudo pro altitudine Cylindri, vel Prismatis aquei, usurpari.

Hactenus casum illum particularem, quo Aqua, Gravitatis vi, ex vase defluit, seorsum consideravimus. Id eo fecimus lubentius, tum quod illum fere solum adhibere soleant Mathematici, quoties agitur de Fluidorum impetu, tum quod Curvæ Hyperbolicæ supra expositam proprietatem, qua Cataractam Aquæ descendentis format, non indignam censeamus contemplatione Geometrarum. Alioqui potuisset iste casus nullo negotio deduci ex Theoremate generali, quod proximo loco proponemus.

Theorema III. *Fig.* 23. Aqua fluente per Canalem plenum quemcunque *A B C D* secundum lineam *E F*, cui sit perpendiculare utrumque Canalis orificium *A B* & *C D*, Motus Aquæ versus Orificium *C D*, sive Motus impedimenti, quod in ipso orificio oppositum sistat Motum totius Aquæ, æqualis est Motui Prismatis aquei sub qualibet Sectione Canalis *C H* & linea directionis, sive longitudine Canalis *E F*, quod moveatur eadem cum velocitate, qua Aqua fluit per istam Sectionem: sive æqualis Motui molis Aquæ, quæ dato quovis tempore effluit ex Canali, cujusque ea sit velocitas, qua percurratur eodem dato tempore spatium æquale longitudini Canalis.

Cas. 1. Sit linea directionis recta quævis *E F*.

Facile demonstratur pars prima eodem modo, quo Theorema primum: Est enim Factum ex quavis sectione Canalis *C H*, & Aquæ velocitate in ea Sectione, quantitas constans.

Pars secunda sequitur ex prima.

Cas. 2. *Fig.* 24. Si linea directionis *A B C D E*, ex pluribus rectis *A B*, *B C*, *C D*, *E F*, ad sese invicem inclinatis sit composita, idem erit Aquæ Motus. Nam Motus Aquæ in toto Canali composito *A B C D E*, conficitur ex Motibus Aquæ in partibus Canalis *A B*, *B C*, *C D*, *D E*, additis sibi invicem. Statuimus autem aquam fluentem secundum rectam *A B*, mutata

mutata ista directione in aliam, qua feratur secundum rectam $B\ C$, nihil ex Motu deperdere. Leges enim illas, quæ in motu corporum solidorum observantur, quoties eorundem directio mutatur, fluida non sequuntur. Alioqui fluidum, mutata directione in aliam priori perpendicularem, penitus sisteretur, quod Experimentis neutiquam deprehenditur Aqua porro ex Vasis foramine exiliens, sive deorsum, sive secundum Horizontis planum, sive recta sursum feratur, eandem obtinet velocitatem. Quod si aliquando vel ratiocinio subtiliori, vel Experimentis innotescet, aliquam Motus imminutionem ex mutata directione proficisci, erit ejusdem ratio habenda.

Si Curva fuerit linea directionis $A\ B$, referetur ad hunc Casum, quippe quæ ex pluribus rectulis confecta concipi queat. *Fig.* 25.

Cas. 3. *Fig.* 26. Si divisus fuerit Canalis $A\ B$ in plures ramos $B\ C$, $B\ D$, $B\ E$, longitudine æquales, eadem ratione invenietur Aquæ Motus, usurpando pro linea directionis longitudinem $A\ B\ D$, compositam ex longitudine Canalis principis $A\ B$, & longitudine cujusvis rami $B\ D$. Perinde autem est, sive Aqua a Canali principe versus ramos, sive a ramis fluxerit versus principem Canalem. Quod si rami fuerint inæquales, inveniendus est Motus Aquæ in singulis ramis, adhibendo pro linea directionis longitudinem confectam ex longitudine cujusque rami, & longitudine principis Canalis.

Nullo negotio deducitur ex Casu secundo.

Cas. 4. *Fig.* 27. Si rami æquales, in quos distributus est Canalis $A\ B$, iterum in Canalem unicum $F\ G$ colligantur, ad Motum Aquæ inveniendum adhibenda est pro linea directionis longitudo integra $A\ B\ D\ F\ G$, confecta ex longitudine principis Canalis $A\ B$, rami cujusvis $B\ D\ F$, & Canalis recompositi $F\ G$. Si Rami sint inæquales, inveniendus est in singulis Aquæ Motus, & eorum Motuum Summa Motui Aquæ in Canali recomposito addendus. Sequitur ex Casu 2, & 3.

Corol. 1. Data longitudine Canalis, & qualibet Sectione ejusdem, erit Motus Aquæ in ratione velocitatis, qua Aqua fluit per istam Sectionem.

2. Data quavis Sectione, & velocitate Aquæ Sectionem istam præterfluentis, erit Motus Aquæ ut longitudo Canalis.

3. Data Canalis longitudine, & velocitate Aquæ in quavis Sectione, erit Aquæ Motus in ratione illius Sectionis.

4. Dato Motu Aquæ, & aliqua Sectione, erit longitudo Canalis in ratione inversa velocitatis.

5. Dato Aquæ Motu, & longitudine Canalis, erit Sectio quævis reciproce ut velocitas.

6. Data velocitate in qualibet Sectione, & Motu Aquæ, erit ista Sectio in ratione reciproca longitudinis.

7. Data longitudine Canalis, & mole Aquæ certo quovis tempore effluentis, erit Aquæ Motus reciproce ut istud tempus.

8. Data Canalis longitudine, & tempore, erit Aquæ Motus ut moles effluens.

9. Dato tempore, & mole Aquæ effluentis, erit Motus ut longitudo Canalis.

10. Dato

10. Dato Motu Aquæ, & longitudine Canalis, moles effluens eſt in ratione temporis.

11. Dato Aquæ Motu, & mole effluente, erit tempus ut longitudo Canalis.

12. Dato tempore, & Motu Aquæ, erit moles effluens reciproce ut longitudo Canalis.

13. Si binæ moles Aquæ motu contrario in directum occurrant, & pares ſint utrinque tum ſuperficies quibus in ſe invicem impingant, tum velocitates quibus iſtæ ſuperficies in adverſum moveantur, fuerit autem altera moles Aquæ guttulæ uni æqualis, altera Aqua omnis Oceano contenta, vel etiam quantitas Aquæ infinita; fieri poteſt ut una iſta guttula Aquam omnem Oceani, vel quantitatem Aquæ infinitam, non ſolum ſuſtineat, ſed poſt occurſum, eadem ac prius velocitate, ipſa in plagam eandem moveri pergat, eadem illam in partes contrarias repellat. Quod eſt mirabile paradoxon in re Hydraulica.

14. Si certa moles Aquæ, per canalem ex tubis duobus cylindricis, Diametro inæqualibus, compoſitum, a tubo ampliore verſus anguſtiorem fluat, & motus Aquæ neque minuatur inter fluendum neque augeatur, ſimul ac prima pars Aquæ tubi minoris initium ingreſſa fuerit, ſtatim tardius fluere incipiet, & continuato effluxu ex tubo latiore in anguſtiorem, gradatim magis retardabitur Aqua in tubo anguſtiore, uſque dum tota in eum tubum pervenerit. Contrario modo res eveniet, fluente aqua a tubo minore verſus ampliorem. Quod eſt alterum Paradoxon in re Hydraulica. Ponitur autem aqua ubique ſibi cohærere.

Oriuntur bina iſta Corollaria ex Caſu 1.

15. Ex Caſu ſecundo datur Methodus æſtimandi Motum Sanguinis in qualibet arteria.

16. Datis quibuſcunque arteriis binis, æqualem Sanguinis molem tranſmittentibus, major eſt impetus Sanguinis in arteria a Corde remotiore quam in propiore. Quod eſt Paradoxon notatu dignum in Oeconomia animali.

17. Ex Caſu tertio oritur alterum Paradoxon in Oeconomia animali, nempe majorem eſſe Sanguinis motum ſive impetum, in arteriis omnibus Capillaribus ſimul ſumptis, quam in ipſa aorta. Item, major eſt in Capillaribus Venis, quam arteriis.

18. Ex Caſu quarto deducitur Methodus definiendi motum Sanguinis in quavis Vena.

19. Ex eodem deducitur tertium in Oeconomia Animali Paradoxon, nempe majorem eſſe Sanguinis impetum in Vena quavis, quam in arteria ei Venæ reſpondente, & proinde majorem eſſe in Vena Cava, quam in aorta.

Problema I. Invenire motum Aeris ex Pulmone effluentis.

Sit $l=$ Longitudo totius ductus aerei, ab Ore & Naribus ad extremos ramos Trachææ.

$q=$ Quantitas Aeris mediocri exſpiratione ex Pulmone emiſſa.

$\mathcal{Q}=$ Aeris copia validiſſima exſpiratione expulſi.

$t=$ Tempus mediocris exſpirationis.

$T=$ Tempus exſpirationis fortiſſimæ.

Inde,

Inde, per Theorema 3, *Cas.* 3. Motus Aeris ex Pulmone effluentis, in exspiratione mediocri $= \frac{q\,l}{t}$

fortissima $= \frac{Q\,l}{T}$

Hoc est, Motus Aeris ex Pulmone exeuntis æqualis est motui molis Aeris, quæ unica exspiratione emittitur, cujus ea sit velocitas, qua percurratur tempore exspirationis longitudo totius Canalis Aerei *Q. E. I.*

Aeris quantitatem exspiratione mediocri emissam Vir Clarissimus, *Alphonsus Borellus*, facto Experimento 18 circiter, vel 20 unciis cubicis definit. Est autem diversa, non solum in diversis Hominibus, sed etiam temporibus diversis, in Homine eodem. Ipse Experimentum in hunc modum institui.

Vesicæ madefactæ a parte inferiore pondus appendebam, & aptato eidem superius tubo vitreo Diametro circiter unciali, naribus obturatis Aerem vesicæ leniter inspirabam, per spatium trium minutorum secundorum, pondere interim in mensa quiescente. Postea Vesicam cum Aere incluso & pondere appenso, sub aquam in vase Cylindrico contentam, demergebam, notata diligenter altitudine, ad quam aqua attollebatur. Deinde, Aere ex vesica expresso, iterum eandem cum pondere in Aquam immittebam. Quod cum esset factum, facile inveniebatur aquæ moles, quæ vasi infusa altitudinem prius notatam conficeret. Experimento decies repetito, & additis sibi invicem quantitatibus singulis inventis, earum decima, sive media moles aquæ vasi infusa, reperiebatur 35 unciis cubicis æqualis. Quæ moles est Aeris vesica contentæ; & adjecta circiter parte duodecima, seu 3 unciis cubicis, ob Aeris condensationem a frigore Aquæ factam, cum tempestas fuerit hyemalis, efficiuntur 38 unciæ cubicæ. Præterea addendum est tantillum, tum propter Aquæ pressionem in vesicam, tum ob Vaporem qui cum halitu emittitur in humorem coactum; quod fiat necesse est ex frigore Aquæ, & vesicæ madidæ contactu. Æstimavi igitur Aeris copiam, leni exspiratione emissam tempore trium minutorum secundorum, numero rotundo 40 unciarum cubicarum.

In exspiratione validissima exspirabam uncias cubicas 125, tempore minuti secundi unius.

Hujusmodi autem exspiratione, cum vehementi Pulmonis contentione ad strangulatum fere continuata, 220 uncias cubicas ex Pectore emittebam. Unde patet, ut id obiter moneam, multo plus Aeris in Pectore superesse, quam unica exspiratione mediocri emitti.

Si ergo ponatur $l = 2$ pedes

$q = 40$ unciæ cubicæ

$Q = 125$ unciæ cubicæ

$t = 3''$

$T = 1''$

Aeris Gravitas Specifica ad Gravitatem Aquæ, ut 1 ad 1000.

Pes Aquæ cubicus $= 1000$ unc. *Avoird.*

Erit Motus mediocris Aeris Pulmone exeuntis æqualis motui ponderis Scrupulorum 4 & Granorum 9, quod percurrat unciam unam minuto secundo; vel motui ponderis Grani $1\frac{1}{3}$, quod eodem tempore conficiat longitu-

dinem

dinem 5 pedum & 7 unciarum. Quæ eſt velocitas Aeris per Laryngem effluentis, poſita Laryngis Sectione $= \frac{1}{5}$ unciæ quadratæ.

Motus maximus Aeris Pectore expulſi æquatur motui ponderis unciæ 1 $\frac{3}{4}$ circiter, percurrentis unciam unam minuto ſecundo; ſive motui ponderis grani 1 $\frac{1}{3}$ percurrentis eodem tempore 52 pedes. Quæ eſt velocitas Aeris in fortiſſima exſpiratione per Laryngem erumpentis.

Corol. 1. Data Aeris copia & longitudine Canalis aerei, motus Aeris eſt in ratione inverſa temporis exſpirandi.

2. Data mole Aeris & tempore, erit motus in ratione directa longitudinis.

3. Data longitudine & tempore, motus eſt ut Aeris copia.

4. Dato motu & Aeris copia, erit longitudo in ratione directa temporis.

5. Dato motu & longitudine, erit Aeris moles directe ut tempus.

6. Dato motu & tempore, erit Aeris moles reciproce ut longitudo Canalis Aerei.

7. Motus Aeris eſt in ratione compoſita ex ratione quadruplicata Diametri cujuſvis homologæ ipſius animalis, & ratione inverſa temporis exſpirandi; vel in ratione compoſita ex ratione ponderis totius animalis, ratione ejuſdem ponderis ſubtriplicata, & ratione temporis reciproca.

Nam pondus animalis, Diametri cujuſvis homologæ Cubus & moles Aeris expulſi ſunt in eadem ratione. Ponitur autem Corpora Animalium Machinas eſſe ſimiliter factas.

Scholium. Longitudinem hic uſurpatam, vel ipſam eſſe concipies Canalis aerei longitudinem, ſi Rami omnes Tracheæ longitudine æquales ponantur; vel mediam inter longitudines diverſas, ſi Rami ſint inæquales.

Problema II. Determinare impetum, ſive impreſſionem quam excipit interna Pulmonum ſuperficies ab Aere exſpirando.

Cum actioni æqualis & contraria ſit reactio; neceſſe eſt, ut, quanto motu urgetur ab interna Pulmonum ſuperficie Aer exſpirandus, tanto viciſſim ab Aere repellatur ſuperficies Pulmonum.

Unde, per Problema ſuperius, impetus dictus in exſpiratione me-

diocri $= \frac{ql}{t}$

fortiſſima $= \frac{Ql}{T}$ $Q.$ *E. I.*

Hinc poſitis iiſdem quæ in ſuperiore ponuntur, impetus mediocris Aeris in Pulmones æqualis eſt motui ponderis drachmæ circiter 1 $\frac{1}{2}$, quod minuti ſecundi ſpatio percurrat unciam unam; vel motui ponderis 19 librarum, conficientis eodem tempore $\frac{1}{1642}$ unciæ, quæ eſt velocitas Aeris in contactu ſuperficiei Pulmonis internæ. Ponimus autem cum Viro Doctiſſimo *Jacobo Keilio* ſuperficiem Pulmonis internam 21900 circiter unciis quadratis æqualem.

Impetus vero maximus Aeris in Pulmones æquatur motui ponderis unciæ circiter 1 $\frac{3}{4}$ moti unciam unam minuto ſecundo; vel motui ponderis 19 librarum, quod partem $\frac{1}{175}$ unciæ conficiat eodem tempore. Quæ eſt Aeris velocitas ad ſuperficiem Pulmonis in exſpiratione vehementi.

Corol. 1. Sequuntur ex hac Propoſitione Corollaria præcedenti ſubjuncta.

2. Impetus

2. Impetus mediocris incumbens in partem superficiei Pulmonis, quæ sit ipsi Laryngis Sectioni æqualis, est motus ponderis $\frac{1}{22}$ grani, conficientis unciæ spatium minuto secundo, vel motus grani $1\frac{1}{3}$ quod eodem tempore percurrat unciæ partem $\frac{1}{1642}$. Impetus autem maximus in parem superficiem est motus ponderis $\frac{1}{131}$ partis grani quod unciam unam; vel motus ponderis grani $\frac{1}{3}$ quod $\frac{1}{173}$ unciæ singulis minutis secundis conficiat.

3. Impetus Aeris in mediocri exspiratione in Pulmones impressus, æquatur motui Columnæ aqueæ percurrentis unciam unam minuto secundo, cujus Columnæ basis est ipsa Pulmonum superficies, interna altitudo autem est $\frac{1}{68658}$ unciæ. Estque Columnæ altitudo pars $\frac{1}{7302}$ unciæ, in exspiratione omnium vehementissima.

4. Impetus incumbens in superficiem parem circulo maximo Globuli Sanguinei in leni exspiratione, est pars $\frac{1}{14}$ ponderis Globuli Sanguinei; in exspiratione vehementi $\frac{2}{3}$ ejusdem ponderis, moti unciam unam minuto secundo. Qua autem ratione Diametros Globulorum Sanguinis dimensus sim, cum usui esse queat ad aliorum Objectorum minimorum magnitudines definiendas, libet obiter exponere. Capillum tenuem, & satis longum aciculæ pluries circumvolvi, ut omnes convolutiones sese invicem accurate contingerent, quod admotum subinde Microscopium luculenter ostendebat. Deinde cum intercapedinem inter extremas utrinque circumvolutiones Circino cepissem, eandem Scalæ, quam vocant, Diagonali applicabam, spatiumque in Scala repertum per convolutionum numerum dividebam. Unde inventa est unius convolutionis latitudo, sive ipsa Capilli Diameter. Postea Capillum eundem, in Segmenta minutula divisum, plano Microscopii, cui Sanguinis parum erat illitum ut Globuli conspicerentur distincti, superinspergebam. Ea cum Microscopio contuerer, reperiebam aliquibus in locis Capilli Segmenta ita commode disposita, ut numerare liceret, quot Globuli Diametro Segmenti opponerentur. Erant autem Segmenta Diametro inæqualia, quod Capillus tenuior versus extremum fuerit, quam propius a Radice, adeo ut jam 7, vel 8, jam 12, 13ve Globuli transversæ Sectioni Capilli responderent. Utroque autem Experimento sæpius iterato, æstimavi tandem mediam Capilli Diametrum parte $\frac{1}{324}$ unciæ, & Diametrum Globuli Sanguinei parte decima Diametri Capilli, sive parte $\frac{1}{324}$ unciæ.

5. Impetus, quem patitur interna Pulmonum superficies ab Aere exspirando, minor est Motu lenissimi roris e Cœlo decidentis.

Scholium. Neglecta est in solutione Problematum duorum præcedentium impedimenti consideratio, quod Aeri egredienti objicitur ex affrictu laterum Arteriæ Trachææ, ejusque ramorum; cum perparvum sit, neque ullo experimento satis accurate æstimari posse videatur. Nec fuimus admodum solliciti de rationibus numerorum exquisite servandis, cum id unum nobis propositum fuerit, ut methodum exponeremus æstimandi, aliquanto certius quam antehac factum, vires eas, quibus agit Aer inter exspirandum in vasa sanguinea superficiem Pulmonis internam perreptantia. Unde dignosci potest, utrum pares sint hæ vires effectis istis producendis, qui iisdem a Doctissimis quibusdam Scriptoribus Medicis tri-

buuntur. Quod liberum esto Lectoris Scientia Mechanica & Anatomica instructi Judicium.

Problema III. Definire impetum Sanguinis in Vena Cava prope dextram Auriculam Cordis; sive motum Sanguinis per omnes Arterias & Venas fluentis, præter Pulmonares.

Sit $q=$ Quantitas Sanguinis una Cordis Systole in Aortam projecti.

$l=$ Longitudo media ductus integri Arterio-Venosi ratione habita ramorum longiorum & breviorum.

$t=$ Temporis spatium inter binos pulsus interceptum.

Inde, per Theorema 3. Cas. 4. impetus quæsitus $=\frac{q\,l}{t}$

Hoc est, Impetus Sanguinis in Vena Cava æquatur motui molis Sanguineæ, quæ una Systole in Aortam projicitur, cujus ea sit velocitas, qua percurri queat integra Arteriarum & Venarum longitudo temporis spatio inter binos Pulsus intercepto. *Q. E. I.*

Si in Corpore Humano ponantur

$q=2$ unciæ *Avoird.*

$l=6$ pedes

$t=\frac{3''}{4}$

Erit impetus Sanguinis in Vena Cava æqualis motui ponderis 12 librarum, quod unciæ unius longitudinem conficiat singulis minutis secundis; seu motui ponderis 2 librarum, quod pari temporis spatio percurrat pedem $\frac{1}{2}$. Quæ est fere Sanguinis velocitas in Cava fluentis. Ponimus autem, ex dimensione Viri Doctissimi supradicti, Cavæ Sectionem dodrantem esse unciæ quadratæ.

Corol. Oriuntur ex hoc Problemate mutatis mutandis omnia Problematis primi Corollaria.

Problema IV. Determinare motum absolutum Sanguinis in Vena Cava; sive motum Sanguinis, per omnes Arterias & Venas fluentis præter Pulmonales, sublata Vasorum resistentia.

Sit velocitas Sanguinis Naturalis, ad eam velocitatem qua Sanguis flueret, dempta omni resistentia, ut 1 ad x. Cumque per *Corol.* superioris Problematis, & *Corol.* 1. *Probl.* 1. Motus Sanguinis sit in ratione velocitatis, erit inde motus quæsitus $=\frac{x\,q\,l}{t}$. *Q. E. I.*

Quod si proportio per Experimentum a Viro Clarissimo supra laudato institutum inventa, ut veræ propinqua, admittatur, erit $x = 2.5$.

Unde positis iisdem quæ in superiore ponuntur, motus absolutus Sanguinis in Vena Cava æquatur motui ponderis 30 librarum, quod minuto secundo longitudinem uncialem percurrat; sive motui ponderis 2 librarum percurrentis eodem tempore pedem $1\frac{1}{4}$. Qua fere velocitate Sanguis, omni resistentia liber, per Cavam deferretur.

Problema V. Motum Sanguinis invenire in Vena Pulmonali prope sinistram Cordis Auriculam; sive motum totius Sanguinis per Pulmonem fluentis.

Præter

Præter notulas in *Probl.* 3. uſurpatas, ſit λ=Canalis Arterio-Venoſi Pulmonici media longitudo.

Unde, per *Theor.* 3. *Caſ.* 4. invenitur motus quæſitus $=\frac{q\ \lambda}{t}$.

Hoc eſt, motus Sanguinis per Pulmonem fluentis æqualis eſt motui molis Sanguineæ, quæ una Syſtole in Arteriam Pulmonalem projicitur, obtinentis eam velocitatem, qua percurratur longitudo Arteriarum ac Venarum Pulmonalium, tempore inter duos Pulſus intercepto. *Q. E. I.*

Si ponatur in Corpore Humano $\lambda = 1\frac{1}{2}$ pes.

Erit motus Sanguinis in Pulmone æqualis motui ponderis 3 librarum, percurrentis unciale ſpatium minuto ſecundo.

Problema VI. Definire momentum Sanguinis abſolutum in Vena Pulmonali.

Eodem Argumento, quod in *Probl.* 4. uſurpatum eſt, invenitur motus quæſitus $=2.5\times\frac{q\ \lambda}{t}$. *Q. E. I.*

Poſitis vero iiſdem quæ ſupra ponuntur, motus abſolutus Sanguinis Pulmonem præterfluentis æquatur motui ponderis $7\frac{1}{2}$ librarum, quod ſingulis minutis ſecundis unciæ unius ſpatium percurrat.

Scholium. Experimento *Keiliano* definita eſt proportio, quam obtinet Sanguinis per Aortam ejuſque ramos fluentis velocitas naturalis ad eam velocitatem qua Sanguis per eoſdem flueret, ſublata reſiſtentia Arteriarum & Sanguinis præcedentis. Eam nos proportionem ad Sanguinem per Arteriam Pulmonalem fluentem tranſtulimus. Quia vel ſublata vel imminuta ſecundum quamvis rationem reſiſtentia, quæ Sanguini per utramque Arteriam fluenti objicitur, neceſſario Sanguis pariter acceleratur in utraque Arteria. Id enim niſi fiat, bini Cordis Ventriculi aut eodem tempore non contrahentur, aut eandem Sanguinis quantitatem non ejicient. Quorum utrumvis, abſque ſumma totius Machinæ perturbatione & diſcrimine, fieri omnino non poteſt.

Corol. Ad tria Problemata præcedentia.

Sequuntur hinc Corollaria Problemati quinto ſubjuncta, mutatis mutandis.

Scholium ad quatuor Problemata ſuperiora.

Notandum Sanguinis velocitatem, tum per Pulmonem, tum per reliquum Corpus fluentis, cum reipſa æquabilis non ſit, hic tamen talem fingi, ut motus Sanguinis medius inveniatur.

VIII. There have been many Methods propos'd, and Engines contriv'd for enabling Men to abide a compe[t]ent while under Water; and the Reſpiring freſh Air, being found abſolutely neceſſary to maintain Life, ſeveral ways have been thought of for carrying this *Pabulum Vitæ* down to the Diver, who muſt, without being ſomehow ſupply'd with this, return very ſoon or periſh. *The Art of living under Water, by Dr.* Halley. *n.* 349. *p.* 492.

We have heard of the Divers for Spunges in the *Archipelago* helping themſelves by carrying down Sponges dipt in Oyl in their Mouths; but conſidering how ſmall a Quantity of Air can be ſuppos'd to be contain'd in

in the Pores or Interſtices of a *Spunge*, and how much that little will be contracted by the Preſſure of the *incumbent Water*, it cannot be believed that a Supply, by this means obtained, can long ſubſiſt a Diver. Since by Experiment it is found that a *Gallon* of *Air*, included in a *Bladder*, and by a Pipe reciprocally inſpired and exſpired by the *Lungs* of a Man, will become unfit for any further Reſpiration, in little more than one Minute of Time; and though its Elaſticity be but little altered, yet in paſſing the *Lungs*, it loſes its *vivifying Spirit*, and is rendred effete, not unlike the *Medium* found in *Damps*, which is preſent Death to thoſe that breath it; and which in an inſtant extinguiſhes the brighteſt *Flame*, or the ſhining of glowing *Coals* or red hot *Iron*, if put into it. I ſhall not go about to ſhew what it is the *Air* loſes by being taken into the *Lungs*, or what it communicates to the *Blood* by the extreme ramifications of the *Aſpera Arteria*, ſo intimately interwoven with the Capillary *Blood-Veſſels*; much leſs to explain how 'tis performed, ſince no diſcovery has yet been made to prove that the ultimate Branches of the *Veins* and *Arteries* there, have any *Anaſtomoſes* with thoſe of the *Trachæa*; as by the *Microſcope* they are found to have with oneanother. But I rather chooſe to leave this Enquiry to the Curious *Anatomiſt*, to whom the Structure of the *Lungs* is better underſtood; and ſhall only conclude from the aforeſaid Experiment, that a naked Diver, without a *Spunge*, may not be above a couple of Minutes encloſed in Water, (as I once ſaw a *Florida-Indian* at *Bermudas*) nor much longer with a *Spunge*, without Suffocating; and not near ſo long without great Uſe and Practice: ordinary Perſons generally beginning to ſtifie in about half a Minute of Time. Beſides if the Depth be conſiderable, the preſſure of the *Water* on the Veſſels is found by Experience to make the Eyes Blood-ſhot, and frequently to occaſion ſpitting of Blood.

When therefore there has been occaſion to continue long at the *Bottom*, ſome have contrived double flexible Pipes, to circulate *Air* down into a Cavity encloſing the *Diver* as with Armour, to bear off the preſſure of the *Water*, and to give leave to his Breaſt to dilate upon *Inſpiration*: the freſh *Air* being forced down by one of the Pipes with *Bellows* or otherwiſe, and returning by the other of them; not unlike to an *Artery* and *Vein*. This has indeed been found ſufficient for ſmall Depths, not exceeding twelve or fifteen Foot: but when the Depth ſurpaſſes three Fathoms, Experience teaches us that this Method becomes impracticable: for though the Pipes and the reſt of the *Apparatus* may be contrived to perform their Office duly, yet the *Water*, its weight being now become conſiderable, does ſo cloſely embrace and claſp the Limbs that are bare, or covered with a flexible Covering, that it obſtructs the Circulation of the *Blood* in them; and preſſes with ſo much force on all the Junctures, where the *Armour* is made tight with *Leather Skins*, or ſuch like, that if there be the leaſt defect in any of them, the whole Engine will inſtantly fill with Water, which will ruſh in with ſo much violence, as to endanger the Life of the Man below, who may be drown'd before he can

be

be drawn up. Upon both which Accounts, the Danger encreafes with the Depth. Befides a Man thus fhut up in a weighty *Cafe*, as this muft needs be, cannot but be very unweildy and unactive, and therefore unfit to do what he is defigned to do at the Bottom.

To remedy thefe Inconveniencies, the *Diving-Bell* was next thought of; wherein the *Diver* is fafely conveyed into any reafonable Depth, and may ftay more or lefs time under *Water*, according as the *Bell* is of greater or leffer Capacity. This is moft conveniently made in form of a *Truncate Cone*, the fmaller *Bafis* being clofed, and the larger open; and ought to be fo poifed with *Lead*, and fo fufpended, that the Veffel may fink full of *Air*, with its greater or open *Bafis* downwards, and as near as may be in a Situation parallel to the *Horizon*, fo as to clofe with the Surface of the Water all at once Under this Couvercle the Diver fitting, finks down together with the included *Air* into the Depth defired; and if the Cavity of the Veffel may contain a Tun of *Water*, a fingle Man may remain therein at leaft an Hour, without much Inconvenience, at five or fix Fathoms deep. But this included Air, as it defcends lower, does contract it felf according to the Weight of the Water that compreffes it; fo as at thirty three Foot deep or thereabouts, the Bell will be half full of Water, the Preffure of it being then equal to that of the whole Atmofphere: and at all other Depths, the fpace occupied by the compreft Air in the upper part of the Bell, will be to the under Part of its Capacity fill'd with Water, as thirty three Feet to the Depth of the Surface of the Water in the Bell below the common Surface thereof. And this condenfed Air, being taken in with the Breath, foon infinuates it felf into all the Cavities of the Body, and has no fenfible Effect, if the Bell be permitted to defcend fo flowly as to allow time for that purpofe. The only Inconvenience that attends it, is found in the Ears, within which there are Cavities opening only outwards, and that by Pores fo fmall as not to give admiffion even to the Air itfelf, unlefs they be dilated and diftended by a confiderable Force. Hence on the firft Defcent of the Bell, a Preffure begins to be felt on each Ear, which by degrees grows painful, as if a Quill were forcibly thruft into the Hole of the Ear; till at length, the Force overcoming the Obftacle, that which conftringes thefe Pores yields to the Preffure, and letting fome condenfed Air flip in, prefent Eafe enfues. But the Bell defcending ftill lower, the Pain is renewed, and again eafed after the fame manner. On the contrary, when the Engine is drawn up again, the condenfed Air finds a much eafier Paffage out of thofe Cavities, and even without Pain. This Force on the auditory Paffages might poffibly be fufpected to be prejudicial to the Organs of Hearing, but that Experience teaches otherwife But what is more inconvenient in this Engine is the Water entring into it, fo as to contract the Bulk of Air (according to the aforefaid Rule) into fo fmall a fpace, as that it foon heats and becomes unfit for Refpiration, for which reafon it muft be often drawn up to recruit it: and befides, the Diver, being almoft cover'd with the Water thus entring into his Receptacle, will not be long able to endure the Cold thereof.

Being

Being engaged in an Affair that required the Skill of continuing under Water, I found it neceſſary to obviate theſe Difficulties which attend the uſe of the common Diving-Bell, by inventing ſome means to convey Air down to it, whilſt below; whereby not only the Air included therein would be refreſh'd and recruited, but alſo the Water wholly driven out, in whatever Depth it was. This I effected by a Contrivance ſo eaſy, that it may be wondred it ſhould not have been thought of ſooner, and capable of furniſhing Air at the bottom of the Sea in any quantity deſired. The Deſcription of my *Apparatus* take as follows.

The Bell I made uſe of was of Wood, containing about 60 Cubick Foot in its Concavity, and was of the form of a Truncate Cone, whoſe Diameter at the Top was three Foot, and at Bottom five. This I coated with Lead ſo heavy that it would ſink empty, and I diſtributed the weight ſo about its bottom, that it would go down in a perpendicular Situation and no other. In the Top I fixed a ſtrong but clear Glaſs, as a Window to let in the Light from above; and likewiſe a Cock to let out the hot Air that had been breathed; and below, about a Yard under the Bell, I placed a *Stage* which hung by three Ropes, each of which was charged with about one hundred Weight, to keep it ſteddy. This Machine I ſuſpended from the Maſt of a Ship, by a *Spritt*, which was ſufficiently ſecured by *Stays* to the Maſt-head, and was directed by *Braces* to carry it over-board clear of the Ship-ſide, and to bring it again within-board as occaſion required.

To ſupply Air to this Bell when under Water, I cauſed a couple of Barrels of about 36 Gallons each, to be caſed with Lead, ſo as to ſink empty; each having a Bung-hole in its loweſt Parts to let in the Water, as the Air in them condenſed on their Deſcent; and to let it out again when they were drawn up full from below. And to a Hole in the uppermoſt part of theſe Barrels I fixed a Leathern Trunk or Hoſe, well liquored with Bees-Wax and Oyl, and long enough to fall below the Bung-hole, being kept down by a Weight appended; ſo that the Air in the upper Part of the Barrels could not eſcape, unleſs the lower ends of theſe Hoſe were firſt lifted up.

The Air-Barrels being thus prepared, I fitted them with Tackle proper to make them riſe and fall alternately, after the manner of two Buckets in a Well; which was done with ſo much eaſe, that two Men with leſs than half their Strength, could perform all the Labour required: and in their Deſcent they were directed by Lines faſtned to the under edge of the Bell, the which paſt through Rings placed on both ſides the Leathern *Hoſe* in each Barrel; ſo that ſliding down by thoſe Lines, they came readily to the Hand of a Man, who ſtood on the Stage on purpoſe to receive them, and to take up the ends of the *Hoſe* into the *Bell*. Through theſe Hoſe as ſoon as their ends came above the Surface of the Water in the Barrels, all the Air that was included in the upper Parts of them was blown with great force into the Bell, whilſt the Water entred at the Bung-holes below, and fill'd them: and ſo ſoon as the Air of

of the one Barrel had been thus received ; upon a Signal given, That was drawn up, and at the ſame time the other deſcended ; and by an alternate Succeſſion furniſhed Air ſo quick, and in ſo great Plenty, that I my ſelf have been One of Five who have been together at the Bottom, in 9 or 10 Fathoms Water, for above an Hour and half at a time, without any ſort of ill Conſequence : and I might have continued there as long as I pleaſed, for any thing that appeared to the contrary. Beſides the whole Cavity of the Bell was kept entirely free from Water, ſo that I ſat on a Bench, which was diametrically placed near the Bottom, wholly dreſt with all my Cloaths on. I only obſerved, that it was neceſſary to be let down gradually at firſt, as about 12 Foot at a time ; and then to ſtop and drive out the Water that entred, by receiving three or four Barrels of freſh Air, before I deſcended further. But being arrived at the Depth deſigned, I then let out as much of the hot Air that had been breathed, as each Barrel would repleniſh with Cool, by means of the Cock at the Top of the Bell ; through whoſe Aperture, though very ſmall, the Air would ruſh with ſo much Violence, as to make the Surface of the Sea boyl, and to cover it with a white Foam, notwithſtanding the great Weight of Water over us.

Thus I found I could do any thing that was required to be done juſt under us ; and that, by taking off the Stage, I could, for a ſpace as wide as the Circuit of the Bell, lay the Bottom of the Sea ſo far dry, as not to be over-ſhoes thereon. And by the Glaſs-Window, ſo much Light was tranſmitted, that, when the Sea was clear, and eſpecially when the Sun ſhone, I could ſee perfectly well to write or read, much more to faſten or lay hold on any thing under us, that was to be taken up. And by the Return of the Air-Barrels, I often ſent up Orders written with an Iron Pen, on ſmall Plates of Lead, directing how to move us from Place to Place as occaſion required. At other times, when the Water was troubled and thick, it would be as dark as Night below ; but in ſuch Caſes I have been able to keep a Candle burning in the Bell, as long as I pleaſed, notwithſtanding the great Expence of Air neceſſary to maintain Flame.

This I take to be an Invention applicable to various Uſes ; ſuch as Fiſhing for Pearl, Diving for Sponges, Coral, and the like, in far greater Depths than has hitherto been thought poſſible. Alſo for the fitting or plaining the Foundations of Moles, Bridges, *&c.* upon Rocky Bottoms ; and for the cleaning and ſcrubbing of Ships Bottoms, when foul in calm Weather at Sea. I ſhall farther intimate that by an additional Contrivance, I have found it not impracticable for a Diver to go out of an Engine to a good Diſtance from it, the Air being convey'd to him with a continu'd Stream, by ſmall flexible Pipes, which Pipes may ſerve as a Clew, to direct him back again when he would return to the Bell.

IX *An Account of a Book omitted*

Joh. Poleni in Gymnaſio Patavino Phil. Ord. Prof. & Scient. S. R. Lond. & Berol. Sod. De Motu Aquæ Mixto, Libb. duo, *&c.* Patavii. 1717

CHAP.

CHAP. VI.

Geography, Navigation and Musick.

Of drawing a Meridian Line thro France; by Monf. Cassini. n. 278 p. 1097.

I. MOnsieur *Cassini* opened the Assembly of the *Academy of Sciences* held at *Paris*, *Nov.* 12. 1701. with a Discourse containing the Observations he had made in his last Voyage, with a Design to determine the Passage of a Meridian Line (taken from a Point in the *Observatory* at *Paris*) from one End of *France* to the other. In the first Part of this Discourse he went back to the most ancient Astronomers, and recounted their Opinions of the Spherick Figure of the Earth, and their Methods to know its Dimension, of which the two most Famous are, first, that of *Eratosthenes* the *Cyrenian*, who lived in the Reign of *Ptolomie Euergetes*, King of *Ægypt*: The second that of *Possidonius* of *Rhodes*, who lived in the Time of *Pompey the Great*. After having enlarged on the Methods used by these two ancient Philosophers, he proceeded to those of the Moderns *Johannes Fernelius*, and some others; And in the last place he related the Method of the late Monsieur *Picard*, of the *Academy Royal*, as the most exact. Then he spoke of his own Observations on the same Subject, of the use he had made of the *Satellites* of *Jupiter*, more fit for this than the Eclipses of the Moon, in that they are more frequent; and said that his Observations had been confirmed by the like made in *China*. He shewed the Method he took to determine the Passage of the *Meridian* taken from a Point in the Observatory at *Paris*. By the means of Triangles which he made through the whole Course of his Journey, and very exact Calculations, he determined the Place of this Meridian, and marked all the considerable Places through which it passed, from *Paris* to the highest Mountains of the *Pyreneans*, which separate *Roussillon* from *Catalonia*; among these Mountains he observed one of a prodigious height, it being 1440 Toises high. But the most extraordinary Observation was that of the Inequality of the Degrees of the Meridian on the Earth; which is such, that Monsieur *Cassini* found that going Southward one degree surpassed another an 800th part, which may give great Reason to doubt of the exact Roundness of the Earth. Upon this occasion he reported two different Opinions: the one Messieurs *Huygens* and *Newton*, the other of a Mathematician of *Strasbourg*, named *Eisenschmidius*. The two former hold that the Earth is flatted towards the Poles, so that it is something of the Shape of an *Holland* Cheese; which they both conclude by Physical and Algebraical Deductions, from an Observation made at Cape *Verd*; that the *Pendulums*, though of the same Length, make their Vibrations there much slower than in the Northern Countries. The other Mathematician holds that the Figure of the Earth is Elliptick, so that it is stretch'd out towards the Poles, and has the Form of an Egg. Monsieur *Cassini* left the Question undecided. The Cities through which

he

he obſerv'd the Meridian of *France* to paſs are *Dunkirk*, *Amyens*, *Aubigney*, *Bourges*, *Aurillac*, *Rodez*, *Alby* and *Carcaſſone*. Monſieur *l' Abbe Bignon* ſaid that the Meridian at *Paris* would be obſerv'd round the whole World, with the ſame Exactneſs as had already been begun to be done from one End of *France* to the other; and that there were Perſons of Courage ſufficient to undertake ſo painful a Voyage; and that nothing hindred them but the War, which began to be kindled every where. He meant by this Perſon Monſieur *de Chaſel*, who having already made ſeveral Voyages into the *Mediterranean*, in quality of the King's Engineer, and collected many Obſervations, was then making a true and exact Chart thereof.

To divide mechanically the Nautical meridian Line, by Mr. Perks, *n.* 345 *p* 3[illegible].

II. The moſt uſeful Projection of the Spheric Surface of Earth and Sea for Navigation, is that commonly call'd *Mercator's*; tho' its true Nature and Conſtruction is ſaid to be firſt demonſtrated by our Countryman Mr. *Wright*, in his *Correction of the Errors of Navigation*. In this Projection the Meridians are all parallel Lines, not divided *equally*, as in the common plain Chart (which is therefore erroneous) but the Minutes and Degrees (or ſtrictly, the *Fluxions of the Meridian*) at every ſeveral Latitude are proportional to their reſpective *Secants*. Or a Degree in the projected Meridian at any Latitude, is to a Degree of Longitude in the Equator, as the *Secant* of the ſame Latitude is to *Radius*.

The Reaſon of which Enlargement of the Elements of Latitude is, to counterbalance the Inlargement of the Degrees of Longitude. For in this Projection the Meridians being all Parallel, a Degree of Longitude at (ſuppoſe) 60 Deg. Lat. is become equal to a Degree in the Equator, whereas it really is (on the Globes Surface) but *half* as much, the Radius of the Parallel of 60 Deg. (that is its *Coſine*) being but *half* the Radius of the Equator. Therefore to proportion the Degrees of Latitude to thoſe of Longitude, a Degree (or Elemental Particle) in the Meridian, is to be as much greater than a Degree (or like Particle) in the Equator, as the Radius of the Equator is greater than the Radius of the Parallel of Latitude, *viz.* its *Coſine*.

In *Fig.* 29. let the Radius CD repreſent the half of the Equator, DM an Arc of the Meridian; MS its Sine, CE its Secant; then is CS equal to its *Coſine*: and $CS : CM :: CD\ (=CM) : CE$, that is, as Coſine : to Radius : : ſo is Radius : to Secant. The Coſines being then, in this Projection, ſuppos'd all equal to Radius, or (which comes to the ſame) the Parallels of Latitude being all made equal to the Equator, the Radius of the Globe, at every point of Latitude (by the precedent Analogy) is ſuppoſed equal to the Secant of Latitude; and conſequently the Elements (Minutes, &c.) of the Meridian muſt be proportional to their reſpective Secants. *Plate* 4.

The Way Mr. *Wright* takes for making his Tables of *Meridional Parts*, is by a continual Addition of Natural Secants, beginning at 1 Minute, and ſo proceeding to 89 Deg. Dr. *Wallis* (in * *Phil. Tranſ.* No. 176.) finds the Meridional Part belonging to any Latitude by this *Series*, putting S for its Natural Sine, *viz.* $S + \frac{1}{3} S^3 + \frac{1}{5} S^5 + \frac{1}{7} S^7 + \frac{1}{9} S^9$, &c. which gives

* *Abr. Vol.* I. p. 572.

the *Meridional part* required. How to find the same Mechanically by means of an easily constructed Curve Line, is what I shall now shew.

1. Prepare a Rular AB (*Fig.* 28.) of a convenient Length, in which let Bo be equal to the Radius of the intended Projection. To the Point o, as a Center (on the narrower Edge of the Rular) fasten a little Plate-wheel wh tight to the Rular, and of a Diameter a little more than the Thickness of the Rular. Let KR (*Fig.* 29.) represent another long Rular to which AR is a perpendicular Line. Place the Rular AB upon the Line AR, with the Center of the Wheel at A Then with one Hand holding fast the Rular KR, with the other Hand slide the end B of the Rular AB by the Edge of KR; so will the little Wheel wh describe on the Paper a Curve Line ACB, to be continued as far as is convenient.

2. Having drawn the Curve ACB, draw a streight Line KR by the Edge of the Rular KR: which Line is the *Meridian* to be divided, and also an Asymptote to the Curve ACB.

3. In this Meridian, (accounting R to be the Point of its Intersection with the Equator,) the Point answering to any Degree of Latitude is thus found. In the perpendicular AR, make RG equal to the *Cosine* of Latitude (Radius being AR,) and from G draw GC parallel to KR, and intersecting the Curve in C. With Center C and Radius $CM = AR$, strike an Arc cutting the Meridian at M; so is M the Point desir'd.

4. In the Curve AC, let c be a Point infinitely near to C, and cm, ($= CM$,) a Tangent to the Curve at c, making the little Angle MCm, to which let the Angle RAr be equal: so is $Rr = Md$ (a Perpendicular from M to cm.) Draw CD equal and parallel to AR, intersecting KR in S. With Center C and Radius CD draw the Arc DM, and its Tangent DE and Secant CE.

5. Because of the like Triangles CDE, Mdm; $CD : CE :: Md : Mm$, that is, as Radius to Secant of the Arc DM, (whose Cosine is $CS = GR$,) :: so is Md ($= Rr$ a Degree or Particle of the Equator:) to Mm the Fluxion or correspondent particle of the Meridian Line RM. Whence, and from what is premised concerning the Nature of this Nautical projection, 'tis evident that RM is the *meridional Part* answering to the Latitude whose Cosine is GR. Or thus; with Center R and Radius AR describe the Quadrant $A\kappa\alpha$, in which let the Arc $A\kappa$ be equal to the given Latitude. From κ draw κC parallel to KR and intersecting the Curve in C, so is $C\kappa$ the Meridional part desir'd, being equal to RM, as is easy to shew.

6. As to the other properties of this Curve, 'tis evident, from its Construction, that its *Tangent* (as CM) is a *Constant Line* every where equal to AR; the Curve being generated by the Motion of the Wheel at the End of the Rular which is its Tangent. And from hence the Curve ACB may, for Distinction, be call'd the *Equitangential Curve*.

7. The Fluxion of the Area $ARMC$ is the little Sector or Triangle MCd, which same is also the Fluxion of the Sector CDM: whence the Areas $ARMC$, CDM are equal, and the whole Area ACB, &c. KMR, being infinitely continued, is equal to the Quadrant $AR\alpha$.

8. To find the Radius of Curvature of any particle, as Cc; from C draw

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an indefinite Line CT perpendicular to CM, (on the concave side of the Curve) and from c another Line perpendicular to cm, which Lines, (because of the Inclination of CM to cm,) will somewhere meet as at T, making an Angle $CTc = MCm$. These Angles being equal, their Radii are proportional to their Arcs: therefore, $Md : Cc :: MC : CT$. But $Cc = dm$ (because of $CM = cm$) so that $Md : dm$ $(:: CD : DE) :: CM : CT$. But $CD = CM$, therefore $CT = DE =$ Tangent of the Arc DM.

9. So that supposing ATt a Curve Line in which are all the Centers of Curvature of the particles of ACB, any point as T being found as before, the Length AT (by the Nature of *Evolution of Curves*,) is every where equal to the *Tangent* of its correspondent Circular Arc DM. The point T is also found by making MT perpendicular to RM, and equal to the Secant CE: for so is the Angle $CMT = MCD$; and the Triangle MCT equal to the Triangle CDE.

10. Let AHh be an Equilater Hyperbola whose Semiaxis is AR and Center R. In the Meridian let RP be equal to the Tangent DE. Join AP, and draw $PH = AP$ and parallel to AR. Compleat the parallelogram $HNRP$, so will the point H be in the Hyperbola, and its ordinate HN $(= RP = DE = CT)$ be equal to the Curve ATt. From whence, and from *Prop.* 3. *Coroll.* 2. of Dr. *Gregory's Catenaria* (*Phil.* * *Transf.* 231.) it appears that the Curve ATt is that call'd the *Catenaria* or *Funicularia*, viz. the Curve into whose Figure a *slack Cord or Chain* naturally disposes its self by the Gravity of its particles.

* Abr. Vol. I. p. 39.

' 11. Hence we have another property of the *Catenaria* not hitherto ta-
' ken Notice of (that I know of,) *viz.* that supposing AR $(= a$, the
' constant Line in Dr. *Gregory*) equal to the Radius of the Nautical pro-
' jection, and RN the Secant of a given Latitude, then is NT the *Cate-*
' *naria's* Ordinate at N, equal to RM the Meridional part answering to
' the Latitude whose Secant is RN.

12. That TA is the *Catenaria* is also demonstrable from Dr. *Gregory's* first *Prop.* Let Tu be the Fluxion of the Ordinate NT; and tu $(= Nn)$ the Fluxion of the Axe AN. Then because of like Triangles TCM, Tut, $CM : CT$ $(= TA)$ $:: Tu : ut$, that is, as CM a constant Line to TA the Curve $::$ so is the Fluxion of the Ordinate, to that of the Axe $(\dot{y} : \dot{x})$ according to *Prop.* 1. *Catenariæ*.

13. From the Premises the Construction and several Properties of the *Catenaria* are easily deducible; one or two of which I'll set down.

1. The Area $ATMR$ is equal to $AOPR$ a Rectangle contained by Radius AR and RP the Tangent answering to Secant $HP = TM$. For because of the like Triangles CMm, CEe; $CM : CE :: Mm : Ee$, that is, putting r, s, t, m for Radius, Secant, Tangent and Meridional part RM) $r : s :: \dot{m} : \dot{t}$ whence $r\dot{t} = s\dot{m}$, and all the $r\dot{t}$ = all the $s\dot{m}$, that is, $AOPR = ATMR$, which agrees with Dr. *Gregory's Cor.* 5. of *Prop.* 7

14. Supposing the former Construction, let be added the Line RH, including the *Hyperbolick Sector* ARH. I say the same Sector is equal to half the Rectangle $ARMQ$ contained by Radius AR and the Meridional part RM, $(= \frac{1}{2} rm)$ For the Sector $ARH =$ Triangle RNH wanting the

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Semisegment ANH. The Fluxion of the Triangle RNH is $\frac{s\dot{t}+t\dot{s}}{2}$ The Fluxion of ANH is $t\dot{s}$. So the Fluxion of the Sector ARH is $\frac{s\dot{t}+t\dot{s}}{2}$ $-t\dot{s}=\frac{s\dot{t}-t\dot{s}}{2}$ 'Tis found before (*Sect.* 13.) that $r:s\ (s:\frac{ss}{r})::\dot{m}:\dot{t}$; whence $s\dot{t}=\frac{ss}{r}\dot{m}$. And because of the like Triangles CDE, Efe, CD : $DE::Ef:fe$. But $Ef=Mm=\dot{m}$, because both Ef and Mm are to Md in the same Reason, *viz.* as s to r; therefore $r:t\ (t:\frac{tt}{r})::\dot{m}:\dot{s}$: whence $t\dot{s}=\frac{tt}{r}\dot{m}$, and $\frac{s\dot{t}-t\dot{s}}{2}=\frac{ss-tt}{2r}\dot{m}=\frac{rr}{2r}\dot{m}=\frac{1}{2}r\dot{m}$, = the Fluxion of the Hyperbolick Sector ARH, whose flowing Quantity is therefore equal to $\frac{1}{2}rm=\frac{1}{2}ARM$ Q. *Q. E. D.*

15. This shews another property of the *Catenaria*, viz. that it squares the Hyperbola; for RM is equal to NT the Ordinate of the *Catenaria*.

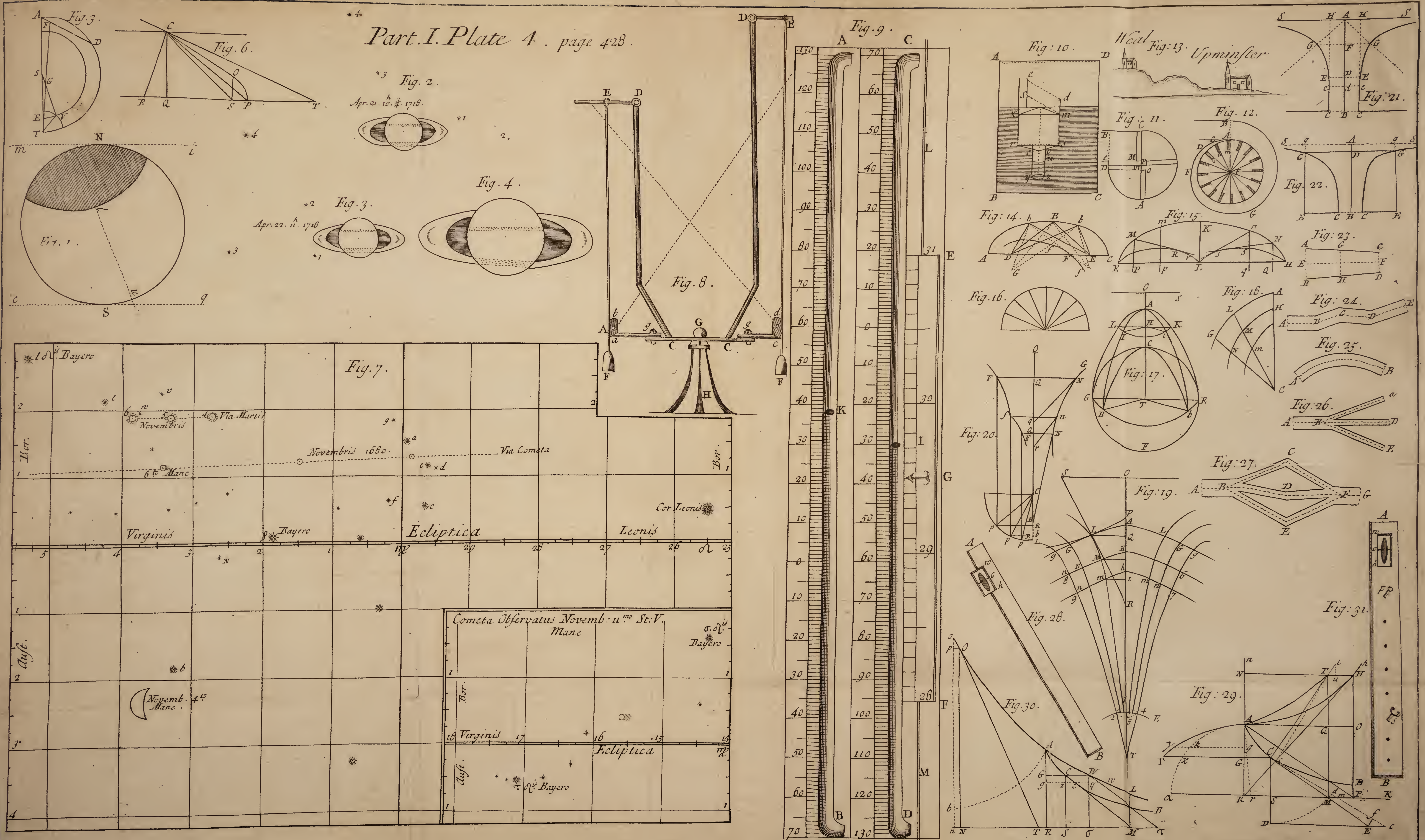
16. In *Fig.* 30. Let AR be Radius, ACB the Equitangential Curve; MRN its Asymptote, in which let M, N, be any two points equally distant from R. Upon M draw ML parallel to AR and equal to the *Difference* of the Secant and Tangent of that Latitude whose Meridional part is RM (by §. 3, 4) Upon N draw NO parallel to AR, and equal to the *Sum* of the foresaid Secant and Tangent. Do thus from as many points in the Asymptote as is convenient, and a Curve drawn equably through the points L - - - A - - - O - - &c. will be a *Logarithmic Curve*, whose *Subtangent* (being constant) is equal to Radius AR.

17. Let no be an Ordinate infinitely near and parallel to NO. $OP=Nn$ the Fluxion of the Asymptote; OT the Tangent, and TN the Subtangent to the Logarith. Curve in O. Then $op:pO::ON:NT$. But $ON=s+t$; therefore $op=\dot{s}+\dot{t}$. $pO=\dot{m}$ (the Fluxion of the Meridian or Asymptote.) So the Analogy is $\dot{s}+\dot{t}:\dot{m}::s+t:NT$. By *Sect.* 13, 14, $\dot{s}:\dot{m}::t:r$. also, $\dot{t}:\dot{m}::s:r$. and thence $\dot{s}+\dot{t}:\dot{m}::t+s:r$. wherefore NT (the Subtangent to LAO) is equal to Radius AR a constant Line, and consequently the Curve LAO is the Logarithmic Curve and its Subtangent known.

18. The same Demonstration serves for LM, (any Ordinate on the other Side of AR) only changing the Sine + into —; and then it agrees with Mr. *James Gregory*'s *Prop.* 3. *pag.* 17 of his *Exercitations*, viz. *That the Nautical Meridian is a Scale of Logarithms of the Differences whereby the Secants of Latitude exceed their respective Tangents, Radius being Unity.* So here RM is the Logarithm of ML, the Difference of the Secant and Tangent of the Latitude whose Meridional part is RM.

19. Supposing the precedent Construction, if through any point C of the Curve ACB be drawn a right Line GCW parallel to MR, terminated with the Logarithmic Curve in W and the Radius AR in G: I say that the same right Line WG is equal to the intercepted part of the Curve Line AC.

20. Let


Part. I. Plate 4. page 428.
Fig. 1.
Fig. 2.
Apr. 21. 10. h. 1718.
Fig. 3.
Apr. 22. 11. h. 1718.
Fig. 4.
Fig. 6.
Fig. 7.
Bayero
Via Martis
Novembris
Novembris 1680.
Via Cometa
Mane
Cor Leonis
Virginis
Ecliptica
Leonis
Bor.
Aust.
Novemb. 4to Mane.
Cometa Observatus Novemb: 11mo St: V. Mane
Fig. 8.
Fig. 9.
Fig: 10.
Fig: 11.
Fig. 12.
Weal
Fig: 13.
Upminster
Fig: 14.
Fig: 15.
Fig: 16.
Fig: 17.
Fig: 18.
Fig: 19.
Fig: 20.
Fig: 21.
Fig. 22.
Fig: 23.
Fig: 24.
Fig: 25.
Fig: 26.
Fig: 27.
Fig: 28.
Fig: 29.
Fig: 30.
Fig: 31.

20. Let $w\,g$ be a Line infinitely near and parallel to WG, and terminated by the same Lines; and CS, $W\sigma$, perpendicular to the Meridian; CS intersecting wg in z, and $W\sigma$ in y. Let CM be a Tangent to AC in C; $W\tau$ a Tangent to AW in W; so is $CM = \sigma\tau$. Because of like Triangles Czc, CSM; and Wyw, $W\sigma\tau$; $CS : CM :: Cz : Cc$: also $W\sigma : \sigma\tau :: Wy : yw$. But $W\sigma = CS$; $\sigma\tau = CM$; $Cz = Wy$; therefore is yw the Fluxion of GW, equal to Cc the Fluxion of the Curve AC. Consequently $GW = AC$. *q. e. d.*

21. It may be noted that this Equitangential Curve gives the Quadrature of a Figure of Tangents standing perpendicular on their Radius. In *Fig.* 29. let $A\gamma\Gamma$ be a Curve whose Ordinates as $g\gamma$, $G\Gamma$, are equal to the Tangents of their respective intercept Arcs Ak, $A\kappa$. Let ΓG be produced to touch the Curve AC in C; then is the Area $A\Gamma G$ = to the Rectangle contained by Radius AR and GC the produced part of the Ordinate; or $A\Gamma G = AR \times GC$. The Demonstration of which, and of the following *Section* I for Brevity omit.

22. If we suppose the Figure ACB, &c. KR (*Fig.* 29.) infinitely continued, to be turned about its Asymptote RK as an Axe, the Solid so generated will be equal to a rectangled Cone whose Altitude is equal to AR. And its Curve Surface will be equal to half the Surface of a Globe whose Radius is AR. So that if the Curve be continued both ways infinitely (as its Nature requires) the whole Surface will be equal to that of a Globe of the same Radius AR.

The Description of the Rular and Wheel, *Fig.* 28. is sufficient for the Demonstration of the Properties of the Curve: but in order to an actual Construction for Use, I have added *Fig.* 31. where AB is a brass Rular; wh the little Wheel, which must be made to move freely and tight upon its Axe (like a Watch-Wheel) the Axe being exactly perpendicular to the Edge of the Rular. s represents a little Screw-pin to set at several Distances for different Radii, and its under End is to slide by the Edge of the other fix'd Rular. p is a Stud for convenient holding the Rular in its Motion.

III. Having had the Honour last Week of making the Tryal of a Musical Experiment before the Society, it may be necessary to give a farther Account of it; that the Theory of Musick, which is but little known in this Age, and the Practice of it, which is arriv'd at a very great Excellency, may be fix'd upon the sure Foundation of Mathematical Certainty. The Propositions on which the Experiments were admitted, were; that Musick consisted in proportions, and the more exact the proportions, the better the Musick: That the proportions offer'd were the same that the ancient *Grecians* us'd: That the Series of Notes and half Notes was the same our Modern Musick aim'd at: which was there exhibited upon Finger-boards, calculated in Mathematical proportion. This was demonstrated upon a Viol, because the Strings were of the greatest length, and the proportions more easily discern'd; but may be accommodated to any Instrument, by such mechanical Contrivances as shall render those Sounds, which the Musick requires.

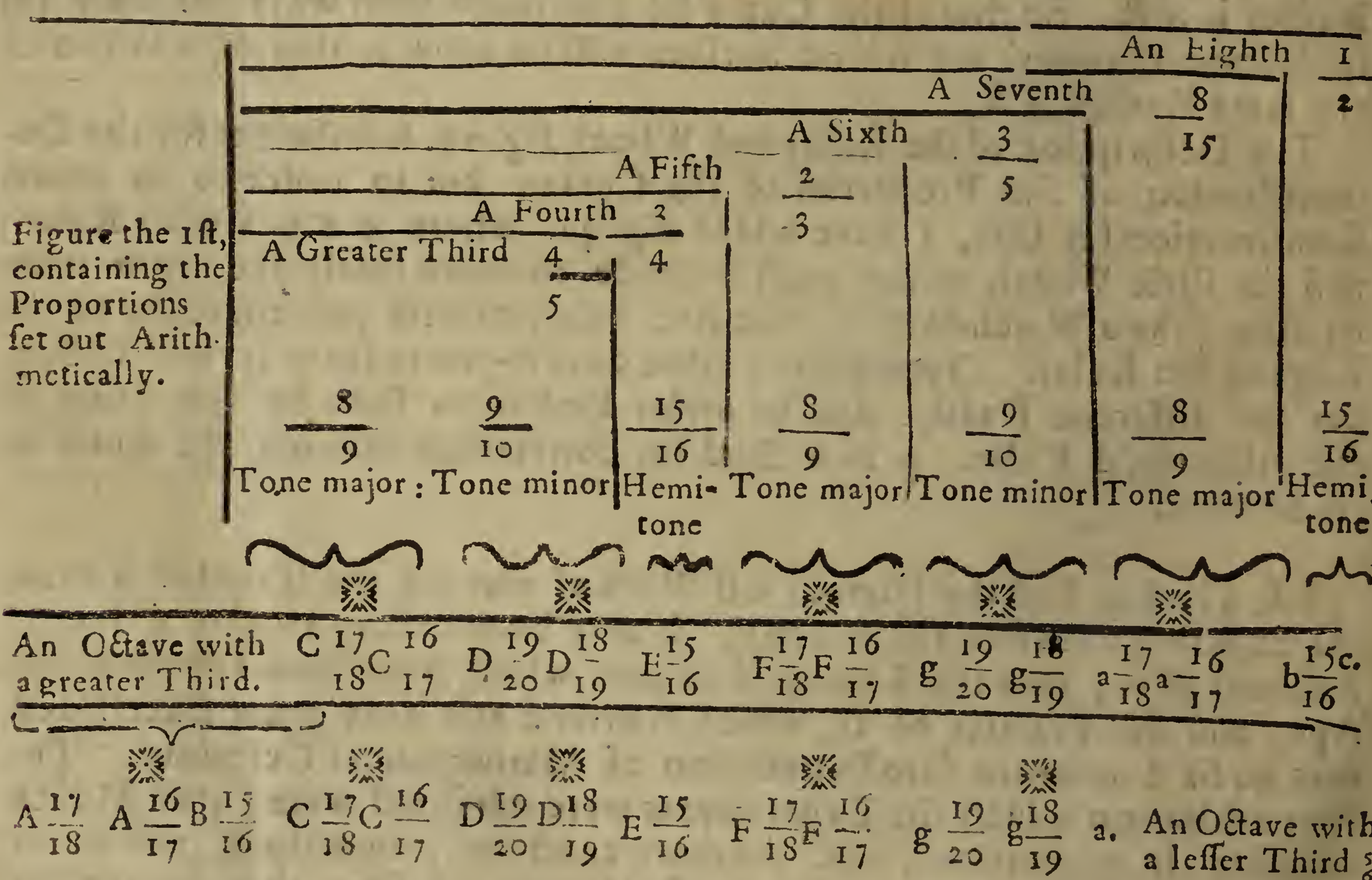
The Theory of Musick reduc'd to Arithmetical and Geometrical Proportions; by Mr. Tho. Salmon. *n.*302. *p.* 2072.

To

To prove the foregoing propositions, two Viols were Mathematically set out, with a particular Fret for each String, that every stop might be in a perfect exactness: Upon these a Sonata was perform'd by those two most eminent Violists, Mr. *Frederick* and Mr. *Christian Stefkins*, Servants to Her Majesty; whereby it appear'd, that the Theory was certain, since all the stops were owned by them to be perfect. And that they might be prov'd agreeable to what the best Ear and the best Hand performs in modern practice, the famous *Italian*, Signior *Gasperini*, plaid another Sonata upon the Violin in Consort with them, wherein the most compleat Harmony was heard.

The full knowledge and proof of this Experiment may be found in the two following Schemes, wherein Musick is set forth, first Arithmetically and then Geometrically: The Mathematician may, by casting up the proportions, be satisfied, that the five sorts of Half-Notes here set down, do exactly constitute all those Intervals, of which our Musick does consist. And afterwards he may see them set forth upon a Monochord, where the measure of all the Notes and Half-Notes comes exactly to the middle of the String. The Learned will find that these are the very proportions which the old *Greek* Authors have left us in their Writings, and the Practical Musician will testifie, that these are the best Notes he ever heard.

Figure the 1st, containing the Proportions set out Arithmetically.



Explanation of the first Figure. Between the 2 lowest Lines, you have the Series of all the 12 half Notes in an Octave, from Are to Alamire, which added together make an Octave or exact Duple Proportion: The several parts also added together make all those inervals of which it is constituted. As for example, the two half Notes from A to A✱ $\frac{17}{18}$, and from A✱ to B $\frac{16}{17}$ make a Major Tone $\frac{8}{9}$; to which if an Hemitone from B to C $\frac{15}{16}$ be added, you have a lesser Third $\frac{5}{6}$.

In

In like manner between the two next lines, you have the series of all the 12 half Notes, in an Octave from C fa ut to C sol fa ut: the two first Tones added together make a greater Third: and so you may add a Tone or Hemitone till you arrive at every interval in the Octave, which is so call'd because eight sounds are required for expressing those seven gradual steps whereby we commonly ascend to it.

It may be also observ'd, that the proportions falling upon the same Notes in two Keys, one finger-board will be sufficient for both.

'Tis acknowledg'd by all that are acquainted either with Speculative or Practical Musick, that every interval is divided into two parts, whereof one is greater than the other: An Eighth $\frac{1}{2}$ into a Fifth $\frac{2}{3}$ and a Fourth $\frac{3}{4}$. Again, a Fifth $\frac{2}{3}$ into a greater Third $\frac{4}{5}$ and a lesser Third $\frac{5}{6}$. Thus also a greater Third $\frac{4}{5}$ must be divided into a Tone Major $\frac{8}{9}$ and a Tone Minor $\frac{9}{10}$. The Lesser Third (to comply with the practice of Musick) is rather compounded of, than divided into a Tone Major $\frac{8}{9}$ and an Hemitone, which is its complement, $\frac{15}{16}$.

Three Tones Major, two Tones Minor, and two of the foresaid Hemitones, placed in the order found in the Scheme, exactly constitute the practical Octave; which is so call'd because it consists of eight sounds, that contain the seven gradual intervals. But it is also necessary to set down the Divisions of the whole Tones, which are the true Chromatick half Notes because there is great use of them in Practical Musick.

To make all our whole Notes, and all our half Notes of an equal size, by falsifying the proportions, and bearing with their imperfections, as the common practice is, may be allowed by such Ears as are vitiated by long custom: But it certainly deprives us of that satisfactory pleasure which arises from the exactness of sonorous numbers; which we should enjoy, if all the Notes were truly given according to the Proportions here assign'd.

It is very easie to satisfie our selves in the Arithmetical Scheme, by those operations which *Gassendus* has set down. As for example, his rule for Addition is, That two Proportions being given, if the Greater number of one be multiplied by the Greater number of the other, and the Lesser by the Lesser, the two numbers produc'd exhibit the compounded Proportions. Thus take a Practical Fifth $\frac{2}{3}$ and a Practical Fourth $\frac{3}{4}$ for the two Proportions given, multiply 3 by 4 and you have 12, then multiply 2 by 3 and you have 6: which compounded proportion of 12 to 6 makes the Practical Octave $\frac{1}{2}$.

Manuduction to the Theory of Musick, Tom. V. pag. 635.

Thus, according to his Arithmetical operations of Addition, Substraction, Multiplication or Continuation, and Division, is our whole System proved, which for the more easie application to Practical Musick, shall be also set forth Geometrically upon the 6 strings of a Viol. *See Figure the 2d.*

This Mathematical fixing of the Frets enables every Practitioner, who stops close to them, to give the Proportions of the Notes in a greater exactness, than can be done upon the Bass-Violin or Violin itself: since they may be set forth more perfectly by a pair of Compasses dividing a line, than the nicest Ear can direct.

Though

Though the Frets for the ſeveral Strings do not ſtand in a ſtrait line, and the places are alſo ſhifted in different Keys, yet the Ear naturally directs the Fingers to them: inſomuch that thoſe perſons, who have all their lives time been accuſtom'd to ſtop upon Frets that go quite croſs the Finger-boards of their Inſtruments, do with very little practice fall right upon theſe. Such is the power of a Muſical Genius, as may be undeniably proved by thoſe that play upon the Violin; who, when they change the Key, fall upon the right Stops, tho' they have no viſible direction where to ſtop, nor time to alter, by the Ear, the Note they firſt pitched upon.

By this Standard of Regular Proportions may the Voice be formed to ſing the pureſt Notes; they are all the ſame in Vocal and Inſtrumental Muſick; if then the Inſtrument which governs the Voice be perfect, the Ear will of neceſſity bring it to perfection. It is a great pity that a good natural Voice ſhould be taught to ſing out of Tune, as it muſt do, if it be guided by an imperfect Inſtrument; and this may be the reaſon why ſo few attain to that melody, which is ſo much valued; but ſince we now know wherein perfection lies, a conſtant practice will come to the attainment of it. The dividing Wholes into Chromatick Hemitones is very neceſſary, but very difficult for the Voice to be broken to: If it learns from an Inſtrument whoſe whole Notes and whoſe half Notes are ſuppoſed to be equal, the ſound muſt needs be very uncertain and unharmonical; whereas the proportions truly fixed, would bring it to a perfection in the niceſt and moſt charming part of Muſick.

The Chromatick Hemitones are the ſmalleſt Intervals our Modern Muſick aims at, tho' the Antients had their Enharmonick quarter Notes, which they eſteem'd their greateſt excellency: Theſe may alſo in time be recover'd, ſince we know their proportions; for as the Diatonick Tone is divided into Chromatick Hemitones, ſo after the ſame manner may the Chromatick Hemitones be divided into thoſe leaſt Enharmonick Intervals, which were ever made uſe of.

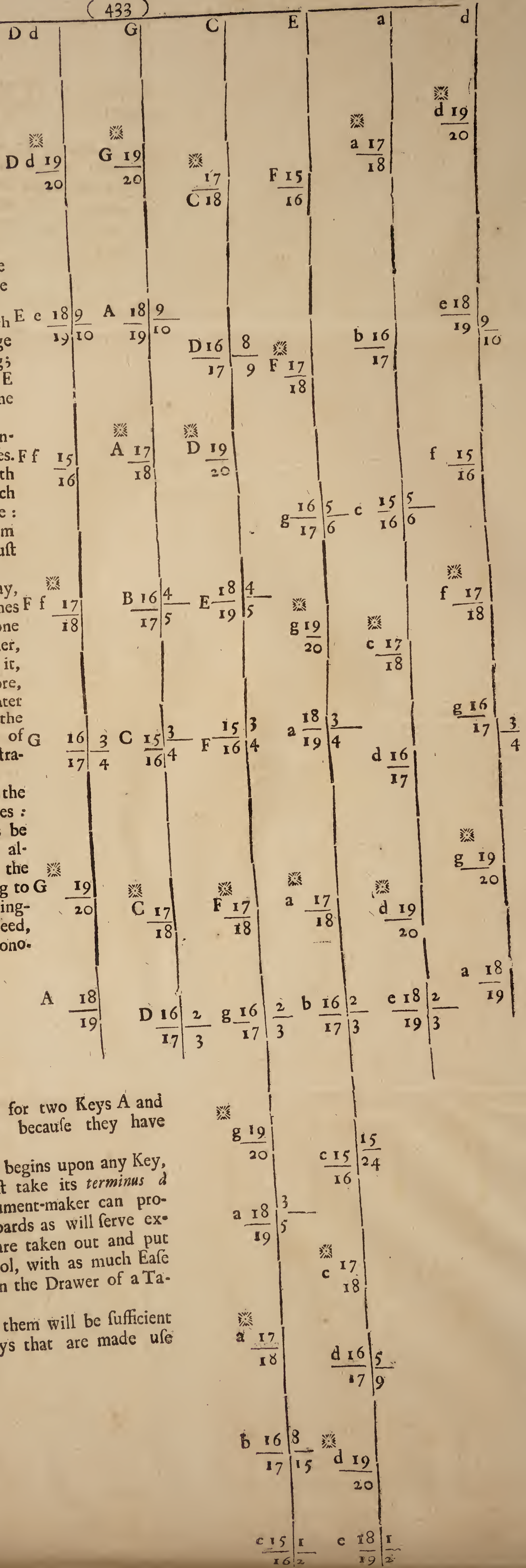
But if we go no further, yet this Experiment demonſtrates the true Theory of Muſick, and brings the practice of it to the greateſt perfection.

IV. *An Account of a Book omitted.*

Linear Perſpective: or a New Method of repreſenting juſtly all manner of Objects. *By Brook Taylor,* LL. D. R. S. Secr. 8*vo. London* 1715.

The

The 2d Figure, wherein the Proportions of Musick are described Geometrically.



The Explication.

THese six Lines represent the six Strings of the Viol in the common Tuning.

The sounding part of each String from the Nut to the Bridge is suppos'd to be 30 Inches long; the two middle Strings C and E are drawn out to 15 Inches, the half of the whole.

'Tis easie to measure every Interval with a Pair of Compasses. Suppose you are to take the 20th part of the String G; 'tis an Inch and a half for the first half Note: If you take the whole Note from G to A, 'tis the tenth part, and must be 3 Inches.

After these are taken away, your String will be but 27 Inches long, so that if you advance one Note, or a Major Tone further, you must take a 9th part of it, which will be three Inches more, whereby you arrive at a greater Third, being the fifth part of the whole String. Thus the Series of all the Notes may be demonstrated.

All the Strings are Unison at the Stops where the Tuning requires: So that though the Proportions be carried on as far as the Frets allow, yet the String is open the same with the Stop of that String to which it is tuned; and accordingly the Series of the Notes proceed, as if they were all upon a Monochord.

This Calculation serves but for two Keys A and C, which are called Natural, because they have no essential Flats or Sharps.

But because the Composer begins upon any Key, and the Series of Notes must take its *terminus à quo* from thence; the Instrument-maker can provide such moveable Finger-boards as will serve exactly for every Key. They are taken out and put in upon the Neck of the Viol, with as much Ease as you pull out and thrust in the Drawer of a Table.

Three, or at most five of them will be sufficient to accommodate all the Keys that are made use of.

THE

Philoſophical Tranſactions

From the Year 1700. to 1720.

Abridg'd and Methodically Digeſted.

PART II.

The *Anatomical* and *Medical* Papers.

CHAP. I.

A Courſe of Anatomy:

Eodem modo quo alii *Anatomici D. Antonio Marchetti* diſſe-ctionem exorſus eſt; *Abdominis* nimirum cutem in crucis formam ſecando, umbilico tamen intacto. *Cuticulam* à cute ſeparabat candelam accenſam ſub cute tenendo, quæ cuticulam in veſicam attollebat, unde facile eam ſcalpello ſeparabat. Cuticula à cute niſi vel actuali vel potentiali cauterio, *i. e.* veſicante ſeparari nequit. Sub *Cute* copioſa *Pinguedo* ſemidigitum craſſa abdomen totum inveſtiebat. Erat autem pinguedo crebris fibrillis veluti fulcris ne diffflueret ſtipata. 1

A Courſe of Anatomy at Padua, by Mr. Ray, *com. by Mr.* Dale, n. 307. p. 2283.

Sub *Pinguedine* panniculus seu *membrana carnosa*, quæ tamen hac in parte carnosa non apparebat; erat etiam & sub membrana carnosa aliqua pinguedo sed parcior. Membranam hanc ille in Brutis duplicem esse asseruit, qiua Bruta cutim totam movere possunt & corrugare, in homine duplex est in fronte, unde & frontem contrahere & corrugare potest, in nonnullis etiam duplex est in Occipite, unde & totum Capillitium commovere possunt. *Verùm alii diversam rationem assignant horum motuum, nimirum quia in Fronte & Occipite membrana carnosa cuti arctè cohæret, & in Musculum degenerat, quod probabilius est.*

Tum *musculos abdominis* aggrediebatur, & primùm *Obliquè descendentes*, qui à medio circiter costarum notharum exorti serratim cum musculo Thoracis *serrato* coaptantur (ut in suturis Ossium). In parte posteriore musculus dorsi latissimus huic incumbit, atq; idcircò aliquousque primùm elevandus est.

Musculi recti eminentiis, seu processibus ossium pubis lato tendine adnectuntur, superiùs à cartilagine primæ costæ nothæ prope cartilaginem ensiformem utrinq; oriuntur tendinibus nervosis.

Oblique ascendentes, à suprema margine ossis *Ilei* radiosis fibris exorti, venam musculam recipiunt à ramis *iliacis*. Secundò *Obliquè ascendentes*, in quibus nihil singulare, ortum habent à summitate ossis Ilii: Tendo ejus duplex musculos rectos veluti amplectitur: Una sc. ejus pars musculo recto incumbit, quæ antequàm dimidium latitudinis musculi superavit, cum tendine oblique descendentis arctissimè cohæret, vel potiùs in unum coalescit, ut nulla arte possit separari: Altera sub musculo recto eodem modo cum tendine musculorum transversorum coalescit. Venam accipit à musculâ dictâ quæ à ramo Iliaco oritur.

Musculi recti duas tantum habuêre inscriptiones nervosas, cùm in aliis 3, in aliis 4 aut 5 habeant, ut observat Veslingius. In his Musculis observavimus anastomoses venæ Mammariæ internæ & venæ epigastricæ.

Musculi transversi à processibus vertebrarum lumborum oriuntur; non autem Musculi oblique ascendentes illis vertebris annectuntur ut ille observat.

Musculos etiam *pyramidales* in hoc cadavere observavimus, qui rectis oblique incumbunt.

Obs. 1. Pinguedo in dorso Fœminarum liquidior & mollior est quàm in maribus.

2. Cutis in iis quæ pepererunt circa Ilia corrugatur, in Virginibus non item.

3. Vena, Arteria & Nervus semper se mutuò comitantur, arteria ad dextram, vena in medio, nervus ad sinistram.

4. Sub musculis supra peritonæum prope lumbos copiosam observavimus pinguedinem, unde in hac parte facilè à peritonæo separantur musculi, verùm prope lineam albam cum tendinibus musculorum arctissimè cohæret, ut nulla arte separari possit.

5. Musculos etiam in originibus seu capitibus suis tendines habere asseruit.

6. Incepit dissecare musculos à capitibus seu originibus, quia ita motus seu usus musculi in motibus faciliùs discernitur.

7. Cavendum

7. Cavendum eſt Chirurgis ne muſculos tranſverſim ad fibras ſecent, quia ita periculum eſt ni nervis (qui ſemper cum fibris parallelis decurrunt) diſſectis, convulſiones oriantur.

8. Si quis velit rectè diſſecare & ſeparare muſculos, debet accuratè obſervare fibras, earumq; ductum ſequi.

9. Muſculi tranſverſi tum in initio, tum in fine latam habent tendinem membranoſam.

Oſtendit tum nobis *vertebras lumbares*, numero quinque; ſingula proceſſus 7. obtinent, ſpinatum unum, tranſverſos duos, obliquè aſcendentes duos, & obliquè deſcendentes duos: Obliquè aſcendentes inferioris cum obliquè deſcendentibus ſuperioris per Ginglymon articulantur: Vertebræ verò ipſæ per Harmoniam: cavitas ſci. ſuperioris gibbum ſeu protuberantiam inferioris recipit.

Os Sacrum ex 6 componitur oſſibus nonnunquam, communiter 5; cum è 6 conſtat, *os coccygis* tria tantùm habet oſſa, cum è quinque quatuor. Os coccygis interiùs curvatur ad commodiorem ſeſſionem.

In difficili partu, *Chirurgus* immittendo digitum in *Inteſtinum rectum*, & retrahendo ſeu reflectendo *Os Coccygis* partum facilitare poteſt: Quod & *Marchettus* ſe feciſſe aſſerit.

Os Sacrum magna habet foramina ad egreſſum nervorum.

Aſſerit ille Oſſa hæc quæ pelvim conſtituunt in fœminis ampliora non eſſe quàm in viris pro ratione corporis, ut alii aſſerunt.

Os Ilium, *Os Pubis*, & *Os Coxæ*, ſeu *Iſchion* in adultis in unum velut os coaleſcunt, in Infantibus diſtincta ſunt, & cartilagine juncta. Omnia hæc tria oſſa femoris acetabulo coeunt, & ſingula partem aliquam cavitatis efformant. In oſſe ilio diſtinxit marginem, coſtam, dorſum, ſinus duos, ſuperiorem unum, ſuper quod nervus è ſummo foramine oſſis ſacri egreſſa tranſiens ad crura deſcendit; inferiorem alium inter eminentias duas, ad commodiorem ſeſſionem.

Nervi egrediuntur ad latera vertebrarum è foraminibus inter duas vertebras formatis.

Oſtendit *Viſcera* & *Inteſtina* in ſitu, *rectum* ſcil. *colon* quod omnia inteſtina circumcinxit; *cæcum* ad dextrum latus digiti parvi magnitudine, quod ille in fœtu & Infantibus nec majorem, nec excrementis repletum unquam inveniri ex ſua obſervatione aſſerebat. Supra cœcum immediate incipit *Ilium* quod & majus eſt & excrementis repletum: deinde *Jejunum*, quod & carnoſius, & vaſis plenius, & magis rubidum, & excrementis vacuum; *duodenum* ad flexuram terminatur.

Pars illa *Meſenterii* cui *colon* annectitur *Meſocolon* dicitur; reliqua pars, cui tenuia inteſtina, *κατ' ἐξοχὴν* *Meſenteriam* appellatur. *Arteria Meſenterica* inferior per totum colon, atq; etiam rectum ramos ſpargit, unde *Arteria hæmorrhoidalis*, meſenterica ſuperior, ad reliqua omnia fere inteſtina.

Splen in hoc cadavere prægrandis ultra naturalem molem: Hoc ille ebrioſitati aſſignabat.

Colon in hoc cadavere peritonæo adhærebat.

Musculi Pyramidales à processibus seu eminentiis ossis pubis utrinq; orti & oblique ascendentes tendinibus suis vicinis in *linea alba* terminantur: inserviunt hi urinæ expellendæ comprimendo *Vesicam* autore *Fallopio*; ubi hi desunt (ut in nonnullis fit) extremitates musculorum rectorum latiores sunt.

Ostendit insuper *venam umbilicalem*, quæ in fissuram *hepatis* inseritur, & *ligamentum* degenerat: *Arterias umbilicales*, quæ peritonæo adnexæ decurrunt ad ramos usq; iliacos arteriæ magnæ: *Urachum*, qui pariter adnexus peritonæo ad fundum vesicæ descendit, eamq; sustentat, quinetiam in homine ligamenti duntaxat usum præstare, nec omnino perforatum esse.

Ostendit præterea *uteri tubos*, *uteri ligamenta rotunda*, *testes muliebres*, & *vasa Spermatica*, nec non *ligamenta lata*.

Hepatis ligamentum latum. Hepar in viventibus & sanis non incumbere ventriculo, ac proinde unguenta, fomenta, & epithemata ventriculi regioni exteriùs applicari.

Ligamenta uteri rotunda perforant peritonæum & omnes musculos, & deinde divisa unum mittunt ramum ad *Clitoridem*, alterum ad genu usque. *q.*

Venas & *arterias gastricas* & *gastroepiploicas* ostendit & desecuit accuratè; verùm in his omnino consentit cum *Veslingio*, quem adi.

Obs. 1. *Bubones* nonnunquam oriuntur etiam in his quæ castæ sunt, verùm in iis sine suppuratione possunt discuti: *Venerei bubones*, nisi *gonorrhæa* succedat, semper suppurantur.

2. *Valvulam* in coli initio observavimus; iliacam passionem *Volvulum* dictum oriri ab inflammatione istius valvulæ quæ impedit ne excrementa descendant, ex propria observatione asseruit; adeo sc. angustatum vidit foramen ut ne cuspidem aciculæ potuerit admittere.

Venæ ventriculi sunt vel propriæ vel communes: Propriæ sunt 1. *Gastricæ sinistræ minores*, 3. aut 4. (quarum prima & brevissima vas breve dicitur) a ramo splenico venæ portæ prope lienem ortæ. 2. *Gastrica sinistra major* seu *coronaria*, quia in summitate ventriculi sparsa coronæ in modum. 3. *Gastrica dextra* seu *pylorica*. Communes sunt, 1. *Gastroepiploica sinistra*, quæ a ramo splenico prope lienem exorta fundum perreptat ventriculi, hinc in ventriculum inde in omentum ramos subinde spargens, inde in omentum vero unam insignem epiploicam sinistram dictam. 2. *Gastroepiploicam dextram* in hoc cadavere à ramo mesenterico ortam prope pylorum, quæ pariter fundum ventriculi perreptat, hinc in ventriculum, inde in omentum obiter ramos spargens, maximo suo ramo seu trunco *gastroepiploicæ dextræ* per anastomosin conjuncta. Est hæc vena insignis, & unum præ cæteris memorabilem emittit ramum *epiploica vena dextra* dictum.

In fundo *vesiculæ felleæ* nulla sunt conspicua vasa quæ *bilem* eo deducant, sed *porositates* quædam quæ bilem transmittunt, & proinde cum separatur vesicula ab hepate humor biliosus manifestè exudat. Quinetiam capillares quædam venulæ ab ipso hepatis parenchymate in membranas vesiculæ sparguntur adeo, ut sine effusione sanguinis ab *Hepate* dividi nequeat. Asserit vesiculam felleam quâ parte hepati conjungitur simplici tantum membranâ constare, alibi duplici.

Meatus

Meatus cysticus ubi in ductum communem terminatur valvulum non habet, sed *ostiolum* tantum, quod refluxum bilis impediat.

Hepar 3 habet sinus, unum in quo jacet cystis fellea, alterum in quem intrat vena umbilicalis, tertium ubi transit venæ cavæ truncus.

Cystis fellea arteriam habuit grandissimam, venas parvulas. Observasse se dixit ubi arteria magna est, ibi venam esse parvam quæ ei respondet & vice versa: *Non credo.*

1. Asserit præterea, ubi meatus cysticus obstruitur, icterum flavum oriri, ubi porus cholidochus, icterum nigrum. *Opiniones* Marchetti *mihi parum probabiles.*

2. Vasa venæ portæ & cavæ in hepate non conjunguntur per oscula, sed per harmoniam aut incumbentiam mutuam vasorum.

3. Vena portæ intra hepatis parenchyma non induit membranam novam.

4. Se vidisse venas lacteas in ipsum venæ portæ truncum insertas.

5. Se nunquam potuisse invenire neque credere dari ullum commune receptaculum chyli: *Experientia mea contrarium evincit.*

6. Se vidisse ramum insignem ductus chyliferi in pancreas desinentem.

7. Se putare usum lienis esse separare bilem atram à sanguine, eamque una cum sanguine ad hepar transmittere per ramum Splenicum, ubi per meatum cholidochum expurgatur in intestina.

8. Se putare venas lacteas chylum exugere ex intestinis, eúmque ad pancreas differre, cujus usus est eum ulterius perficere & exaltare, partemq; excrementitiam in intestina per novum vas *Virsungianum* ablegare.

9. Se vidisse venas lacteas in mesocolo ad intestina sparsas; quod proculdubio verum est. *Veræ nobis visæ.*

10. Venas hæmorrhoidales externas à vena cava non oriri, sed ramos esse venæ portæ: Ejus autem surculos extremos cutim etiam ipsam perforare, & in tubercula sub cuticulâ desinere; & his applicantur hirudines. Oritur Hæmorrhoidalis vena aliquando à ramo splenico, aliquando à mesenterica, sæpissimè in ipsa divaricatione venæ portæ. Vena hæc ramos suos spargit per totum mesocolon.

Mesocolon à mesenterio tenuitate suâ differt.

Arteriæ satis amplæ à cœliaco ortæ tres aut quatuor rami lienem ingrediuntur.

Venæ Splenicæ plures rami per totum lienis parenchyma sparguntur contra *Sylvium*, qui asserit *eas osculis suis duntaxat in lienem hiare, substantiam verò ejus non penetrare.*

Obs. Cum quis ex morbo diuturniore moritur, lien nigricat; si violenter moritur, rubicundior est.

Novum vas pancreatis & *porus cholidochus* eodem in loco *duodenum* perforant; aliquando diversis foraminibus in intestinum exeunt, ut in canibus fit.

Ostendit nobis in mesenterio nervorum plexum; *cui usui qui?*

Porus cholidochus in hoc cadavere mihi videbatur esse amplissimus.

Ren sinister in hoc & aliis omnibus major est dextro & superiùs situs, & à trunco venæ cavæ remotiùs, unde & emulgens longior est. *Huic rei rationem*

rationem sane probabilem dedit, quia hepar *in dextro latere incumbens tum illum deprimit, tum augmentum ejus prohibet.*

In dextro latere duas habet hoc cadaver arterias emulgentes, unam consueto loco in sinum renum ingredientem, aliam in superiore extremitate.

Uretres in hac fœmina amplissimi, quod ille omnibus fœminis commune esse asserit, quia humidiores sunt, & plus mingunt.

Glandula renalis dextra ab ipso venæ cavæ trunco venam accipit; sinistra vero ab emulgente. Glandulæ interius cavitatem habent. Dextra ipsi reni incubuit.

Arteria spermatica utraque ab ipso aortæ trunco infra emulgentes sibi mutuo proxime oriuntur. Venarum altera ab emulgente, altera nimirum dextra duplici trunco, uno ab emulgente, altera trunco venæ cavæ, qui paulo post in unum conveniunt, exoritur.

Affirmavit se vidisse venas lacteas in prægnantibus ad uterum sparsas; per quas probabili conjectura aquosum illum Serum in quo infans natat ad uterum differri putat. *Venas hasce lacteas in Ove prægnante, facillimus inveni.*

Mesenterium à tribus superioribus lumborum vertebris oritur.

Monstravit vasa seminalia, quæ ad testes quidem descendunt, eos vero non intrant, sed supra ligamenta lata ad convexum testiculi latus decurrentes, partim in tubos uteri sparguntur, partim in uterum ipsum.

Tubi uteri ad utrumque fundi angulum siti cornibus uteri in animalibus respondent, & sunt omnino cavi, adeo ut ab utero ad extremitates eorum usque possit stylus immitti: Tunica eorum interior albicat, inque iis sæpius reperitur humor serosus albidus, qui semen muliebre esse creditur.

Testiculi muliebres epididymidibus carent; una extremitate nervosis ligamentis utero annectuntur; substantiam habent molliorem laxioremque testiculis masculorum. Unus horum exulceratus cavitatem habuit.

Quòd testes tàm in maribus quàm in fœminis ad generationem nihil conducant, memorabili imprimis experimento probavit: Canis nimirum masculi testes execuit epididymidibus integris relictis, deinde canem fœminam in cubiculo conclusit per tres annos, nec ullum admisit ad eam canem cum salaxasset præter castratum hunc, qui canem iniit & cum eâ sæpius implicatus est. Triennio hæc ter peperit, una vice 7 catulos, altera 9, tertia 5. Re satis explorata fœminam dimisit. Alias duas vel tres historias huic parallelas nobis narravit; unam de Equo castrato, relicta tamen epididymide una, qui equas sæpius imprægnavit, fuitq; in venerem admodum proclivis: Alteram de cane quem ipsius servus execuit: Tertiam de homine quodam rustico, qui ob bubones venereos utrumq; testem amisisset, epididymide unica duntaxat manente, qui tamen uxorem duxit, & tres masculos filios genuit. Credit ergo ille testes non alii usui inservire quàm quem *Aristoteles* adfert, nimirum ut sint pondera impedientia ne Spermatica vasa implicentur; & revera vasa seminalia in eos non terminantur, nec transeunt, sed epididymides solùm.

Uteri

Uteri cavitas perangusta est & minima, verùm tunica uteri spissior densiorq; quàm ego credidissem.

Ligamenta uteri rotunda non sunt in uterum perforata, verùm vasis deferentibus in masculis quoquomodo respondent.

Orificium internum uteri in gravidis lentâ & viscosâ materiâ obseratur, *ut nos sæpiùs in bove observavimus*, adeò ut nihil omninò in uterum penetrare possit: Unde nihil seminis in uterum projici possit, adeoque nec superfœtatio fieri. Narravit tamen nobis se audivisse de muliere quadam rustica in montibus vicinis degente, qui tribus mensibus postquam unam peperisset, prolem aliam denuò peperit.

Os ipsum uteri Tineæ piscis ori persimile, corpus uteri Cucurbitæ tonsoris.

Vagina uteri ampla est atq; intus rugosa, in meretricibus verò longo veneris & assiduo usu rugæ istæ abolentur, & omnino levis evadit.

B. N. Ad hanc uteri vaginam vasa quamplurima (*venæ* sc. & *arteriæ*) tendunt, à ramis iliacis interiùs sive hypogastricis orta, miris plexibus & anastomosibus juncta, quæ in superficiem vaginæ sparguntur, & probabile est oscillis suis sive capillaribus extremitatibus in cavitatem ipsius hiant, in eamq; effundunt Sanguinem menstruum, quanquam se nunquam horum vasorum orificia potuisse invenire asserit *Marchettus*; nec mirum. Nonnulli ex his ramis in cervicem etiam uteri sparguntur.

In pudendo demonstravit nobis Labia, Clitoridem in supremo rimæ angulo, Alas seu Nymphas in superiore etiam parte, Urethram seu meatum urinarium, & Circulum membranaceum qui pudendum à vagina uteri distinguit, quiq; in virginibus membrana *hymene* dicta occupatur totus, excepto foramine in medio per quem menstrua defluunt.

In defloratis etiam apparet hic circulus qui pudendum hoc loco coarctat, ponè quem vagina laxior & amplior est.

Vagina hæc ultra interiùs uteri orificium inferius percurrit, unde si membrum virile longius quàm par est fuerit, ultra orificium interius uteri in hunc sinum sperma projicit, unde uxorem imprægnare nequit.

Ait se observasse etiam in fœminis utero gerentibus uterum fuisse duos ferè transversos digitos crassum.

Obs. 1. Ratio cur virgines coitu liberantur à morbo illò nostratibus (*The Green Sickness*) est quia membrum virile distendit nonnihil vaginam uteri & frixione sua reserat orificia venarum, adeòq; menses affluere facit.

2. Locus ubi fœminæ calculo laborantes à Chirurgo debent secari est in superiore vulvæ parte prope labia, Stylum in urethram immittendo, & super eam secando in carnoso vesicæ collo.

3. Urachus in homine (ne in fœtu quidem dum adhuc in utero matris latitat) perforatus non est ex observatione *Marchetti*, sed usum tantum ligamenti ad sustentandam vesicam præstat.

4. Se nunquam in medio ureteris hærentem invenisse calculum, sed semper vel prope infundibulum, vel prope vesicam.

*

1610

16to Decembris. Ostendit Musculos pectoris, & primò pectorales dictos, qui inserviunt adducendo brachio ad pectus, horum & insertionem q. in *Vesling*. deinde Musculos serratos anticos minores, qui inserviunt humero antrorsum adducendo, & sub pectoralibus siti sunt in processus coracoidos inserti.

Tum musculos serratos anticos majores, qui inserviunt scapulæ antrorsum deorsumque ducendæ; in basin ὠμοπλάτυ inseruntur.

Pòst, intercostales externos, qui ab inferiore latere costæ superioris orti in superiorem marginem costæ inferioris inseruntur; & tandem intercostales internos qui à superiore margine costæ inferioris orti in inferiorem marginem costæ superioris terminantur. Horum musculorum fibræ se invicem obliquè intersecant in crucis Andreanæ formam: Neutrorum scil. Fibræ ad costas perpendiculares sunt sed obliquæ.

Notavimus venas & arterias mammarias; externas, quæ ab axiliaribus oriuntur; & internas, quæ à Subclaviis ortæ, & intra cavitatem Thoracis aliquousq; progressæ in duos dividuntur ramos; unus musculos Thoracis perforat & in mammas distribuitur, alter deorsum tendens usq; ad medium recti musculi ibidem cum vena epigastrica extremitatibus suis per anastomosin conjungitur.

Dixit se observasse singulos venæ hujus capillares ramulos in singulos mammarum tubulos desinere, & proinde se putare lac non à chylo sed à sanguine generari.

Musculi subclavii à claviculis, ubi acromio junguntur, orti, in costam primam, ubi cartilagini sui committuntur, desinunt.

Observavimus Sphincterem & levatores ani dictos musculos, qui ab infimo ossis sacri, ubi coccygi committitur, oriuntur omnes.

In ulceribus ani & fistulis cavendum est Chirurgis, nè fibras Sphincteris transversim secent, quoniam ita amittitur facultas retinendi excrementa.

Vena cephalica dissepit & distinguit musculos pectorales & deltoides.

In collo primùm observavimus platysma myodes carnosum, scil. panniculus hoc in loco in musculum degenerat, qui mento affixus caput deorsum trahit.

Deinde Musculos Mastoideos dictos: Tum musculos digastricos, qui medio suo tendine Styloceratohyoides musculos perforant.

Ostendit musculos ossis hyoidis, quorum sex sunt paria: 1. Sternohyoides. 2. Coracohyoides. 3. Styloceratohyoides. 4. Thyreohyoides. 5. & 6. Geniiohyoides internum & externum.

Præparavit insuper columellam cum musculis ei famulantibus quorum duo sunt paria, nimirum pterygostaphylinum internum & externum.

Ostendit musculos Cartilaginis Scutiformis, quorum 3 sunt paria. 1. Sternothyreoides. 2. Cricothyreoides. 3. Hyothyreoides.

Musculos Cartilaginis arytænoidis, quorum 4 sunt paria: 1. Thyreoarytænoides, 2. Arytænoides seu Sphinster, 3. Cricoarytænoides laterale, 4. Cricoarytænoides posticum.

Musculos

Plate I. Part II. P. 15.

Musculos pharyngis, quorum 3 paria: 1. Stylopharyngæus, 2. Sphenopharingæus. 3. Cephalopharyngæus, qui potius carnosum æsophagi initium sunt, quam musculi; etiam Musculus æsophagæus dictus qui gulam constringit.

1. In Angina Spuria inflammantur tonsillæ, in legitima Musculi Laryngis, sed præcipuè Arytænoides.

2. In Angina Legitima ipsius Parens fecit incisionem in laryngem inter duos annulos superiores, fistulâ argenteâ in vulnus admissâ, per quam patiens inspirabat exspirabatque, adeoq; eam curavit. Oportet autem ut Chirurgus incisione facta dividat paulum & diducat curiosè Musculos Sternohyoides & Sternothyreoides.

3. Dixit se observasse ramulum ductûs Thoracici sive chyliferi ad pericardium tendentem, per quem immisso tubulo inflavit pericardium, unde non absque ratione conjectabatur lympham in pericardium derivatam esse.

4. Pulmo humanus ungulæ bovinæ specie externa similis.

5. Asperæ arteriæ rami seu bronchiæ intra pulmones cartilaginibus angularibus carent.

6. Valvulæ venæ cavæ tricuspides dictæ sunt, arteriæ venosæ mitrales, quia ambæ junctim acc. piæ mitram episcopalem quoquo modo representant; valvulæ venæ arteriosæ sigmoides dictæ sunt, aortæ semilunares.

Pericardium in hoc cadavere præter naturam diaphragmati in sua cuspide erat adnexum.

Observavimus glandulam geminam infra laryngem sub musculis Sternothyreoides ad utrumque asperæ arteriæ latus, quæ in Bronchocele (cui obnoxii sunt Alpium & montium altissimorum incolæ) mirum in modum intumescunt.

Observavit insuper ductum Thoracicum unum ramulum mittere ad glandulam parotidem.

18*vo Decembris.* Præparavit Musculos dorsi nimirum 1. Musculos trapezios seu cuculares dictos à figurâ, *de quibus abunde Vesling.* 2. Musculos rhomboides in basin Scapulæ desinunt. 3. Levatores Scapulæ, patientiæ & pauperum musculi dicti, quia pauperes cùm eleemosyna eis negatur, Scapulas levant dicentes, *Oportet patientes esse.* 4. Musc. latissimum dorsi, in summitatem cubiti inseritur, & ab officio *Aniscalptor* dicitur. 5. Serratos posticos minores, qui superiores. 6. Serratos posticos majores, qui inferiores. 7. Musc. longissimum dorsi qui discurrit per dorsi longitudinem, initio musculis sacrolumbis unitus, singulis costis duas ansulas seu tendines nervosos largitur, qui se mutuò decussant in crucis formam, ansulæ sci. exteriores sursum tendunt, interiores deorsum. 8. Musc. Sacrolumbos qui præcedentibus initio juncti interiùs, & processibus vertebrarum spinatis ad collum usq; protenduntur, singulis costis ansulas pariter donando, verùm exteriores hujus ansulæ carnosæ sunt, & non tendinosæ, quemadmodum præcedentis. 9. Musc. Semispinatus.

Præparavit insuper Musculos capitis & colli; & primò Splenicos dictos, quia Splenem bubulum repræsentant: in occiput inseruntur, atque etiam

(quod non habetur apud *Vesling* aliosve) tendinem satis validum à reliquâ sui parte divisum ad processum transversalem 2dæ cervicis vertebræ mittunt. 2. Complexos, adeo dictos, quòd quasi à diversis musculis compositi videntur. 3. Rectos majores sive externos. 4. Rectos minores sive internos, a primæ vertebræ tuberculo exortos. 5. Obliquos superiores. 6. Obliquos inferiores. 7. Musculos mastoideos. 8. Longos. 9. Scalenos. 10. Transversales. 11. Spinatos, de quibus consulantur auctores. Tandem præparavit Musculos Sacros & Musculos quadratos.

Monstravit musculos faciei. In fronte membrana carnosa in musculum degenerat, ibi incipiens duplicari ubi desinunt capilli.

Obs. 1. Musculi labia obliquè moventes seu dividentes, Sardonii dicti, in morbo illo risu Sardeo nimirum debent secari.

2. Si caput inungatur pinguedine supra Cranium humanum nascente capillos abunde producit.

3. Qui è Febri maligna moriuntur, iis intestina post mortem livida aut viridi-cœrulea apparent.

4. Non est pericranium à periosteo diversum, verùm periosteum in capite dicitur pericranium, potestque in plures, v. g. 7 vel etiam 10 plagulas dividi.

Musculi temporales, ob tutelam & ut in situ contineantur, membranâ propriâ conteguntur, a nonnullis fallò pro pericranio habitâ. Cavendum est ne hæc membrana lædatur: Siquidem vulnus ei inflictum, non rarò convulsiones excitat, unde & hujusmodi vulnera lethalia habentur.

19no Decembris. Præparavit musculos faciei: Nasi duos, triangulares sci. & obliquos: Oculi, sphincterem palpebrarum: Labiorum, elevatores sci. labii superioris, quorum duo paria; unum ab angulo interiore oculi ortum labiis & naribus commune. Musculi ab osse jugali nati, ideoque Zygomatici dicti, in risu Sardeo dissecandi sunt. Observavit in nonnullis hos Musculos deesse. Constrictor sive Sphincter labiorum Musculus nonnullis Basiatorius dictus. Depressores Labii inferioris ab imo mento exorti, admodum spongiosi ubi pili crescunt. Aliud par, quod Labium inferius & superius simul deorsum abducit, in angulos musculi Sphincteris sive Oris insertum.

Musculos maxillæ inferioris, scil. Temporalem, Massetarium, pterygoidem internum & pterygoidem externum, qui maxillam sursum trahunt omnes; diagastricos deinde, qui depriment.

Not. 1. In cranio perforando trepano, cavendum est a suturis; nam si Dura Mater (quæ per suturas cum pericranio committitur) lædatur, periculum ingens est nè ægrotus convulsus moritur.

2. Cerebrum humanum ingens est, corporis magnitudinis respectu habito.

3. In Cerebri ventriculis observavi duo corpora Hippocampi & Bombyces Arantio dicta ob similitudinem.

4. Cerebrum non pulsat per se, sed Arteriarum respectu: Nam si animalis vivi cranium aperias Cerebrumq; denudes, & ex una parte Menyngem piam amoveas cum vasis eidem intertextis, videbis alteram partem pulsare,

alteram

alteram verò nudam membranâ non item. Asserit seipsum hoc expertum esse, cerebrumque post amotionem Cranii plùs horæ quadrante pulsasse.

5. Observavimus nervorum par 4. seu Fallopianum, qui a posteriore Cerebri parte exorti, ad latera basis Cerebri reptantes, juxta tertium par exeunt.

6. Observavimus plures nervos è 7. conjugationibus non simplices esse, quamvis ex uno foramine exeunt, sed revera divisos & multiplices, nimirum tertium & quintum par ex 4. utrumque nervis constat, sextum ex 8. vel 10. verùm omnes illi simul sumpti non adeo ampli sunt *ac ego putaveram.*

7. Glandula pituitaria major est in homine & solidior, quàm ego in animalibus observare solitus sum.

8. Sub infundibuli membranâ duo nobis ostendit corpuscula alba viciæ magnitudine, testiculi figurâ, quæ fratrem suum primùm invenisse dixit, *verum habentur picta apud Veslingium.*

9. In foraminibus Narium maximis observavi 4. corpora aut etiam plura, oblonga spongiosa,membranâ tecta, quæ (ut probabile est) Mucum ne defluat impediunt.

10. Pia Mater composita videtur ex tunicis venarum & arteriarum, quæ eam crebræ perreptant.

Post aggressus est oculi dissectionem, in quo Musculos 6. notavit. 1. Elevatorem, Superbum & Hispanum dictum. 2. Depressores, humiles & Capucinos. 3. Adducentes, bibitorios & Germanos. 4. Abducentes, meretricios. 5. Obliquantes, & 6. Trochleatores, amatorios dictos.

Anterior tantùm tunica humoris crystallini aranea dicitur, posterior pro parte hyaloidis habetur.

Observavit Musculos in singulos ingredientes nervos; de Musculis oculi loquor.

Vesiculæ seminales ad vasa deferentia, immediate supra glandulas prostatas veluti alæ utrinque adjacent, originem seu radicem suam juxta glandulas habentes.

Capsulæ Seminales nihil aliud sunt quàm vasa deferentia dilatata immediatè supra glandulas prostatas.

Meretrices in coitu habent artem Vulvas coarctandi, os coccygis protrudendo introrsum, adeòque coitum jucundiorem reddendi.

21*mo Decembris.* Observavimus in uno cadavere Arteriam Spermaticam inferiùs à trunco Arteriæ magnæ ortam ascendere & emulgentem supergredi.

Vidimus manifestè Capsulam Spermaticam perforatam in Urethram, atque etiam in vesiculas seminales, adeò ut in utramque facilè stylum admittat. Foramen illud, quòd in urethram exit per tuberculum in ipso vesicæ collo,seu carunculam in urethræ initio, valvulam habet quæ impedit nè Sperma involuntarie exeat, aut in capsulas regrediatur.

Asserit ille se nunquam invenisse semen in glandulis prostatis, neq; agnoscere ulla foramina per quæ Semen in urethram exeat. *Ego aliter sentio, & puto Sperma in hisce glandulis contineri etiam in homine.* Putat ille glandu-

las ideo tantùm factas esse, ut vesicæ collum comprimant, adeoque conducant ad Semen cum impetu ejaculandum.

In Urethra summa, in glande sci. penis propè extremitatem, canalis se dilatat & foveolam efficit, in qua si materia aliqua acrior aut putrida stagnet, sive Sperma sit, sive urina, acerrimos dolores creat, & pustulas causat.

Gonorrhæa flava vehementissimos excitat dolores.

Musculos manûs dissecuit. Ii autem sunt. 1. *Deltoides.* 2. *Coracoides,* qui humero attollendo inserviunt. 3. *Rotundus major.* 4. *Rotundus minor,* qui humerum deprimit. 5. *Spinatus* inferior. 6. *Spinatus* superior. 7. *Infrascapularis* sive *demersus*; qui humerum circumrotare creduntur. 8. *Biceps,* præcipuè notabilis ob duplex initium, quorum alterum in sinu vel foveolâ ossis humeri capiti insculptâ, velut nervus arcûs in fibulâ, tendine suo immittitur. 9. *Brachiæus*; qui cubitum flectunt. 10. *Musculus longus.* 11. *Brevis*; qui simul juncti. 12. *Anconæus*; simul cubitum extendunt. 13. *Quadratus.* 14. *Teres*; qui pronatores dicuntur. 15. *Supinator longus.* 16. *Supinator brevior.* 17. *Musculus palmaris,* qui per totam manûs volam expanditur. 18. *Flexor Carpi externus.* 19. *Flexor Carpi internus.* 20. *Extensor Carpi externus.* 21. *Extensor Carpi internus.* 22. *Flexores primi* internodii, *Lumbricales* dicti, à tendinibus flexorum 2di internodii, orti & carnosi: 23. *Flexores secundi* internodii, *Perforati* dicti. 24. *Flexores tertii* internodii, *Perforantes* nominati. 25. *Abductor* minimi digiti. 26. *Annularis.* 27. *Medii.* 28. *Indicis* indicator dictus, præcipuè notabiles. 29. *Adductor Indicis.* 30. *Medii.* 31. *Annularis.* 32. *Auriculares,* musculi in formam crucis intra digitos collocati, atq; *Interossei* dicti. 33. *Flexor* primi internodii pollicis. 34. *Flexor* 2di internodii, qui in 4 partes dividi potest. 35. *Adductor.* 36. *Abductor* pollicis. 37. *Flexor* 3tii internodii. 38. *Extensor* primus pollicis. 39. *Extensor* 2dus. 40. *Extensores* digitorum, qui articulos singulos velut investiunt.

25to Decembris We saw the Operation of cutting a Child out of the Womb, performed in a Carcass by *Marchetti* the younger: This is called *Partus Cæsareus.*

He told us that himself had taken a Child out of the Mother's Womb, after she was dead, which lived two or three Days.

Incisio facienda est in uno latere, cavendum à *linea alba* & locis ei vicinis propter musculorum tendines, qui ibi omnes conveniunt, & si secentur difficiliùs coalescunt: In illo etiam latere, seu ibi, ubi infantis caput existere, seu jacere deprehenditur.

In incisione facienda magnam adhibere oportet cautionem, sensim & leniter secando, nè intestina vulnerentur: Postquam Chirurgus Musculos omnes & peritonæum perforaverit, de reliquo debet immittere duos digitos, adeoque omnes musculos & peritonæum attollere, atque digitos sursum versus scalpellum dirigendo, secare. Ita cum aperit *uterum* summopere cavendum est pariter ne fœtum vulnerat.

Postquam

Postquam fœtus eductus est, vulnus consuendum est, acu per omnes musculos & membranas adducto; & fila connectenda sunt in singulis puncturis. Uterus ipse nullo modo consuendus est.

Hoc facto in uterum injici debet decoctum Saniculæ, Consolidæ, & aliorum vulnerariorum; item Vinum maximè austerum. Vulneri autem externo abdominis primò applicari debent lintea albumine ovorum madefacta, deinde Emplast ra, ut Diapalma, &c. quòd si vulnus ad suppurationem veniat, immittenda est in inferiorem partem turunda.

Dixit se nunquam potuisse observare ossa pubis in partu separari, nam & ipse in partu difficillimo ei parti manum imposuit, nec potuit sentire ullam disjunctionem aut oblongationem. Habet etiam argumentum ex Hippocrate contra hanc sententiam desumptum à callo, Qui inter ossa fracta aut luxata existere solet, & in futurum ossium separationem impedit.

CHAP. II.

The HEAD.

FIG. 1. Shews a Bone taken from the Falx, or first Process of the *Dura Mater*, of a Man who died of violent Headachs. *An unusual Bone in the Head, and a Distemper'd Optick Nerve, by Mr.* Cheselden, n. 387. p. 287.

Fig. 2. Shews the Optick Nerves, the right Nerve being wasted and discoloured; the Eyes both appear to be very good. I had not an Opportunity of enquiring into the Case of this Person; but suppose it must have been a *Gutta Serena*. I open'd another Eye of a Man who died of that Distemper; in which I found that part of the Nerves which is within the *Cranium* crush'd flat by the interior Lobes of the Brain, their Ventricles being full of Lymph.

II. We had a very remarkable Fracture of the Skull, last Campaign at our Hospital at *Ghent*; it was in the interior part of the *Squamose* Bone, and occasioned by a Splinter of a Fellow Soldier's Piece bursting, that struck him there. Some time had pass'd, before the Accidents made us suspect a Fracture, and obliged us to make a Triangular Incision upon the Temporal Muscle; a Fissure was discover'd, which indicated the Necessity of the Trepan. It was apply'd twice, the first not making room sufficient to extract a large piece of the internal Table very much depressed. After this all the Accidents disappear'd; but twelve Days after the Operation, Rigors, cold Sweats, an intermitting Pulse, and some other Signs of an approaching Death, did make us despair of the Recovery of our Patient. He died the 15th from the Operation, and about the 20th from his Wound. His Skull was open'd, and in it three very remarkable Fissures observed. The first had, notwithstanding the Sagital Suture, cross'd from one Parietal-Bone to the other, as far as the Coronal Suture on that side opposite to the Wound; another had gone cross the Coronal Bone; and the *A remarkable Fracture in the Skull, by Mr.* Amyand. n. 317. p. 173.

third

third was on the Parietal Bone on the ſide of the Wound, pretty near the *Sutura Squamoſa*; but what is moſt ſingular, is, that none of theſe Fiſſures did reach that, upon which the Trepan had been applied. An *Empyema* was found in the *Thorax*, and a conſiderable Impoſthume in the Liver.

A Bullet lying Thirty Years in the Head. By Dr. Fielding. *n.* 320. *p.*317.

III. At the firſt *Newberry* Fight, in the Time of the late Civil Wars, Dr. *Rob. Fielding* was ſhot by the Right Eye on the *Os Petroſum*, by the Orbit of the Eye to the Skull, which was likewiſe broke, with great Effuſion of Blood from the Wound, Mouth and Noſtrils.

The Surgeon carefully probing the Wound for the diſcovery of the Bullet, but failing of his Intention, on the third day after the Shot, plac'd him Horizontal to the Sun; by which means depreſſing the broken Skull with the Probe, he could ſee the Palpitation of the Brain, but could not diſcover the Bullet.

When the Doctor began to grow old, his Mouth cloſed up, and ſo continued for the ſpace of half a Year, till many Fractures of Bones were come out of the Wound, Mouth, Noſtrils; and afterwards whenſoever a Scale of Bone was to come out, his Mouth would cloſe, inſomuch that ſeveral Years after he prognoſticated to ſome Friends, that a Bone was then coming out, which continued ſo for ſix or ſeven Weeks; at which time finding an itching in the Orifice of the Wound, with his Finger he felt a Bone, upon which he made known to ſome Friends then preſent, that they ſhould ſee him open his Mouth, and taking out a Bone no bigger than a Pins Head, he immediately open'd his Mouth.

At the ſecond *Newberry* Fight it heal'd up, no Art could keep it open. After this, for the ſpace of Ten Years, or more, a Flux of Sanious Matter iſſued out of the Right Noſtril, and then ceaſing there, it flow'd from the Left Noſtril for ſome Years. At length, for the ſpace of two Years or thereabouts, upon riding, the Doctor would ſometimes find a pain on the Left Side, about the Almonds of the Ear, which he attributed to Cold; but more eſpecially after riding in a cold dark Night, which occaſion'd a kind of Deafneſs too; and having ſtop'd his Ear with Wool to recover his Hearing, one Day, either Writing or Reading, ſuddenly an Huffe came in the Ear, which made him ſtart, and the manner not to be expreſs'd, unleſs you can imagine a *Vacuum*; this happen'd about *March* or *April*, 1670. Upon this, all that ſide of the Cheek hung looſe as tho' Paralytick, and under the Ear might be felt a hard Knob.

After this, Tumour upon Tumour appear'd upon that ſide under the Jaw-Bone, which occaſioned his conſulting ſome Phyſicians, two at one time, one of which ſuſpected the Bullet, which, conſidering the Shot, they thought not credible. At length the Tumours coming to the Throat, if he held up his Head a little, it ſeem'd as if one with a Hook did pull down the Jaw-bone; and if any thing touch'd the Throat, 'twas as painful as if prick'd with a handful of Needles. Being at laſt perſwaded to make ſome Applications, a ſmall hole appeared, after that another, and a third

third near the *Pomum Adami*; by these the Bullet was discover'd, and cut out in *August*, 1672.

IV. Canis superiori parte bregmatis terebrata, Membranam duram non sine aliqua Sanguinis jactura denudavi. Statim prius linteis carptis cohibita hæmorrhagia, ablutaque Sanguinis aspergine, apparebat Membranæ istius, ejusque Sinus longitudinalis qui per Marginem foraminis decurrebat, motus Systalticus, cordis vibrationis quæ citatior ordinario erat æmulus, eique rithmo ad amussim respondens. Hanc mecum astantes vibrationem per spatium unius horæ quadrantis conspexerunt, quo elapso, eam hamulo acuto prehendere tentabam, ut ita, ea super ipsum suspensa & cultro divisa, cerebri subtus motum satius expiscarer & ediscerem: factâ autem ejus perforatione (quam acute persentiscere canis videbatur, cujusvis autem Convulsionis expers) insequuta est hæmorrhagia, ut prius, brevique similiter restricta. Eloto Sanguine, particulari isto foramine à punctura hamuli in Membrana facto, comparuit Cerebri motus systalticus, parum id Sanguinis qui in eo hærebat, extus propellens.

An Experiment to discover the true Cause of the Motion of the Dura Mater. *By Mr.* Ridly. *n.* 287. *p.* 1480.

Postea cuspide forficis obtuso caute in aperturam Membranæ vulneratæ immisso, eam transversim parte ejus à sinus longitudinalis transitu maxime remota dissecui; quo facto se protrusit cerebrum Pia Matre textum per istam aperturam, adhuc manente motu ejus ad tactum satis forti, licet membrana ipsa, ob vim restitutivam suarum fibrarum à plaga accepta diminutam, ad oculos obscurius vibrari videbatur. Toto hoc tempore vegetus erat canis, levibus tantum tremoribus cum quodam horrore per universum corpus affectus.

Post horas aliquot hisce observandis transactas, quo tempore variis modis excruciatum fuisset animal, multumque sanguinis effudisset, tandem ut omnem vim pulsificam quæ vel huic membranæ inesse, vel ei aliunde donari supponi potest ratione fibrarum suarum, quarum beneficio motum primo à cerebro sibi impressum validius absolvit, adimerem, guttulas aliquot oli Vitrioli, unde nigro tingebatur colore, in eam leniter illinebam; Hinc nulla, saltem exigua & obscura ejus percepta est vibratio; non obstante autem hac ejus ad impulsum cerebri obsequendum ineptitudine, admoto digito satis distinctus cerebri pulsus sentiebatur.

Peractis his, adhuc alacri manente cane, cum cultri mucronem ad unius unciæ crassitiem in cerebri molem intrudissem, vehementer se agitabat canis, & in horribiles corporis jactationes, & pedum anteriorum & posteriorum convulsiones incidebat; quo tempore immisso ut prius in aperturam præfatam digito, fortius se movebat cerebrum quam antea; Postea specillum profundius adigebam, unde sensus dolorifici indicia maxima dedit; tandem cultro integro usque ad latus cranii oppositum trajecto, horrendi exoriebantur spasmi; & jam quo omnis de cerebri motu dubitatio amoveretur, digitis nostris tum ego, tum adstantes pari modo quo aliàs, indentes, Systolen Diastolenq; ejus cum maximo renixu factam persensimus. Quid amplius ad veram genuinamque motus hujus partis causam proximam eruendam

endam detegendamq; desiderari potest haud video, modo ad phænomena in hoc experimento occurrentia, sedulo attendatur.

An extraordinary Case of an Apoplectick Person. By Dr. Arch. Adams. *n.* 313. *p.* 40.

V. I open'd the Head of a Woman who died of an Apoplexy, and in the Left Ventricle of the Heart, I found four or five Ounces of clotted Blood; in the Right Ventricle no Blood at all, but every thing as usual; and all the Nerves which commanded the Right Side of the Body were as strong as any I ever observ'd in a sound Animal, especially in the Origin, and as far as I could trace them in their Course. It was my Opinion, that whichever Ventricle the Obstruction was in, the Nerves and Muscles corresponding to that side were affected, but here the contrary appear'd plainly; for altho the Obstruction was in the Left Ventricle, the Sense and Motion of the Right Side were entirely lost, and the small Remains of either were observable in the Left Side.

An unusual Cancer. By Mr. Jon. Kay. *n.* 277. *p.* 1069.

VI. My Father had a Cancer, which took its Rise from the *Os jugale*, and in process of time spread itself over the whole Cheek; and notwithstanding the Endeavours of the most eminent Surgeons, ulcerated his Eye round, which I saw him take out with his own Hand; and afterwards extended itself to his Ear, and through his Cheek into his Mouth, and across the upper part of his Nose, and perforated the Bone there. It likewise overran the side of his Forehead, fouling the *Os frontis* which came away in pieces, leaving the *Dura Mater* bare as broad as Half a Crown, which running thro the Perforation of the *Cranium*, in a few days putrified and expos'd the Brain itself, and several Portions of it came away fresh and untainted; and what is most strange, he perfectly retained his Senses, and rose every day to dress the Ulcer himself, till a considerable quantity of the Brain was come away. And when he was confined to his Bed, his Speech first fail'd, and he died about four Days after, his Brain being totally consumed, and nothing remaining in the *Cranium*, but a small quantity of black putrid Matter. He had neither Spasmus nor Convulsion of any part all the time of his Illness.

An unusual Blackness of the Face. By Mr. Yonge. *n.* 323. *p.* 425.

VII. A Girl 16 Years old, had a few hot Pimples risen on her Cheeks, which Bleeding and a Purge or two cured. She continued very well till about a Month afterward; when her Face, so far as is usually covered with a Vizard Mask, suddenly turned black like that of a Negro. This surprizing Accident much amazed and frighted the Girl, and the Passion and Terror of Mind encreased to a great degree, even to Distraction, and then they demanded my Assistance.

By some compos'd Anti-Hysterical Remedies, the Violence of her Fits was much pacified. I also directed a Lotion for her Face, which took off the Discoloration; but it returned frequently, with no Regularities, sometimes twice or thrice in Twenty four Hours, sometimes five or six times. It appears insensibly to the Girl, without Pain, Sickness, or any Symptoms of its approach, except a little warm Flush just before it appears. It easily

easily comes away, and leaves the Skin clear and white, but smuts the Cloth that wipes it from the Face; it feels Unctuous, and seems like Grease, or Soot and Blacking mixt. It hath no taste at all, which is to me very strange, that a fuliginous Exudation should be insipid.

She never had the *Menses*, is thin, but healthful; the Blackness appears no where but in the prominent part of her Face. The Blackness is since divided into a few dark cloudy Specks, which appear but seldom, and nothing so livid as formerly. *ib. p. 432.*

VIII. *Alex. Palmer*, of the Parish of *Keith*, in the County of *Bamff*, in the North of *Scotland*, about Fifty Four Years of Age, observed, when about Twenty Seven, a little hard Swelling in the Muscle of the lower Jaw on the Left Side, without any Hurt or manifest Occasion; which at first went on slowly, but afterwards it proceeded more quickly, and the older it grew, it still came on the faster, until it increased to a prodigious Bulk and Weight. From the first Appearance of this Tumour to the total Excision of it, there were about Twenty Seven Years. He had excessive Pains and Uneasiness in it, and at last it mightily extenuated and emaciated him, who was otherwise a strong and robust Man. *An extraordinary Wen cut off the Cheek. com. by Dr.* Bower. *n. 354. p. 713.*

This Excrescence was of the natural Colour of the Skin, and seem'd to be an *Atheroma*, being a glandulous Substance, with several big Blood-Vessels in it, and had Hair growing on it, as on the other Parts of the Body, as may yet be seen. It was almost round, and very hard, and was as sensible as any other Part of the Body; for, when the poor Man was working in the Fields, some six or seven Years ago, he accidentally made a great Gash or Wound in it with a sharp Iron, which was very painful; but was cured by a Surgeon, after the manner of an ordinary Wound; the Cicatrice is still to be seen in it.

This Excrescence having grown so big, was attach'd to the Muscle under the Left Eye, call'd *Obliquus minor* or *inferior*, to the Ear and its Muscles, and to the Muscle of the lower Jaw, named *Deprimens*. By reason of its great Bulk and Weight, it could not hang down freely without some Support, therefore it rested on the top of the Shoulder, which made a considerable Dimple in it, that is yet very observable; besides, it was holden up by the Man's Hand in the Day time, and laid on a Pillow in the Night season.

Some three or four Days before the total Excision was made, the Patient observed this Tumour begin to mortify at the lower end, which made him so uneasy, that he took a Knife and cut off a good part of it. This occasion'd a great Hæmorrhage, so that he reckon'd there was lost a *Scots* Pint, or four Pounds of Blood, before it could be stopt. The Patient at last applied himself to Mr. *Gordon*, Surgeon of the Place, who made a total Extirpation of it, on the 19th of *January*, 1717.

He made a close Ligature, taking in the Basis of the Excrescence, and all the loose Skin, and contracting it as much as possible, he cut it entirely off with a sharp Rasour. There gush'd out of the Excrescence, after it

 was

was cut off, and was lying on the Ground, as near as could be guess'd, two Pounds of Blood; for it was nourish'd by several large Blood-Vessels, perhaps by some Branches of the *Carotide* Artery much inlarged, and other Blood-Vessels coming from the Ear, and the Muscles of the Eye and lower Jaw above mentioned. When Mr. *Gordon* brought it to us, which was full three Months after it was cut off, we cut off with a Knife about a quarter of an Inch broad of the Basis of it; and in that small Space we observed four big Blood-Vessels. The Basis, as it now appears, is five Inches Diameter, which seems too large for the whole side of the Face: So that after the Exsection, I think the loose Skin has turn'd backwards, which may make the Basis now appear so big.

After all this Blood was lost, the Excrescence was weighed, and was full Nineteen Pound Weight; so that before his own Incision, and this Operation, it behoved to be several Pounds heavier; which is a most prodigious Weight to be depending on such a Place. This Tumour was of a Spheroidical Figure, and when measured, was Thirty four Inches about by the longest way, and Twenty eight by the broadest.

The Hæmorrhage, which was great, was stopped by the Vitriolic Powders and other Astringents, and the ordinary Dressing was used. So this great Cure was completed in six Weeks time, and the Patient entirely recover'd, and goes about his Business, to the great Admiration and Astonishment of every Body. The Lid of his Left Eye continues still downwards a little, as does that same side of the Mouth, which was occasion'd by the great Weight depending on that side of the Face, but it may be expected they may come again to their right Posture: for the Head, at first after cutting, inclined much to the Right side, by reason of the great Weight on the Left Cheek having been removed; but it now begins to stand upright since he is perfectly recovered. Tho' the Skin, and even a deal of the musculous part of the Cheek and lower Jaw was cut away, yet it is grown up again, and is of the ordinary Colour of the Skin, and like the other side of the Face, so that there grows Hair on that side of the Face as well as on the other, which he ordinarily shaves; and this is as surprizing as any thing in the whole Affair.

Observations on Incisions of the Cornea. *By Dr.* Gandolphi. *n.* 322. *p.* 387.

IX. In a blow upon the Eye, in the Month of *April*, 1709. there was a light Contusion on the outside of the Part, with very little Alteration to appearance; but a Vessel being broken within, pour'd forth a considerable quantity of Blood. The Eye also lost its Transparency, and almost its Sight; which was so very weak, that it could scarce perceive the greatest Light when objected to it. The *Cornea* appear'd all over red, but without any Inflammation or Blood-Vessels; it receiving its colour from the Blood pour'd in upon the *Aqueous Humour*. The Patient had been let Blood thrice; and the 8th Day I caused the *Cornea* to be open'd near the middle; my Design being to make a large Orifice, I determined not to make it at the bottom of the *Cornea*. The Orifice being made, there came forth some drops of the *Aqueous Humour* mixt with Blood. The *Cornea* still appear'd as red as before,

before, and was not fo even as we could have wifh'd. This Circumftance made me refolve to make a fecond Orifice immediately, as large as the former, but lower. There run out fome drops of the *Humour*; and the Eye appear'd not fo red and convex as before. The *Humour* continued coming out of the Orifice for fome time. We applied nothing to the Eye but a Comprefs (or Stupe) dipt in a Mixture of four Ounces of *Plantain* Water, and two Ounces of a Vulnerary Water.

The Day after the Operation, the upper part of the *Cornea* was tranfparent, the lower part not fo red, and the whole Membrane appear'd to have recover'd its natural Convexity. It feems that all the extravafated Blood had quite run out, had the lower part of the *Cornea* been open'd, and remain'd fo for fome time. I obferved the Alterations of the Eye for three Days together; in which time the extravafated Blood feem'd fome times to fpread over the whole Cavity of the *Cornea*. We judged that the motion of the Patient himfelf, had open'd anew fome Blood-Veffel, or had mixed the extravafated Blood with the *Aqueous Humour*; for we did not perceive all that time that there was any frefh Effufion of more Blood.

The 5th Day after the two firft Incifions, I caufed a third to be made at the bottom of the *Cornea*; there run out fome drops of the *Humour*, and continued fo to do for fome time; and two Days after, the Eye recover'd its natural Tranfparency. The *Pupil* was now very much dilated; but by little and little it contracted again, but not to its ufual fmalnefs. The *Iris* all this while kept its motion; fo that we cannot fufpect that the Lancett, in making the Incifion on the *Cornea*, any ways touch'd upon the *Iris*, becaufe the *Pupil* continued exactly round: And a ftroke that is able to divide the continuity of the Parts of the Eye, and caufe a fuffufion of Blood, is but too capable of depriving the *Iris* of its natural Power of Contracting.

The *Pupil*, which before the Blow was one Line in Diameter, when the *Iris* was contracted, is at leaft two Lines in Diameter at prefent. The tranfparency of the *Humours*, and Convexity of the *Cornea* are the fame as before. The Sight is now reftored, and there remains no other alteration than what neceffarily follows from the like Dilatation of the *Pupil*.

From hence we may draw the following Remarks:

1. Incifions are made on this Part without any Pain.

2. The Orifices unite again without any Scar; which has been before obferved, but is known to very few.

3. We find that Plants of a difcuffive Quality have an ill Effect, the Patient finding himfelf much worfe after ufing a Cataplafme made of *Cervile* and *Parfley*. Thefe Plants, which are excellent in refolving Extravafated Blood in the Mufcular Parts, have an ill effect when applied to the Eye, by caufing Pain, and rendring the Sight more difturbed. We had twice Experience of this, and the Patient affured us both times, that he found himfelf much better from the ufe of the firft Medicine.

When there is a confiderable Effufion of Blood in the Eye in couching of a Cataract, and no Orifice is made in the *Cornea* to let it out, it may fo

alter the Tranſparency of the *Vitreous Humour*, as to cauſe a loſs of Sight; which ſometimes follows from this Operation.

I made the Inciſion higher on the *Cornea* than it ought to be, becauſe the Perſon that perform'd the Operation, having never before made the like, and deſiring to make an Orifice large enough to diſcharge eaſily the *Aqueous Humour*, I thought it proper to make it near the middle of the *Cornea*, that the Point of the Lancett might not touch upon the *Iris*; which would have been of much worſe Conſequence than a Scar. The Effuſion of Blood, that ſometimes happens in Couching of Cataracts, is diſcuſſed again, either by external Applications, or the help of Nature; but when the Effuſion is very conſiderable, this Operation may be neceſſary to prevent worſe Conſequences.

As for the Scar, that ſometimes follows from an Inciſion of the *Cornea*; I remember I have read in an ancient Phyſical Author, that we need not fear it. But if we practice Inciſions on Eyes affected with Inflammations, Ulcers, or Defluxions, which very much dilate the *Retina* and Veſſels, an Eſchar forms itſelf much more eaſily in theſe Caſes, and conſequently we ought to uſe the greater caution; which was not ſo neceſſary in my Patient, who had no kind of Inflammation on the Eye or *Cornea*.

Obſervables in a human Ear. By Dr. Arch. Adams. *n.* 311. *p.* 2415.

X. The boney Cavity of the Ear is covered at each end by a Membrane; the former is called the Membrane of the Drum, and the other is directly oppoſite to it; the outer is ſtronger than the inner, ſo I call them with ſubmiſſion. They are joined together by the handle of the *Malleus* adhering to the outer, and the upper part of the Stirrup to the inner; which by the intervention of the *Incus* and the Orbicular Bone, make a Chain, and they ſeem to be acted and re-acted by theſe ſmall Bones reciprocally.

Whether Artiſts had any reſpect to this Original, when they firſt deviſed Drums, I cannot ſay; but nothing can more nearly repreſent the Natural, than the Artificial does; the Skins of this anſwering to the Membranes of that, the Wooden Cylinder to the Boney Cavity; the ſound of the Drum would be flat without a Hole in the ſide, and Nature has given a paſſage from the Palate to the Ear. The Skins of the Drum would leſſen the Sound, if they were not kept on the ſtretch; ſo would thoſe of the other flag, if the handle of the Hammer and the Stirrup kept them not on the Tenſe.

This inner Membrane is cloſely ſtretch'd before the Labyrinth, the *Foramen rotundum*, and the paſſage into the *Cochlea*, (I omit the *Foramen Ovale*, becauſe the Foot of the Stirrup exactly ſhuts it) that ſo the ſound may be the bigger upon its approach to the Nerves. The Stirrup is generally broke in diſſecting the Ear, particularly that Cover which goes over the Bone on each ſide; but if it be carefully open'd, the Stirrup is entirely cover'd with a Membrane, which forms a Cavity flatly Oval, and the inſide is Excavated.

XI. *Accounts*

XI. *Accounts of Books omitted.*

1. CONsilium Ætiologicum de Casu quodam Epileptico: Quo respondetur Epistolæ Doctissimi viri *Thomæ Hobart*, M. D. Annexa disquisitione de Perspirationis Insensibilis materia, & peragendæ ratione. Auctore *Gulielmo Cole*, M. D. *Coll. Med. Lond.* Socio. *n.* 287. *p.* 1485.

2. De Aure humana Tractatus: In quo integra Auris fabrica, multis novis inventis & Iconismis illustrata describitur; omniumq; ejus partium Usus indagantur. Quibus interposita est Musculorum Uvulæ, atque Pharyngis nova Descriptio & Delineatio. Auctore *Antonio Maria Valsalva*, *Imolensi*, Philosophiæ & Medicinæ Doctore, in *Bononiensi* Universitate ad Incisionem & Ostensionem Anatomicam Professore conducto, necnon Nosocomii Incurabilium Chirurgo. in 4*to*. *Bononiæ*, 1704. *n.* 299. *p.* 1978.

3. Dissertatio Epistolaris de Glandulis conglobatis duræ Meningis humanæ, indeq; ortis Lymphaticis ad Piam Meningem productis. Auctore *Antonio Pacchiono*. 8*vo*. *Romæ* 1705. *n.* 328. *p.* 208.

CHAP. III.

The Neck. The Thorax.

I. *A very large Tumour in the fore part of the Neck. by Dr.* Douglas. *n.* 305. *p.* 2214.

I Lately had the Opportunity of opening a Woman about 50 Years old, who had a very large Tumour, or hard Swelling, in the fore part of her Neck, possessing all the Space between the whole Extent of the lower Jaw and the upper part of the *Sternum*, with a considerable rising in its middle; laterally its point inclining to the Left side, tho' the biggest part of the Tumour was on the Right. The Skin on the *Apex* of this protuberating part was thin and shrivell'd, of a Colour different from the rest, and lookt as if the Swelling would have broke in that Place.

The Skin was exceeding thin, having no Fat under it, only in a Cavity between two Lobes, to be afterwards described, on its Right-side, there was a small appearance of some; for the Skin being less stretcht there, the Cells of the *Membrana adiposa* were not quite emptied. The fleshy Fibres of the *Latissimus colli* were scarcely visible. The *Mastoidæus* and *Coraco-hyoidæus* were extremely thin, and in their Ascent they adhered very firm to the subjacent Tumour. The *Sterno-hyoidæus* and the *Sterno-thyreoidæus*, that run up the fore part of this Swelling, were distended so thin, that it was difficult to separate them from it, especially the last named. The Right *Carotid* Artery, in its Ascent to the Head, ran along the outer edge, which increasing, did much obstruct the Current of the Blood that way.

The

The internal *Jugular*, the *Par Vagum*, and the Intercoſtal Pair went alſo over ſome part of this Swelling in their Deſcent to the *Thorax.* Two of the Lymphatick Glands of the *Jugular* Vein were ſwelled to the Bigneſs of little Eggs, being placed at ſome diſtance one from another, with a Hollow between, where ſome Fat was found; theſe two Lobes made the Tumour very uneven alſo on its Right ſide.

Theſe Muſcles, the *Jugular* with the two Glands adhering to it, and the reſt of the forenamed Veſſels being removed on both ſides, I could eaſily obſerve the Bigneſs, the Figure and the Circumſcription or Limits of this preternatural Tumour, with all its Adhæſions to the adjacent Parts. In Magnitude it ſeemed to exceed that of two Fiſts joined together. Its Figure was almoſt triangular, with a broad Baſis under the Chin, ſloping a little on each ſide, as it deſcended to the upper part of the *Sternum*, where its point was pretty narrow; its Surface was made uneven, by three Riſings, of which the largeſt was turned to the Left Side; the other two being placed on the Right, as above remarked. It adhered by membranous Filaments to the *Maxillar* Glands, to the *Digaſtrick* Muſcle, and to the *Stylohyoidæus*; under which, on the Right ſide, a ſmall portion of it, in the Form of a Nipple, did intrude it ſelf as it were under the Tongue; in the upper and fore part it alſo adhered to the *Os hyoides.*

Laterally it was connected to the *Levator Scapulæ*, and lower down to that part of the *Cucullaris* that terminates into the *Clavicle*; backwards to all the fore part of the *Aſpera Arteria*, between its third or fourth Cartilaginous Ring and the *Os pectoris*, as alſo to that Muſcle of the Head called *Rectus internus major*, and to ſome part of the *Scaleni*; its lower part was engaged under the *Jugulum*, or lunated part of the Breaſt-bone, to which it adhered. It was eaſily freed from its Connexions to all theſe different parts, but not ſo from the *Glandulæ Thyreoidææ*, to which it adhered after a far different manner; for where the *Thyreoidal* Glands are joined to one another, a little below the *Cartilago Cricoides*, on the fore part of the rough Artery, there was no ſeparating of it without cutting its Subſtance; whence it plainly appears, that the Union of theſe Glands was the Root or Beginning of this Tumour: And yet, which is very remarkable, the Glands themſelves kept their uſual Figure, and were no larger than ordinary.

This Tumour was hard and very firm, being exactly of the Conſiſtence of a Cow's Udder when boyled, yet in a few places it was ſoftiſh, containing a liquid and thick Juice. Its Colour was chiefly of a whitiſh Yellow, only in ſome Places it was exceeding red, from its having a greater ſtore of Blood Veſſels, and in others it was very white. I pared off all the ſoft part, and the hard Subſtance that remained I boiled, and then cleared it very well, having left ſticking to it at one Corner a ſoft Cartilaginous Body, which poſſibly, had the Patient lived longer, would have acquired the ſame degree of Induration. It very much reſembles a piece of white unpoliſhed Rock Coral; but whether it may be reckoned oſſeous, or if it be rather the Viſcid Humour of the Glands hardened and concreted into

to this irregular Chalky or Gravelly Subſtance, or whatever elſe it may be, I leave to better Judgment. Fig. 3. I remember about two Years ago I found in the *Proſtates* of a very old Man a great many Bodies, like white Peas, being of a Subſtance exactly like this, only ſmoother on the outſide; ſome of theſe were in the Body of theſe Glands, others adhered by ſmall Roots to the Muſcular Membrane that inveſts them. Fig. 4.

The firſt Appearance of this large Swelling was about twenty Years ago, cauſed by the breaking of a Vein, as the good Woman uſed to expreſs it, in a hard and very difficult Labour. It increaſed very ſlowly, not arriving to any conſiderable Bulk till a few Years before ſhe dyed; it was never very painful, being a true Schirrus: Many things by ſeveral Perſons had been uſed and applied unſucceſsfully. Its Bigneſs at Length became very troubleſome, in impeding her Swallowing and free Breathing, and at laſt it quite choaked her, by compreſſing the Wind pipe, upon which it lay.

I obſerv'd that the *Uterus* was entirely Schirrous, and diſtended to that degree that it filled up the whole Capacity of the *Pelvis*. Part of the *Colon* and *Ileon* adhered ſo firmly to it, that there could be no Separation without tearing. Both the *Ovaria* and the *Tubæ* grew cloſe to it; and indeed the Confuſion and Mixture of all theſe Parts was ſo great, that if the *Ovaries* had not been ſwelled here and there with Hydatical Tumours, I could not have diſtinguiſhed them. The Neck of the Womb was preſſed down ſo low, that upon a very gentle dilation of the *Labia* it offer'd itſelf to view, being extreamly hard, but yet ſmooth and even, and ſo cloſely ſhut, that I could paſs nothing without cutting. It had ſqueezed the *Veſica Urinaria* ſo cloſe againſt the *Os Pubis*, that it could contain but little or no Urine; which obliged her to make it often, and with pain. The preſſure of this part backwards was ſo great upon the *Inteſtinum Rectum*, that the evacuation of *Fæces* had been obſtructed for the ſpace of five Weeks before ſhe died.

Indeed there was obſerved to come away *per Anum* for ſome conſiderable time, a great deal of *Pus* and ſlimy Matter, but that proceeded from the *Uterus*; for the Acrimonious Humour, which was wont to be diſcharged *per Vaginam*, having been pent up within its Cavity, by the cloſe Conſtriction of the *Collum Uteri*, had corroded, and eat its way through the Subſtance of the Womb into the *Rectum*, by which it had its vent. Which deplorable Caſe I have more than once obſerved in Diſſection.

The thickneſs of the Womb was near two Inches, and in its bottom there was a great deal of this Humour, white and thick, which upon touching made the ends of my Fingers white and rough, by ſhrivelling the *Cuticula*, as if I had waſhed them with a ſtrong Solution of ſome Acid Lixivial Salt. Thus the Cauſtick Salt lodged in Soap affects the Hands of thoſe Women that waſh Linnen. It was very hard to take the *Uterus* out of the *Pelvis*, by reaſon of its ſo cloſe adheſion to the neighbouring Parts. The *Fæces Alvinæ* contained in the Guts, were but few, by reaſon

ſhe

ſhe could not ſwallow any thing ſolid for a long time, but very hard, and in ſeveral diſtinct Clots.

A Tumour in the Neck full of Hydatides. *By Mr.* Hewnden. *n.* 308. *p.* 2344.

II. A Gentlewoman in *London*, aged Twenty five Years, had a large wenny Tumour, the Baſis taking its Origin from all the lower hinder part of the Skull, ſtretching down the Neck near each Jugular, extending it ſelf almoſt as low as both *Scapulas*; on the upper part was a Phlegmon. The Radix being ſo large, I put on a tranſverſe Cauſtick the length and breadth of the Tumour, intending to ſeparate the *Cutis* from the Membrane of the *Cyſtis*; but it being ſo thin where the Phlegmon was, obliged me to divide the *Cyſtis*, out of which I ſav'd about Sixty *Hydatides*, of the bigneſs of a ſmall Walnut; ſeveral more were broken. Theſe *Hydatides* ſwam in a Liquor of the conſiſtence of Whites of Eggs. In this *Cyſtis* I found a large quantity of Atheromatous and Steatomatous Matter, at the Baſis a large *Sarcoma*; the greateſt part I cut off, but fearing to hurt the Muſcles of the Neck, deferred it to the next dreſſing, intending to take the reſt of the *Sarcoma* and Radix of the *Cyſtis* away by Cauſtical Medicines, which I applied with Succeſs, they coming off without making an Eſchar, the Radix being of a Cartilaginous Subſtance. Searching with my Probe to find an Interſtice, it dropt into one; and touching ſome Membranous or Nervous Body, cauſed the Patient to cry out furiouſly. Into that Interſtice I put a piece of *Roman Vitriol*, which came out the next Day all diſſolv'd, with ſome of the Radix. By continual applying the Vitriol, I extirpated all the Radix, and healed the whole *ſecundum Artem*.

Seven Years before that Operation, this Tumour was ſo big, and ſubſided of itſelf.

When I began with Cauſtical Medicines, the firſt I us'd was *Precip. rub.* with which I covered the whole Radix, which came off, and no Eſchar; but it ſalivated the Patient Five Weeks.

A Schirrous Tumour included in a Cyſtis. *By Mr.* Rich. Ruſſel. *n.* 337. *p.* 276.

III. *Auguſt* 18. 1713. I was ſent for to Mrs. *Smith*, who had been reduc'd very low by a Fever, which from her Cough, ſharp Pain under her Breaſt, and other Symptoms, was judg'd Pleuritick. But upon having a Diſcharge from her painful Breaſt, of a thin Gleet, all Symptoms vaniſhed.

When I ſaw her firſt, the Liquor diſcharg'd by a ſmall Pin-hole near the *Papilla*, was little more than would have wet a Handkerchief four times double. Examining of the Breaſt, I found a large Tumour, that lay deep, yielding to my Fingers, and paſting like Dough. I ſearch'd the Abſceſs with my Probe, and twiſted out with it a Matter like Saw-duſt or Bran, mix'd with Hair. Upon laying open the Breaſt, I ſeparated a Cyſtick Tumour, which weigh'd eight Ounces, and contain'd a ſolid Matter like the above-mention'd, mix'd with a Body like Hair.

Upon inquiring into the manner of its coming, ſhe told me, that Eight and Thirty Years ago, ſhe receiv'd a Bruiſe in that Breaſt by a Fall from a Horſe, which was attended with great Pain and Fluxion, inſomuch that the

the Veins of her Breaſt appear'd Varicous and Turgid, as in a Cancer. But her Pain ceaſing, they ſunk, and left an Indolent Tumour in her Breaſt, ſuppos'd by her Surgeon to be a true *Schirrus.* Since which time it hath always continued nearly in the ſame State, without pain, increaſing very little in Magnitude, but obſtructed in ſuch a manner, that ſhe could not nurſe her Child with that Breaſt. The *Tunic* was pretty thick, nouriſhed with very ſmall Veſſels, but had form'd a *Schirrus* of the Glands it adher'd to, by keeping up a Diſtention of Parts, till there was a Coheſion of their Membranes and Veſſels.

I make no doubt, but this was a Body of diſeas'd Glands, which had ſuffer'd a Colliquation by ſome extravaſated Fluid, and that the Membrane of the Tumour was their proper *Tunic.* After this manner all our Tunicated Tumours ſeem to be form'd; for when an Obſtruction proceeds to Extravaſation, there is a Liquor pour'd out which conſiſts of ſuch Particles, that by degrees makes a Colliquation of the Glandulous Fleſh, which is not very ſenſible of Pain; and by degrees the *Capſula* becomes diſtended with a Matter of a very different Conſiſtence, which gives the Name to the Tumour, either *Steatoma*, *Atheroma*, or *Meliceris.*

Thus pour Oyl of Olive on Spirit of Nitre, and your Oyl firſt becomes a little hardned, then of the Colour and Conſiſtence of Marrow, till by degrees it is hardned into a white Fat, reſembling that of Animals. The Poſſibility of this Colliquation and Digeſtion we may the eaſier be induc'd to believe, if we conſider how often we find the Glands of the *Viſcera* petrified, without any degree of Pain, or the Membrane in any Meaſure deſtroy'd.

A Schirrous Tumour in the Breaſt which prov'd Mortal. By Mr. Tho. Greenhill. *n.* 300. *p.* 2009.

IV. Mr. *J. D.* was ſuppoſed to die of a Conſumption; foraſmuch as Fourteen Months before he had been violently ſeized with an Inflammation of his Lungs, accompanied with a ſharp Feaver, difficulty of Breathing, Cough, acute Stitches, and Pleuritick Pains, with a ſpitting Blood, *&c.* He was bled largely in the beginning, and often repeated it during his Sickneſs. But about *Eaſter*, there appeared a Tumour on the Breaſt Bone, Pap, and Pectoral Muſcle, of the Left Side, with a fulneſs under the *Axilla*: From whence there was conjectured to be a collection of Purulent Matter in the Cavity of the *Thorax*, and that the *Sternon* was foul. The firſt from the aforeſaid Tumours, and his ſpitting a bloody and purulent Matter, and the latter from the riſing and inequality of that Part. So ſoon as I had open'd him, and divided and removed the common Teguments of the *Thorax*, I found, inſtead of a riſing of the Bone with Carioſity, only an oblong Tumour, about four Fingers in length, and two in breadth, and a proportionate thickneſs, weighing about three Ounces; it extended itſelf perpendicularly on the Superficies of that part of the *Sternon* which joins with the *Cartilago Enſiformis.* I ſeparated it with my Knife, eaſily, from the Breaſt bone, and found it to be of that Sort of Wens or enciſted Tumours called *Atheroma*, containing a pappy Subſtance like ſodden Barley. Next appeared a very large Tumour on the Left Side of the *Thorax*,

covering the whole Pap and Pectoral Muscles forwards, with a fulness under the *Axilla* of the same Arm. Then opening the *Thorax*, I found the same Tumour comprehending the *Intercostals, Deltoides, Subclavian*, and *Subscapulary* Muscles, and the whole Axillary and Mamillary Glands; which being obstructed, and its Vessels replete with a creamy pappy Matter, more thick and white than the former, there was produced such an induration of the aforesaid Glands and Muscles, which compose the upper part of the Breast, that it may more properly be esteemed a *Schirrus*.

The same Tumour on the outside of the Breast was somewhat bigger than ones Hand, extending itself from the Clavicle to the lower part of the Pap; and laterally from the Basis of the Muscle quite under the Arm pit. Internally it possessed a third part of the Cavity of the Breast, crouding the Left Lobe of the Lungs to the Right Side, and in its upper part firmly growing to it; which it likewise did every way to the Intercostal Muscles. It was about the bigness of a Penny Loaf; and the whole Tumour being considered together, might reasonably be allowed to weigh between three and four Pounds; which being cut into, there ouzed out of it, like an expressed Sponge, a great quantity of thick, white and pappy Matter. And what is more particularly remarkable, there was form'd a large Sink or *Pelvis*, in the middle of the Axillary Gland, which contained a thinner and discolour'd Matter, and had a free Communication to the Vessels of the Lungs in the upper part of it, where I told you before it was united; and from hence it was that he generally found ease when he had somewhat emptied it by large Expectorations, and that he could so exactly perceive, when any thin Rheums or Matter flow'd to the Part: And it was here only that the Lungs were black and replete with stagnated Blood, and some Globules of the aforesaid Matter in its *Vesiculæ*. The rest of the Lungs were pretty clear from any Ulcers or Matter, but of a Sublivid Colour, and strictly adhered on both sides to the *Pleura*, but particularly on the Left Side, all about the Schirrous Tumour. The *Vesica Fellis* or Gall Bladder was full of Stones, of the bigness of a Runcival Pea, and consisted most of odd Angles, and were formed of a thick viscous Sediment of Gall (which we found in it) from an obstruction of its Vessels, or Jaundice, which he had some Years before: They were in Number Twenty two, some Triangular, Quadrangular, Quincuncial, *&c.* There was a *Marasmus* of the External Parts, the washing of the Gall, and emptying of all the *Viscera* and Blood Vessels in general.

An Apostemation in the Lungs cur'd by Dr. Wright. *To Mr.* Cowper. *n.* 285. *p.* 1372.

V. 1. Mrs *Jane Terry* fell ill of the Small Pox in *May* 1701. She was about Eighteen Years old, of a fresh Complexion, and pretty Fleshy. Her Relations apprehending she might have the Small Pox, removed her to a Nurse's House, where she had the distinct Sort very kindly; her Case proceeded so very well (as they conceived) that no Physician was called to her, till they began to shell; only for some Days before she had a little difficulty in her breathing, which gradually increased till she began to raise

raiſe Blood, which was about the ſeventh Day from their firſt appearing. This raiſing of Blood had increas'd every Day for three Days before I ſaw her; ſhe cough'd and brought up a viſcous Phlegm, ſuch as our Patients vomit when their Stomachs are very foul; only, as meer Phlegm is white, this was all of it as red as Blood: It was not ſtreak'd with Blood, or had a mixture of white Phlegm with it, but was ſo deeply colour'd, that it ſeem'd to be all Blood, only it would not flow as Blood does whilſt it is hot; nor did it coagulate as Blood does when it is cold, but hung from the Baſons when it was pour'd out, as vomited Phlegm does; and in this it differ'd from all the bloody Expectorations I have ſeen, excepting Mr. *Jones* at *Myn Heer Meyſters*'s in *Kenſington*, who cough'd the ſame bloody colour'd Pituita, but in much leſs quantity; for Mrs. *Terry* rais'd above a Pint in Twenty four Hours, for ſome Days, and though a leſs, yet a conſiderable quantity afterwards. Mrs. *Terry*'s afforded a very ſtrong ſmell, but Mr. *Jones*'s had no odour. After ſome Weeks ſhe recover'd again, regain'd her Fleſh, which was waſted in her illneſs; the *Menſes* return'd, and ſhe continued very well from *July* till near *Chriſtmas*.

In *January* ſhe gave this Account of her ſelf. That about three Weeks before *Chriſtmas* ſhe perceived her ſelf a little ſhort breath'd, which increas'd daily, with a fullneſs and weight in her Left Side; that ſhe lay well on the Left Side; but when ſhe turn'd to lie on her Right, ſhe felt as if a Weight fell from the Left to the Right Side, which gave her a ſhortneſs of Breath, and made her Cough. Thus it continued increaſing till *Chriſtmas*, when ſhe began to raiſe a conſiderable quantity of ſtrong ſtinking Pus; ſhe ſaid ſhe eat her Victuals well enough all this time, and was not Feveriſh. When I ſaw her towards the middle of *January*, 1701. ſhe raiſed a conſiderable quantity and often of ſtinking offenſive Pus, which was as fluid as the Pus of other Parts. Her Fleſh was a little abated, but ſhe was at no time Feveriſh; ſhe eat and ſlept pretty well, and had the Catamenia duly. I preſcribed ſuch Medicines as abated that Purulent Expectoration ſeveral times, and ſhe often gave me hopes of her Recovery, ſhe continuing to have the *Menſes* regularly, and being ſtill free from an Hectick; but upon every little Cold, ſhe again raiſed that fetid Pus in a conſiderable quantity. She generally continued pretty free from Coughing ſeveral Hours together, till ſhe perceived ſomething of a fullneſs in her Breaſt, which would oblige her to Cough; and after ſhe had once begun to raiſe, ſhe could not ceaſe till ſhe had brought up two Spoonfuls or more of that fetid Pus. This ſhe did chiefly in the Morning, Afternoon, and at Night. I did apprehend ſhe had an abſceſs in the Left Lobe of her Lungs, and made her lie upon the Bed, with her Head reaching to the Chamber Floor, leaning upon her Left Arm. In this poſture ſhe could at any time, after a little Cough, ſet the Pus a running out of her Mouth, as you have ſeen ſeveral times, till the whole which was therein contain'd, was diſcharg'd.

Then ſhe would get up, and ſeem to be as well as another Perſon, till it was almoſt fill'd again. In the beginning of *May*, Dr. *Torleſſe* and Dr.

Pitts favour'd me with their Affiſtance. They ſaw her lye in the Poſture I have deſcrib'd, and ſaw more than two Spoonfuls of ſtinking Pus or Corruption run out of her Mouth, after a little Coughing. This made it ſo apparent that there was an Ulcer in her Lungs, that they immediately approv'd of what I had propos'd to her and her Relations, of the Neceſſity of making an Apertion in her Side, where we could apprehend the Lungs grew to it ; for that ſeemed unqueſtionable, from the Poſture of diſcharging it, and ſome little Pain ſhe felt in her Side. Her Relations readily conſented to what theſe Gentlemen had propos'd, and deſir'd your Affiſtance. About a Week before, the Pus had begun again to encreaſe, and the Day before theſe Gentlemen ſaw her, ſhe was taken in the Afternoon with a Chilneſs, after which her Pulſe became a little quicker, and ſhe a little Feveriſh, as I believe, you remember ſhe was, when you applied the Cauſtick ; this Feveriſh State encreaſed every Day, and after ſome Days a Raſh appear'd, which laſted about fourteen Days before it was quite got off, and left her in a Hectick, with Redneſs in her Cheeks towards Evening, Night-Sweats, continual Looſeneſs, extreme waſting of her Fleſh, and at length a Swelling in her Legs, though ſhe kept her Bed. We felt ſome little Knots betwixt the ſeventh and eighth Rib, which with other Circumſtances made us conclude the Adheſion was in that Part, and would have laid the Cauſtick there, but that it would certainly have ſpread to the Glands of her Left Breaſt, which made us lay it between the ſixth and ſeventh Rib, (*ſurſum numerando*) ; As ſoon as you could you took it off, and with your Knife gently paſs'd through into the Cavity of her Breaſt, whence iſſued a bloodiſh Water, but no Pus, by bending your Probe you found the Adheſion reach to the lower Edge of the ſeventh Rib, and before the Eſcar was ſeparated, the Pus began to flow at every Dreſſing, and ſo continued, gradually abating, till the Ulcer was cured ; during which, a Part of the Inſide of that Rib, above an Inch long, exfoliated, and after that another leſſer Piece of the outſide of the Rib. Towards the latter end of the Cure, ſhe complained very much of a Pain at the *Cartilago Enſiformis*, ſo great that ſhe ſometimes pluck'd out the hollow Tent, which we conceiv'd was occaſioned by its preſſing upon the Nerve. During the firſt ſeven or eight Days of her Raſh, ſhe rais'd very little, if any, of that Pus, nor did it diſcharge any of itſelf then by the Orifice, nor was there a Collection of it in her Breaſt, which made me apprehend, that the Fever did ſo alter the State of her Blood, as not to permit it, to ſeparate its Impurities into the Abſceſs. For ſix Days before the Fever began, ſhe had the *Catamenia* very orderly ; by *Auguſt* ſhe was cured, her Side heal'd up, and would not endure it to be converted into an Iſſue ; by *October* ſhe recovered her Fleſh, and the *Catamenia* return'd, which had been wanting ever ſince *May* ; and now, as you lately ſaw her, ſhe is plump, fleſhy, clear and freſh Complexioned, has little or no Cough, and no fœtid or tabid Expectoration ; and ſeems, and I believe is, perfectly cured, having for many Months taken no Medicine.

There are several Circumstances in this Case, which I cannot forbear making some Remarks on.

That there was an Ulcer in the Lungs, and that it has admitted of a Cure, contrary to the general Opinion of Physicians. That this Ulcer did contain at least two Spoonfuls, and must have been as large as a Hens Egg. That this Abscess arose from a Collection, with an indiscernible (if any) Fever, and so continued from *Christmas* to about the 10th of *May*. The tender Membranous or Vesiculary Composition of the Lungs seem to justify this Opinion, that it is almost impossible for them to heal, when there is a considerable Diminution of them; the continual and indispensable Necessity of their Motion, very much hindring the Coalition of the *Vesiculæ*.

Several Parts of the Body afford a proper Cement to unite and repair them, when hurt or diminish'd. Carious and broken Bones send forth a *Callus*; when the Skin is consum'd by Ulcers or Burns, the Parts afford a Cicatrix, which pretty well supplies the Defect of the Skin. The Lungs separate a viscid Pituita, which will be expanded into Fleaks like a Membrane; Mr. *Stringer*, *Sarah Deeping* and some other Patients have brought up great Quantities of them, and a little Boy * at Mr. *Tolley*'s in *Kensington* cough'd up several Pipes, form'd exactly like the *Bronchia*, and its Divarications, and at first View seem'd to be the internal Membrane. This Child, two Years before, had an Ulcer in the Right Side of his Lungs, and they adhered to his Back; when I separated them I found a Cicatrix near three Inches long, but very little (if any) Defect in his Lungs. I am of Opinion this Pituita or Mucus doth serve to re-unite the Parts of the Lungs, when there is a Solution by an Ulcer.

* n. 263. p. 545. *Abr. Vol.* III. p 63.

Consumptive Persons generally flatter themselves, that they have no Ulcer in their Lungs, because they do not feel a Soreness, as in Ulcers of other Parts. This Opinion keeps them from making a timely Application to Physicians, whilst they might receive a speedy and easie Cure. When you touch'd the Sound or Ulcerated Parts of her Lungs with your Probe or Finger, she discovered no Sense of feeling you, which may confirm the Opinion of Physicians and Anatomists, that the Lungs have little if any Sensation. When you touch'd her Heart with your Finger, tho' I believe for not the twentieth part of a minute, she grew very much disordered, pale and ready to faint, which shews Nature cannot suffer the least Alteration in its Pulsation, without great Prejudice and Inconveniency.

It is the Opinion of some Physicians, that the Fever which attends Consumptive Patients arises from some Particles of the *Pus*, which being receiv'd into the Blood, and circulating with it, cause that Effervescence, which we call an *Hectick*. This Patient had no Fever from *Christmas* to *May*, and then unhappily came a continued Fever with a Rash, which left a Febricitation every Afternoon, with those Symptoms which attend a Hectick.

I have

I have observ'd for many Years, that if I could preserve my Consumptive Patients from that Hectick Fever, or relieve them who already labour'd under it, I could cure them, tho' the Expectoration was very plentiful and foul. I doubt not, but if some of our Faculty applied themselves more particularly to the Cure of Consumptions, so many thousands would not dye of it yearly in this City, as appears by the Bills of Mortality.

I do not doubt but some Part of her Lungs do adhere to her Side, and it's probable a little part of them do not receive the Air in Inspiration, but I believe that Defect is very inconsiderable, because she can run up Stairs, and is no more disordered in her Breath than most other People.

The easy Discharge of the *Pus*, by her lying down in that Posture, did undoubtedly very much preserve her Lungs, and prevented its breaking through the Abscess, into the Cavity of her Breast, and putrifying her Lungs to a greater degree. *Pus generat Pus*, is a noted Aphorism, and the Air Bladders of the Lungs are so very tender, that they must have yielded to the Pressure of the *Pus*, had it lain long in the Abscess, and been only discharg'd by violent Coughing. By lying in a proper Posture, Sir *Tho. Proby*, *Sarah Deeping*, and other Persons have prevented a greater Solution in their Lungs, and either prolonged their Lives many Years, or recovered their Healths by proper Medicines.

An Answer to the foregoing Account, by Mr Cowper, ib. p. 1386.

2. Nothing occurs to me so remarkable that you have omitted in relating Mrs. *Terry*'s Case, as your Prognostick of her Recovery, when you so often encouraged me to expect Success in her Case. You may remember, that the Matter or *Pus* which first flowed from her Side was so offensive in its Scent, as obliged the By-standers to quit the Chamber, insomuch that the Nurse usually at the time of Dressing, and afterwards, was wont to burn Rosemary, *&c.* to suppress the Stench. So putrid was the *Pus* that it tarnished that end of the Silver Probe I past into the Cavity of the Abscess, as it did the Top of the Silver Syringe in making Injections. There seems no room to doubt that the *Pus* which then flowed from her Side came from the same Cavity the *Pus* did she before coughed up, when the Liquor that was injected at her Side came into her Mouth; which she frequently complained of, and particularly of the bitterish Taste of the Tincture of Myrrhe I sometimes used in the Injections.

About two or three Years since, I saw a Boy in the ninth or tenth Year of his Age, who (some time before) after a continu'd Fever was pursu'd with an intermitting one; a Cough follow'd, in which he brought up (at short Intervals) no small Quantity of thick purulent stinking *Pus*, which Discharge (I think) continued on him not less than fourteen or fifteen Months before I saw him: His Physicians order'd him Issues in his Back, which I made as usual: He had then an healthy Aspect, his Cheeks florid, and was very brisk and active: When he just came from Play he was bid to take a Bason in his Hand and cough as he was wont, which he did,

did, wherein I saw him discharge at his Mouth not less than 4 or 5 Ounces of the sort of *Pus* above-mention'd: This his Mother told me he had been wont to do twice every Day; nor did he appear any ways disorder'd after, but return'd to play immediately. His Physicians sent him into the Country whence he came, where in about a twelve month I heard he dyed, but was not acquainted with his Circumstances after: What Success the Operation we practised on Mrs. *Terry* would have had on this Boy, I dare not determine; tho' I cannot but think it might have been safely done to him and another Patient I was since call'd to, but I could not obtain the Consent of the Physician that was consulted.

Another Instance (in which a considerable Part of the Lungs was obstructed, and consequently became useless, some time before Death) was in a Girl of sixteen, who had been Scrophulous not less than 9 Years; the Glands about her Neck and Throat being very much indurated as well as distended, her Lips and Nose were also swoln: About a Year and a half before her Death she coughed up seven or eight Ounces of fœtid *Pus*, in less than 24 Hours. On changing the Air of this Town for that of the Country, together with the use of Balsamick Pectorals, she recovered a healthful Appearance in her Face; but continued somewhat Asthmatick. On taking cold (as 'tis call'd) her Appetite as well as Digestion fail'd her, she grew Feverish, and dyed after a few Days Indisposition.

On opening the *Thorax*, I found the Lungs cleaving to the *Pleura* of the Left side, in such manner that they could not be separated, without one of those Parts borrowing from the other. A Portion of one of the Left *Lobes* of the Lungs being cut off, sunk in Water; from which Part 'twas likely the Matter came which she formerly coughed up, tho' the Ulcer was then closed, and no Appearance of Matter was to be seen in that or any other Part of her Lungs. The Lymphatick Glands at the Divarecation of the Windpipe had by their Intumescence so compreft the Canal of the Left side, that it wanted more than two thirds of its proper Passage for the Air.

In these, and some other Instances I could produce, it's evident that considerable Parts of the Lungs may be obstructed, and the Person survive: but Mrs *Terry*'s Case demonstrates the Possibility of their Recovery when Part of their Lungs are totally obstructed, as must happen in such large Abscesses. But how the remaining sound Parts of such diseas'd Lungs become capable of transmitting the whole Mass of Blood from the Right Ventricle of the Heart to the Left, in equal Time and Quantity with the Blood that circulates in the rest of the Parts, seems not easily accounted for, when indeed it exacts our Wonder that it is done in a natural State, when all the Passages of the Lungs are open and free. Since I had often found Water, injected by the *Arteria Pulmonalis*, return readily from the Lungs again by the *Vena Pulmonalis*, I was tempted to try if melted Wax, when very hot, would not do the like. Which succeeded in two young Cats Lungs: for after injecting the Wax (mixt with Oil of Turpentine, and tinged with Vermilion) by the *Arteria Pulmonalis*, I found it had fill'd the

the Pulmonick Vein with the Left Auricle, insomuch that some of the Wax had reacht the Left Ventricle of the Heart: I don't remember this Experiment succeeded, but that some of the Wax was extravasated, and came into the *Bronchiæ* and Wind pipe at the same time.

In preparing a Human Heart, by filling its Ventricles, Auricles and Trunks of its large Blood Vessels with Wax, I found, on injecting the Pulmonick Arteries and Veins with Wax differently tinged, that the Wax pass'd from the Veins to the Arteries without coming into the *Bronchiæ*, or being extravas'd, tho' the Wax was not injected with near so much Force as might be. I must confess I was never so fortunate to make Wax pass thro the Arteries to the Veins in Human Bodies or Quadrupeds, unless in their Lungs, as above noted, and the *Spleen* and *Penis*; Nor do I remember it has happen'd in those Parts, but when the Wax has been impell'd with great Force, tho' I have constantly observ'd the Communication of Arteries and Veins, of the *Spleen* and *Penis*, more open than in other Parts except the Lungs. I wish Dr. * *Morland* had told us in what Part of the Human Body Dr. *Areskin* had made Wax pass from the Arteries to the Veins, so as to demonstrate their Continuation to the naked Eye, because I have hitherto found the naked Eye unable to discover the Extremities of the Arteries and Veins, when the Blood it self was moving in them, in the transparent Parts of the *Omentum* or *Mesentery* of the Quadrupeds, or in the Lungs of Frogs or Lizards when living; or after Death, when the Blood has been retain'd in their Lungs in the following manner. On making Incision into the Bodies of these Creatures their Lungs will start out, and be distended with inspired Air; on which make what haste you can to pass a Ligature (*i. e.* a Waxt Thread) and tye it firmly toward the upper part of the Lobe, as near the Heart as you can.

* n. 283. p. 1292. *infrà* Ch. VIII. *Tract.* 11.

Whence will appear that the Communications between the Arteries and Veins of the Lungs are more open than those of other Parts, at least in the Feet of Frogs: And till it can be shewn that melted Wax can be as easily injected from the Arteries to the Veins of other Parts in a Human Body and Quadrupeds; I shall be inclin'd to think the Communications between the Pulmonick Arteries and Veins in general are more open than the Arteries and Veins of other Parts, except the *Spleen* and *Penis*.

This patent Communication of the Arteries with the Veins of the Lungs shews how those Vessels transmit the Blood in equal Time and Quantity with the Blood that moves in the rest of the Blood Vessels of the whole Body in a healthful State.

Hence it is, when any of the Blood Vessels of the Lungs are streightened or totally compress (either or both which Circumstances must happen in Mrs. *Terry*'s Case) the remaining unobstructed Blood Vessels are forced to discharge more than they were wont, and in time those Vessels become sufficiently dilated to supply the Defect. The like happens in the Communicant Branches of the Arteries of any Part, when some considerable Branch or Trunk is ty'd up, as in the Operation for curing of an * Aneurism.

* n. 280. p. 1191. *infrà* Ch. V. *Tract.* 1.

VI. I send you herewith the Account and Figure of a *Polypus*, which I took out of a Child of about a Year old. It's first observable Disorders were a quick Pulse, and a Difficulty of Breathing. In about four Days the Gums were observ'd to be swell'd, for which they were cut, and all the Symptoms disappear'd for five or six Hours, after which they return'd. Notwithstanding Blood-Letting and the Application of other Remedies, the Difficulty of Breathing increas'd, the Pulse became still lower and quicker, and in four Days more the Child dyed. *A Polypus taken out of the* Vena Pulmonalis. *by Mr.* Cowper.. 270. p. 797.

The Body was open'd, and the *Viscera* of the lower Belly were found well constituted.

In the *Thorax* the *Thymus* exceeded the natural size even in Children. The fore part of the Lungs appear'd to be well dispos'd, but the back Parts were very hard, and very much inflam'd. Making the Incision on the Diseas'd Part, purulent Matter followed the Knife in such Quantity from divers Cells, that it fill'd the Wounds as fast as made, and Pieces cut from it sunk in Water. But as we approached nearer to the Parts unaffected, the Pieces became gradually more buoyant, till at length we came to the fore part which floated.

The Cavities of the right Auricle and Ventricle of the Heart were fill'd with a *Polypus*, which was continued into the superiour and inferiour Trunk of the *Vena Cava*.

Opening the *Vena Pulmonaris* at the *Basis* of the Heart I found it there compleatly fill'd with a *Polypus* (or Coagulation of Blood) which was continued into all the large Branches of it in the Lungs, and were easily drawn out, and when display'd, appear'd as express'd in Fig. 5. Plate 1.

2. This *Polypus* affords us a better Idea of the Contrivance and Structure of the Pulmonary Vein than any Figures of that Vessel yet publisht: For tho' in different Subjects of the same Species we meet with frequent Varieties in the Distribution of the Blood-Vessels, especially of the Veins; we no where find a more constant Regularity and Uniformity than in the Trunks and large Branches of the Pulmonary Vein; of which I have added two Figures, (Fig. 6. and 7.) drawn after a Preparation of that Vein injected with Wax, and freed from the Lungs of an Adult Humane Body. The Original Preparation is to be seen among the Anatomical Collections of the Accurate Dr. *Tyson*. *The Structure of the Vessel by the same,* ibid.

Among many very considerable Parts of the HumaneBody, not ill exprest in the Tables published by *Bidloo*, and overlookt, or not known by him, this Trunk of the Pulmonary Vein is one; unless he may be allow'd to call it the Left Ventricle of the Heart, as he has done Tab. 22. Fig. 7. A.

The Left Auricle of the Heart (*vid. Fig.* 6. 7.) in Human Bodies being much less than the Right, it was necessary that the part of this Vein next the Basis of the Heart should be very large; (*ib.* A A B) lest the sudden strong Motion of the Systole should cause the Refluent blood

to recoil in the Branches of this Vein (*ib.* D D E E) and prevent a ready Supply in the fucceeding *Diaſtole* of the Heart. But the Weight of fo much Blood lying in the Trunk of this Veffel (*ib.* A A. B) does effectually prevent its Retroceffion in the lateral Branches within the Lungs; (*ib.* DD. EE.) and the more, becaufe the Orifices of thofe Branches (*ib.* DD.) are not diametrically oppofite (at A A Fig. *ib.*) to the Mouth of the Veffel on the Bafis of the Heart (*ib.* B) its lateral Branches making Acute Angles with the Trunk, as reprefented Fig. 6.

Offifications or Petrifactions in the Coats of the Aorta, by Mr. W. Cowper, *n* 299. p. 1970.

VII. The Offification or Petrification in the great Artery, at its Rife from the Heart, has been fo commonly found, that fome think it is conftant; how it may be in fome Animals I cannot be certain; but in Humane Bodies, I am well affured, whenever it happens it is a Difeafe, and does in fome meafure incommode thofe Parts in the due Execution of their Office, as the following Cafes will evidence: But that this Paper may be of fome Ufe, I fhall fet down the Symptoms before Death, which may help our Conjectures when the like offers again. A fpare Man about 30, who languifht with an Ulcer in the Thigh, attended with a *Caries*, or Rottennefs of that Bone at its Articulation with the *Tibia* and *Patella* call'd the Knee, where all thofe Bones were affected, at length fell into a true *Phthifis*, and coughed up no fmall Quantity of *Pus*; fome Months before his Death, I frequently faw him, when he would often offer me his Wrift, to feel his unequal Pulfe, which was wont to amufe him; the Artery there miffing fometimes one, fometimes two Strokes in 6 or 7: At firft he told me he obferved it mift but one in ten, but at length thofe Stops became more frequent, efpecially on any Agitation of the Body or Mind. Tho' a *Polypus* in any of the great Veffels about the Heart may induce that Symptom, yet the Continuance of it fo long before Death, fhews it owing to fome other Caufe, as appear'd on opening the Heart and Great Artery of this Perfon.

Fig. 1. Plate 2. A A. The Trunk of the Great Artery opened and difplay'd.

a a a. The three Semilunary Valves of the *Aorta*, which hinder the Blood from returning to the Heart, after it is expell'd thence by its *Syſtole* or Contraction; thefe Valves in this Cafe were fomewhat thicker, and not fo pliable as naturally, and did not fo adequately apply to each other, as is expreft Fig. 4. a a a. Whence it hapned fometimes that the Blood in the Great Artery (A A A Fig. 1.) would recoil, and interrupt the Heart in its *Syſtole*. But this ftubbornnefs of thefe Valves was owing to a Bony or ftony body, markt b. Fig. 1ft, which appear'd much plainer when the Valves were dry, a is reprefented in the Figure beneath, markt with an *: a a. the two Valves pinn'd out and dry'd, b the Petrifaction or ftony Body at their Junction. In this Inftance I obferv'd the Left Ventricle of the Heart, expreft at G G. (D D. e e. f f. Fig. 1ft, to be a little dilated from its natural Size, but was not by two parts in three fo big as the Left Ventricle of the Heart of one I diffected in the

Prefence

Plate II. Part II. P. 34.

M. V^{dr} Gucht Sculp

Presence of Dr. *Sloane*. The Symptoms, some Years before the Death of this Person, who was about forty Years of Age, were extraordinary Shortness of Breath, especially on any Fatigue, with an Intermission of one Stroke in three of the Pulse; his Posture of sitting up was more eligible than any other; he complain'd of great Faintness, and now and then Pain about the Heart; the extreme Parts often cold, which towards his Death increased more and more on him; his Legs and Arms being gangreen'd some Hours before; insomuch that the Corps was very offensive in opening, tho' 'twas done within 24 Hours after he expired, in the Month of *November*.

Upon opening the Chest, the Heart, particularly its Left Ventricle, was found larger than that of an ordinary Ox, and fill'd with Coagulated Blood. The Valves of the Great Artery A A. Fig. 1. were Petrify'd, insomuch that they could not approach each other, as exprest Fig. 2. and 4. But an Orifice, represented at Fig. 5. remain'd always open by the Petrifactions b b, Fig. 3. and a a, Fig. 5, which had clogg'd these Valves, and hindered their Application to each other, as in a Natural State is represented in Fig. 2. and 4. a a a.

The Explication of the Symptoms in both these Cases is obvious enough; for tho' the Person first instanced did not die of the same Disease with the last mentioned, yet the Symptoms in his Illness plainly shewed what must follow, from the Disorders of these *Valves*, as they are rendred more or less useless: For as their Office is to prevent the Return of the Blood into the Heart, in its *Diastole*, by exactly shutting up the Passage of the *Aorta* (as the Flaps in Water Engines) so if by any Accident they are hinder'd from doing their Duty, as they were by these Petrifactions, the Consequences must be, not only a Regurgitation of Blood into the Heart, but they baulk its impulsive Force, when the Muscular Fibres (which are in these Valves) cannot contract to prepare the Passage for the Blood of the Left Ventricle, when to be expelled into the *Aorta*. Hence the Intermissions of the Pulse in the first Instance may be accounted for. In the latter Instance, these Valves were wholly useless, the Circulation became more difficult, as appear'd by the Refrigeration of the extreme Parts, Gangreens, *&c*. In both these Cases the Left Ventricle of the Heart was dilated proportionably to the ill constitution of these Valves, which clearly shews these Valves give that assistance to the Heart in its Office that it cannot be without, and that it gradually suffers according to their Indisposition.

In an Elderly Gentleman about 72, who had sometimes Intermissions in his Pulse several years before his death, I found divers Petrifications in the Mitral and Semilunary Valves of the Left Ventricle of the Heart.

The Explanation of the Figures.

Fig. 1. Shews the Left Ventricle of the Heart open'd, *&c.*

A A A. The inside of the *Aorta* slit open to the Left Ventricle.

B B. The Bulbous Trunk of the *Vena Pulmonalis* divided through, and pinn'd aside to shew

a a a. The three Semilunary Valves of the *Aorta*, which hinder the Blood from returning to the Heart.

b. A small stony Body at the Conjunction of two of the Semilunary Valves, exprest at the * below this Figure.

a a. Parts of the two Valves dryed.

b. The Petrifaction, as it appears in the dryed Valves.

C. Part of the lower Trunk of the *Vena Cava*, cut off immediately above the Liver.

c c c. The Left Auricle open'd and pinn'd out.

D D The Sides of the Left Ventricle divided and drawn aside, to shew its inside, d d e e f f G G.

d d. The Mitral Valves of the Left Ventricle of the Heart, or *Arteria Pulmonica* divided and turn'd aside.

e e. The *Carneæ Columnæ*, whence spring the Tendons fasten'd to the Valves, d d, exprest Fig. 3. d f.

f f A Transverse Cord or Tendon, by which the *Columnæ Carneæ* are drawn nearer to each other in the *Systole*, or Contraction of the Heart; when the Blood is expell'd into the *Aorta*; whereby the Tendons (express'd f f Fig. 3 and 5) draw the Mitral Valve laterally; by which Means its Orifice g c. Fig. *ibid.* is not only closed to prevent the Return of the Blood by the *Vena Pulmonalis*, but at the same time it opens a Passage for the Blood of the *Arteria magna*, by withdrawing the Mitral Valve, d Fig. 2. from the Orifice of the *Aorta*, a a a g.

G G. The internal Surface of the Left Ventricle, where it is somewhat smoother as it leads to the *Aorta*.

g g. The Trunk of the Coronary Vein divided when filled with Wax.

h h. The Coronary Artery in like manner divided.

i. One of the Trunks of the *Vena Pulmonalis*.

k k k. The three Orifices of the Trunks of the *Vena Pulmonalis*, as they open into the Bulbous Trunk express'd at BB.

H. The Cone of the Heart.

Fig. 2. A. Part of the *Aorta* next the Heart.

a a a. The three Semilunary Valves, as they appear next the Heart in a Natural State, when the Heart is in *Diastole*, and the Blood hinder'd by these Valves from returning to its Left Ventricle.

b b. Part of the Basis of the Heart cut off.

e e. The two *Columnæ Carneæ* of the Left Ventricle.

d. The Mitral Valve.

f f. The Tendons springing from the *Carneæ Columnæ*, and inserted into the upper and middle Parts of the Valve, as well as to its lower Margin; which is better exprest in the following Figure.

g. The Orifice of the *Aorta* compleatly clos'd by the Application of these three Valves to each other.

Fig. 3. Shews the same Parts exprest in the preceding Figure, as they appear'd when the Valves of the *Aorta* were Petrified: The same Letters also directing to the Parts already explain'd, except a.

a. Part

a. Part of one of the Valves which was not cover'd with the Petrifaction.

b b b. The Petrifaction on the rest of the Valves.

† A small Petrifaction on the Mitral Valve.

h h h. Some of the Transverse Tendons which draw the *Carneæ Columnæ* to each other, when the Heart is in *Systole*, for the more effectual closing the Orifice of the Mitral Valve, exprest here at g.

Fig. 4 and 5. Shews the Parts represented in the two preceding Figures, as they appear view'd towards the Heart, when dry'd and display'd.

A A The Trunk of the *Aorta*.

a a a. Fig. 4. The Semilunary Valves in a Natural State, when the Blood in the Arteries presses them close to each other.

b b b b. The Trunks of the two Coronary Arteries cut off.

a a. Fig. 5. The Semilunary Valves Petrify'd.

c. The Orifice of the Mitral Valve next the *Vena Pulmonalis*.

d d d. The Internal Surface of the Mitral Valve leading into the Left Ventricle.

e e e. The *Columnæ Carneæ*.

f f. The Tendons.

g g. The transverse Tendons which draw the fleshy Columns to each other, when the Heart is in *Systole*.

VIII. Fig. 9. Tab. 1. Shews the Beginning of the *Aorta*, or Great Artery from the Heart of a Woman, who died of a Dropsy. *A.* is the Aorta. *B B.* Two Chalk-stones; which possest the Place of the Semilunar Valves. The Left Ventricle of the Heart was dilated to twice its Natural Magnitude. We suppos'd these Stones occasion'd the Dropsy by obstructing the Valves, and hindering the regular Distribution of the Blood. *The like by Mr. Cheselden, n. 337. p. 281.*

Fig. 10. Shews a Bone taken from the Ventricles of the Heart of a Man who died Hydropic and Tabid. In this Body the whole Pericardium adher'd to the Heart.

IX. Before the Body was removed from the Bed whereon it lay some Hours after Death, the Blister in the Neck had discharged not less than a Quart or three Pints of Serum, before I began the Dissection. *Dissection of one dying of an* Asthma, *n.* 336. p. 534. *by Mr.* Cowper.

In the Abdomen was a small Quantity of Water, such as is usual in those that dye of Chronical Diseases: The Parts in the Lower Venter were in a Natural State, except the Kidneys; of which the Right was very much contracted, even to a third part of its Natural Size, and had two large Hydatides, or Bladders of clear Water on its Surface. The Left Kidney was also lessen'd, but not so much as the Right. Its Surface like that was unequal, but had no Hydatides in it. The Ureter of the Left Kidney was very much contorted at its Rise from the *Pelvis*, where

its

its sides were petrifi'd, insomuch that its Canal was almost render'd impervious for the Passage of the Urine.

Nothing was found in the Bladder of Urine, but divers stones of unusual Figures, as if they had been pieces of a large Stone, broken to Bits, in whose Center a Nucleus had been lodg'd. The Gall Bladder was fill'd with gall Stones. Nor was the Stomach, which he complain'd of (*i. e.* for want of Appetite) any other ways disorder'd: but a little redder, having more Blood in its Vessels than is usual; its Muscular Fibres being stronger than we generally find them in the Stomachs of healthy Persons.

The Cavity of the Thorax or Chest was fill'd with water on both sides; insomuch that the Lungs were not above the third part of the natural Magnitude. The Pleura or Membrane that lines the Cavities of the Thorax was very much thicken'd by the Serum or Water; from whence it descended by the Muscles of the Back into his Legs. The Valves of the left Ventricle of the Heart were petrifi'd in several Places, especially those call'd *Mitrales*. There were some stony bodies found on the *Bronchia*, at and near the rise from the Lungs.

A Child with the Intestines, Mesentery, &c. in the Cavity of the Thorax. by Sir Cha. Holt. *n.* 275. *p.* 992.

X. The Child was uneasy and restless from its Birth, and constantly labour'd under a difficulty of Breathing. Its illness was like what hath been seen in other Children, neither could they perceive it reliev'd by any thing administred to it; tho' by advice of a Learned Physician, but it lay groaning and pining till it died. When the Child was undrest, an odd sort of Working was observ'd in its Breast, and a Crawling perceiv'd round the Ribs and Breast, on both sides, as if a Knot of small Eels or large Earth Worms had been pen'd up within the Cavity. This Relation seemed strange, but upon the dissection we found sufficient reason to believe the account.

When we had opened the Abdomen, there appeared none of the *Viscera* belonging to the Belly, except the Liver, the Kidneys, *Vesica Urinaria*, and *Intestinum Rectum*. We at first imagined that the other Intestines might be covered by the Liver, which, tho' it be commonly large in Children, in this exceeded the usual size; but upon turning of it up towards the Diaphragm, we only found, under its concave part, the Stomach, not lying in its natural posture, for the *Pylorus* was drawn by the *Duodenum* cross the *Vertebræ* of the Back towards the *Fundus Ventriculi*, and part of the *Duodenum* passed through a Foramen in the Diaphragm, placed on the Left side of that through which the *Gula* descends, which occasion'd the *Pylorus* to lye almost *sub Fundo Ventriculi.* We then resolved to trace the *Rectum* from the *Anus* upward, not doubting but that it would lead us to the Mesentery and Intestines. The *Rectum*, we found, lay in an oblique line from the *Anus* to this new *Foramen*, and was received into it with a Portion of the *Duodenum*. This *Foramen* seemed to be formed by Nature, *a primo Ortu*, for transmitting those Guts into the *Thorax*; for had it been made by any Force, its sides would

would have appeared wounded, or lacerated; but on the contrary, round this Orifice there was a ſmooth Verge, as is ſeen in *Foramine Venæ Cavæ pervio*, or that, *per quod Gula deſcendit.* A Sketch of the Diaphragm with its Perforations I have attempted.

When we took off the *Sternum*, we ſaw the Meſentery *cum adjunctis Inteſtinis* in the Cavity of the *Thorax*, and lying upon the Heart and Lungs. There was no *Omentum* ſpread over the Inteſtines; that, as I remember, was wholly miſſing, as was likewiſe the *Mediaſtinum*. Moſt part of the *Duodenum* lay in the *Thorax*, and all the reſt of the Guts, except the *Rectum*, which (as is already related) aſcended in an oblique line from the *Anus*, and its upper end was inſerted into this Orifice. We enquir'd how this Child, according to the Common Notions of Nutrition, could be nouriſhed? That it was Nouriſh'd ſeems plain, becauſe it daily receiv'd Food, and regularly voided Excrement, as we are aſſur'd by thoſe who attended upon it. For ſalving this Quære, we propos'd to enquire, what communication there was between that Gland, or Glands, in the middle of the Meſentery (commonly called *Pancreas Aſellii*) and the *Receptaculum Chyli* placed between the Internal Lumbar Muſcles, called *Pſoas*; but upon the moſt accurate Search we were capable of making, there was none to be found; for the whole Meſeraick Membrane, and Inteſtines, lay perfectly looſe upon the Heart and Lungs, abſolutely diſengag'd from any manner of Communication with any other part.

That Vermicular Motion, which ſhewed itſelf on the Ribs and Breaſt, we aſcribed to the Periſtaltick Motion of the Guts; and the *Dyſpnæa*, we thought, might be occaſioned by Preſſure made on the Lungs by the Inteſtines and Meſentery, which likewiſe ſo fill'd the *Thorax*, that there wanted room for the Lobes of the Lungs to move freely in, and by conſequence Inſpiration and Expiration would be performed with Difficulty. *See the Figure; where a ſhews the* Foramen *through which the* Vena Cava *paſſes.* *b the* Foramen *thro which the* Gula *deſcends.* *c the* Foramen *through which part of the* Rectum *and* Duodenum *went into the* Thorax. *Fig.* 11. *Plate* 3.

XI. The Subject I found theſe Glands in, was a Boy of about 4 or 5 Years Old, that died of a general *Atrophy*, or Conſumption of all the Muſcular Fleſhy parts of the Body, occaſioned without all doubt from the numerous Glandulous Swellings ſcattered up and down the whole Meſentery; which by compreſſing the Lymphatick Veſſels, called in this place *Vaſa lactea*, prevented the acceſs and ſupply of the Chyle, ſo neceſſary for the continued nouriſhment and increaſe of the Parts. For without the conſtant Recruit of this whitiſh Balſamick Liquor, the Maſs of Blood will in a ſhort time be unfit to perform any of thoſe good Offices, which a freſh acceſſion of Chyle qualifies it for. *Of the Glands in the human Spleen, by Dr. Douglas. n.* 349. *p.* 499.

In a piece of this *Spleen* we might ſee, without the aſſiſtance of a Glaſs, ſeveral round whitiſh Bodies of a pretty hard Conſiſtence, and abundance of ſmall white and ſofter ſpecks; but both of the ſame nature. Theſe, to me at leaſt, appear to be ſo many diſtinct Glands become viſible;

which

which in a Natural State are only to be seen by a fine Glass, as the curious *Malpighius* first observed in his Treatise *de Liene, Cap.* V. *De quibusdam corporibus per Lienem dispersis.* Where is a Case which was the very same with mine.

Anatomical Observations by Mr. Chiseden, *n.* 357. *p.* 282.

XII. Fig. 12. Plate 1. Shews three Spleens taken from one Body.

Fig. 13. Two Spleens taken from a Man.

Fig. 14. Two Spleens taken from a Woman.

N. B. That in all these three Cases of the Spleen, each had its proper Vessels, but the Arteries only are there exprest; and that the Spleens in each Body taken together were but equal in Magnitude to the one we usually meet with.

An Ureter double of two Thirds of its Length next the Kidneys, and distended by Stones pass'd thro' it.

The *Tubæ Fallopianæ* impermeable; and without *Alæ Vespertilionis*; the outer Ends being connected to the *Testes.*

A Heart with the *Vena Azygos* inserted into the Right Auricle; and the descending *Cava* coming round the Basis of the Heart, above the Aorta and Pulmonary Vessels, to enter the Auricle at the lower Part with the ascending *Cava.*

The Left Ventricle of the Heart of a prodigious Bigness. by Dr. Douglas, *n.* 345. *p.* 326.

XIII. I lately open'd a young Man that died of the Palpitation of the Heart, whose violent Beating and prodigious subsultory Motion, some Months before his Death, was not only easily felt, by laying the Hand on the Region of the Heart; but seen to rise and fall by lifting up the Bed-Cloaths that covered it. And, which is almost incredible, at some times the Tremblings and Throbbings made such a Noise in his Breast, as plainly could be heard at some Distance from his Bed side. This was accompanied with frequent *Deliquiums*, sometimes slow, sometimes swift, and often intermitting. In the Dissection of this morbid Heart, I observed the following remarkable Particulars.

1. That the *Pericardium* or *Capsula Cordis* was very thick, and firmly adhered or grew by a fibrous Connexion to all the outer Surface of the Heart.

2. Instead of the Water called *Liquor Pericardii*, there was only in some Places about the *Basis* of the Heart, a mucilaginous clear Substance like a Gelly.

3. In the right Auricle laid open there was nothing preternatural. The ascending and descending *Cava* opened into the same as usual. The *Vestigium* or Mark of the *Foramen ovale* with its semicircular *limbus* was very plain. And the *Orificium* of the *Vena Cordis Coronaria* was extreamly large, yet its Valve was less than usual.

4. In the right Ventricle laid open, the *Valvulæ* called *tricuspides* were configurate after the usual manner. The sides of this Cavity were thin and full of small fleshy *Columnæ* as they commonly are, with great variety of Furrows and little Holes. The three *Sigmoide* or *Semilunar Valves* in the

Mouth

Mouth of the *arteria pulmonalis*, were as they always are in a Natural State.

5. The Left Auricle was not much bigger than ordinary: but its muscular Appendage, called the *Bulb* of the *Pulmonary Vein* by the late Mr. *Cowper*, was extraordinarily dilated and enlarged beyond any thing that I ever saw.

6. The Left Ventricle, whose Capacity in a natural State is always less than the Right, was here considerably larger. And if the Experiment had been made before Dissection, of filling both with any Liquor, this had certainly contained three times more than the other.

7. The *Valvulæ*, called *Mitrales*, placed at the Orifice of this Ventricle, are much thicker in Substance than ordinary; and the two fleshy Columns, called by *Nicolaus Massa*, almost 200 Years ago, *duo parvi musculi*, which send out abundance of small Tendons to be inserted into those Valves, were proportionably augmented in Bigness.

8. The *Semilunary Valves* in the Mouth of the *Aorta*, or of that great *Vena pulsatilis* that dispenses the Blood to all the several Parts of the human Body, were very much preternaturally affected; as would easily appear upon comparing them with those in the Orifice of the *Pulmonary Artery*, in which they are thin and very broad, so as to be able to shut the Cavity of that Vessel, and hinder the Blood from returning back into the Ventricle, and likewise transparent: but in this they are very thick, contracted as it were, and furled together, and of a whitish Colour; and in all Appearance, if the Person had lived longer, they had turned boney or undergone a Petrification.

This uncommon Structure of the Heart being thus demonstrated, let us endeavour to account for the following Phænomena. The first is the Palpitation of the Heart, which was the chief Symptom and Complaint of the sick Person. The second is the preternatural Dilatation and Enlargement of the left Ventricle. It is not improbable but the firm Adhesion of the *Capsula Cordis membranosa* to the Substance of the Heart occasioned that uncommon Trembling and Throbbing thereof; its free and easy Motion being hindered by that thick *involucrum*, which surrounded it so close on each * side.

* *v.* Lower *de Corde.*

As for the Dilatation of the left Ventricle and muscular Bag of the *Pulmonary Vein*; that is altogether owing to the ill Configuration of the *Valves* we have now described: for as the great Artery or *Aorta* arises out of this Ventricle, it has three Valves, which separating, give Passage to the Blood from the Ventricle into the Vessel; and in a natural State, they shut that Passage, and so prevent the Blood from recoiling into the same, if it should endeavour to return. But in this Case, by reason of its contracted Narrowness and Thickness, not being able to close or shut the Passage, the Blood flow'd back again into the Cavity which it had gradually enlarged and dilated to the Bigness we see. Besides, the *Muscular Valves* not being duly qualify'd to perform their Office, the Blood recoil'd into the Auricle, which it had distended in like manner. This constant Re-

gurgitation or Reflux of the Blood, is besides sufficient of it self to cause this extraordinary Trembling.

Of some Influence of Respiration on the Motion of the Heart, hitherto unobserved. By Dr. Drake, *n.* 281 *p.* 1217.

XIV. Tho' divers accurate Treatises of the *Heart*, and its Action have been written by Learned Men of several Nations, especially by two of our own Country, Dr. *Harvey*, and Dr. *Lower*, yet there remain several Doubts and Difficulties about it (in my Opinion) not sufficiently accounted for. Dr. *Lower* has so well accounted for the *Systole*, or Contraction of the Heart from the *Mechanical* Structure of it, that he seems almost to have exhausted the Subject, and had he been as happy in discovering the true Cause of the *Diastole*, he had left little room for the Industry and Sagacity of others about this *Viscus*. But having judiciously and solidly explain'd the *Systole*, he contents himself to ascribe the *Diastole* to a Motion of *Restitution*, which Account gives me no Satisfaction: Because the *Systole* being the proper, and (as himself confesses) the only Motion of the Heart, a State of *Contraction* seems to be the natural State, and consequently without external violence, it should have no *Diastole* at all.

This will appear more plain, if we consider the Circumstances of it, and its Motions, as a Muscle, with respect to orher Muscles. That Contraction is the proper Action, and State of all Muscles, is evident from Experience of Fact, as well as Reason. For, if any Muscle be freed from the Power of its *Antagonist*, it is immediately *contracted*, and is not by any Action of the Will, or Spirits to be reduc'd to a State of *Dilatation*. Thus, if the *Musculi Flexores* of any Joint be divided, the *Extensores* of that Joint being by that Means freed from the contrary Action of their *Antagonists*, that Joint is immediately extended without any Consent of the Will, and in that State it remains; and so *Vice versâ* if the *Extensores* be divided. From whence it is plain, that the Muscles have no restitutive Motion, but what they derive from the Action of their *Antagonists* by which they are ballanc'd. Thus likewise the *Sphincters* of the *Gula*, *Anus*, and *Vesica*, having no proper *Antagonists*, are always in a State of Contraction, and suffer nothing to pass 'em, but what is forced thro' 'em by the contrary Action of some stronger Muscles, which, tho' not properly to be call'd *Antagonists*, yet on all necessary Occasions perform the Office of such.

That the Heart is a Muscle, furnish'd and instructed for Motion like other Muscles, is (in my Opinion at least) demonstrated beyond Contradiction, by Dr. *Lower* and others. And, as it is a *Solitary* Muscle without any proper *Antagonist*, and not directly under the Power of the Will, nor exercising *Voluntary* Motion, it approaches nearest to the *Sphincter* kind, which only has these Conditions in common with it. But in constant and regular Alternations of Contraction and Dilatation, it differs exceedingly from all the Muscles of the Body. This *reciprocal Æstus* of the Heart has given the Learned abundance of Trouble; who, finding nothing peculiar in the Structure, which shou'd necessarily occasion it, nor any *Antagonist*, whose Re action should produce it, have been extreamly perplext to find out the Cause of it.

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The Learned Dr. *Lower* having by ſound Arguments, drawn from the Structure and Mechaniſm of the Heart, eſtabliſht the certainty of its *Muſcular* Motion, reſts ſatisfied, without taking notice of any Aſſiſtance, that the Heart receives from any other Part, except from the Brain, by the means of the eighth Pair of Nerves.

The Accurate *Borellus* in his *Oeconomia Animalis* computes the *Motive* Power of the *Machine* of the Heart to be equal to, or to ſurmount that of a Weight of 3000 *l.* The *Obſtacles* to the Motion of the Blood through the *Arteries* he eſteems equivalent to 180,000 *l.* which is 60 times as much as he rates the Force of the Heart at. Then deducting 45,000 *l.* for the Adventitious Help of the *Muſcular Elaſtick Coat* of the *Arteries*, he leaves the Heart with a Force of 3,000 *l.* to overcome a reſiſtance of 135,000 *l.* that is, with 1 to remove 45. This ſtupendous effect he contents himſelf to aſcribe to the *Energy* of *Percuſſion*. But, had he proceeded in his Calculation to the Veins, which he allows to contain conſtantly a quantity of Blood, quadruple to the Contents of the Arteries, and to which this *Energy* of *Percuſſion* does either not reach at all, or but very languidly, he might probably have ſeen a neceſſity for ſome other Expedient to remove ſo inſuperable a difficulty. But not to inſiſt rigorouſly on the exactneſs of this Calculation, we may allow a much greater Deduction than would be juſtifiable, without leſſening the difficulty. But this account I have taken notice of purely for the ſake of the Calculation, which may be of uſe in the Sequel, the account it ſelf being in other reſpects more defective than Dr. *Lower*'s, to which we will return.

Part 2d. Prop. 67. Prop. 73. Prop. 76.

The Dr. appears (to me) to have overlookt ſomething of very great moment and importance in the explication of the Action of the Heart. For, though it ſhou'd be granted, that the *Muſcular Fibres* of the Heart, acted by the Nerves, are the immediate Inſtruments of its *Conſtriction* or *Syſtole*, yet it muſt not be denied, that the *Intercoſtal* Muſcles and *Diaphragm* are of great ſervice to aid and facilitate this Contraction, by opening a Paſſage for the Blood through the Lungs, which denied wou'd be an invincible obſtacle. Neither do they promote it that way only. The manner how they farther aſſiſt the Heart in its Contraction will appear manifeſtly, if we conſider the different Poſture, Scituation and Capacity of the Blood Veſſels of the Lungs in the ſeveral times of *Elevation* and *Depreſſion* of the *Coſtæ*.

The *Pulmonary* Artery riſes from the *right* Ventricle of the Heart, and runs into one Trunk, till it comes to the *Aſpera Arteria*; where it is divided, and ſends a Branch along with each diviſion of the *Aſpera Arteria*, according to all the minuteſt ſubdiviſions, of which it is likewiſe ſubdivided, accompanying all the *Bronchi*, in their whole progreſs through the Lungs. The *Pulmonary* Vein, which empties it ſelf into the *Left* Ventricle of the Heart, ſpreads it ſelf on the *Aſpera Arteria* and *Bronchi*, in the ſame manner that the Artery does. The neceſſary conſequence of this diſpoſition is, that this Artery and Vein being coextended with and faſten'd to the *Bronchi*, muſt needs ſuffer ſuch alte-

ration of *Superficial* Dimensions, as the *Bronchi* do in the *Elevation* or *Depression* of the *Costæ*.

While the Ribs are in a state of *Depression* (whether before Commerce with the external Air or after) the *Annular Cartilages* of the *Bronchi* shrink one into another, and by that means their *Dimensions* are exceedingly contracted. In conformity to this condition of the *Bronchi* the *Pulmonary* Artery and Vein must likewise, either by means of their *Muscular* Coats contract themselves to the same *Dimensions*, or lye in *Folds* or *Corrugations*, which is less probable. On the other hand, when the Ribs are elevated, and the *Diaphragm* bears downward; the Air rushing into the Lungs, shoots out the *Cartilaginous* Rings, and *Divaricates* the Branches of the *Trachea*, and by them extends and divaricates the several divisions of the *Pulmonary* Artery and Veins, and thereby lengthens and enlarges their Cavities. This enlargement of their Cavities is very considerable, not only upon the score of the addition, which they receive in length thereby, but also upon the account of their *Divarication*. For whereas, when the Ribs are deprest, and the Lungs subside, the Blood Vessels are not only contracted, (as I have already observ'd) but their Branches, which are exceeding numerous, approach one another, and lie in juxta-position, by which their Cavities are very much compress'd and streighten'd: when the Ribs are elevated, and the Lungs turgid with Air, not only the Fibres, by which their Coats in the opposite State were contracted, are extended; but those innumerable Vessels, which lying before in lines almost parallel upon one another, compress'd one another, making an *acute* Angle at their Junctures, are divaricated and separated from each other, and make an *obtuse*, whereby their Channels are widen'd.

Thus a passage is open'd to the Blood, from the *Right* Ventricle of the Heart to the *Left*, through the Lungs, to which it cou'd not otherwise pass; and the opposition, which the Blood, contain'd in that Ventricle, must otherwise necessarily have made to its constriction, is taken off, and the *Systole* thereby facilitated. Nor is that all. For the *Diastole* being caus'd (as I shall in the sequel shew) by the Force of the Blood rushing into the Ventricles, this Ampliation and Extension of the *Pulmonary* Artery is a sort of *Check* or *Counterpoise* to it, and prevents an endeavour towards two contrary Actions at once, which must necessarily frustrate both. For the Heart being a *Springy*, *Compressible* Body, whose proper Action, which is Contraction, depends on the influx of certain Fluids into its Fibres, or Substance; and containing besides a Fluid in its *Ventricles*, or great Cavities, in one of which is the Mouth of this Artery; the action of this Vessel must in great measure resemble that of a *Syringe*, whose extremity is immers'd in Water, the Enlargement or Expansion of the Channels of the Artery answering the drawing of the *Embolum*, as the Constrictive Motion of the Muscle of the Heart does the Pressure of the *Atmosphere* upon the *Surface* of the Water, the one making way for the fluid, and the other forcing it to follow,

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where the refiftance is leaft. In this fenfe we may allow a fort of Attraction to the *Pulmonary* Artery, depending wholly upon the Action of the *Intercoftal* Mufcles and *Diaphragm*, which we muft therefore confefs to be very ferviceable and inftrumental in promoting the *Syftole* of the Heart.

But the Learned Author has much lefs fufficiently accounted for the *Diaftole*, or Dilatation of it, which he afcribes to a Motion of *reftitution* of the over-ftrain'd Fibres, which yet he confeffes are made for *Conftriction* only. 'Tis true, he immediately after joyns the *influx* of the *Blood* as a concurrent caufe; but from the flight notice that he takes of it, 'tis plain, that he did not fo much as dream of any great fhare it had in that action. His words are thefe. *Quin &* (*ut obiter hoc monеam*) *cum omnis motus contractione perficiatur, & Cordis Fibræ ad conftrictionem folum factæ fint, apparet quoque Cordis motum* totum *in* Syftole *pofitum effe; cumque Fibræ ultra tonum fuum in omni conftrictione ejus tendantur, idcirco ubi nixus ifte abfolvitur, motu quafi* reftitutionis *Cor iterum relaxatur, & fanguine a Venis* influente *rurfus diftenditur; a* nullo *enim cordis motu, nifi* tenfionem fuam remittente, *& ab* irruente *fanguine* Diaftole *ejus libratis adeo viribus fuccedit.* *De Corde Pag.* 75.

If Contraction be the fole Action of thefe Fibres (as this Great Man confeffes it to be) and as indeed it is of all *Mufcular* Fibres, I wonder how fo judicious a Writer came to flip into fuch an abfurdity, as to call their Diftenfion (vulgarly but improperly called Relaxation) a Motion of *Reftitution*. For from the Nature of thofe Fibres, and their difpofition in the Structure of the Heart, the Natural State of the Heart appears manifeftly to be *Tonical*, and its Dilatation a State of Violence; and confequently the Conftriction is the *true* motion of *Reftitution*, and the State to which it will *fpontaneoufly* return, when the Force is taken off, which is the work of the *Intercoftal* Mufcles and *Diaphragm*. Thus we are left ftill to feek for the true caufe of the *Diaftole*, which feems to me to be the main and moft difficult *Phænomenon*, relating to the Heart and Circulation of the Blood.

But in Mr. *Cowper*'s ingenious *Introduction* to his *Anatomy of Human Bodies*, I find the Share which Dr. *Lower* hints the Blood to have in that Action, further profecuted, and improv'd into the main Inftrument of the Dilatation of the Heart, wherein I agree entirely with him. But as to the manner, and Reafons of its being fo very Inftrumental, I can't be fo perfectly of his Mind. *The Heart* (fays he) *of an Animal bears a great Analogy to the Pendulum of thofe Artificial Automata, Clocks and Watches, whilft its Motion is performed like that of other Mufcles, the Blood doing the Office of a* Pondus. This Explication, being but a Simile without a diftinct Application to Particulars, is befide fo very fhort, that I can at beft but conjecture at the Meaning.

By the Blood's doing the Office of a *Pondus*, I fuppofe he means, that the Blood contributes in the fame manner to the Motion of the Heart as the Weights do to that of the Pendulum of a Clock. If fo, the Blood, according

cording to him, muſt be the Inſtrument of *Conſtriction*; and *Dilatation* muſt be the Natural State, or Spontaneous Motion, to which it wou'd, when under no Violence, return; the contrary of which, I preſume, will appear e'er I have done. But if he means, that the Blood in its Reflux, by gravitating on the Auricles and Ventricles, dilates and expands them, acting therein as a Counterpoiſe to its Contraction as a Muſcle, I could wiſh his Deſign had not bound him up to ſo narrow a Compaſs, and that he had given us an Explanation at large of ſo abſtruſe and ſo important a Phænomenon. Becauſe the Specifick Gravity of the Blood ſeems to me a Cauſe by no Means alone adequate to the Effect, which it is here ſuppos'd to produce. For, if the Blood acts only as a Weight by meer Gravitation, then that Part of it only which deſcends from the Parts above the Heart can be employ'd in that Action. This at the largeſt Computation can't amount to five Pound Weight, and muſt, according to the Computation of *Borellus*, force a Machine, that is able to overcome a Reſiſtance of 135,000 *l.* I leave every Man to deduct what he ſhall upon Examination find reaſonably to be deducted, and yet ſhall reſt ſecure, that it is not to be effected in the leaſt with ſo ſmall a Weight.

But neither does the Refluent Blood gravitate in any ſuch Proportion, as I have here aſſign'd. For to make a true Eſtimate of its Gravitation, we muſt conſider the Circumſtances of the Liquor ſuppos'd to gravitate; in which it very much reſembles Water inclos'd in a recurve Tube, of which if the Length of the two Legs be equal, it may be ſuſpended in the Air full of Water, with the Extremities downwards, without loſing a Drop; altho' the *Diameter* of thoſe Legs ſhou'd be very unequal. The Caſe of the Arteries and Veins is pretty near a Parallel to a Tube, ſo fill'd and inverted. For, if the Arteries and Veins be continued Tubes, (as by the Microſcope they are made to appear) then, ſuppoſing their Contents to have no other Determination of Motion, than their own Weight wou'd give 'em, the contain'd Fluids muſt be Counterpoiſes to each other. For the Veins and Arteries being join'd at the ſmaller Extremities, and the larger of both terminating in the ſame Parallel Line, it is impoſſible, according to the Laws of *Hydroſtaticks*, that the Contents of either ſhou'd overballance t'other. How far then muſt it fall ſhort of forcing the natural Power and Reſiſtance of ſo ſtrong a Muſcle as the Heart by meer Gravitation?

The Blood indeed has a *Progreſſive* Motion thro' its Veſſels, wherein it differs from Water, in a recurve Tube, in the Experiment above ſtated. But, if the natural Gravitation of the Blood contributes nothing to the Dilatation of the Heart, this Progreſſive Motion will not be found much more ſufficient. For, as this Motion is deriv'd entirely from the Heart's Conſtriction (as all Accounts hitherto derive it) cou'd the Blood be ſuppos'd to re-act upon the Heart, with all the Force firſt impreſs'd upon it by the Heart, it wou'd be inſufficient, unleſs we will ſuppoſe the *Force communicated* to be ſuperiour to the *Power Communicant*, which is abſurd. But when the juſt and neceſſary Deductions, for the Impediments which the Blood

Blood meets with in its Progrefs thro' the Veffels, fhall be made, the remaining Force will be found fo exceeding weak, that to prop the Blood thro' the Veins may be a Task alone too great for fo fmall a Power, without charging it with the additional Difficulty of forcing the Mufcle of the Heart.

Alphonfus Borellus cafts up the Force of the Heart, and the *Mufcular* Coat of the Arteries, to be together equal to a Weight of 3,750 *l.* and allots 'em a Refiftance equal to 180,000 *l.* to overcome which is 45 to 1. To make up for a Difproportion, by his own Confeffion, incredible to thofe who have not confidered the Matter as he had done, he flings into the Scale the additional *Force of Percuffion,* which he leaves *indefinite,* and thinks fufficient to *force any quiefcent finite Refiftance whatfoever.* But as this Account and *Hypothefis* are a Pofthumous Work, I fhould fufpect, that thefe Papers were left unfinifht by *Borellus;* or at leaft, that in many Places the laft Hand was never put to 'em. For, neither in this Place, nor any other of this Work, does he account for any more than the *Syftole* of the Heart, and the Refiftance which is made to the Progreffive Motion of the Blood in the Arteries only. This alone he found to exceed the Power of the Heart fo prodigioufly, that he feems to fhuffle it off his Hands, with a general and precarious Solution, as a Difficulty that he was defirous to be rid of. For, having afcrib'd this *ftupendious* (as he himfelf calls it) effect to the *Energy of Percuffion,* he takes no Care to fatisfie his Reader any farther about it, or to refer him, or give him the Expectation of Satisfaction any where elfe; although he has an exprefs Treatife on the *Force of Percuffion,* which was written preparatory to this, and to which he often refers in other Places of this Work. But what confirms my Sufpicion, that this Part was intended for a farther Revife by the Author, is, that he has left the Progrefs of the Blood thro' the Veins, and the *Diaftole* of the Heart, abfolutely untouch'd, tho' they are Difficulties of a much greater Magnitude, than this which he has attempted to account fo flightly for. For in thefe he is excluded the Benefit of *Percuffion,* and has yet a greater Refiftance to overcome without it. Omiffions of this kind are fo unufual wirh this Author, where-ever he knows himfelf to go upon fure Grounds; that it is to me an Argument, that he doubted the Sufficience of his *Percuffion,* and referv'd thefe important *Phænomena* for farther Confideration, without plunging himfelf into fuch an Abfurdity as to afcribe to *Percuffion* any fuch *Energy* as to be able (fo broken as it returns to the Heart) by its Re-action to force that Power, from whence only it was firft deriv'd. But tho' the *Hypothefis* of *Borellus* may in this cafe be found precarious or infufficient (a Misfortune that has befallen him in divers Particulars) his *Theory* holds ftill good. At leaft it ought to be allowed in Juftice to his great Abilities and Exactnefs, till fome body convicts him of fome material Error in his Calculations.

Suppofing then the Force of the Heart, and of the Mufcular Coat of the Arteries, as likewife of the Refiftance, which they muft overcome, to be computed with any degree of Accuracy, there remains yet fuch a pro-

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digious Difproportion to be accounted for, as requires fome more powerful Agent, than any yet affign'd, to make up the Deficiency.

What Affiftance the Heart receives from the Action of the *Thorax* towards the facilitating its Contraction, without which Affiftance there could have been no *Syftole*, has been already fhewn. But neither the *Intercoftal* Mufcles, or *Diaphragm*, which are fo inftrumental in that Part of its Action, can contribute any thing to the *Diaftole*; becaufe they ferve only to enlarge the Cavity of the *Thorax*, and thereby to open the Paffage to the Blood from the Heart; and promote its Conftriction. Whatever therefore the Force is, that dilates the Heart, and is the Caufe of the *Diaftole*, muft be equal to that of the Heart, the *Intercoftal* Mufcles and *Diaphragm*; to all which it acts as an Antagonift. I take no notice of the *Serratus Major Anticus*, and other Mufcles; which have an obfcure Share in the *Elevation* of the *Coftæ*, becaufe as much may reafonably be deducted upon the Account of the *Obliquus externus Abdominis*, and other Mufcles; which, having their Infertions on fome of the lower Ribs, are as inftrumental towards the Depreffion of 'em, and fo ballance the Account. But the chief Ufe of thefe is in violent Refpiration. In ordinary Refpiration their Share is fmall.

Such a real Power (which may in the leaft be fufpected of any fhare in this Action) is hard, perhaps impoffible, to be found in the Machine of any Animal Body; and yet without fome fuch Antagonift, it is as impoffible the Circulation of the Blood fhou'd be maintain'd. All the Engines yet difcover'd within the Body confpire towards the *Conftriction* of the Heart, which is the *State* of *Quiefcence*, to which it naturally tends. Yet we find it alternately in a *State* of *Violence*, that is, of *Dilatation*; and this upon Neceffity, becaufe upon this *Alternation* depends all Animal Life. Some fufficient Caufe external muft therefore be found, to produce this great Phænomenon, which Caufe muft be either in the Air, or Atmofphere, becaufe we have no conftant and immediate Commerce with any other *Mediums*.

Some great Phyficians obferving this, and, that depriv'd by whatfoever means of Communication with the external Air, we became inftantly extinct, have imagin'd, that in the Act of Infpiration certain purer Parts of the Air mixed with the Blood in the Lungs, and was convey'd with it the Heart, where it nourifh'd a fort of *Vital Flame*, which was the Caufe of this reciprocal *Æftus* of the Heart. Others not quite fo grofs, rejecting an *Actual* Flame, have fancied that thefe fine Parts of Air mixing with the Blood in the Ventricles of the Heart, produc'd an *Effervefcence* which dilated it. But thefe Fancies have been long fince exploded and condemned upon ample Conviction, and 'tis a Point yet undetermin'd, whether any Air does mix with the Blood at all in the Lungs or not. But fuppofing that fome Air may infinuate it felf into the *Pulmonary* Vein, it can no other Way dilate the Heart than by an Effervefcence in the Left Ventricle, which wou'd not dilate the Right. But this Opinion is contradicted by *Autopfie*, and too laborioufly confuted by others, to be brought upon the Stage again here.

There remains therefore only the groſs Body of the Atmoſphere to be conſider'd, which is undoubtedly the true Antagoniſt to all thoſe Muſcles, which ſerve for ordinary Inſpiration and Conſtriction of the Heart. This will appear more evidently, if we conſider not only the Power, but the Neceſſity of its Action upon *Animal* Bodies, as well as the want of other ſufficient Agents.

The Heart is a Solitary Muſcle of very great Strength, and the *Intercoſtal* Muſcles and Diaphragm, which likewiſe have no Antagoniſts, are a vaſt additional Force, which muſt be ballanc'd by the contrary Action of ſome equivalent Power or other. For, tho' the Action of the *Intercoſtal* Muſcles be voluntary, that does not exempt them from the Condition of all other Muſcles, ſerving for voluntary Motion, which wou'd be in a State of perpetual Contraction, notwithſtanding any Influence of the Will, were it not for the Libration of Antagoniſt Muſcles. This Libration between other Muſcles is anſwer'd by the Weight of the incumbent *Atmoſphere*, which preſſes upon the *Thorax* and other Parts of the Body. And, as in all other voluntary Motions the Influence of the Will only gives a Prevalence to one of two Powers before equilibrated, ſo here it ſerves to enable thoſe Muſcles to lift up a Weight too ponderous for their Strength not ſo aſſiſted; and therefore as ſoon as that Aſſiſtance is withdrawn, the *Coſtæ* are again depreſs'd by the meer *Gravitation* of the *Atmoſphere*, which wou'd otherwiſe remain elevated thro' the natural Tendency of thoſe Muſcles to Contraction. This is evidently proved from the *Torricellian* Experiments, and thoſe made upon Animals in Mr. *Boyle*'s Engine; where, as ſoon as the Air is withdrawn, and the *Preſſure* thereby taken off, the Intercoſtal Muſcles and Diaphragm are contracted, and the Ribs elevated in an Inſtant, and can't by any Power of the Will be made to ſubſide, till the Air is again let in to bear 'em forcibly down.

As in the Elevation of the *Coſtæ*, the Blood, by the Paſſage that is open'd for it, is in a manner ſollicited into the Lungs, ſo in the Depreſſion of 'em, by the Subſidence of the Lungs and Contraction of the Blood Veſſels, both which are Conſequent thereof, the Blood is forcibly driven, as it were with an *Embolum*, thro' the Pulmonary Vein into the Left Ventricle of the Heart. And this, together with the *general Compreſſion* of the Body by the Weight of the *Atmoſphere*, which ſurrounds and preſſes upon the whole Surface of it, is that Power which cauſes the Blood to mount in the Veins, after the Force impreſs'd upon it by the Heart is broken and ſpent, and which is ſufficient to force the Heart from its natural State to Dilatation. He that is able to compute the Weight of a Column of Air, equal to the Surface of the whole Body, will readily grant it a Power ſufficient for the Effects, which are here aſcrib'd to it. And, when he conſiders, that the Bodies of Animals are compreſſible Machines, he will find that it muſt of Neceſſity affect them in the manner here laid down. But tho' our Bodies be entirely compos'd of *Tubuli*, or Veſſels fill'd with

Fluids, yet this Preſſure, how great ſoever, being equal, cou'd have no effect upon 'em, if the ſuperficial Dimenſions were not eaſily variable; becauſe being compreſs'd on all Parts with the ſame Degree of Force, the contain'd Fluids cou'd not any where begin to recede, and make way for the reſt to follow, but would remain as fixt and immovable as if they were actually ſolid. But by the Dilatation of the *Thorax*, room is made for the Fluids to move, and by the Coarctation of it freſh Motion is impreſt, which is the main Spring whereby the Circulation is ſet and kept going. This reciprocal Dilatation and Contraction of the Superficial Dimenſions of the Body ſeems ſo neceſſary to animal Life, that there is not any Animal ſo imperfect as to want it, at leaſt none to the inward Structure of which our Anatomical Diſcoveries have yet reach'd. For, tho' moſt kinds of *Fiſh*, and *Inſects*, want both moveable Ribs and Lungs, and conſequently have no dilatable *Thorax*, yet that want is made up to 'em by an *Analagous* Mechaniſm anſwering ſufficiently the Neceſſities of their Life. Thoſe Fiſhes which have no Lungs, have Gills, which do the Office of Lungs, receiving and expelling alternately the Water, whereby the Blood Veſſels ſuffer the ſame Alteration of Dimenſions, that they do in the Lungs of more perfect Animals. The Lungs or Air Veſſels of Inſects are yet exceedingly more different in Structure, Diſtribution and Scituation from thoſe of perfect Animals, than thoſe of Fiſhes are, and yet in their Uſe and Action agree perfectly with both, that is, *receiving* and *expelling* the Air, and *varying* the *Dimenſions* and *Capacities* of the Blood Veſſels. Theſe having no *Thorax*, or ſeparate Cavity for the Heart and Air Veſſels, have the latter diſtributed thro' the whole Trunk of their Bodies, by which they communicate with the External Air thro' ſeveral *Spiracula* or *Vent Holes*, to which are faſten'd ſo many little *Trachea*, or Wind-pipes, which thence ſend their Branches to all the Muſcles and *Viſcera*, and ſeem to accompany the Blood Veſſels all over the Body, as they do in the Lungs only of Perfect Animals. By this Diſpoſition in every *Inſpiration*, the whole Body of theſe little Animals is inflated, and in every *Exſpiration*, compreſs'd, and conſequently the Blood Veſſels muſt ſuffer a Viciſſitude of Extenſion and Contraction, and a greater Motion muſt thereby be impreſs'd upon the Fluids contain'd in them, than the Heart, which does not in theſe Creatures appear to be Muſcular, ſeems capable of giving. The only Animal that is exempted from this neceſſary Condition of *Breathing*, or receiving and expelling alternately ſome Fluid into and out of the Body, is a *Fœtus*. But this, while included in the Womb, has little more than a *vegetative* Life, and ought ſcarce to be reckon'd among the Number of *Animals*. For, were it not for that ſmall Share of *Muſcular* Motion, which it exerciſes in the Womb, it might without Abſurdity be accounted for as a Graft upon, or Branch of the Mother.

In order to account for the Motion of the Blood thro' the Veſſels, it will be neceſſary to obſerve, that the Pulſation of the Heart in a *Fœtus* is ſo very weak and obſcure, and the Motion of the Blood ſo

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extream flow and languid, as to be fcarce, if at all, perceivable, as has been experienc'd in the Diffection of Puppies before Refpiration had. To produce fuch a feeble Palpitation, and creeping Motion, no greater Force feems to be requir'd than may be deriv'd from the Communication between the Veffels of the Mother and *Fœtus* in the *Placenta.* I am not ignorant, that divers Anatomifts (whom the Crowd have implicitly follow'd) have abfolutely rejected all Communication between thefe Veffels. But with Submiffion to great Authorities, I think they have acted arbitrarily, and without fufficient Warrant from Reafon or Experiment. For neither are the Arguments which they bring againft it conclufive, nor the Office which they affign to the Umbilical Veffels in lieu of it, proper, or natural to thofe Veffels, or the Reality of the Fact made out by any fubftantial Reafons. Thofe that reject this Communication ufually do it in favour of one or both of thefe Opinions, that the Arteries of the *Uterus* do depofite a Nutritive Juice, or a Juice impregnate with Air in the *Placenta,* which is fuck'd in by the *Umbilical* Vein, and convey'd to the *Fœtus,* for the neceffary Ufes of Nutrition and Life. Now thofe that patronize either of thefe Opinions lead Nature an unneceffary Dance. For if the *Maternal* Blood does really contain any fuch *Nutritious,* or any fuch neceffary Aerial Particles, why fhould they be feparated and extravafated, to be with Difficulty receiv'd into the Umbilical Vein, and again mixt with the Blood, when they might more eafily have been imparted by the plain fimple way of Transfufion from the Arteries of the Mother to the Veins of the *Fœtus.* And, that this is the Courfe which Nature takes in this Cafe, I am perfuaded from the Eafinefs and Simplicity of the Method, which readily performs what might be perhaps in vain expected from t'other, and wou'd over and above find 'em, what they feem to grope fo blindly about for, a firft Mover of the Blood in a *Fœtus.*

Boyle of the Elafticity of Air.
Pechlinus de Aeris & Alimenti defectu.

Thofe that contend for the Conveyance of a Nutritious Juice thro' the Umbilical Vein from the *Placenta,* are forc'd upon two Difficulties next to Abfurdities. For firft, they are oblig'd to make this Vein, which, as all other Veins, feems dedicated to the Re-conveyance of Blood only, the proper and immediate Channel, through which a very different Liquor is to be carried; and next to give a Power of Attraction or Suction to it; becaufe the Nutritious Juice, which it is thus deftin'd to carry, is both Vifcous and Stagnant, and has neither Force to drive, nor Subtilty to penetrate, or infinuate it felf into the Capillary Veins; and therefore muft be drawn or fuckt as Milk is from the Breaft, to which the *Placenta* and its Nutritious Juice are by the Favourers of 'em exprefly compared. But if this were the fole ufe of the *Placenta,* and Umbilical Veffels, why were the Umbilical Arteries fent along with the Vein? Their Bufinefs is not to bring any thing back to the *Fœtus,* nor can they contribute any thing to the Benefit of the Mother; for the Uterine Arteries bring all to the *Placenta,* the

Umbilical Vein carries it to the *Fœtus*, and the Uterine Veins convey back again the Surcharge of the Mothers Blood; the Umbilical *Arteries* only have nothing to do, and are ſuperfluous and impertinent, which is contrary to the conſtant Practice of Nature. Yet if *Autopſie* did in the leaſt countenance this Hypotheſis, ſome Defence might ſtill be made; but we find in the Umbilical *Vein* of a *Fœtus* nothing but *Florid* Blood, ſuch as in all probability it received immediately from the *Arteries* of the Mother without any Mixture. And therefore I can't help concluding, that this Opinion engages its Favourers in ſome Abſurdity, without Neceſſity and without Proof.

They that from the *Placenta* ſupply the Body of the *Fœtus* with *Air*, are as much diſtreſs'd as t'other, for they are forc'd to beg the Queſtion twice, which, even when granted, will not anſwer their Ends. Firſt, they ſuppoſe, that an intimate Mixture or Confuſion of *Air* with the Blood, is neceſſary for the Support of animal Life, a *Poſtulatum*, which perhaps the former Part of this Diſcourſe may have render'd unneceſſary; and next that the *Fœtus* is ſupplied with *Air* from, and its *Blood* mixt with it in the *Placenta*. But here again they fetch a Compaſs without Neceſſity or Proof. For if a Mixture of *Air* were neceſſary to a *Fœtus*, why ſhould it be ſeparated from the Mother's Blood, and not rather both communicated together, ſince it is ſo much more eaſie and commodious. But neither does the *Placenta* ſeem to be inſtructed and provided for the Separation of *Air*, but of a much *groſſer Fluid*, deſtin'd to ſome other Uſe, which *Autopſie* confirms. Yet were both theſe Opinions true, they are however defective, and the Circular Motion of the Blood unprovided for.

By the way of *Transfuſion* this great Phænomenon is naturally accounted for, and the Ends, for which the other two Hypotheſes were devis'd, might both be anſwered with more Eaſe. For the *Hyſterick* Arteries tranſmitting their Blood immediately to the Umbilical Vein, may very eaſily tranſmit ſuch Nutritious Juices or Aerial Particles as are contain'd in the Blood, along with it, without depoſiting 'em by the way. By this means ſo much of the Impulſe of the Mother's Blood is preſerv'd, as ſuffices to maintain that languid Circulation, which a *Fœtus* enjoys. For the Blood being driven thro' the Arteries of the *Uterus* into the Umbilical Vein, is convey'd directly to the *Sinus* of the *Porta*, and thence by a ſhort and direct Paſſage through the *Cava* to the Heart; where paſſing through the *Foramen Ovale* to the Left Ventricle, and through the *Canalis Arterioſus* from the Right and Pulmonary Artery, 'tis all deliver'd without coming at the Lungs, to the *Aorta*, and from thence again by the Umbilical Arteries to the Veins of the *Uterus*, making a ſort of *Epicycle* to the main Circulation in the Mother. As this Opinion is favour'd by the Structure and Diſpoſition of the Blood Veſſels on both Parts, ſo there is nothing in it difficult to be conceiv'd, or repugnant to Experience. Late Diſcoveries have made it appear, that the Arteries and Veins are continued

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Tubes, and that the latter contain nothing but what they receive from the former, and no Reaſon appears why we ſhould think this Method to be varied in the *Placenta*. On the other Hand, if the Arteries of the *Uterus* were continued to the Veins of the ſame Part, and thoſe of the *Fœtus* in like manner, without communicating with each other, their Confluence in the *Placenta* ſeems to be altogether impertinent and of no uſe, and the Umbilical Arteries and Vein fram'd for no other Service or Purpoſe, than to give the Blood room for an idle Sally. Mr. *Cowper* has the Honour to re-eſtabliſh this old, but long exploded Truth. For by pouring *Mercury* into a Branch of the *Uterine Artery* of a Cow, that went into one of the *Cotyledones* of the *Uterus*, he fill'd thoſe Branches of the Umbilical Veins, which went from that *Cotyledon* to the Navel of the *Fœtus*; which with a Part of the *Uterus* he keeps prepar'd by him.

It would be a weak Objection to alledge that the Obſervation and Experiment being made on the *Uterus* of a Cow, the Inference would not hold from thence to a Woman, the one being *Glanduliferous*, and the other *Placentiferous*; ſince every one of theſe *Cotyledones*, or *Uterine Glandules*, is in all reſpects a little *Placenta*, and all the Difference between 'em is in Number, Name, and Magnitude. Why *Ruminants* differ in this Particular from other *Viviparous* Animals, is beſide the Subject of our preſent Enquiry. But the great Flux of Blood which follows upon drawing the *Placenta* from Women (which is frequently ſo great as to coſt 'em their Lives) is as plain a Demonſtration to Reaſon of the Continuity of the Veſſels, as Mr. *Cowper*'s Experiment is to the Eye.

I have heard it objected by very learned men, that if there were ſuch a *Continuity* of Veſſels, and ſuch *Transfuſion* of Blood, the *Fœtus* muſt neceſſarily periſh through loſs of Blood, upon the ſeparation of the *Placenta* from the *Uterus*, but that on the contrary no viſible flux of Blood does follow while the *Fœtus* continues wrapt in the Membrane, in which condition it may be kept alive ſome hours. To this it may be anſwer'd, that the Circulation in the *Fœtus* being deriv'd from the Mother, may be ſuppos'd wholly to ceaſe upon the cutting off the communication between them, till it is again renew'd more forcibly by *reſpiration*. But if we allow the Motion already impreſs'd upon the Blood to be ſufficient to keep it going a little while, yet it muſt needs be ſo exceeding languid, that the meer reſiſtance of the external Air muſt be more than enough to hinder any Efflux of Blood from a *Fœtus* before Reſpiration. How long Life may be preſerv'd without an *actual* Circulation of the Blood, is a queſtion not of this place. But we have been convinc'd by many and notorious Obſervations and Experiments, that Life has been recover'd a long time after all tokens of Reſpiration, Circulation, or even Life it ſelf have diſappear'd, ſo that we can't think the firſt ſolution either impoſſible or improbable.

I expect to be told, that in the early days of *Geſtation* in *Viviparous* Animals

Animals there is no *Placenta*, or any Adhesion of the *Umbilical* Vessels to any part of the *Mother*, and consequently no such *Transfusion*; and that in *Oviparous* there is no *continuity*, or *communication* of Vessels of any kind, during the whole time of *Incubation*. But these Objections carry neither the Weight nor Difficulty along with them, that they may be suppos'd to do; for in those days there is neither *Blood* nor *Blood Vessels*, and consequently there can be no *Circulation* of the Blood; and the *Embryo*, of what Species soever, is no more than a *Vegetable* at that time; nor does the *Fœtus* of any *Viviparous* Creature enjoy any *Circulation*, or shew any signs of Animal Life, till after those Vessels, as well as others requisite to the Circulation, are compleated.

It must be confess'd, that *Oviparous* Animals are denied the Benefit of this Communication: But that want is sufficiently compensated by a peculiar Mechanism, which directly answers the ends of *Respiration*, and the *Pressure* of the *Atmosphere* upon the *Fœtus*: There is at the *Obtuse* end of an Egg a small Cavity fill'd with Air, which is the Succedaneous Instrument to the *Respiratory* Organs. For as soon as the Contents begin to be warm'd by the *Incubation* of the Hen, or *any analogous* Heat of *Furnace* or *Dunghil*, the several Humours of the Egg require a *Fermentative* Motion, and the *Air* contain'd in the *Cavity* or *Vesicle* at the *obtuse* end of the Egg is rarefied, and the Vesicle extended and enlarg'd, and consequently the other contents are comprest; to which the *Fermentative* Motion naturally resists. But both Bodies being as well *compressible* as *dilatable*, and both having an *Expansive* Motion imprest upon them by *Incubation*, the compression and renitency will be mutual; but varied in degree, according as either, through the variation of Circumstances, shall prevail. By this means, an Alteration of Compression and Dilatation will be produc'd in both answering the *Respiratory* Motion, by which a Motion will be communicated, which, as soon as the Organs by which it shou'd be regulated are compleated, will in the Body of the *Pullus* be *regular* and *circulatory*.

Fabritius ab *Aquapendente*, and after him our Great Dr. *Harvey*, have assign'd divers uses to this Cavity or Air Vesicle, the Extravagance of which have perhaps deterr'd others from enquiring so much into the Use, as the importance of it requir'd. But though I can't agree to that *Perspiration*, *Refrigeration*, and *Respiration*, which they make it the Instrument of, yet perhaps the *Air*, that was inclos'd in that *Cavity*, may through the augmentation of the Body of the *Pullus*, and its own *Rarefaction* (which is at last so great as to occupy half the Shell) break the *Membrane*, which separated it from the *Pullus*, and thereby give so much *Respiration* as to form the *chirping Voice*, which is often heard before the breaking of the Shell, and with it give an Addition of Strength to enable it to break the Shell. But how it should respire sooner is to me inconceivable.

The incomparable *Harvey* enquires, *Why a* Fœtus, *taken out of the* Uterus *with the* Membranes *entire, shall live in Water some hours without communi-*

communication with the external Air; *where if it be taken out and suffer'd once to breath, it can't afterwards survive a moment without the benefit of* Respiration. Granting the Fact to be as he has deliver'd it, which yet is not so in all cases, the main difficulty is grounded on a Mistake, which from the stating of the Question I find this Great Man to have slipt into. For he thinks, * that a *Fœtus* is sooner suffocated after having once breath'd, than if it had not breath'd at all, and that by breathing it had contracted something which rendered it more perishable. The Dr. observing a *Fœtus* to live longer without Respiration, and to dispence better with the want of Air while included in the Membranes entire, than it could afterwards; infers thence, that the Air does in the first Act of Inspiration impress upon the Lungs some quality, which renders it ever after more indispensably necessary. But allowing his Observation, I must yet deny his Inference to be good: For deprive a *Fœtus* of means of respiring, and then take it out of the Membranes, and it shall be as soon suffocated, as if it had respired before. This proves, that this Necessity of intercourse with the Air by way of the Lungs is not the Offspring, but the Parent of Respiration, and that that Learned Man was drawn into a Fallacy of *Non causa pro causâ.* *Harv. de Gen. Anim. Cap. de Partu.

The reason of this Necessity is the pressure of the External Air upon the Surface of the Body, from which it was defended by the interposition of the Membranes and the Humours contain'd, which are not so compressible, as the Body of the *Fœtus* itself. So soon therefore as the *Fœtus* is excluded, and expos'd to the immediate contact of the ambient Atmosphere, the Vessels and all the Cavities of the Body must necessarily be so compress'd, that the Fluids can't have room for Motion, and consequently the *Fœtus* could have no Life, if Nature had not contriv'd by the Motion of the *Thorax* to remove and admit that pressure alternately, and thereby to impress a Motion on the Fluids, which is the Spring of Life. But this Motion of the *Thorax* being any way suppress'd, the equal pressure of the Atmosphere on all parts occasions a total Cessaion of Motion, which is Death.

XV. Uti non gravate concedimus doctos Viros & olim exstitisse, & hodie reperiri non paucos qui nullâ instructi disciplinâ Mathematicâ medendi Artem tamen feliciter & cum laude exerceant; ita vicissim fateri æquum est eam doctrinam in Praxi expediendâ non inutilem, ad naturam vero & causas Morborum explorandas plane esse necessariam. Corpora enim Animalium, cum partim solidis canalibus partim fluido constent per eosdem jugiter propulso, Machinas esse patet, ac proinde opus esse, ad eorum Fabricam, Vires, Actiones, & agendi Impedimenta sive Morbos rite perspiciendos, rei Mechanicæ peritiam. *Of the Force of the Heart, by Dr* Jurin. *n.* 358. *p.* 863.

De quibus tamen multa traduntur etiam à Mathematicis Scriptoribus adeo parum accurata, secumque invicem & cum ratione pugnantia, ut nobilissimæ scientiæ non modo commendationem non addant & dignitatem, sed etiam contemptui & hominum indoctorum ludibriis

briis eandem objiciant. Quis enim, non ipse doctrinâ Mathematicâ imbutus, cum videat, Exempli gratiâ, Cordis Humani vires jam ponderi 3,000 librarum pares, jam 180,000 pondo superantes, jam vero ad uncias 5 vel 8 deductas; Aerem quoque ex Pulmone inter exspirandum propulsum modo 100, modo 50,000 librarum vi; Quis inquam, qui istas conclusiones legerit discrimine tam immani à se invicem remotas, & tamen omnes demonstrationibus suis munitas; si forte se à risu temperet, non tamen inutilem plane & ineptam pronuntiaverit ad explorandas Corporis facultates Scientiam Mechanicam? Sed meminerint oportet æqui rerum Judices neutiquam mirandum esse, si quandoque in difficili Problemate vel summa Ingenia allucinentur, neque errores, siqui forte inciderint, Arti ipsi sed Artifici imputandos. Quod ut Exemplo manifestius declaretur, libet celeberrimi Problematis de Cordis viribus indagandis solutionem novam proponere. Utque facilius mihi temeritatis opinionem detraham, qui ejusmodi inceptum post *Alphonsum Borellum* aggredi ausim, utque viam simul Lectori expediam ad æquam certamque sententiam in tantâ scriptorum dissensione ferendam, primo loco ostensurus sum, quæ in *Borelli* demonstratione reprehendi debeant, deinde Virorum Doctissimorum, *Morlandi*, & *Keilii* solutiones, cum eâdem philosophandi libertate ad examen revocabo.

1. Nobis, quidem, longe præcipuum videtur *Borellianæ* solutionis vitium, quod Cordis Potentiam per pondus iners & quiescens exposuerit. Cor enim cum & ipsum inter contrahendum movetur, & corpora opposita, Sanguinem nempe & Arteriarum tunicas, in motum impellit, patet ejus Potentiam non aliâ ratione sciri posse quanta sit, quam ut motus hujus quantitatem cognitam teneamus. Motus autem quilibet cum pondere quiescente comparari non magis potest, quam Linea cum Rectangulo.

2. Quod in ipso Experimento à Circulatore instituto, neutiquam constet pondus illud suspensum fuisse à solâ Musculorum vi contractrice; quum etiam vis illa, quâ tum Musculi adhibiti, tum genæ quoque, & ipsa forsitan ligamenta divulsioni sui ipsorum & fibrarum ruptioni obstiterint, quâque Musculi etiam ex cadavere exsecti pondera satis magna sustinent, venire in subsidium potuerit.

3. Quod vires Musculorum pondere æqualium a *Borello* pares statuantur: quod profecto dubium admodum videtur, præsertim ubi Musculi sunt figurâ dissimiles.

4. Quod integram Cordis Potentiam, quanta maxima exeri potest cum summâ fibrarum contentione & molimine, ad singulas Systoles adhiberi posuerit. Quum ipse Circulator, si pondus suspensum vel continenter, vel alternis vicibus brevissimâ quiete interpositâ, sublevare contenderet, non ita longo tempore plane succubiturus labori fuisset.

5. Quod Sanguinis & Arteriarum resistentiam sexagecuplam statuerit totius Potentiæ Cordis, loco ejus Potentiæ, quæ ad systolem peragen-

dam à Corde impenditur, quæque forte totius Potentiæ minima pars eſt.

6. Quod in eâ ratione ſexagecuplâ definiendâ errorem inſignem admiſerit. Nam in *Prop.* 60. loco rationis, quam obtinet Summa Potentiarum *P* & *Q* ad Summam *R* & *S*, adhibuit rationem, quæ eſt inter Rectangulum ex Potentiis *P*, *Q*, confectum, & Rectangulum ex *R*, *S*. Quod errati ſi per Propoſitiones ſubſequentes corrigatur, habebitur in *Prop.* 73. reſiſtentia longe major, quam ab ipſo *Borello* definita eſt, nempe pondus librarum 1,076,000, loco librarum 180,000, idque ſecundum poſitiones ab ipſo Viro Clariſſimo uſurpatas.

7. Denique quod pondus illud librarum 180,000, quum à Cordis Potentiâ libris 3,000 æquali ſuperetur, miraculi cujuſdam aut monſtri loco Lectoribus obtrudat; & Vim Percuſſionis, quaſi quendam Θεὸν ἀπὸ μηχανῆς in auxilium advocet. Reipsâ enim nihilo plus hic ineſt prodigii, quam ubi pondus 3,000 librarum pondus aliud 180,000 librarum, ad ſubſexagecuplam diſtantiam à centro Libræ inæqualium radiorum appenſum, in æquilibrio ſuſtinet.

Proximus ſequitur Vir Doctiſſimus *Joſephus Morlandus*, qui in Diſquiſitionibus de Cordis vi ſermone Anglicano editis, Methodum peringenioſam expoſuit Potentiam Cordis ad Experimentum revocandi. Hic autem, præter delictum ſupra in *Borello* reprehenſum, quod Cordis vires cum pondere quieſcente contulerit, nobis videtur eo quoque nomine notandus, quod integram Cordis actionem in tunicas Arteriarum diſtendendas impendi poſuerit. Cor enim non ſolum Arterias tendit, ſed Sanguinem quoque certâ velocitate per totum Arteriarum & Venarum tractum propellit.

Supereſt, ut Viri acutiſſimi *Jacobi Keilii* ſolutionem, in Tentaminibus Medico-Phyſicis ad Oeconomiam Animalem pertinentibus, non ita pridem cum publico communicatam, expendamus. Qui primus omnium auſus eſt Potentiam Cordis à *Borello* definitam, ac magno Scriptorum conſenſu exceptam & laudatam, non ſolum rejicere, ſed aliam eidem infinito prope diſcrimine minorem numeris diſertis expreſſam ſubſtituere. Hunc autem cenſemus, præterquam quod primum illud *Borellianæ* ſolutionis vitium imitatus ſit, in ſequentibus etiam a vero aberraſſe.

Quod Corollarium *Newtonianum*, quo utitur ad Cordis vires definiendas, aut male intellexerit, aut certe non ſatis apte uſurpaverit. Pondus enim illud ab *Archimede Britannico* determinatum, quo Motus aquæ ex vaſe effluentis generari poteſt, nequaquam generat Motum aquæ; quippe quæ gravitatis vi cadendo ipſa Motum ſuum acquirat. Sed hoc pondus per datum tempus cadendo, Motum concipit Motui aquæ eodem dato tempore effluentis æqualem. Præterea ponit Vir Clariſſimus velocitatem Sanguinis ex Corde effluentis perpetuo æqualem per totam Syſtoles durationem, quam nos inſigniter inæqualem fieri in ſequentibus oſtendemus.

In Methodo illa ſimpliciore, quam poſtea adhibet Vir Doctiſſimus, præter delicta hactenus reprehenſa alia etiam bina admittit.

Adſumit enim Vires Cordis in diverſis Animalibus eam inter ſe rationem obtinere, quæ eſt inter pondera eorundem; quod infra falſum eſſe demonſtrabimus. Tum ponit velocitatem Sanguinis ex ſectâ Iliacâ Arteriâ profluentis, eandem eſſe quâ ex Corde in Aortam emittitur. Atque cum omnis fere ſanguis ex Corde expulſus per Iliacam alteram reſectam emittitur, patet ejus velocitatem tanto eſſe majorem in Iliacâ quam in Aortâ, quanto ſectio Iliacæ circularis à ſectione Aortæ ſuperatur. Præterquam quod velocitas æquabilis, quâ Sanguis per Aortam fluit, longe diſtet ab eâ velocitate, quâcum exit ex ipſo Corde. Similiter fere redargüi poteſt & illa Methodus, quâ uſus eſt Vir Cl. ad rationem definiendam inter velocitates diverſas Sanguinis, reſiſtentiâ nunc oppoſitâ, nunc ſublatâ, per Aortam profluentis. Sed cum iſto Experimento non altera ſolum, ſed utraque velocitas major æquo reperiatur, unde ratio, quæ eſt inter ipſas, non magnopere perturbetur, poterit ſatis tuto proportio ab ipſo expoſita, tanquam veræ propinqua, uſurpari.

Curſu hactenus expedito, erit modo nobis ipſis, ſummâ adhibitâ cautione progrediendum. Et primo quidem loco ad ambiguitatem præcidendam neceſſe eſt, ut id, quod quæritur, quale ſit, accuratius paulo declaretur.

Cordis Virium, ſive Potentiæ, nomine ſignificamus vel ipſum Cordis Motum, dum in contractionem agitur, vel Motum ponderis cujuſlibet, quod Sanguini objectum ex Corde proruenti & velocitate idoneâ delatum in partes contrarias, Sanguinis effluxum, adeoque ipſam Cordis contractionem æquali vi librare valet & ſiſtere. Potentiam iſtam, cum à priori vix ſperandum ſit ut definire poſſimus, quod neque fabricam Cordis interiorem, neque cauſæ contrahentis naturam, aut vires ſatis habeamus exploratas, relinquitur, ut eandem per effecta, ſive à poſteriori, æſtimemus.

Cordis actio in Ventriculorum ſuorum contractione omnis conſiſtit. Ventriculi autem inter contrahendum in ſanguinem impingunt, eique Motus ſui partem communicando, eundem magna vi, qua datur porta, urgent, & expellunt. Sanguis hoc modo in Arterias, Aortam & Pulmonalem, protruſus, impetu in omnes partes facto, partim in tunicas Arteriarum ex Syſtole ſua prægreſſâ collapſas & flaccidas, partim in Sanguinem priorem tardius fluentem impingit. Unde gradatim extrorſum truduntur Arteriarum tunicæ, & Sanguis antecedens curſu celeratur. Quod ſi animo concipiantur Arteriæ ſectionibus tranſverſis minimis diſtinctæ, primâ Sanguinis portiunculâ ex Corde in primam ſectionem irruente, partim diſtenditur iſta ſectio, partim Sanguis eâdem antea contentus in ſectionem proximam detruditur, eamque diſtendit, atque iſta actio per ſuccedentes Arteriarum ſectiones continuatur. Deinde ſecunda, & tertia ſanguinis portiuncula, & cæteræ deinceps, in primam Arteriæ ſectionem incidunt, eamque paulo magis dilatant, & ſanguinem eâdem contentum in proximas ſectiones ſucceſſive propellunt; idque fieri pergit, donec omnis ſanguis ex Ventriculis fuerit

rit ejectus. Cæterum id utique observandum est Arterias, quo magis contractæ & flaccidæ fuerint, eo minus dilatationi obsistere, quanto autem magis fuerint dilatatæ, tanto fortius ulteriori distractioni reniti; atque idcirco Vim Sanguinis ex Corde prorumpentis primo magis impendi in distentionem Arteriarum, quam in Sanguinis præcedentis protrusionem, sub finem vero magis propelli Sanguinem antecedentem quam distendi Arterias, quippe quæ jam rigidæ factæ majorem dilatationem vix admittant.

Sanguis autem ex Corde profiliens, cum, uti dictum est, Motus sui partem Arteriarum tunicis, partem Sanguini præcedenti communicat, ipse necessario de pristinâ celeritate remittit; adeoque dum Ventriculorum contractionem moratur, novum ab iis impulsum excipit, ejusque partem, eâdem ratione atque antea, tunicis Arteriarum & præcedenti Sanguini impendit, unde iterum retardatur, & alium Ventriculorum ictum suscipit, & sic deinceps, donec omnis ex Ventriculis fuerit expulsus.

Præter causam supra expositam, superest alia, quâ Sanguis ex Corde effluens gradatim retardatur, adeoque novos successive impetus excipit ex Ventriculis sese contrahentibus. Nam Sanguis in Arteriam Aortam influens, etiamsi nulli omnino resistentiæ occurrere ponatur, adeoque nullam pati Motus sui imminutionem, tamen, cum ex lato in angustum fertur, longitudine perpetim crescit, donec totus in Aortam pervenerit; cumque sectio Aortæ non minuatur, necessario minuitur Sanguinis velocitas. Motus enim Sanguinis est in ratione compositâ, ex ratione Sectionis Aortæ, velocitate in eâdem, & longitudine Columnæ Sanguineæ, per Theorema nostrum III. * *de Motu Aquarum fluentium.* Cum vero ea Sanguinis portio, quæ jam pervenerit in Aortam, gradatim retardetur, retardabitur inde Sanguis iste qui adhuc Ventriculo continetur, & hinc retardabitur ipsius Ventriculi contractio. Unde Ventriculi perpetuò aliam atque aliam Motus sui partem Sanguini contiguo, his de causis perpetim retardato, communicabunt. Patet vero isthinc, ut id obiter notemus, alium esse Motum Sanguinis ex Corde erumpentis, alium ejusdem jam ex Corde expulsi, & intra Arterias fluentis. Item ictum sive impulsum Ventriculorum in Sanguinem impressum, qui alioqui unicus esset futurus, & puncto temporis transigeretur, tamen causarum dictarum vi, quibus Sanguis perpetim retardatur, per totam Cordis Systolen continuari.

* *v* *Part* I. *Ch.* V. *Tract.* VII. *supra.*

Ventriculum itaque alterutrum Cordis Sanguinem impellentem licebit spectare, ut datum corpus cum datâ celeritate impingens in aliud corpus quiescens, cui Motus sui parte communicatâ ambo corpora communi velocitate deferuntur. Æquatur autem Potentia ejusdem, vel Facto ex pondere Ventriculi & velocitate ejus initiali, priusquam in Sanguinem impingat; vel Summæ Motuum ipsius Ventriculi ac Sanguinis ex eodem profluentis, & Motûs qui tunicis Arteriarum & Sanguini præcedenti communicatus est; vel etiam, si abesse

ponatur omnis Arteriarum & Sanguinis præcedentis resistentia, Summæ Motuum ipsius Ventriculi & Sanguinis effluentis.

Theorema I. *Motus, quo Machina cava inæqualiter contractilis in contractionem agitur, æqualis est summa Factorum ex singulis Machinæ particulis ductis in velocitates respectivas.*

Patet ex Mechanicâ.

Corol. 1. Machinæ Motus minor est Facto ex pondere Machinæ ducto in velocitatem earum Machinæ partium, quæ omnium celerrime moventur inter contrahendum.

2. Motus Machinæ æquatur Facto ex pondere ejusdem, ducto in velocitatem aliquam mediam inter velocitates earum Machinæ partium quæ omnium celerrime, & earum quæ omnium tardissime, moventur.

3. Si Machinæ plures similes similiter sese contrahant, velocitate mediâ, vel æquabili vel inæquabili, similiter tamen auctâ vel imminutâ in omnibus Machinis; Motus, quo Machina quæque in contractionem agitur, rationem obtinet compositam ex ratione quadruplicatâ Diametri homologæ ipsius Machinæ, & ratione inversâ temporis, quo Machinæ contractio perficitur; vel rationem compositam ex ratione ponderis Machinæ, ratione ejusdem ponderis subtriplicatâ, & ratione temporis inversâ.

n. 359. *p.*929. *Theorema* II. *Si ex Machinâ cavâ inæqualiter contractili,* A B C D, *aqua per Machinæ contractionem exprimatur, Motus aquæ ex orificio* A *prosilientis æquatur Summæ Factorum ex Sectionibus quibusvis transversis omnium aquæ filamentorum* A B, A C, A D; *singulis ductis in longitudines & velocitates respectivas.*

Plate 1. *Fig.* 8. *Demonstratio.* Loco filamentorum aquæ, concipiatur Machina tubis minimis, inæqualiter amplis, A B, A C, A D, in orificium A desinentibus, tota constare.

Est aquæ Motus in quovis tubo æqualis sectioni cuivis ipsius tubi, ductæ in velocitatem aquæ per sectionem istam fluentis, & longitudinem tubi, per *Theorem.* 3. *De Motu Aquar. fluent.* Proinde Summa Motuum aquæ tubis in omnibus simul sumptis, sive Motus aquæ ex Machinæ orificio prorumpentis, æqualis est Summæ Factorum ex omnium tuborum sive filamentorum aquæ sectionibus, ductis in longitudines, & velocitates, respectivas. *Q. E. D.*

Corol. 1. Motus aquæ effluentis minor est Facto ex orificio A, velocitate aquæ exeuntis, & longitudine filamenti aquæ omnium longissimi. Est enim Factum ex orificio & velocitate aquæ effluentis æquale Summæ Factorum ex sectionibus filamentorum singulis ductis in velocitates respectivas; & Summa horum Factorum, ducta in longitudinem filamenti omnium longissimi, major est quam Summa eorundem ductorum cujusque in suam longitudinem.

2. Motus aquæ æquatur Facto ex orificio A & velocitate aquæ exeuntis, ducto in longitudinem aliquam mediam inter longitudines filamentorum longissimorum & brevissimorum: vel æquatur Facto ex quantitate

quantitate aquæ dato tempore effluentis, & longitudine mediâ prædictâ, applicato ad tempus illud datum.

3. Si Machinæ plures similes aquâ plenæ similiter contrahantur, sive æquabili velocitate mediâ, sive inæquabili, similiter tamen in omnibus Machinis auctâ, vel imminutâ; Motus, quo aqua ex Machinæ cujusque orificio prorumpit, rationem habet compositam ex ratione quadruplicatâ Diametri cujusvis homologæ ipsius Machinæ, & reciprocâ temporis ratione, quo peragitur Machinæ contractio: vel rationem compositam, ex ratione ponderis Machinæ, vel molis aquæ, sive Machinâ contentæ, sive ex eâdem expulsæ, ratione ejusdem ponderis, vel molis, subtriplicatâ, & ratione temporis reciprocâ.

Problema. *Invenire Potentiam Cordis.*

Sit p = Pondus Ventriculi sinistri, sive quantitas Sanguinis eidem ponderi æqualis.

S = Superficies interna ejusdem.

l = Longitudo media filamentorum Sanguinis ex eodem prodeuntium.

s = Sectio Aortæ.

q = Quantitas Sanguinis Ventriculo sinistro contenti.

t = Tempus, quo Sanguis ex Corde expelleretur, sublatâ Arteriarum & Sanguinis præcedentis resistentiâ.

v = Velocitas variabilis, quâ Sanguis ex Corde prosiliens per Aortam flueret, sublatâ resistentiâ.

x = Longitudo variabilis Aortæ à Sanguine ex Corde effluente percursa.

z = Tempus, quo longitudo x percurritur.

Inde velocitas media variabilis Sanguinis Ventriculo contigui, sive media velocitas ipsius Ventriculi $= \frac{sv}{S}$.

Motus Ventriculi (per *Theor.* 1. *Cor.* 2.) $= p \times \frac{sv}{S}$.

Motus Sanguinis effluentis (per *Theor.* 2. *Cor* 2.) $= sv \times \overline{l + x}$.

Horum Summa, sive Potentia Ventriculi $= sv \times \frac{p}{S} + l + x$. Est autem $v = \frac{x}{z}$. Unde per Methodum *Newtonianam* inversam, elicitur Potentia Ventriculi $= \frac{sx}{z} \times \overline{\frac{p}{S} + \frac{x}{2} + l}$. Sed cum $z = t$, erit $sx = q$.

Hinc Potentia Ventriculi $= \frac{q}{t} \times \overline{\frac{p}{S} \frac{q}{2s} + l}$.

Simili ratione invenitur Potentia dextri Ventriculi $= \frac{q}{t} \times \overline{\frac{\pi}{\Sigma} + \frac{q}{2\sigma} + \lambda}$.

Literis

Literis autem Græcis eadem ſignificantur in dextro Ventriculo, quæ Latinis in ſiniſtro.

Hinc tota Cordis Potentia

$$= \frac{q}{t} \times \frac{p}{s} + \frac{\pi}{\Sigma} + \frac{q}{2S} + \frac{q}{2\sigma} + l + \lambda. \quad Q. E. I.$$

Si ponatur

$p = 8$ unc. Avoird $= 13.128$ unc. cub.
$\pi = 4$ $= 6.564$
$S = 10$ unc. quadrat.
$\Sigma = 10$
$l = 2$ unc.
$\lambda = 1\frac{1}{2}$.
$q = 2$ unc. Avoird. $= 3.282$ unc. cub.
$s = 0.4185$ unc. quadrat. } Ex *Keillianis* Experi-
$\sigma = 0.583$ } mentis.
$t = 0.1''$

Erit Potentia Ventriculorum æqualis motui ponderum ſubſcriptorum, nempe,

	lib.	unc.
Ventriculi ſiniſtri	9	1
Ventriculi dextri	6	3
Cordis totius	15	4

Quorum ponderum ea eſt velocitas, quâ percurratur longitudo uncialis ſingulis minutis ſecundis.

Cor. 1. Quoties Pulſus fit celerior; aut minuitur reſiſtentia, aut Potentia Cordis augetur, aut minor ſolito Sanguinis copia ſingulis contractionibus ex Corde expellitur.

2. Si Pulſus ſolito tardior fiat; neceſſe eſt, vel augeatur reſiſtentia, vel Cordis Potentia minuatur, vel major Sanguinis moles ex Corde ejiciatur.

3. Auctâ reſiſtentiâ, neceſſario vel Pulſus retardabitur, vel augebitur Cordis Potentia vel Sanguinis quantitas ſolito minor ex Corde exprimetur.

4. Imminutâ reſiſtentiâ, vel Pulſus acceleratur, vel major Sanguinis copia quâque Syſtole ejicitur, vel Cordis vires minuuntur.

5. Auctis Cordis viribus, neceſſario vel augebitur reſiſtentia, vel Pulſus accelerabitur, vel plus Sanguinis ex corde ejicietur.

6. Viribus Cordis imminutis, vel minuatur neceſſe eſt reſiſtentia, vel Pulſus tardior fiat, vel minus Sanguinis ex Corde exprimatur.

7. Cum minor Sanguinis moles ex Corde projicitur; vel acceleratur Pulſus vel Cordis vires minuuntur, vel augetur reſiſtentia.

8. Cum plus Sanguinis ex Corde exprimitur; vel Pulſus tardior fiet, vel augebitur Cordis Potentia, vel reſiſtentia minuetur.

Schol. 1. Ventriculorum ſuperficies internas, cum factu difficillimum videatur, ut accurate determinentur, aut etiam ratio habeatur imminutionis, quam inter contrahendum patiuntur, contenti fuimus præterpropter

propter æſtimare: cum ſive eaſdem 12, ſive 8 unciis quadratis ſingulas æquales ſtatueris, perparva reperiatur Potentiarum facta mutatio. Quod etiam obſervari poterit de longitudine mediâ filamentorum Sanguinis. Præterea differentias, quâ Arteriæ ambæ, earumque rami proximi à Corde progredientes, ſectione augentur, ut æſtimatu perdifficiles & pene inſenſibiles, negligimus. Alioqui eſſet Cordis Potentia tantillo minor ſtatuenda, quam quæ ſupra definita eſt.

2. Determinavit vir celeberrimus, *Jacobus Keillius*, velocitatem Sanguinis, reſiſtentiâ ſubmotâ, ex Corde effluentis, eam circiter, quâ percurrantur pedes, $6\frac{1}{2}$ ſingulis minutis ſecundis. Ponit vero ille celeritatem Sanguinis per totam Syſtolem æquabilem, quam nos inſigniter inæqualem fieri, & perpetim à Syſtoles initio retardari ſupra oſtendimus. Hanc ſi cui definire libuerit, ſubſtituenda eſt, in quartâ Æquatione ſupra poſitâ, Potentia Ventriculi proxime inventa, & ipſi x valor quivis tribuendus, ut eliciatur v, ſive velocitas eidem reſpondens. Ita, cum initio Syſtoles ſit $x = 0$, ſub finem vero $x = \frac{q}{s}$, determinatur inde ea Sanguinis velocitas initio Syſtoles, quâ pedes $14\frac{1}{4}$; in fine autem quâ $4\frac{1}{4}$, minuti ſecundi ſpatio percurrantur. Pariter in dextro Ventriculo, velocitas Sanguinis initialis pedes circiter $10\frac{5}{6}$, ultima vero 3 pedes eodem temporis ſpatio conficiet.

Adhibuimus hactenus eam Hypotheſin, quâ Muſculi Cordis Ventriculos conſtituentes Motum omnem, quo adiguntur in contractionem, Momento temporis concipiunt. Quod ſi ponamus Motum iis communicari non unico quidem Momento, ſed tantillo tamen temporis ſpatio, quod cum totâ Syſtoles duratione comparatum rationem obtineat admodum exiguam; erit Cordis Potentia paululo major ſtatuenda, quam quæ ſupra determinata eſt. Si vero ſtatuatur iſte Motus, procedente Syſtole, in ratione temporis augeri; erit totus Motus in fine Syſtoles acquiſitus duplo major quam ſupra poſuimus, ubi nulla reſiſtentia Sanguini ex Corde profluenti objicitur: Ubi autem ſolita adeſt reſiſtentia, erit idem quintuplo major; quod inſtituto calculo facile patebit. Pari ratione poterit calculus noſter ad aliam quamlibet Hypotheſin, quâ Ventriculorum Motus in duplicatâ vel ſuperiore quâvis ratione temporis augeatur, accommodari. Potentia vero in fine acquiſita ſuprapoſitâ elicietur longe major, nempe ex ratione duplicatâ Potentia tripla, ex triplicatâ quadrupla, ex quadruplicatâ quintupla, & ſic in infinitum.

Nobis autem videtur ſecunda Hypotheſis, quâ Ventriculi parvo admodum temporis ſpatio Motum omnem concipiunt, cæteris longe veriſimilior. Quum neceſſe ſit, ut aliquid temporis impendatur ad Motum quemlibet generandum; neque videatur adeo tarde increſcere Ventriculorum Motus, ut non celerius augeatur, quam ſecundum temporis rationem. Motus enim Muſculorum impetu ſolo Fluidorum quorumcumque, quæ ex Sanguine proveniunt, perfici nequit; quum Brachio alterutro Motum exercere poſſimus Motu Sanguinis per vaſa Cor-

poris

poris universa profluentis longe majorem. Relinquitur ergo, ut Musculorum fibræ Ventriculos Cordis constituentium, rarescentiâ quâdam liquorum in easdem influentium, in Motum impellantur. Hæc autem, quoties vim magnam concipit, plerumque subita est, & fere instantanea. Adde quod Ventriculorum Motus secundum hanc Hypothesin longe minor efficitur, quam in tertiâ. Non solet autem sapientissimus Artifex, Rerum Conditor, in operibus suis plus virium adhibere, quam quantum sufficit ad finem propositum consequendum.

Cæterum sive admittatur ista Hypothesis, sive alia quæcunque ex supra dictis verior censeatur, poterunt omnia Corollaria nostra eodem jure ex Problemate deduci. Quæ utrum aliquid adjumenti afferant ad Morborum Historiam explicandam Medico sagaci considerandum permittimus. Facile autem ex morbi cujusque Naturâ sciri poterit, utrum aucta sit vel imminuta resistentia. Augeri vero credibile est vel imminui Cordis vires auctis vel imminutis Musculorum reliquorum viribus, quamvis aliter statuisse video virum celeberrimum, *Laurentium Bellinum.*

Theorema III. *Totus Motus resistentiæ, quæ Sanguini ex Corde erumpenti durante Systole objicitur, sive totus Motus, qui Sanguini præcedenti & Arteriarum tunicis communicatur, toti Cordis Potentiæ quamproxime æqualis est.*

Dem. Peractâ Cordis Systole, quæ pars Aortæ & Arteriæ Pulmonalis Cordi proxima est, perstat plena Sanguine per totam Systolem Arteriarum. Nec enim patitur earum fabrica & nexus, quo Cordi conjunctæ sunt, ut tunicis in sese penitus collabentibus totæ occludantur, neque potest earum cavum Sanguine vacare. Alioqui enim, contrahentibus sese reliquis Arteriarum partibus, Sanguis iisdem contentus retro in vacuum impelleretur, motu & inutili & motui Sanguinis naturali contrario. Tum etiam Valvulæ Semilunares non tenderentur versus Ventriculos, adeoque Sanguis ex Auriculis in Ventriculos expressus, etiam in Diastole Cordis, in Arterias protruderetur.

Hinc patet Sanguinem proxime ex Corde expulsum Systole peractâ immotum in Arteriis persistere, adeoque tum omnem Ventriculorum Motum excepisse, tum eundem totum partim Sanguini antecedenti, partim tunicis Arteriarum communicasse. *Q. E. D.*

Theorema IV. *Motus, qui in Systole Cordis communicatur Sanguini præcedenti, est ad Motum tunicis Arteriarum communicatum, ut tempus Systoles Cordis ad tempus Diastoles quam proxime.*

Dem. Quum sanguis per vasa Corporis universa, si partes Arteriarum Cordi propiores exceperis, æquabili cursu deferatur; necesse est, ut tum Motus affrictu Sanguinis ad vasorum latera deperditus, tum Motus Sanguini redditus à Systole sive Cordis sive Arteriarum, æqualibus temporibus æqualis sit. Qui autem Motus à Systole Arteriarum Sanguini communicatur, idem est præcise, qui prius à Cordis Systole Arteriarum tunicis fuerat impressus, cum Arteriæ eodem impetu quo distractæ fuerint etiam restituantur. Et Systole Arteriarum cum Cordis Diastole duratione convenit. Unde patet Propositum. *Q. E. D.*

Cor.

Cor. Si ponamus cum viro doctiſſimo *Jacobo Keillio*, Syſtolen Cordis peragi tertiâ parte temporis inter Pulſus binos intercepti; erit Motus Sanguini præcedenti communicatus totius Potentiæ Cordis pars tertia: Motus vero Arteriis communicatus prioris duplus, ſive duæ partes tertiæ totius Cordis Potentiæ.

Theorema V. *In diverſis Animalibus Potentia Cordis rationem obtinet compoſitam ex ratione quadruplicatâ Diametri cujuſvis homologæ Animalis, & ratione inversâ temporis, quo Cor contrahitur: vel rationem compoſitam, ex ratione ponderis vel ipſius Cordis vel integri Animalis, ratione ponderis ejuſdem ſubtriplicatâ, & ratione temporis reciprocâ.*

Facile demonſtratur vel ex *Corol.* 3. *Theor.* 1 & 2. vel ex Potentiâ Cordis Problemate præcedente definitâ.

Cor. 1. Si ponatur Cordis Potentiam rationem obtinere ponderis vel ipſius Cordis, vel integri Animalis, vel Sanguinis copiæ in toto Animali; erit Animalis longitudo in ratione temporis, quo Cordis Syſtole perficitur, ſive in ratione inversâ frequentiæ Pulſuum.

2. Si ratio longitudinis integri Animalis major fuerit ratione inversâ frequentiæ Pulſuum, neceſſe eſt major ſit ratio Potentiæ Cordis ratione ponderis ejuſdem.

Schol. Quum conſtet Experimentis Puerorum Pulſus non eſſe tanto frequentiores Pulſibus virorum quanto pueri virorum longitudine ſuperantur, concludendum eſt, vi ſecundi Corollarii, Potentiam Cordis Virilis majorem obtinere rationem ad Potentiam Cordis Pueri, quam eſt ratio ponderum. Et par eſt ratio in cæteris Muſculis. Nam ſi Corporis robur rationem ponderis ſequeretur, poſſent Pueri æqualia itinerum ſpatia eodem tempore cum viris conficere.

Simili ratione ac Motum ſanguinis ex Ventriculis Cordis erumpentis ope *Theorem.* 2. determinavimus, poterit quoque Urinæ Motus ex Urethrâ profluentis determinari. Nempe ſi ponatur Urethræ Veſicæ longitudo 12 unciis æqualis, & binæ unciæ Urinæ minuti ſecundi ſpatio emittantur, erit Motus Urinæ effluentis æqualis Motui ponderi libræ $1\frac{1}{2}$, quod uncialem longitudinem ſingulis minutis ſecundis percurrat. Quoniam vero Urina non ſolis Veſicæ Urinariæ viribus contractivis, ſed etiam Diaphragmatis & Muſculorum abdominalium ope in ſubſidium vocatâ, expellitur, nequit Veſicæ Potentia ex Motu Urinæ profluentis æſtimari.

2. Epiſtolam *D. Jurin*, in Actis Philoſophicis nuper publicatam, legi; in quâ, ea quæ à me traduntur de viribus Cordis, infirmare conatur vir Doctiſſimus. Cum æſtimatio virium, quibus Cor Sanguinem expellit, à *Borellio* facta, fere omnibus valde incredibilis videbatur, non me temerarium, non *Borelli* nomini injurioſum, non orbi literato ingratum facturum exiſtimavi, ſi ad verum propius accedere tentarem. In quo Tentamine, non accuratam virium Cordis definitionem mihi propoſitum erat dare, ſed potius methodum, quâ hæ vires forte inveniri poſſent, indicare; & Geometriæ peritiores ad Problematis valde deſiderati inveſtigationem incitare.

Remarks by Dr. James Keill, *n.* 361. *p.* 995.

Præcipuum quod *Borellio*, D. *Moreland*, mihi objicit vitium eſt, quod in Potentiâ Cordis æſtimandâ, quam rationem ad pondus iners, vel corporis gravitatem obtineat, determinare ſuſcepimus. " Sed Cor, " inquit, cum & ipſum inter contrahendum movetur, & corpora op- " poſita, Sanguinem nempe & arteriarum tunicas, in motum impellit, " patet ejus *Potentiam* non aliâ ratione ſciri poſſe quanta ſit, quam ut " motus hujus quantitatem cognitam teneamus. *Motus* autem quilibet " cum pondere quieſcente comparari non magis poteſt, quàm linea " cum rectangulo." At à nemine certe noſtrum, quod ſcio, eſt *Motus* Cordis cum pondere quieſcente comparatus. Potentiam autem Cordis, ſeu vim Cordis motricem & Sanguinem impellentem cum pondere conferre, quid prohibet non video. Quamquam enim inter *Pondus* & *Motum* corporis ſolidi nulla ſit relatio, vis tamen motrix, ſi in fluidum agit, ad vim gravitatis quandam certe rationem habet. Et revera vis corporis motrix, certam in fluido motûs quantitatem in dato tempore efficiens, æqualis eſt ponderi, quod vi gravitatis cadendo, in eodem tempore, eandem motûs quantitatem ſibi acquirit. Hinc vis, quâ ex orificio aliquo aqua exprimitur, certo ponderi æqualis eſſe dicitur: quia pondus datum, & vis aquam exprimens æquales motus in temporibus æqualibus generant. Hic genuinus Corollarii *Newtoniani* ſenſus mihi videtur eſſe, nec ab hoc ſenſu diſcrepant, quæ de Cordis viribus explicui. Verba *Newtoni* ſunt, *Vis, qua totus aquæ exilientis motus generari poteſt, æqualis eſt ponderi, &c.* quæ non ſatis attendiſſe videtur *Jurinius*, cum dicit *Pondus autem illud quo motus aquæ ex vaſe effluentis generari poteſt*, &c. Sed ſi hâc re à nobis peccatum eſt, cum ſummis certe hujus ſæculi Geometris *Hugenio* & *Newtono* peccavimus, quorum uterque vim fluidorum per vim gravitatis exponit. Nec in *Corol.* prædicto id ſolummodo facit *Newtonus*, ſed in aliis etiam locis oſtendit Methodum, quâ ratio reſiſtentiæ Medii, id eſt, actionis fluidi in corpus ſolidum, ad vim Gravitatis vel centripetam inveniri poteſt, ut videri licet in *Prop. 4ta* & *5ta* Libri ſecundi, eorumque Corollariis. Alia profecto eſt actio fluidorum in ſolidum, & alia ſolidorum in ſe invicem. Fluidum, datâ velocitate motum, datum pondus ſuſtinere poteſt, cum fluidi partes ſibi mutuo continuò ſuccedentes in pondus impingunt, adeoque vis fluidi eſt revera ponderi æqualis; ſed cum Solidorum non par eſt ratio, eorum Vis cum Gravitate comparari nequit.

Me inſuper reprehendit Vir ingenioſiſſimus, quòd velocitatem Sanguinis è corde detruſi, per totam ſyſtolem æqualem poſui, quam ille valde inæqualem eſſe demonſtravit. Verum à me nuſquam Sanguini æqualis data eſt velocitas, ſed pro ſummâ omnium velocitatum mediam poſui. Sed utrum æqualis vel inæqualis eſt Sanguinis è Corde ejecti celeritas, nondum ſatis mihi conſtat; certe quæ pro æquali velocitate ſtat ratio, ea mihi in præſens firmior videtur.

Quod in alterâ methodo quæ ſubjuncta eſt, illi diſplicet, eſt certe aſſumptio illa quæ à *Borellio*, aliiſque viris doctis ſæpius uſurpata eſt, nempe, quod ſimilium muſculorum vires ſunt in ratione ponderum.

Aliam

Aliam virium rationem in Theoremate *5to* ſtabilire conatur *Jurinius*: ſed cum ex communi omnium Theorematum Principio oritur demonſtratio, communi etiam eorum fato involvetur: Si enim principium illud fallax eſt, (ut mihi videtur) nec ad caſus ad quos adhibetur, congruit; corruunt certe omnia, quæ hâc baſi innituntur. Supponit Vir Clar. Vaſorum tunicas in Sanguinem intus contentum impetu irruere, & motûs ſui partem Sanguini ictu communicare: & hîc in motu Cordis, vult Ventriculum tanquam ſolidum Corpus, datâ velocitate motum, in Sanguinem impingere, & ictu motûs ſui partem illi impertire: quæ ſuppoſitio nec Sanguinis nec Cordis, nec Aeris è Pulmone expreſſi motui competit, nec, ullâ minimorum ictuum reiteratione, horum motibus ita accommodari poteſt, quin quæ inde deducentur concluſiones pro incertis & omnino falſis haberi debeant.

Cum inter Sanguinem & Cordis intimum nullum intercedit ſpatium, ſed eſt alter alteri contiguus, non ictu hoc in illum, ſed preſſu agit: nec ullam in initio ſuæ contractionis celeritatem ventriculi habent, ſed ſe contrahendo velocitatem tempore acquirunt, tanquam gravia cadendo, vel ut fluida rareſcendo, ex quo forte omnis vis Cordis oritur. Adeoque non æquabilis eſt motus contractionis, ut vult Vir Doctiſſimus, ſed eſt motus inſtar cadentis acceleratus. Idem igitur eſt diſcrimen inter ictum quo Cor Sanguinem ferire vult *Jurinius*, & preſſuram quâ Cor revera in Sanguinem agit, quod eſt inter actionem corporis ſolidi moti & vim gravitatis: ſed ipſo fatente hæc comparari nequeunt, adeoque preſſura ſeu actio cordis in Sanguinem per ictum nec à Viro laudato expoſita eſt, nec unquam exponi poteſt. Hanc ſententiam confirmat ipſa Cordis potentia a Viro Cl. inventa. Si enim pondus datâ velocitate motum cordis potentiæ æquale eſſet, tunc Sanguis omni vi Cordis in pondus illud directe impulſus motum ponderis temporis momento deſtrueret: ſed quocunque magno impetu ponderi occurrat Sanguis, nunquam illi omnem motum in inſtanti eripiet, adeoque eſt hoc pondere potentia Cordis minor, nec recte per motum ponderis vires Cordis exponuntur.

Fluidorum vires in corpora ſolida, ubique eodem prorſus modo quo ſolidorum vires in ſe invicem, *Jurinius* æſtimat & perpendit, cum tamen maxima interſit differentia; & ab hoc capite fluit quicquid eſt in illius Propoſitionibus erroris. Ubi enim corpus ſolidum, cujus partes firmiter inter ſe cohærent, in aliud impingit, unaquæque corporis particula ſimul & ſemel ſuam alteri vim impertit: at res aliter ſe habet in fluidis, in queis nulla eſt partium cohærentia, nulla fluidi pars, niſi in ipſo tactu, in corpus ſibi oppoſitum agit: idcirco cum columna aquæ adverſus corpus ſolidum ſurſum vertitur, partes columnæ à corpore remotiores nullam illi vim imprimunt. Corpus etiam ſolidum unicum ſolummodo ictum alteri communicat: at columna fluidi in corpus ſibi oppoſitum continue agit, & minima columnæ pars minimo temporis momento, ictum infinite parvum illi imprimit, eodem prorſus modo quo gravia cadendo agunt, quibus igitur fluidorum motus recte comparatur.

paratur. Porro omnis motus corporis solidi in alterum directe impingentis in temporis momento destrui potest: sed motus solidi vim fluido imprimentis, non nisi gradatim imminuitur, & in dato tempore evanescit, pari ratione, quâ Gravitas in corpus sursum missum vim suam exerit. Ex quibus satis abunde constat, inter vim fluidi in motum acti, & vim gravitatis magnam esse affinitatem, & unam per alteram recte exponi posse; vim autem corporis solidi ad vim gravitatis referri non posse. Cumque hanc differentiam non satis attendisse videtur Doctissimus *Jurinius*, à vero multum aberrasse mihi videtur. Si igitur sepositâ suâ de Vasorum ictu hypothesi, & vi pressuræ, quâ Natura utitur, pro Principio adhibitâ, alia Theoremata de Cordis & Sanguinis motu & viribus, elegante suâ demonstrationis methodo, construere dignabitur, sese dignum, mihi certe gratum, nec eruditis inutile præstiterit.

Reply'd to, by Dr. Jurin. *n.* 362. *p.* 1039.

3. Queritur primo Vir Clarissimus, quod sese una cum Doctissimis Viris *Borello*, & *Morlando*, tanquam Cordis Motum cum pondere inerti conferentem, injuste perstrinxerim. Ego certe, cum prius notassem Motum quendam Sanguinis & Arteriarum ex Cordis Vi oriri, dixi tandem sciri non posse Cordis Potentiam quanta sit, nisi Motus hujusce quantitatem cognitam teneamus: Motum vero quemlibet cum pondere quiescente comparari non magis posse, quam Lineam cum Rectangulo. Quibus verbis id significare volui, Doctissimos Viros non quidem diserte Motum Cordis cum pondere quiescente comparare, sed ipsos, cum Cordis Potentiam per pondus exponerent, nullam ostendisse rationem, quâ Motûs quantitas ex Cordis Potentiâ oriundi posset æstimari. Ex hâc Objectione, si recte assequor mentem Viri Clarissimi, ita sese expedire conatur. Cordis Potentia in pressione consistit, eamque æquabiliter in Sanguinem impendit, eodem prorsus modo, quo Gravitatis vis deorsum pondus impellit, & actione perpetuâ in motum accelerat. Proinde, cum Cordis Potentia ponderi per Corollarium *Newtonianum* definito æqualis est, ea Motum eundem durante Systole in Sanguinem imprimet, quem pondus istud eodem tempore cadendo per Gravitatis vim comparabit. Ita vero cum mentem suam exponit Vir Cl. sublatam iri penitus Objectionem istam nostram confitemur; si nimirum Cordis Potentia prædicto ponderi æqualis sit, eademq; consistat in æquabili pressione per totam Systolen continuatâ. Atqui ex duabus istis Propositionibus posteriorem neutiquam probare conatur vir doctissimus, sed Hypotheseos loco ponit; quamvis nos rationibus quibusdam adductis contrariam Sententiam conati sumus verisimiliorem reddere; nempe, quod Cordis Potentia nequaquam æquabiliter agat in Sanguinem per totam Systolen, sed cum totas vires exiguâ temporis particulâ collegerit, inde uno impetu in Sanguinem irruat, eumque ex Ventriculis expellat, eo modo quem in Dissertatione nostrâ Epistolari fusius exposuimus. Priorem vero Propositionem, etiam concessâ Viro Cl. istâ Hypothesi, falsam esse mox demonstrabimus.

Corollarii

Corollarii *Newtoniani* ſenſum quod attinet, nolumus Lectori moleſtiam nimiam faceſſere, cum neque putemus ejus intereſſe uter *Newtoni* mentem rectius acceperit; neque ita perſpicue ſententiam ſuam expoſuerit Doctiſſimus Adverſarius, quin periculum ſit, ne aliquem ei ſenſum affingamus, quem ipſe forſitan, ſi poſſet adhuc ſe defendere, foret repudiaturus. Id vero adnotaſſe operæ pretium erit, quod cum loquatur *Keillius* de Vi quâ ex Orificio aliquo aqua exprimitur, *Newtonus* nullum omnino verbum in iſto Corollario poſuerit, quo Aqua per vim aliquam exprimi ſignificetur; ſed pondus ſolum determinaverit æquale iſti vi, quâ totus aquæ effluentis Motus generari poteſt, ſive quod Gravitatis Vi cadendo Motum comparare poteſt Motui aquæ eodem tempore effluentis æqualem.

Quod autem Corollarium illud, ſi non male intellexerit Vir Cl. certe non ſatis apte uſurparit, facile perſpiciet Lector Eruditus, qui animum adverterit, quid interſit diſcriminis inter effluxum aquæ ex foramine in fundo vaſis ſemper pleni quomodo à *Newtono* conſideratur in eo Corollario, & effluxum Sanguinis ex Corde in Aortam. In caſu enim priori aqua jam totam velocitatem comparavit, & per datum temporis ſpatium æquabiliter effluit ex foramine. At Cordis Vis per Hypotheſin *Keillianam* applicatur Sanguini in Ventriculo quieſcenti, & eum primo temporis momento velocitate infinite parvâ verſus Aortam propellit; continuatâ vero æquabili preſſione tandem ei finitam velocitatem imprimit, eamque perpetim auget, donec omnem Sanguinem ex Ventriculo expulerit.

Rurſum in caſu *Newtoniano* conſideratur Motus non quidem totius aquæ Cataractâ contentæ, quæ omnis in motu conſtituta eſt, & diverſa velocitate verſus exitum tendit, ſed aquæ ſolum in ipſo foramine poſitæ & jam exilientis. Vis autem Cordis toti Sanguinis moli Ventriculo contentæ Motum imprimit, totamque Aortam verſus propellit.

Denique negamus pondus quinque unciarum, à Viro Cl. determinatum, poſſe eam Motus quantitatem durante Cordis Syſtole per Gravitatis Vim comparare, quam Cordis Potentia producit, conceſſâ etiam ei Hypotheſi iſtâ, quod Cordis Potentia in æquabili preſſione conſiſtat. Per hanc enim Hypotheſin erit Motus à Ventriculi ſiniſtri Potentiâ productus, ex Calculo noſtro * [*Tranſ.* Numb. 359. p. 932, 934.] æqualis Motui Ponderis Octodecim librarum circiter, quod ſingulis minutis ſecundis longitudinem uncialem percurrat. Motus autem, quem pondus quinque unciarum durante Cordis Syſtole, ſi tollatur omnis Arteriarum & Sanguinis præcedentis reſiſtentia, ſive decimâ parte minuti ſecundi, per Gravitatis vim comparabit, æquabitur fere Motui Ponderis duodecim librarum, quod ſuprapoſitâ velocitate moveatur. Quod ſi cui libuerit adſumptâ hâc Hypotheſi verum pondus definire, quod Cordis Potentiæ æquale eſt, is poſito Calculo eliciet pondus unciarum circiter ſeptem cum ſemiſſe. Hoc enim durante Syſtole Cordis eundem

*v. ſupr. p. 61, 62.

fere

fere Motum cadendo comparabit, quem producit ipſa Cordis Potentia.

Sed inquiet forſitan aliquis diſcrimen modo expoſitum inter Motum à *Keilliano* pondere acquiſitum, & Motum ex Potentia Cordis oriundum inde proficiſci potuiſſe, quod forte minus accuratæ fuerint poſitiones illæ, quibus Characteres Algebraicos in Calculo noſtro ad numeros revocavimus. Cui dubio ut occurramus, & oſtendamus ſimul nos longe majus diſcrimen inventuros fuiſſe, niſi contigiſſet ut poſitiones iſtæ *Keillio* faverent; operæ pretium erit caſum aliquem ſimpliciorem adſumere, quo data moles aquæ, per datum orificium, dato tempore, per vim aliquam ſive preſſionem æquabilem exprimatur, quæ ſunt conditiones ab Adverſario poſitæ ad Potentiam Cordis definiendam. In eo autem caſu demonſtrabimus neque Motum aquæ effluentis, neque Motum toti tandem moli aquæ per vim illam impreſſum, Motui aquæ in Corollario *Newtoniano*, neque Vim eam ſive preſſionem, ponderi per iſtud Corollarium definito, æquari. Quod ſi præſtare licuerit, corruat funditus neceſſe eſt tota demonſtratio *Keilliana.*

Adſumemus igitur Cylindrum aquæ datum, tubo Cylindrico infinitæ longitudinis contentum; eritque pro orificio iſta ſectio tubi ad quam pertingit utralibet aquæ ſuperficies, alteri autem ſuperficiei Vis applicabitur ope Emboli eâdem Diametro cum ipſo tubo. Perfluat jam dato tempore data quævis aquæ quantitas per dictam ſectionem tubi; tum alia quantitas æqualis per foramen pari Diametro factum in fundo vaſis, quod more *Newtoniano* uſque plenum conſervatur: & primo loco diſpiciamus, utrum pares futuri ſint in utroque caſu Motus aquæ effluentis.

Exponatur tempus effluxus aquæ per rectam *A C*, velocitas autem æquabilis, qua aqua effluit ex foramine in fundo vaſis per rectam *A B*. Unde moles aquæ effluentis ex foramine, cum ſit in ratione temporis & velocitatis conjunctim, exponetur per Rectangulum *A B C D*; & Motus ejuſdem exponetur per ſolidum Parallelepipedon, ex eodem Rectangulo ducto in altitudinem *A B*, quippe qui ſit in ratione compoſita ex rationibus molis & velocitatis.

In caſu altero, ubi aqua per tubum Cylindricum fluit, tempus, ut prius, exponetur per eandem rectam *A C*; velocitas autem aquæ erit in ratione temporis, quippe cum vis adhibita, ex Hypotheſi, in datam aquæ molem æquabiliter agat, & proinde repræſentabitur per rectam
Plate I. Fig. 9. mutabilem *F G*, rectæ *A F*, ſive tempori ab initio effluxus, proportionalem. Molecula autem aquæ, particulâ temporis *F H* prædictam Sectionem præterfluens, exponetur per Rectangulum ex ipsâ *F H* ductâ in exponentem velocitatis *FG*; vel ſi evaneſcere intelligatur rectula *F H* per Trapezium *F G I H*, & moles aquæ toto tempore *A C* præterfluens ſignificabitur per Triangulum Rectangulum *A C E*. Et quoniam ex Hypotheſi moles iſta moli aquæ in caſu priore effluenti æqualis eſt, erit Triangulum *A C E* æquale Rectangulo *A B D C*; unde *C E*, ſive velocitas acquiſita in fine temporis *A C*, dupla erit velocitatis *C D* ſive *A B*, quâ aqua ex

foramine

foramine in fundo vasis effluebat. Motus autem aquæ particulâ temporis *F H* præterlabentis, cum sit in ratione molis & velocitatis conjunctim, exponetur per Prisma evanescens, quod fit ex Trapezio *FG IH* ducto in velocitatem *FG*: Unde totus Motus aquæ toto tempore *A C* præterfluentis exponetur per Pyramidem, cujus basis est Quadratum rectæ *C E*, cujusque altitudo perpendicularis est ipsa *A C*. Quæ Pyramis cum sit ad Parallelepipedon casu priore definitum, ut 4 ad 3, erunt quoque Motus aquæ effluentis in utroque casu in eâdem ratione, & proinde inæquales, quod primo loco demonstrandum susceperamus.

Proximum est, ut ostendamus Motum tandem impressum toti aquæ tubo contentæ non esse æqualem Motui in exemplo primo determinato. Hic autem, cum tota ista moles aquæ per positiones supra scriptas neutiquam definita sit, adsumemus eam æqualem moli expositæ per Rectangulum *A B C D*, quæ in casu primo effluit ex foramine, quæque in secundo sectionem dictam præterfluit. Unde cum totus Motus ei tandem impressus sit, in ratione molis & velocitatis in fine acquisitæ, idem exponetur per Parallelepipedon ex Rectangulo *A B D C* ducto in rectam *C E*. Hoc autem est ad Parallelepipedon, primo casu definitum, ex eodem Rectangulo & rectâ *C D*, ut altitudo *C E* ad altitudinem *C D*, sive in ratione duplâ. Porro, cum molem aquæ tubo contentæ per quodvis aliud Rectangulum, loco Rectanguli *A B C D*, exponere licuisset, patet inde Motum hunc posse quamlibet rationem ad Motum primo casu definitum obtinere, & idcirco nequaquam eidem esse æqualem. Quod erat secundo loco demonstrandum.

Superest, ut ostendamus Vim in hoc casu adhibitam ponderi per Corollarium *Newtonianum* definito non esse æqualem. Hæc autem Vis & vis Gravitatis agens in istud pondus, cum ambæ sint æquabiles, erunt in ratione Motuum ex iisdem dato tempore productorum. Quos cum inæquales esse modo demonstratum sit, erint illæ vires itidem inæquales. Quod erat demonstrandum postremo.

Pergit Vir Cl. ad alterum illud vitium, quod ego in ejus solutione reprehenderam, nempe quod velocitatem Sanguinis ex Corde effluentis æquabilem posuerit, quam insigniter inæqualem fieri à me demonstratum est. Negat autem se æquabilem velocitatem Sanguini tribuisse, sed pro summâ diversarum omnium velocitatum velocitatem mediam usurpasse. Præterea nondum satis sibi constare dicit, utrum æqualis vel inæqualis sit Sanguinis ejecti velocitas, sed quæ pro æquali velocitate stat ratio, eam sibi firmiorem videri. Utrum vero, qui velocitatem Sanguinis inventurus molem Sanguinis expulsi ad orificium Aortæ applicat, nullâ factâ mentione neque diversarum velocitatum, neque velocitatis mediæ, velocitatem Sanguinis æquabilem ponat, penes æquum Lectorem sit Judicium. Idem quoque facile æstimabit, utrum Vis aliqua sive pressio fluido in vase quiescenti applicata, quæ est Hypothesis Viri Doctissimi, id fluidum primo temporis momento, eâdem velocitate quâ in fine, propulsura sit.

Postquam ita satisfactum putat Vir Cl. iis Objectionibus, quas contra priorem suam Methodum attuleram, jam ad alteram illam faciliorem vindican-

vindicandam accedit. In hac Ego animadverteram Virum Cl. adſumere iſtam Propoſitionem, quod Vires Cordis in diverſis Animalibus ſint in ratione ponderum, item ponere velocitatem Sanguinis ex ſectâ Iliacâ Arteriâ profluentis æqualem ei, quâ Sanguis ex Corde in Aortam emittitur: quas ambas poſitiones falſas eſſe nobis demonſtratum eſt. Vitium poſterius non defendit Vir Cl. prius vero tuetur *Borelli* & aliorum Doctorum Virorum auctoritate, qui aſſumptionem iſtam ſæpius uſurparunt. Ita quidem, & nos ejuſmodi aſſumptionem in *Borello* reprehendimus, neque valet cujuſquam auctoritas contra legitimam demonſtrationem. Supereſt ergo Viro Cl. ad examen revocanda noſtra demonſtratio. Hanc autem fallaci quodam Principio inniti putat; quo cum omnia Theoremata noſtra ſuperſtructa ſint, communi ruinâ omnia involvit. Ait enim me ponere, quod Ventriculi Cordis, tanquam ſolidum corpus datâ velocitate motum, in Sanguinem impingunt, eoque ictu Motus ſui partem eidem communicant. Quam Hypotheſin Motui neque Sanguinis, neque Cordis, neque Aeris ex Pulmone expreſſi, competere cenſet Vir Clariſſimus.

Quod Pulmonem attinet, quoniam hoc obiter attingere voluit Vir D. agnoſco me conſideraſſe Pulmonem inter contrahendum tanquam datâ velocitate impingentem in Aerem contentum; idque conſulto feciſſe profiteor. Quum enim tum *Bellinus*, tum alii multi Viri Doctiſſimi, quos inter eminèt Cl. Adverſarius, multa protulerint de Vi illâ, quâ Aer inter expirandum in Sanguinem Pulmones præterfluentem agit, ejuſque moleculas diſſolvit; quam ſolutionem ipſo expirationis initio cenſent accidere; mihi propoſitum erat hanc ipſorum ſententiam ad trutinam revocare. Videbam autem, quod, ſi aerem per Vim æquabilem ſive preſſionem expelli ſtatuerem, Motus aeri à Pulmone impreſſus initio exſpirandi, ſive reactio aeris in Pulmonem, adeoque in Sanguinem præterfluentem, pro quantitate infinite parvâ habenda erat, adeoque nihil omnino eorum effectuum, quæ ipſi adſcribebantur, præſtare poterat. Ita vero ſi feciſſem, jure queſturos putabam *Bellini* ſequaces, quod inique ſecum ageretur; quippe cum rejiceretur ipſorum ſententia propter demonſtrationem ex Hypotheſi arbitrariâ & eâdem omnium adverſiſſimâ deductam. Malui igitur ex illâ Hypotheſi demonſtrationem deducere, quæ omnium maxime ipſis faveret, maximamque Motus quantitatem exſpirandi initio aeri tribueret. Hæc autem erat, quâ ponebatur Pulmo initio exſpirationis datâ velocitate in Aerem impingere.

Cæterum in Potentiâ Cordis definiendâ iſtam quidem Hypotheſin, quâ ipſius Ventriculi, omni impetu momento temporis concepto, tanquam ſolidum corpus datâ velocitate præditum, in Sanguinem irruunt, primo loco propono, tanquam omnium ſimpliciſſimam, ex eâque ſolutionem deduco. Atqui deinde conſidero tum eam Hypotheſin, quâ Ventriculi Cordis Motum omnem ſuum particulâ temporis admodum parvâ concipiunt, quæque mihi veriſimillima videtur, tum ipſam Hypotheſin *Keillianam*, atque alias infinitas, iiſque omnibus ſolutionem

meam

meam accommodo. Adeo ut, ſive iſtud Principium incertum & fallax, ſive verum & ſtabile reperiatur, nihil exinde ſolutionis noſtræ certitudini detrahatur.

Non tamen videmus aliquid argumenti allatum, quo minus iſtam poſitionem nobis adhibere, pari jure atque Viro Cl. contrariam illam de Vi ſive preſſione uſurpare, licuerit. Nihil ſane ſpatii inter parietes Ventriculorum & Sanguinem intercedere non diffitemur, & tamen quare res ictu peragi nequeat nondum liquet. Certe, ſi Cubo Globum contingenti ictus imprimatur, Cubus partem Motus ſibi impreſſi Globo communicabit pari facilitate, ac ſi ſpatium inter eos interceſſerit.

At hæc ſunt corpora ſolida, & ubi de fluidorum Motu agitur, longe alia res eſt. Diſcrimen ſane inter ictus corporum ſolidorum & actionem ſive ſolidi in fluidum, ſive fluidi in ſolidum, fuſius exponit Vir Cl. quod diſcrimen cum me minus advertiſſe cenſeat, ex eo fonte fluere pronunciat quicquid Erroris in meis Propoſitionibus continetur. Ego vero differentiam iſtam ut recte traditam a Viro Cl. lubens admitto, & aio me communem illam doctrinam neutiquam ignoraſſe, cum nihil frequentius in Mechanicis ſcriptoribus occurrat, ſed caſus quoſdam novos expoſuiſſe, quibus ea doctrina cum adhiberi nequiret, alia erat ineunda ratio atque hactenus fuerat uſurpata. Ea tribus verbis abſolvi poteſt. Nam, ut exemplo facillimo utamur, quieſcere ponatur Cylindrus aquæ datæ longitudinis in dato tubo, & moveatur per iſtum tubum Cylindrus alius ſolidus pari diametro, ac datâ velocitate in Cylindrum aqueum impingat. Quid inde futurum eſt? Nempe totus Cylindrus aquæ eo ictu in motum ciebitur, pari ratione, ac ſi fuiſſet & ipſe ſolidus Cylindrus: alter vero Cylindrus Motus ſui partem momento temporis deperdet, & ambo Cylindri communi velocitate per tubum deferentur. Simili modo res eveniet, ſi Cylindrus aqueus per tubum fluens Cylindro ſolido quieſcenti impegerit. Quod ſi Cylindrus aqueus datâ velocitate per tubum feratur, eique occurrat Cylindrus ſolidus aliâ velocitate, ita ut quantitates Motuum Cylindri aquei & ſolidi utrinque pares ſint, jam momento temporis deſtruetur utriuſque Cylindri Motus, pariter ac ſi duo ſolida corpora æquali Motu prædita ſibi mutuo occurrant *.

Quod autem nos amicè admodum hortatur Vir Candidiſſimus, ut ſepoſitâ noſtrâ de Vaſorum ictu Hypotheſi, & Vi preſſuræ, quâ Naturam uti cenſet, pro Principio adhibitâ, Theoremata alia conſtruamus; id profecto, niſi gravi morbo impeditus perfunctorie prorſus evolviſſet noſtram Diſſertationem, dudum à nobis præſtitum animadvertere potuiſſet. Quum enim ponimus Motum Cordis in ratione temporis augeri, eâdem utique Hypotheſi utimur, ac ſi Vim preſſionis adhibeamus. Hoc autem poſito, Motum ex Cordis Potentiâ oriundum determinavimus, duplo ſcilicet majorem quam ubi Ventriculorum ictu res peragitur. Calculum vero ipſum, ut ſatis facilem & priori noſtro ſimilem, Lectori reliquimus inſtituendum. Quæ autem ſequuntur Theoremata & in iis Theorema quintum, quod rejiciendum ſtatuit Vir Cl.

** v. infra, Part* I. *Ch. V. Tract.* VII.

K tanquam

tanquam ex Hypothesi de Ventriculorum ictu deductum, neutiquam pendent ex istâ Hypothesi, sed ex ipsa Hypothesi Doctissimi Adversarii pari facilitate demonstrantur.

CHAP. IV.

The Abdomen.

I. *A Triple Bladder, by Mr.* Bussiere, *n.* 268. *p.* 752.

IN his best Health, the Patient could not make his Water in a full and continual stream, the Urine running out by little and little, and with great Efforts of Inspiration, chiefly when there was but little Quantity of it in the Bladder, which did fatigue him very much, tho the passing of the Urine through the Neck of the Bladder was not painful; except the two or three last Years of his Life, because of a thick *Mucus*, which then discharged with the Urine. That *Mucus* growing in greater Quantity of late, made him apprehensive it had been caused by a Stone in his Bladder; upon such Thoughts he apply'd himself to one, in order to be searched by him, who accordingly introducing his Catheter, and meeting with some Resistance in the *Urethra*, did force the Catheter through the Membranes, and made such Dilaceration in them, that the Patient lost immediately a great Quantity of Blood; which bleeding continuing for ten Days, without his seeking for any Help, brought him under very great Torments, by reason the Blood being grumulous in the *Urethra*, could not be forced out but by very violent Efforts and acute Pain, which caused a Mortification in the Part, of which he died.

The next Day after his Death, I did open the Body, in which the natural Urinary Bladder was found lying on the left side of the *Pelvis* upon the *Ilium* Bone, then searching what should be the Cause of such unnatural Situation, we did find one large and round Bag, lying under the *Pubis* upon the *Rectum*, filling up all the Cavity of the *Hypogaster*; then in order to examine the thing more exactly, I did dissect the *Penis* and the *Rectum*, and having taken them out of the Body, and laid them down upon a Table, the first thing I did was to lay open the *Urethra*, to examine whether there was any Carnosity, as the Surgeon who first introduced the Catheter had suspected; but there was none: and that *Ductus* was as plain and sound as could be, except the Dilaceration which the Catheter had made in it; then having introduced a Conductor into the Bladder, I divided it quite; and first it was observed, that the round Bag, which was made up of two Bladders, or rather two Cystis's, divided one from the other only by a Membrane, that which was next to the true Bladder was something bigger than the Bladder, the other which was lying on the right side being much lesser;

each

Fig. 1.

each of thefe two Cyftis's had its Orifice open in the Neck of the natural Bladder, which was longer than it is naturally, as you may fee in the Figure. None of the *Ureteres* were inferted into any of thefe Cyftis's, but they were inferted into the Neck of the true Bladder, by the Orifices of the two Cyftis's, infomuch that the Urine could be equally received by them and the Bladder.

Secondly, It was obferved that the Glandules of the true Bladder were extraordinarily big and red,that Colour being, very likely,the Effect of the Inflammation caufed by the Dilaceration of the *Urethra.* I have often obferved that a thick *Mucus,* which runneth out of the Bladder, and which fome think to be the Matter of an Impofthume or Ulcer in the Kidneys, is only produced by thofe Glandules of the Bladder grown fcrophulous; when that *Mucus* groweth thick and clammy, it caufeth the fame Pain on the Neck of the Bladder as if it were a Stone. The Glandules of the great Cyftis were very fenfible, but very fmall, they were not at all fenfible in the fmaller Cyftis.

Now it is eafie by the Defcription of thefe Bladders, to give a Reafon of the Symptoms that Gentleman did undergo, for by the Situation of the great Cyftis, it is plain that the Water could not come out but by the Force of Infpiration, its own Mufcles being not able to force it out, and confequently could not come out but by little and little; and thefe Efforts of Infpiration were to be the greater, when there was but little quantity of Urine, becaufe it requir'd greater Force to make it afcend from the Bottom of the Cyftis, which could not be done but with great Labour and Fatigue.

The Pain he fuffer'd in the late Years of his Life, was caufed by the Thicknefs of the *Mucus* running out of the Glandules of his Bladder, his Kidneys being very good and found.

A A. The Body of the true Bladder, 1, 2, 3, 4, 5, 6, its Glandules.

B B. The greater Cyftis. *Plate* 3. *Fig.* 1.

C C. The fmaller Cyftis, 1, 2, 3, its Wrinkles.

D. Part of the true Bladder overturn'd.

E. The Neck of the Bladder.

F F F F. The two *Urethra's.*

G. The Infertion of the SpermatickVeffels in the *Urethra.*

H H. The *Proftates.*

I I. *Veficulæ Seminales.*

K K. *Vafa deferentia.* *L.* The *Urethra.* *M.* The *Erectores* Mufcles. *N.* The *Penis.*

II. *Feb.* 168¾. I injected into the *Jejunum* of a Dog, that had for a Day before but little Meat, about 12 Ounces of a Solution of Indigo, in Fountain Water, and after three Hours opening the Dog a fecond time, I obferved feveral of the Lacteals of a blewifh Colour, which on ftretching the Mefentery did feveral times difappear, but was moft eafily

Experiments for tranfmitting blue Liquors into the Lacteals, by Dr. Mufgrave, n. 275 *p.* 996.

easily discerned when the Mesentery lay loose; an Argument that the blewish Liquor was not properly of the Vessels but of the Liquor contained in it. A few Days after this, repeating the Experiment in another Company, with the Solution of *Stone Blew* in Fountain Water, and on a Dog that had been kept fasting 36 Hours; I saw several of the Lacteals become of a *perfect Blue Colour*, within very few Minutes after the Injection: For they appear'd so, before I could sow up the Gut.

About the beginning of *March* following, having kept a Spaniel fasting 36 Hours, and then Syringing a Pint of a deep Decoction of *Stone Blue* with common Water into one of the small Guts; and after 3 Hours, opening the Dog again, I saw many of the Lacteals of a *deep Blue Colour*, several of them were cut, and afforded a *Blue Liquor*, (some of the Decoction,) running forth on the Mesentery. After this, I examin'd the *Ductus Thoracicus*, (on which, together with other Vessels near it, I had, upon my Return, made a Ligature,) and saw the *Receptaculum Chyli*, and *that Ductus* of a *Bluish Colour*; not so Blue indeed, as the Lacteals, from the Solution mixing, in and near the *Receptaculum*, with *Lympha*; but much Bluer than the *Ductus* uses to be, or than the Lymphatics under the Liver (with which I compared it) were.

The Entrance into the Lacteals (which is much the narrowest Part of all the way from the Mouth to the Mass of Blood) being thus beyond Exception proved wide enough to admit so gross a Body as *Stone-blue*, we may here in part explain the Admission of Liquors, (as of Diuretic-waters, *&c.*) into the Vessels in *prodigious Quantities*, in a very little time.

The same Wideness of the *Pylæ Lacteæ*, makes them easie to receive (together with proper Vehicles) those grosser Bodies, which afterwards compose the grumose part of the Blood, the Cartilages and Bones. And this open Entrance being allow'd, it will no longer seem impossible, that with our Nourishment, Eggs, or *Animalcula* themselves should enter these Vessels, there being no manner of Question, but that of both the one and the other, some are much less in Bulk, than the greatest Particles of Indigo in the Decoction above-mentioned seen in the Lacteals. Add to this the many Species there are of little Insects, and their great Fertility; so many and so great, that of the People of the Animal Kingdom, a very small Proportion (perhaps not a quarter Part) comes within view of the naked Eye; and then, we shall be the better able to account for the great variety, as well as Numbers of Insects, observed in the Juices of the Body Animal.

But the chiefest Use of the * Wideness of the Lacteal Orifices, is in deducing from thence the Reception of gross Matters (such as are the Effects of Indigestion, *&c.*) which afterwards in the Blood and *Genus Nervosum*, many times produce most severe Distempers.

* *vid.* Lister *de Med. Font. Angl. exercit alt. Lond.* p. 48.

III. Ha-

III. Having obſerved the extreme Swiftneſs with which the Drink paſſes ſometimes, as they find that drink *Medicinal* Waters, he thence conjectured that it did not always go the Way, which *Anatomy* ſhews us it takes ordinarily; and that therefore it ought to have another ſhorter Paſſage which is not yet diſcovered. A ſtrong Proof of this his Conjecture is, that thoſe who purge with an Infuſion of *Caſſia* render in a very ſhort time by Urine a Tincture as black almoſt as the Infuſion they have taken; which would not conſtantly happen, if the Drink took always the ordinary Way. He then took Pains to diſcover this unknown Paſſage for the Urine, and he perſuades himſelf that he has found it. To make this *Syſtem* the better apprehended, he began with the Explication of the uſe of the Drink, which is to help the Digeſtion, and to ſerve for a *Vehicle* to the *Chyle*: He ſaid, that the *Urine* is nothing elſe but the Drink it ſelf, which having ſerved for this purpoſe is afterwards caſt out more or leſs loaded; that thoſe who drink much without eating, as when they take the Waters, render their Urine very ſuddenly, and that without Colour. On the contrary, they that drink little and eat much, render theirs ſlower and high coloured; and laſtly, they who both eat and drink very much, renders theirs at firſt, one Part leſs coloured, and afterwards another Part high coloured later.

A new Syſtem for the Paſſage of the Drink and Urine by Monſ. Morin, com. by Mr. Blondel, n. 278. p. 1101.

From whence, and from what he had before ſaid, he inferr'd that the Drink, beſides the ordinary Paſſage which it has to the Bladder by the emulgent Veins Kidneys and Ureters, has likewiſe another by the Pores of the Stomach, and of the Bladder. He called thoſe the firſt Urines which paſs by this new way, and the ſecond Urines thoſe that paſs the ordinary way. He afterwards proved the Poſſibility of this new Syſtem by Experiments. He ſaid, that having taken the Ventricle and Bladder out of a dead Body, and filled them with Water, it run all out through the Pores; and turning them inſide outwards, the Water that was put in them run through them after the ſame manner; and that laſtly, letting them ſwim in Water, it eaſily ſoaked thro into them. From which he concludes that in a living Body, it ought to paſs with much greater Facility by the Tenſion of the Stomach, for the Aliment like a Spunge ſoaks up the Liquor in which it ſwims, and ſo ſwells up the Stomach; which in its Turn again preſſing the Food, ſqueezes out the Liquor from it, and forces it to filter thro' its Pores. With this Preſſion it is eaſy to conceive, that the Drink muſt paſs eaſier through the Pores of the Stomach than the Water, which was put into the Stomach taken out of a dead Body; and that this Liquor re-entring into the Bladder makes the firſt Urine: It is evident likewiſe, that this Preſſion is never ſtrong enough to preſs out all the Liquor from the Stomach, and ſo there remains enough to carry on the Aliment and Chyle, after which it comes away high loaded and coloured, and makes what he calls the ſecond Urines.

He

He added, the Paſſage of the Drink into the Capacity of the lower Belly did not cauſe the Dropſy, that Liquor, aided by the Preſſure of the Parts that encompaſs it, finding an eaſie Entrance into the Bladder, and none into the Inteſtines, becauſe of the thick *Mucus* that lines them. The Eaſineſs of this Paſſage is the cauſe that Mineral Waters run away ſo ſuddenly by the firſt ways, and by the ſecond; but much more by the firſt, when there is but little Nouriſhment in the Stomach; for there runs more or leſs Urine by the firſt ways than by the ſecond in proportion to the Aliment taken, and to the Surplus of what is neceſſary for the Digeſtion, Reſpect being had likewiſe to what paſſes inſenſibly by Tranſpiration.

This Syſtem being ſo laid down, he gave the Reaſons of two conſiderable *Phænomena*.

The firſt was, the different Colour of Urine that is made at different times: which proceeded from hence, that thoſe that paſs by the firſt ways, are but little charg'd, whereas the other that paſs by the ſecond ways, having ſerv'd for a Vehicle to the Chyle, and circulated with the Maſs of the Blood, are charg'd with the Volatile and Sulphurous Salts, and other Excrements of the Blood, and conſequently more colour'd.

The ſecond *Phænomenon* was the Red, Greeniſh, and ſometimes Blackiſh Colour of the Urine of thoſe that are purg'd with the Infuſion of *Caſſia*. This, according to him, is becauſe the Tincture paſſes by the firſt ways, as was experimented in the Stomach of a dead Man, where this Liquor paſſes indeed more ſlowly and in a leſs Quantity, but always of a Greeniſh Red. It is the ſame of the Red Tincture of the Urine after eating Beetes; of the Violet Brown, which is obſerv'd after Drinking Mineral Waters; of the Smell of Violets after the taking of Pills of Turpentine, and of the ſtrong Smell after Aſparagus, all which comes from the firſt Urines being charg'd with that Colour and Smell, which is not taken away by any thing that's mixt with it, whereas the ſecond Urine, which carries the Chyle and Aliments, has no other Colour nor Smell than Urine ordinarily has. He did not pretend to give this Syſtem as new, but as explained more exactly, and in ſome meaſure demonſtrated by the Experiments he reported.

Part of a Dog's Gut cut and cur'd, by Mr. John Shipton *n.* 283. *p.* 1299.

IV. Præſente *D. Gul. Pleahill, Societ. Chirug. Londin. Cuſtode*, adjutrices etiam manus commodante *D. J. Dobyns*, Juvene in Anatomicis pariter ac Chirurgicis Operationibus optimè verſato,

Cane ex more ligato, vulnus abdomini ſatis amplum infliximus; proximam inteſtini Ilei partem extraximus; vaſa meſaraica, quæ eam irrigabant, ligavimus; inteſtinum tranſverſim duobus locis forfice diſſecuimus; ejuſque portionem duos circiter digitos longam abſtulimus; tum inteſtini vulnus ſuturâ pellionum abdominis interſciſſâ conjunximus; emplaſtro obteximus; vinculiſque ſolutis, faſciâ obligavimus.

Primo


Fig. 1.
Fig. 2.
3
4
5

Primo ingreſſu titubare æger, ceu vertigine correptus, debiliſque admodum eſſe videbatur; eadem nocte bis vomuit: Poſt aliquot dies ſuturas aliquatenùs laxatas, ſubmiſſis feſtucis & circa fila contortis, arctius conſtrinximus. Poſteà medicamentis faſciiſque abjectis, intra tres ſeptimanas vulneri ipſe lambendo cicatricem induxit.

Obſervatione forſan non indigna videbitur inteſtini ſimul ac abſciſſum fuit retractio; quod ex utraque parte reſiliens arcteque ſe occludens Sphincterem eleganter exhibuit.

Canem deinde poſt aliquot ſeptimanas, quibus ſanus admodum vegetuſque degebat, ſuſpenſum aperuimus, partemq; inteſtini, qua conſutum erat, in hypochondrio ſiniſtro (ſatis longe ab externo vulnere, quod hypogaſtrio dextro inflictum erat) peritonæo firmiter annexum & in ſacculum ampliatum (*D D D*) invenimus: Omentum etiam (*G*) ei adhærebat, ut & inteſtina ibidem pluribus in locis (*** *e e*) ſibi invicem adnata erant. Ad cicatricem ab interna parte (*c*) accuratius luſtrandam, inteſtinum juxta longitudinem incidimus, quo ex uno latere vulneris labia adducta, (*D D D*) & peritonæo connexa (*f*); ex altero eadem diviſa, proximiſq; adjacentium inteſtinorum partibus agglutinanata fuiſſe apparebat; ita ut exteriores eorum tunicæ, internum ab iſto latere efformantes parietem, inteſtinalem fiſtulam continuarent, cibiſq; devehendis commode inſervirent, *e e*. v. Fig. 9.

Quanquam autem hæc in brutis experimenta bene ſucceſſerint, tamen ſi quiſquam tam inhumanus foret ut ſcienter, vel tam imperitus aut infælix ut inſciè, inteſtinum tenuius homini præcideret, nullus dubito, quin certum ei æque ac perfoſſo corde lethum inferret. Tenuia enim hominum inteſtina tantâ fibrarum carnearum compage haud gaudent, qualicanes natura providè ſtipavit; qui durioribus cibis veſci ſoliti, majori inteſtinalis fiſtulæ motu atq; calore, ad chylum elaborandum, fæceſque (in his plerumque duriores) eliminandas, indigere videntur.

Figuram inteſtini unâ cum vicinis partibus Clariſſ. D. *Cowper* pro ſingulari ſua humanitate accuratiſſime delineare dignatus eſt. *Plate* 4.

A a a. Inteſtini Ilei pars ſuperior ventriculum verſus. *B b b.* Ejuſdem pars inferior. *C.* Cicatrix vulneris inteſtini ab interiori parte. *D D D* Inteſtini diviſi labia. *E.* Orificium inteſtini ſuperius. *F.* Orificium inferius. *e e e.* Partes inteſtinorum adjacentium exteriores, portionis inteſtini hoc loco deficientis vicem ſupplentes. *f.* Peritonæi pars inteſtino adnata. *G.* Omentum inteſtino etiam connexum. * * Inteſtini, qua cum altero coaleſcebat, ſeparati veſtigia. *H H.* Aortæ Truncus. *I.* Arteria Cœliaca. *g.* Gaſtrica dextra. *h.* Gaſtroepiploica dextra. *i.* Hepaticæ. *k.* Pylorica. *l.* Gaſtrica major. *m m.* Arteria Splenica. *K.* Arteria Meſenterica ſuperior. *L.* Phrenicæ. *O.* Venæ Portæ Truncus. *P P.* Arteriæ & Venæ per Meſenterium diſſeminatæ.

V. A.

An unusual Colick, by Dr. Davis n. 275. p. 965.

V. A person, aged between 50 and 60, had been for 3 or 4 years last past troubled with Gripes, which generally return'd once a month, or thereabouts; his Body for the most part costive. Last *April* coming from *London*, he heated himself very much with his Journey, having walked a good part of the way, and as soon as he came home, had a return of his Colic pains, which continued upon him for 18 days, notwithstanding the methods commonly used in that case, during which time he had no Stool, besides what the first and second Clysters brought away; his complaint upon his seizure was of a pain in his right side in the *R gio Iliaca.* Some time before he died, his Belly swelled much, and was as tense as any Drum, he vomited for 2 or 3 days at the beginning, which left him, and returned not till just before he dyed, which was at the expiration of the 18th day, at which time he brought up 2 or 3 mouthfuls of black Choler; but never during his whole illness vomited any excrement. Dr. *Shaw* who lived upon the spot, prevail'd to have an hasty Inspection of his Abdomen. He found some black Choler in the Stomach, the *Duodenum* and the rest of the *Intestina tenuia* void of excrement, but incredibly distemper'd with Wind, and tracing the Canalis of the Guts as far as the *Cæcum*, found that of a blackish colour; and from thence for about a yard in length, the Colon mortified and so rotten, that the Excrements had made their way thro' it at several places, into the Cavity of the Abdomen; about 2 inches of the mortified Gut was fastened to the *Peritonæum* on the right side. This part of the Colon was so extended with excrements of a soft consistence, that they when taken out filled two Chamber-pots; at the extremity of the mortification, towards the *Rectum*, the obstruction which occasioned all these misfortunes offered it self to view very plainly; for about ten inches of the Colon was doubled, as if you had taken a piece of Tape, and folded it; the two contiguous surfaces of the duplicature adhered so firmly together, that you could not separate them without tearing the exterior coat of the Intestine. Upon separating this coalescence, there fell from that part a whitish Mucus, the adhesion was about 3 inches broad; the middle of the Duplicature, which made the acute angle, and where the excrements stopped, was smaller, and the Membranes thinner than in any other part of the Gut; from thence towards the Rectum the Colon was sound, and void of excrements, occasioned by the frequent use of Clysters.

An extraordinary Effect of the Colick by Monf. St. Andre. n. 351. p. 580.

VI. A Gentleman that came to Town in good Health, meeting with some Friends, drank a great deal of new bottled Oat-Ale, after some Pints of Wine. These Liquors fermented so violently in his Stomach and Intestins; that he was taken with a violent *Cholick* the same Night.

In the morning an Apothecary was sent for, who administred a Clyster, and took some Ounces of Blood to relieve the Patient, who complain'd of a great Pain in his left Side. The Clysters being repeated

peated the Night following, as alſo the next Morning; and the Patient growing worſe, the Apothecary, without Order of any Phyſician, gave him a violent Vomit: which operated Eight or Nine Times: This added Fewel to the Fire; and the Patient having from that Time been in a deſperate Condition, two eminent Phyſicians were call'd, who order'd that the Clyſters ſhould be repeated: But they not prevailing, I was ſent for about ſix Hours before the Patient died: I found him complaining of a violent Pain in all the Region of the *Abdomen*; a frequent Inclination to vomit; having a great Difficulty of breathing, together with a very ſlow Pulſe; his Belly being as hard as a Stone, though not ſwell'd.

This laſt Indication made me conclude, that the Diſeaſe was a violent Contraction of the Abdominal Muſcles, which had overcome the *Diaphragm*, and that probably the *Inteſtins* might be forc'd into the *Thorax*. Upon which I order'd a Fomentation of hot Milk, adding to every Quart a Drachm of *Liquid Laudanum*, which in theſe Maladies gives great Relief: But before it cou'd be got ready, the Patient expir'd in a violent Convulſion.

In opening this Body, I found the Abdominal Muſcles ſo much contracted, that it was almoſt impoſſible to penetrate them with a very ſharp Scalpel.

Upon Examination, I found the Stomach empty, and ſome Parts of the *Duodenum*, but the *Jejunum* and *Ilium* ſo much diſtended with the fermented Oat-Ale, that the *Ilium* had four Inches of Diameter, and the *Colon* above eight. The *Ilium* was alſo pretty much inflam'd in its inferior Part; and all the Valves of the *Colon* were obliterated by the great Diſtention of that *Inteſtin*. But the greateſt Diſaſter was, the Dilatation made in the *Diaphragm* as I ſuppos'd; made juſt upon the Chink which remits the intercoſtal Nerve to the *Viſcera* of the *Abdomen*, through which a Portion of the *Colon* was forc'd, and the greateſt Part of the *Omentum* and *Pancreas*.

Theſe tender Parts being choak'd ſoon inflamed, a Mortification of them following; and a Rupture of the *Pancreatick Vein* caus'd an internal *Hæmorrhage*, which fill'd all the left Cavity of the *Thorax*, inſomuch that the whole left Lobe of the Lungs was compreſs'd almoſt under the *Muſculus Scalenus*. The Quantity of extravas'd Blood was very great, and it was not in the leaſt coagulated.

VII. Acceſſit curandus nuper Vir Ruſticus, & ex multo-tempore extenuatus, laborans torminibus ventris, atque Hypochondriorum Spaſmodicis contractionibus, nil fere dejiciebat per inferiora, & continuo languore exolutus ſpiritu tandem defecit. Aperto cadavere, invenimus inteſtina craſſa, præcipue ubi in vicinia Duodeni flectuntur, mediis protuberantiis calloſis invicem, & cum adjacentibus conſolidata, ut obſcurum eſſet unum ab altero disjungere, & quod pejus erat, eorum

A pain in the Belly caus'd by a concretion of the Inteſtines, by Dr. Meſaporitus, *n.* 303. *p.* 2118.

 ſubſtan-

ſubſtantiam ad inſtar duriuſculæ cartilaginis concretam, & craſſam, ut nulla fere cavitas amplius ſupereſſet.

An extraordinary coſtive Caſe, by Mr. Sherman. *n.* 302. *p.* 2111.

VIII. 1. *Thomas Phillips* of *Eaſthorp*, near *Keldon* in *Eſſex*, was as well in every reſpect as a Child could poſſibly be till he was a year and a quarter old; at which time a very ſtrange and almoſt continual rumbling in his Inteſtines ſeiz'd him, the conſequence was a violent Looſeneſs, for which all the Phyſicians near the place could find no Remedy: But at laſt, when they deſpair'd of the Childs Life, the Looſeneſs terminated in ſuch an unuſual Obſtruction, that he did not go to Stool for a fortnight or three weeks together, and from three weeks it proceeded gradually to the intervals of 17 or 18 weeks, and ſo continued till he came to be about the age of fifteen, when his Body reſumed its natural temper, which laſted 4 or 5 years, but then the Obſtruction returned, and continued, or rather increas'd till he dyed; for it was cuſtomary with him in the laſt years of his Life not to evacuate any manner of Excrement under the interval of nineteen or twenty weeks, and ſometimes (twice at leaſt) he had no diſcharge for one or two and twenty weeks together. He lived to be near 23 years of Age, and walk'd about almoſt to the hour of his Death; for he was ſuddenly ſeiz'd with very ſick Fits (but could not vomit), two or three of which Fits carry'd him off in a few hours; and when he dy'd it was nine weeks after he had any Stool. He died about a year after I wrote to you firſt concerning him. I ask'd him whether he did Vomit often, or had at any time any Excrementous Taſtes in his Mouth, or did Sweat much, or made more Urine than in proportion to his drinking, or whether he found any eaſe when he did either Vomit, Sweat or Urined; all which queſtions were anſwered in the Negative. When he did go to Stool, he evacuated very many times in a day, and ſeveral days together, until he had emptyed himſelf. But his Mother aſſures me, that throughout his whole Life he never diſcharg'd any other than very thin Excrement.

Before his time of Evacuation came about, he was of an extraordinary bigneſs many weeks before his going to Stool, unleſs when he could break Wind, which he often endeavour'd to do, by laying his Body on the edge of a Table or Stool, but could not often by ſo doing produce the deſired effect. He declined the uſe of all Medicines for many years before he dy'd, contenting himſelf with going to Stool once in three or four months, or nineteen or twenty weeks, as above mention'd. But that which amuſes me as much as any thing is, that he generally had a pretty good Stomach, and eat and drank as the reſt of the Family did; nay, till the time that his Body came to be very full, he could do the work of a Man at Plow, or ſuch like Husbandly labour, for Agriculture was his employment. I was very deſirous to have open'd his Body, but could by no means obtain it from his Mother.

2. It's

2. It's not improbable, if the *Abdomen* of this Person had been opened, but some of its Contents would have been found not unlike those I have mentioned, in my Explication of the 34th Table of Prints published by Dr. *Bidloo*, where I take notice of a young Gentlewoman I dissected, in whom I observed the *Omentum* so lessened, that at first it appeared doubtful, if that part had ever been existent in that subject; but on strict examination the little Remains of it resembled a *Congeries* of small Glandules, stuffed with a Suet like matter. The whole Canal of the Intestines, even from the *Pylorus* to the *Anus*, was distended with Excrements, and the Surfaces of all the small Guts adhered so strictly to each other, that they could not be parted without tearing their external Membrane, to which the *Omentum* contributed by its adhesion: The whole *Compages* of the Intestins very much resembling that of the external surface of the Brain covered with the *Pia mater*, so that the Mesentery in that subject could not be seen till this external inclosure was divided. By this disorder, 'tis certain, the Peristaltick motion of the Guts must needs be very much lessened, if not quite hindered. The *Peritonæum* also in that case was very much thicken'd, and had several preternatural white bodies set at various distances on its internal surface; the like appeared on the Stomach, which very much resembled the Miliary Glands on the back part of the *Aspera Arteria*.

A Note by Mr. Cowper. *ibid.*

IX. When the Skin with the other Integuments were taken off, I observ'd that part of the Omentum had thrust itself through the Annular Holes of the Abdominal Muscles on the left side and there formed an Epiplocele or Hernia omentalis, as large as a Walnut. The *Omentum* reached as low down as the *Pubis* and Inside of the *Ilia*, to which it was ty'd, and by Fibrous Connections it adhered to all the *Peritonæum* below the Navel. All the Fat on the *Omentum* and Guts was firm, and hard like Tallow. The Intestines and Stomach were quite empty, without either Wind or *Fæces*.

Dissection of one dying of an Ulcer in the right Kidney. by Dr. Douglas. *n.* 325. *p.* 32.

The Left Kidney was much larger than ordinary, being near eight Inches long; its Surface being divided into several distinct Lobes, as in a *Fætus*. The Right Kidney was full of a fœtid purulent Matter: All its inner Substance was wholly wasted; and its external or cortical Part was stretched so very thin, that a small touch of the Finger could easily break through it. All the Fat and Glands about the Kidney last mentioned were hard, obstructed, indurate, and big, which made a great compression on the *Musculus Psoas* and the *Musculus Quadratus lumborum*. The Ureter proceeding from this Right Kidney, was covered with a Crust or Bed of indurate Glands; and besides, its capacity was straightned and contracted in several Places. The Cavity of the *Vesica Urinaria* was very small; its Substance so very thick and hard, that I could not even by the help of a Blow pipe distend it any wider; Its Inside seem'd excoriated with several little fleshy

Caruncles, or red Excrescences, here and there. There was a remarkable Corrosion in all the Inside of the *Urethra.* All the Upper and Convex Part of the Liver adhered firmly to the *Peritonæum* that covers the Diaphragm, and to the same Membrane where it covers part of the *Musculus Abdominis transversalis:* Its Substance was so very tender and soft, that it seemed to be almost rotten. The Gall Bladder was extreamly large and full; the Biliose Liquor it contained, being of a whitish yellow colour.

Between the *Tunica vaginalis* and *albuginea* of the Left Testicle, there was a large hydatical or watery Tumor; and upon the last named Coat of the same Testicle, there were several chalky Concretions, about the bigness of a Barley Corn each. In the Right Auricle of the Heart there was a large *Polypus,* that fill'd up its Cavity; extending itself a great way into the Ascending and Descending Trunks of the *Vena Cava.* All the rest of the *Viscera* were as they should be in a Natural State.

About a Year and half ago, the Patient began to decline. His first Complaints being a Heat, Sharpness or Pain in making Water; constant Desire to Urine, though in great Misery after. When the Water stood a while, there appeared a greasy Substance on its Surface, not unlike the Cream or Ice that is found on the top of *Aqua calcis vivæ*; sometime after, it deposited a purulent Matter in great quantity, but without any offensive smell: The Water when made was thick and whitish, but when the Corruption settled to the bottom of the Pot, it became clear. He seldom complained of any great Pain in his Back or Loins; whence they concluded the Ulcer was in the Neck of the Bladder, though the vast discharge of Matter was an Argument of the contrary: But always was on the Rack when he rose up after sitting, and it was a great difficulty to him to get up, which perhaps was occasioned by the Weight and Pressure of the Kidney and adjacent indurate Glands, lying on the head of the *Psoas* Muscle and *Quadratus lumborum.*

He had often a total Suppression of Urine; but was much relieved by *Sal Succini* and *Corn: Cervi.* He took several Doses of *Cantharides* with Camphire, without any ill effect from the Fly, but had little Relief as to his Distemper. For Three Weeks past he was seized with a violent Looseness, which at last, in spite of all means, carry'd him off.

An Hydropical Case, by Mr. Young, *n.* 333. *p.* 426.

X. Mrs. *Dyer* was about 30 Years old, a Mother of several Children, and very heathful till last *January,* when, after frequent watching upon an extraordinary occasion, she was vexed with a Pain in her Belly, like the Cholick, but proved the Dropsey *Ascites*; and grew so fast in despite of all I could do to help it, that *March* the 9th, being almost suffocated, I was forced to tap her by an hollow Needle in the usual Place, and to repeat the Operation so often as she filled:

And

And by that way discharged the several Quantities of Water, at the times here under-mentioned.

	Pints.		Pints.
March 9th I drew	9	*July* 21	16
14	8	30	16
April 2	12	*August* 6	14
16	10	17	14
May 17	14	26	13
31	14	*Sept.* 1, 6, & 22	$11\frac{1}{2}$
June 14	14	*Octob.* 1	3
24	14	30	15
July 7	17		$102\frac{1}{2}$
	112		112
			$214\frac{1}{2}$

All the while I was pumping that out, I endeavour'd by all the means I could to stop the Leak within, but in vain: She dy'd *Novemb.* 4. 1711. and opening her Belly, we found the following remarkable and incredible Things. From the Belly issued fourteen Pints of a greenish *Serum*, mixt with a very purulent Matter, not a little fetid. The *Intestines*, especially the *Colon*, almost every where livid, and adher'd in many Places to the *Peritonæum*, altho' they had been so long immersed in Water. The *Omentum* was also black, and almost consumed. The *Liver*, which I expected to be indurated, was free of all Faults, only two Superficial Ulcers on the Left Lobe. Both that and the *Peritonæum* (which are usually full of *Hydatides* in Dropsical Persons) were wholly free of them; but on the Stomach and Guts were many such.

But we were mightily surpriz'd to find a great Bladder distended like that of an Ox, to fill up almost the whole Region of the Liver and Ventricle, and adhering to the adjacent Parts so firmly, that we could not separate them without Difficulty, and get it out whole. Our Surprize, at such a prodigious Appearance, turned into Astonishment, when we found it the Gall Bladder, and that by its Distention it had torn the Liver asunder; one part of which adhered to the Left side of this monstrous *Cystis*, and another Part behind it, towards the Back; and both expanded with it, and fastened to it, like as the temporal Muscle to the Skull.

The whole weighed ten pounds and twelve Ounces. It had no Passage to let out the Matter it contained, altho' we squeez'd it hard to that purpose; nor could we find any by Probes: So that we were forced to make way by a Knife, and so let out of it seven Pints of a black Liquor like Coffee; which having stood one Night in a Bason, near a Quart of thick yellow *Fæces* subsided. The Liquor in this Bladder, and what we found in her Belly after her Death, added to what was evacuated before by *Paracentesis*, amounts to 235 Pints. Besides the prodigious Quantity of Matter which fill'd this great Bag, we

we found ſeveral Pieces of Membranes like Gut or Bladder cut into Pieces. During the whole time of her Sickneſs, ſhe ejected by Urine near as much as ſhe drank; and yet by Computation ſhe leaked into the *Abdomen* near a Pint every 24 Hours, from *March* to *November*. When her Belly was near full, her Thighs and Legs us'd to ſwell, and grew diſcolour'd, like an approaching Gangrene, but both went off after Tapping, by the Help of Friction, and a warm Lotion, *&c.*

Another by Mr. Laſage, *n* 299 *p.* 1977.

XI. I was called, ſome time ago, to open a Maiden Lady 52 Years of Age, who complain'd, about ſix Weeks before, of a Circonſcript hard Swelling on the *Hypogaſtrica regio,* on the Right Side; from that time her Belly grew by degrees to an exorbitant Bigneſs, the great Weight whereof was the moſt conſiderable Symptom, and at laſt ſuffocated the Lady. The Body was mightily emaciated, and the Legs ſwelled few Days before her Death. I expected Water, but there was only a viſcous darkiſh Humour, to the Quantity of 18 Gallons; after the Evacuation of that Matter, I was no leſs ſurprized to perceive a large Heap of Veſicles ariſing from a thick Membrane covering the Guts, it being the *Peritoneum* ſeparated from the Muſcles. I took it out, to examine the better thoſe Veſicular Bodies diſpoſed on the outward Surface of that Membrane, as alſo them that were on its inſide, towards the Guts. The Veſicles were of different Magnitude; ſome of the largeſt had been broken and ſunk, others were broken and almoſt empty, and the others very much diſtended and full; the Matter of all of them was of the ſame nature with the extravaſated Humours. What was contained in the leſſer ones proved to be of different Colour and Conſiſtence, not unlike Gelly, White of Eggs, Gall and Honey; in ſome it was much like the Humour of a true *Meliceris.* There was but little Matter extravaſated in the Cavity of the *Abdomen*; moſt Part was contained betwixt the *Peritoneum* and the Muſcles. The Right Kidney was affected with a particular Dropſie; all the *Viſcera* beſides were in a natural State, two *Polypus*'s were found in the Heart, and two pretty big Stones in the Gall Bladder.

A Diſſection of an emaciated Body dying of an Hydrops, by Mr Vaughan *n.* 281. *p.* 1245.

XII. I diſſected a poor emaciated Creature, that died of an Hydrops, out of whom I took about 20 Gallons of Liquor meaſured.

A Caſe of the Iliaca Paſſio, *by the ſame,* ibid.

XIII. A Youth whom I attended about 30 Hours, was afflicted with the *Iliaca paſſio,* very terrible for the time; he was aged about 14 Years, of a Sanguine Conſtitution. About 3 or 4 Hours before he dyed, I adminiſtred a Terebinthinat Clyſter, which gave, during its ſtay, immediate Eaſe; he ſo continued about an Hour, then this Diſeaſe returned again as ſevere as ever: He ſoon after dyed: but before ſome time he voided ſome of his Clyſter by Vomit. The Caſe being not very uſual, I applied my ſelf to his Friends, in order to Diſſect him; which accordingly I did the next Day. Finding the Liver only

only ſomething larger than ordinary, I immediately made ſearch from the *Ventriculus* quite to the lower end of the *Inteſtinum rectum*: The *Ventricle* was conſiderably extended; a little ſpace from the Gut *Ileon*, in the *Jejunum*, I found the excrements had made a breach, and ſome quantity was evacuated. I then proceeded to the Gut *Ileum*, I found a conſiderable part of it very livid, not in the leaſt extended; the Colon was much like a Contuſed Wound about 3 or 4 days old. I inſpected into the center of it, where I found it ſomething freſher, not ſo livid as the outſide. About the beginning of the *Inteſtinum rectum* I found another large ruption, where there was more excrements voided.

XIV. I ſhall here give a true and exact account of a Humane *Allantois*, as it appeared in two ſubjects, ſtill by me. One of which I obſerved ſeveral years ago, and the other in *March*, 169$\frac{8}{9}$. *Of the Human Allantois, by Dr.* Hale. *n.* 271. *p.* 835.

Moſt of the Antients indeed allow of one; not from their experience of it, but becauſe they took it for granted, that Men and other Animals were alike, in the *Viſcera*, *Membranes*, *Veſſels*, &c. Hence *Hippocrates* ſays, that Twins lie in *Sinus's*, and that the *Uterus* has *Cornua*. *Galen* deſcribes the *Navel ſtring* to conſiſt of four Veſſels, beſides the *Urachus*, and the *Allantois* to be like a Pudding reaching from one *Cornu* of the *Uterus* to the other. In ſhort, notwithſtanding the Antients might ſometimes diſſect Humane Bodies, and although, [1] *Herophilus* and *Eraſiſtratus* did open live Men and Women, yet it can't be found what great uſe was made of thoſe opportunities: For the Antients Accounts of many parts, particularly of the *Urachus* and *Allantois*, (as to its Name, Figure, Site, *&c.*) agree only to their appearance in *Brutes*. I ſhall ſay nothing of the *Allantois* in Brutes, ſince it is granted by moſt Anatomiſts, to be in theſe Animals, and ſufficiently deſcribed by Dr. *Needham*, who alſo firſt diſcovered part of the *Allantois* in *Humane* ſubjects, but neither he, nor any other, has taken the right method of finding it entire, and 'tis no wonder they could not truly deſcribe what they never ſaw. [2] Dr. *Needham* ſays, that after the *Amnios* is cleared, and left fixed to the *Umbilical Rope*, you may divide by the fingers, or knife, the remaining part of the *Involucra* into two Membranes. The *exterior* he truly calls the *Chorion*, the *interior* he takes to be the *Allantois*. But by theſe ways of ſeparation, you will preſently tear the *Allantois*, and be able to diſcern only ſome ſmall pieces of it. Beſides, the *Allantois* is at firſt ſight ſo like the *Amnios*, that [3] many who ſuppoſe the *Amnios* double, and that its Coats are eaſily ſeparable, have taken theſe pieces of the *Allantois*, for broken parts of one of the Coats of the *Amnios*. Whereas having firſt found the *Hole* whence the Urine came forth (if the *Allantois* is not too much torn) you may blow up the *Allantois* with a Pipe to its full dimenſions, and then ſee its true Shape, the *Fundus*, the *Crevix*, the inſertion there of the *Urachus*, its relation to the other Membranes, *&c.* Be the *Allantois* never

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[1] *Celſus.* *Præf.*

[2] *cap.* 3d *7th.*

[3] *Diemer. lib.* 1. *cap.* 31. *Horn. parag.* 69.

ſo much torn, yet this way you may eaſily ſeparate many Inches of it, from the *Chorion*, and *Amnios*. Which eaſie ſeparation demonſtrates a diſtinction of Membranes, ſince no double Membrane can be divided by the breath alone.

[1] *Lib.* 1. *cap.* 31.

Indeed *Hoboken* and [1] *Diemerbroeck* make it a very eaſie thing to ſeparate the *Allantois* from the other Membranes, only by the fingers; but 'tis plain from their deſcriptions that they never ſaw one entire. Amongſt other miſtakes, *Diemerbroeck* ſays that the Urine of a *Fœtus* lies between the *Urinary* Membrane and the *Chorion*; as though not contained in a diſtinct Bladder, but in a Cavity made partly from the *Chorion*, partly from the *Urinary* Membrane. I confeſs *De* [2] *Graaf* tells us, that by blowing with a Pipe into a Hole made through the *Chorion*, all the Membranes of the *Secundines* will appear diſtinct. He has alſo delineated an *Allantois*, with the other Membranes, *&c.* as he ſays he found them. Yet this [3] *Figure* muſt have been drawn from his own fancy, and not from any preparation, for theſe reaſons. 1ſt, Becauſe, by this way of ſeparation, you can only part the *Allantois* from the *Chorion*, but never ſee its true dimenſions, nor any appearance of a Bladder; for a Bladder, as the *Allantois* is, can be ſhew'd only by blowing into its Cavity, or by finding it full. Yet in this *Figure* no ſign can be obſerved, where 'twas blown up, and tied, *De Graaf* alſo ſpeaking of making a Hole only in the *Chorion*. Nor can this *Allantois* be ſuppoſed full of Urine, becauſe 'tis not of the Shape of a full *Allantois*, and our Author himſelf calls it only the inflated part of the *Allantois*. However, I can't conceive how the *Allantois* could remain partly filled with Air (any more than it might with Urine) ſo long as till this *Figure* was drawn, unleſs ſome *Hole* was tied up, whence the Urine came forth, and the Air was blown in. 2dly, Becauſe in this *Figure* the *Umbilical Rope* ſeems to run through both *Amnios*, and the *Allantois*, to its inſertion on the *Placenta*. Whereas the *Allantois* is no where perforated by the *Umbilical Rope*, nor does it any where paſs through the *Amnios*, but only runs under it, at the place of its inſertion on the *Placenta*. If the *Navel-ſtring* could be allowed to enter the *Amnios* at [4] I, and to paſs under it to the *Placenta*, why ſhould it not appear (which it does not) under the *Amnios*, as well as the thin ſubſtance of the *Allantois*? Again, according to *De Graaf*'s poſition of the *Secundines*, (which is the reverſe of *Figure* 1ſt, where the *Navel-ſtring* lies under the *Allantois*) nothing could hinder a plain view of the place where the *Navel-ſtring* is ſet on to the *Placenta*. This will be eaſily apprehended, by ſuppoſing the part H in my 1ſt *Figure* to lie uppermoſt, the *Fundus* G and *Navel-ſtrings* being turned over; for then the *Strings* will run over the *Allantois*, as in *De Graaf*'s Cut, and its inſertion appear plain on the *Placenta*, which yet can't be diſcovered in his *Figure*. I am ſure the whole is irregular, and I take it to be fictitious. As for the *Urinary* Membrane G it ſeems to be the *Allantois* of a *Colt* (where [5] *Needham* ſays, the *Umbilical Rope* runs through the *Uri-*

[2] *de Mul. Org. cap* 15.

[3] *cap.* 15. *Tab.* 22*d.*

[4] *Tab.* 22*d De Graaf.*

[5] *vid. B. Tab.* 2*d.*

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nary Membrane) not lefs abfurdly added to the *Secundines* of a *Humane Fœtus* than the *Secundines* of a *Whelp* are to a like *Fœtus* by *Vefalius*.

Laftly, 'Tis moft evident that *De Graaf* knew nothing of the true Shape of this Membrane, and that he had never feen one entire, becaufe [1] he confents to [2] *Needham*'s defcription of it as true, which yet is falfe in feveral particulars. For 1ft, the *Urinary* Membrane does not cover the whole *Fœtus* (as he affirms) but only that part of it, which refpects the *Chorion*, and does not lie on the *Placenta*; for the *Allantois* can be extended at fartheft but to the edges of the *Placenta*, where the *Amnios* and *Chorion* are fo clofely joined by *Fibres*, that no Membrane can come between them. Wherefore 2dly, the *Allantois* is not every where faftned to the *Chorion*. And confequently 3dly, the *Allanteis* can't be of the fame Shape that the other Membranes are of, nor be like the *Allantois* of a *Colt*, which contains the *Fœtus* in the *Amnios*; all which neverthelefs [3] *Needham* afferts. In fhort, Dr. *Needham* had feen only pieces of the *Urinary* Membrane, but never an entire one, and fo could only guefs at the Shape, *&c.* of it, from what he had obferved in *Mares*, and *Glanduliferous* Animals. He might have made a better guefs at the figure, fite, *&c.* of a *Human Allantois* from that of a *Whelp*, which does not every where encompafs the *Fœtus*, as he obferves. *Bidloo*, in moft of his *Figures* of the *Secundines*, letters fome *Veftigia* of the *Urinary* Membrane, but in any of thefe *Figures*, you only fee broken pieces of one, fo confufedly placed, that no *Idea* of its Bignefs, Shape, or Situation can be formed from them. I muft confefs that oftentimes the Membranes of the *Secundines* are fo torn, that no Art can exhibit an entire *Allantois*. However, among the many *Secundines* that have come under the hands of Anatomifts, feveral no doubt muft have been entire enough for a fuller difcovery than they have made, had it not been by their ways of proceeding (*viz.* by knife, fingers, or blowing under the *Chorion*) impoffible to difcover any thing plain, or fatisfactory, even in the faireft fubjects.

[1] *De Graaf. cap.* 15.
[2] *cap* 3*d de format. Fœt.*
[3] *cap.* 7*th.*

I come now to anfwer the objections of thofe who ftill deny an *Urinary* Membrane to a *Humane Fœtus*.

The difficulty of finding this *Membrane* is by no means an argument againft the exiftence of it: But a Woman that dies big with Child is fo fair a fubject for the difcovery of *three Membranes*, that I wonder [4] *Parey* having fuch an opportunity could find but two, if he was fo careful as he fays he was. Dr. *Tyfon* obferved *three Membranes* fome years ago, in a like fubject. After the *Chorion* was divided, and laid afide, he faw *two Bladders*, containing Liquors of different colours, which he preffing one towards the other, did not mix, but remained diftinct. This obfervation fully fatisfied that great Anatomift, as to the exiftence of an *Allantois*; and its figure, texture, fite, *&c.* might alfo have been difcovered by him, had not the lefs curious Spectators been impatient to pafs on to other parts of the Diffection.

[4] *Lib.* 3*d. cap.* 34.

Some deny a *Urinary* Membrane to a *Human Fœtus*, because they suppose the *Urachus* to be impervious, and that therefore there would be no passage for the Urine, and consequently no need of an *Allantois*. [1] *Needham* indeed says that he could never find any sign of a Cavity in the *Urachus*, yet is of opinion, that by blowing from the *Bladder*, the Air might be forced through a *Humane Urachus*, as easily as he has often done it through that of a *Whelp*. I don't understand why Dr. *Needham*, and [2] others should insist so much upon an apparent Cavity in the *Urachus*, or expect that Air should necessarily pass through it upon blowing, and think that otherwise it cannot be fit for the assigned Office; since many Bodies, as Membranes, *&c.* will not admit Air, *&c.* yet let Water pass freely through them. It will not seem strange, that Water should pass through the substance of the *Urachus*, if we consider that the Cavity of the *Urachus* to the *Navel* is open, as appears by Inflation, or Injections, (to say nothing of those who are mentioned to have made Water by the [3] *Navel*) and that the rest of the *Urachus* is pervious, tho' not plainly hollow, (the Urine rather soaking gently, than running through its more strait Tubes) may be gathered from hence, 1st, That the substance of the *Urachus*, (as well as the Cavity of the *Allantois*) is always found turgid with a Liquor, that in colour, taste and smell, seems Urinous. 2dly, That since the *Mucous* Coat of the Intestines is demonstrated to be vascular by Mr. *Leeuwenhoeck*, therefore the Mucous substance of the *Urachus* may also be Vascular. 3dly, That Urine may as easily ouze through these Mucous Vessels, as other fluids run through Vascular *Cartilages*, and *Bones*, &c. or the Chyle into *Lacteals*, (whose Orifices, as [4] *Leeuwenhoeck* observes, will scarce admit of Particles so big as the 1000000000 part of a grain of sand) the great Cavity of the Intestines being open at the same time; or as easily as grosser parts of the *Semen* pass the Tubes of the *Testicles*, whose Cavities are not more perceptible. I am sure the Urine is more assisted in its motion by the *Detrusor Urinæ*, &c. than any of these Fluids can be by the *Heart*, or other *Muscles*.

[5] Others will not admit of an *Urinary* Membrane, they thinking it would be useless, because they imagine, that when the Bladder is full, the Urine must be discharged at its *Cervix*, and not at its *Fundus* by the *Urachus*. But in answer to this, the Urine can never pass through the *Cervix*, and *Urethra*, unless the *Abdominal Muscles* contract; because we never void Urine naturally, but by the help of these Muscles, nothing less being able to force open the *Sphincter Vesicæ*. Now it being more than probable that these Muscles never act before *Respiration*, no Urine can pass through the *Sphincter*, before the Child breathes. No reason can be given why the *Abdominal Muscles* of a *Fœtus* should voluntarily contract, since neither the Quantity nor Quality of the [6] Urine can excite to such an action. For when the Bladder is too full of Urine, it will ouze through the lax spongy substance of the *Urachus*, being gently pressed by the *Detrusor* alone. There would arise many inconveniencies

1 *cap.* 3d *cap.* 4th *cap.* 7th.

2 *Parey Lib.* 3d. *cap.* 34 *Mauriceau Lib.* 2d *cap.* 4th.

3 *Fern. Lib.* 6. *cap.* 13. *Senn. Lib. Prac.* 3d *part* 8. *sect.* 2d *cap.* 10. *Lauren. Lib.* 8th *Quest.* 7.

4 *Vol.* 2d. *Epist.* 68. *pa.* 335.

5 *Barth. Lib.* 1. *cap* 36. & 37. *Riolan. Lib.* 6. *cap.* 3. & 4. *Parey Lib.* 3d *cap.* 34.

6 *Barth. Lib.* 1. *cap.* 37. *Need. cap.* 3. *pag.* 81.

veniencies from the voluntary contraction of the *Abdominal Muscles* of a *Fœtus*, as voiding *Fœces* as well as *Urine*, into the *Amnios*, which would be more prejudicial than [1] Sweat, *&c.* Yet if we should suppose the *Abdominal Muscles* of a *Fœtus* to act, the Urine will however pass where it can most easily, *i. e.* through the *Urachus*, which is partly open, and altogether of such a Texture, as in no wise can hinder the passing of the Urine, much less be able to resist a considerable force, as the [2] *Sphincter Vesicæ* can. Besides, the *Urachus* is not only thus qualified for the admission of Urine, but when the Mother lies down, 'tis almost upon a Level with the *Urethra*, and what has once passed the *Urachus*, cannot return by reason of the Length, Situation, and peculiar Structure of it. Lastly, the [3] *Pudendi Clausura* sometimes happening in both Sexes, demonstrates that then at least, the Urine can't pass through the *Urethra*.

[4] *Dioni* not finding any *Allantois*, nor an *Urachus* plainly pervious, thinks there is no need of either, on another account. For he supposes that the Blood, which serves for the Nutrition of the *Fœtus*, is depurated from all Excrement. But I cannot apprehend, what should make this portion of the Blood and Chyle freer from Excrement, than the rest of the *Massa Sanguinea*. There is indeed no portion of it, which does not contain parts unfit for *Assimilation* and *Nutrition*. Our Author would have been convinced of this error, had he ever opened *Abortions* of five Months old or upwards, their Bladders being always full of *Urine*, and some Fœces constantly in the Intestines. 'Tis difficult to determine when this separation of *Urine* first begins, but I am apt to think it much sooner than is generally supposed. *Fig.* the 3d is the *Allantois* of a very small *Abortion*, which I have still by me. Since all the parts are perfectly formed before *Impregnation*, not very long after Impregnation they may begin to perform their Offices. No doubt they begin as soon as there is occasion for any separation, and a separation of Urine is necessary, when the *Fœtus* is first nourished by the *Umbilical Arteries*.

The Existence of an *Allantois* is denied by [5] some who grant an *Urachus*, but will have it convey the *Urine* to between the *Amnios* and *Chorion*. [6] *Diemerbroeck*'s opinion is somewhat like this, only he would have the Urine lodged between the *Urinary* Membrane and the *Chorion*. These men don't consider that the Urine in this case would get into the *Amnios* as well as the *Succus Nutritius* of the *Chorion*, whether imbibed from the *Uterus* by the *Chorion*, or separated by its *Glands*. Such a *Succus Nutritius* of the *Chorion* is granted by the [7] Maintainers of the fore-cited opinions, as well as by those who deny an *Allantois* altogether, or [8] suppose it to have a different Figure, *&c.* from what *Diemerbroeck* assigns it. The Transudation (or Filtration through the Membranes) of this *Succus* seems most likely in *Mares* and *Sows*; for in a [9] *Mare*, the *Chorion* is not joyned to the *Uterus*, till she is half gone, and in a [10] *Sow* it does not adhere to the *Uterus* till near the end of her going

[1] *Harv. de Humor. pag.* 547.

[2] *Galen de usu part. Lib.* 15.

[3] *Senn. Prac. Lib.* 4. *part* 1. *sect.* 1. *cap.* 3 *Mauric. Lib.* 2. *cap.* 3.

[4] *pag.* 145.

[5] *Aquapend. Lib.* 1. *cap.* 7. *Fallop.*

[6] *Lib.* 1. *cap.* 31.

[7] *Harv. de Hum. & membra. Needh. cap.* 2. 3. *cap.* 7. *Barth. Lib.* 1. *cap.* 36. *Graaf de mul. org. cap.* 15.

[8] *Needh. Graaf.*

[9] *Harv. de Membran.*

[10] *Need. cap.* 2. *cap.* 3. *&* 7. *Graaf de mul. org. cap.* 15.

going with young. But tis most evident that the Urine of a Humane *Fœtus* is not contain'd between the *Chorion* and *Amnios*, nor between the *Chorion* and *Allantois*, from the close connexion of these Coats to one another; also from the observation of Midwives, who often find a Bladder of Water (they call it a By-Water) offering itself before the Child, whereas the Humor of the *Amnios* is little, and of the *Chorion* much less, and of another Colour, *&c.* at the time of Birth. This By-Water is taken notice of, as an argument for an *Allantois* by Mr. *Cowper*, to whose assistance we owe that the Figures belonging to these Papers appear correct.

* Harv. de memb. & humor.

The great * *Harvey* will not allow an *Allantois* even to Brutes, and fancies the *Allantois* and the *Chorion* to be the same Membrane, that has two Names, the first from its Shape, the other from its Office, or Number of Vessels. Yet it is plain from *Galen* and all the Antients, that they meant two distinct Membranes by the *Allantois* and *Chorion*. Dr *Harvey* thinks that a *Fœtus* does not void Urine, but that the Bladder contains it, till the time of Birth. What was offered against *Dioni*'s Opinion may serve for an answer to this also. Because 'twas impossible for this diligent Anatomist not sometimes to observe an *Urinary* Bladder, he has thought of ways to explain such *Phænomena* without granting an *Allantois*. In † Sheep and Does, he had seen as it were a certain Process between the Umbilical Arteries full of Urine. This Process is no doubt the *Allantois*, though * *Bartholinus* calls it the *Urachus*. Again, he thinks what is called by others an *Allantois* (if it is not the *Chorion*) is some Coat accidentally formed from a Reduplication of the Membranes: because (since every Membrane is double) Nature may, upon a streight, lodge the Urine between a Duplicature. Yet he does not tell us how his Duplicature is to be filled, he allowing no *Urachus*. But in short, this *Urinary* Bladder can be no Duplicature of the other Membranes, since in all Animals it differs from them, as to Figure, Texture, and in having an *Urachus*, which no other Membrane has. And since every Animal that has a Bladder, must have a like necessity for a Receptacle of Urine till born, since also the *Urachus* is ever alike inserted in the same *Species* of Animals, and the *Urinary* Bladder constantly the same, as to Shape, Texture, Situation, *&c.* the *Urachus* and *Allantois*, with its † By-Water, can be no accidental or preternatural things.

† Harv. de memb. & humor.

* Lib. 1. cap. 37.

† Harv. de Hum.

Fig. 1. Represents the Secundines of Twins, to shew the *Allantois*, and its Relation to other Membranes, *&c.* after the Parts were prepared and dried.

Plate 5.

A A A A Part of the *Chorion* expanded.

B B B a Line, expressing the Edges of the *Placenta*.

C C C the *Amnios*, which is united to

D the *Allantois*, at

E E E the Line of *Union*.

F the *Cervix* of the *Allantois*.

G A Hole at the *Fundus* of the *Allantois*, whence the Urine came forth, and where the *Allantois* was blown up.

H Part of that half of the *Allantois*, which lies under the Line of Union, and immediately covered the *Fœtus's*, unleſs it is ſuppoſed that the *Amnios* is continued under the *Allantois*.

I I Two Stiles or Probes thruſt under the *Amnios*. They ſupport the *Allantois*, and keep open the *Aperture* * * * * of the *Amnios*, whence the *Twins* came forth.

K Part of the *Placenta*, with ſome Blood-Veſſels injected.

L L L L the Arteries of the *Navel-ſtring* filled with Red-Wax.

M M The Umbilical Veins filled with Green Wax.

N a Communicant Artery, by the means of which all the Arteries of both Navel-ſtrings were filled at once, and the Veins were filled by one Injection in like manner.

O a Pin that keeps out the *Amnios*, where from the Edge of the *Placenta* it runs partly to the Line of Union, or Adhæſion, and partly over the *Placenta*.

P Part of the *Chorion* at the Edge of the *Placenta*, where it runs under the *Amnios* on the *Placenta*.

Q a Pin that by a Thread helps to pull open the Aperture of the *Amnios*.

R R R the *Urachus*, lying between the Arteries.

a a a Fibres or Veſſels which faſten the *Allantois* to the *Chorion*.

Fig. 2. Exhibits a ſide-view of the ſame Preparation, that the Inſertion of the *Urachus, &c.* may be better ſeen.

N. B. That *A* and all the ſame Letters in theſe three Figures denote the ſame Parts in every one.

S ſhews the Courſe of the *Urachus R* at *F* in pricked Lines.

T part of the *Amnios* raiſed from the *Placenta*, to diſcover the *Placenta K* and *V*.

V that Part of the *Allantois* which is below the Line of Union, near its Neck *F*.

Fig. 3. Shews an entire *Allantois* of a very ſmall Abortion.

N. B. This *Allantois* was eaſily ſeparated from the other Membranes, between which it lay; and the *Amnios* remained an entire Bladder or Membrane under the *Allantois*.

Now ſome object, that what is called the Line of Union can be no real thing. As to this I don't know whether the *Allantois* of Twins may not require ſuch a Conjunction to ſuſtain, and keep ſteady a greater Quantity of Urine: nor can I reſolve whether the *Allantois* of Twins (like that of a ſingle *Fœtus* in Fig. 3.) may not be diſtinct, and ſeparable from the *Amnios*, but was not diſcovered by me to be ſo, thro' want of Skill or Care. However, the Reaſons why ſuch a Line was figur'd are theſe. 1. Although I uſed more Force, with equal Care, to ſeparate the *Allantois* in this Place than in any other (where nevertheleſs the Separation was very eaſie) yet I could not divide theſe Membranes

branes farther than that Line. 2. This Line ſeeming ſo regular as to divide the *Allantois* into two equal Parts, I could not take it to be the Effect of Chance, or my Separation. 3. The Part *H* below the Line *E E* was alike in Tranſparency to that Part of the *Allantois D* above it. Whereas had the *Amnios* been joined to the *Allantois* (as the Objectors ſuppoſe) the *Allantois* below this Line muſt have appeared thicker than that part above it, ſince the *Amnios* alone is much thicker than the *Allantois*. 'Tis eaſie indeed to conceive the *Amnios* running an entire Bladder, or Membrane, under the *Allantois*, and perhaps it may be ſo; but I think it diſingenuous to conceal what I did obſerve, or to make out by Fancy, what I cou'dn't diſcover in Fact.

Others have thought this *Allantois* to be an *Amnios* of one of the Twins belonging to theſe Secundines. This Objection, though it may ſeem plauſible, yet it is of no Force. For firſt, this *Allantois* is much finer to the Touch, as alſo much more tranſparent, than the other *Amnios*; which ſtill remains ſtiff, whilſt the much thinner *Allantois* ſinks under the leaſt Blaſt of Air, notwithſtanding the ſtiles *I I* which aſſiſt it. 2dly, This *Allantois* had two viſible *Urachus*'s, and is of an Oviform Figure, ſomewhat like the common Cuts of a Man's Bladder (for a true Cut of a Humane Bladder I never yet ſaw, for it ought to be made much bigger (as it really is) at its *Cervix*, *&c.*) Alſo this *Allantois* no where touches the *Placenta*, unleſs at the Neck *F*. But on the contrary, the *Amnios* is of the ſame irregular Figure, as the Poſition, Motion, *&c.* of the *Fœtus* require. Likewiſe, it covers the whole internal Surface of the *Placenta*. 3dly, They who make this Objection muſt ſuppoſe ſome Hole in this Bladder, and in the *Amnios*, through which one Umbilical Rope may paſs from the *Placenta* to the *Fœtus*. But ſuch a *Foramen* would be preternatural, becauſe the Navel-ſtring only runs from the *Placenta* to the *Fœtus*, under a Coat taken from the *Amnios*, and lies with the *Fœtus* in the Cavity of the *Amnios*, that is no where perforated. 4thly, The Hole at the *Fundus G* was ſcarcely wide enough to receive the end of a Man's Finger, whereas the Twins did not want ſix Weeks of their full Time. Since therefore a *Fœtus* of near eight Months could not poſſibly paſs this Orifice, this Bladder could not be an *Amnios*.

Nothing in theſe *Secundines* is preternatural, only ſome things were not before obſerved. Hitherto Anatomiſts have not allowed Twins to lye in a common *Amnios*, but ſuppoſed each *Fœtus* to have a diſtinct *Amnios*. The Reaſon of this Opinion might be, that ſome, denying any Urinary Membrane, called every Membrane they found (except the *Chorion*) an *Amnios*, and theſe finding two Membranes in the *Secundines* of Twins, ſuppoſed them to be two *Amnios*'s. That others granting an *Allantois*, but not diſtinctly diſcovering it, but only two Membranes, alſo imagined them to be two *Amnios's*. Both of theſe taking that for the *Amnios*, which might really be an *Allantois*. But ſince one *Chorion*, and one * *Placenta* (the || *Placenta* and *Chorion* being ever of the ſame Number) generally ſerve Twins (nay, ſometimes

three

* *Mauriceau Lib.* 2. *cap.* 4. *Need. cap.* 2. *Barth. Lib.* 1. *cap.* 36. *Diemerb. Lib.* 1. *cap.* 30.

|| *Diemer. Lib.* 1. *cap.* 31. *Graaf cap.* 15.

three *Fœtus*'s) why ſhould it ſeem ſtrange, that one *Amnios* (at leaſt ſometimes) and one *Allantois* ſhould ſerve the like Number?

I am not ignorant that * *Mauriceau*, and || *Diemerbroeck* think there is an abſolute Neceſſary for every *Fœtus* to lie in a diſtinct *Amnios*, and that otherwiſe Twins in the ſame Membrane would grow together, and make a Monſter. * *Aquapendens* further ſays, that all *Ova Gemellifica* produce ſome other ſort of *Monſter*. Yet 'tis moſt certain that *Ova Gemellifica* do exclude two perfect Chickens, however not both alive. || The Great *Harvey* indeed thinks it poſſible, that ſuch an *Ovum* may produce a Monſtrous Chick, if its *Vitelli* are contained in the ſame Membrane, *&c.* yet does not poſitively ſay it muſt be ſo. For my Part I can't ſee any more Reaſon why Twins in one *Amnios* ſhould grow together, than that the Hands or Heels of the ſame *Ftœus* ſhould grow to its own Body. How can the Humors that lubricate a ſingle *Fœtus*, and help it to move, joyn two together? ſince the Humours are the ſame, and the Parts of the ſame *Fœtus* as tender as thoſe of Twins are, and lye as cloſe to one another, as Twins do. 'Tis very obſervable, that among all the Monſters we read of, there are very few, which ſeem to be made of two entire Bodies joined together, and that moſt of theſe upon * Diſſection were found to have but *one Heart*, *one Liver*, whence 'tis moſt plain, that theſe Monſters (and no doubt all others) were originally Monſters in the *Ova* before Impregnation, and not ſo from want of the *Amnios*. Yet * *Diemerbroeck* does not a little boaſt of having firſt (as he thinks) found the reaſon why Twins muſt lie in diſtinct *Amnios*'s. But ſince the Matter of Fact (ſometimes at leaſt, as in theſe *Secundines* there was only one *Amnios*, and two regular *Fœtus*) is not true, his Argument for a Neceſſity of two *Amnios*'s for two *Fœtus*'s, will never prove valid, even where Twins, and two *Amnios*'s are found. Indeed any Part may be made to grow to any Part, as we ſee in the Cure of Hair-Lips, *&c.* but then the Fibres muſt be firſt broke, before there can be any Union. Now I can't conceive what ſhould naturally break the Fibres of the Twins in the *Uterus*. But although 'tis evident from what has been ſaid, that Twins may lie diſtinct in the ſame *Amnios*, yet there muſt be as many *Urachus*'s as *Fœtus*'s. In theſe *Secundines* I ſaw two running over the *Placenta*, to the Neck of the *Allantois*, which I communicated to ſome Phyſicians, before the Parts began to grow dry. The *Urachus* paſſes under the *Amnios* as the other *Umbilical Veſſels* do, and runs from that Part where the *Umbilical Rope* is ſet on to the *Placenta*, ſtreight to the *Cervix F. S* deſcribes the Courſe of that *Urachus* marked *R* at *F* in the ſecond Figure. The other *Urachus* lay about a quarter of an Inch laterally beyond that marked *R* in the ſame Figure. I mean by two *Urachus*'s, two long roundiſh Bodies, of a depreſſed Figure, they ſeemed as big as a Knitting-Needle, and were of a darker Subſtance than the *Placenta* on which they lay. They appear'd in every reſpect, like that part of the Navel-ſtring, which is allowed by all Anatomiſts

* *Maur. Lib.* 2. *cap.* 3, and 4.
|| *Diem. Lib.* 1. *cap.* 31.
* *pag.* 19.
|| *Exercit. de Generat.*
* *Parey, Lib.* 25. *Cap.* 2.
* *Diem. Lib.* 1. *Cap.* 30. & 31.

to

to be the *Urachus*, and in like manner ſhrunk in two or three Days, from a Mucous Subſtance to a meer Membrane. Theſe two are the only entire Urinary Membranes that I have prepared. Yet in the *Secundines* that have come to my Hands, I have ever found three diſtinct Membranes eaſily ſeparable.

A Boy that never made Water, by Dr. Richardſon *n.* 337. *p.* 167.

XV. *John Worſnape*, a poor Boy at *North Bierley, Ebor.* lived till he was ſeventeen Years old, and never made Water, yet was in perfect Health, vigorous and active. He had conſtantly a *Diarrhœa* upon him, but without much Uneaſineſs. The Obſtruction muſt have been in his Kidneys, for he had never any Inclination to make Water. The Serous Part of the Blood, which ſhould have been thrown off by Urine, was diſcharged by the *Cœliack* and *Meſenterick* Arteries, by the Mediation of the Glands into the Guts. He died of a Fever.

A Boy making Water by his Navel, by Mr. Yonge, *n.* 323. *p.* 423.

XVI. I had once a Boy about 6 Years old brought me, that piſs'd off almoſt all his Urine from his Navel.

One ſwallowing Iron, &c. by Mr. Amyard, *n.* 31?. *p.* 12c.

XVII. 'Tis now, *Jan.* 20. 1708. about 5 Weeks ſince an Idiot from his Infancy died at *Oſtend*, the Place of his Birth, in the 33d Year of his Age; his Death having been preceded with twelve Days continual remitting Fever, and a conſiderable Tumour and Pain about the Region of the Liver. His Brother, in whoſe Houſe he had been a conſtant Dweller, being deſirous to know the Cauſe of it, deſired Mr. *Ricks*, an eminent Surgeon (who at that time had the Cure of me) to open him; but he ſent his Son, likewiſe Maſter Surgeon of the ſame City, with his Servant; who did the Work in the Preſence of the Brother of the Deceaſed and a Nun, brought thither by the deſire of diſcovering the Cauſe of her Spitting or Vomiting of Blood, to which this Fellow had been very liable, (as well as to bloody Stools) for 6 Weeks before his Death. A large Abſceſs or Impoſthume was found in each Lobe of the Liver, whoſe Bulk did far exceed the ordinary Stint. The Stomach being open'd was found extreamly contracted and ponderous; and indeed it was thought no Wonder, when upon the aperture of it, was found a Bundle of the things following cloſely involved and embraced by the Stomach, *viz.* Nine Cart-wheel Nails, and ſix leſſer; a large and long Iron Screw; two pair of Compaſſes, the one having a Circle two Inches in Diameter; a middle ſiz'd Key; a large Iron Pin as big as my Thumb, and four Inches long, with a Ring at the end of it; another of Braſs, but much leſs; the Handle of an Iron Spring-Knife (ſwallow'd as 'tis believed intire, but the ſides and two pieces making up the Spring of it, found aſunder; the Pegs of the Knife, tying thoſe ſeveral Pieces together, were not found;) the upper and lowermoſt end of a Braſs Pommel, inſervient to a Sea-coal Grate, weighing nine Ounces; a broad Piece of Lead weigh-

ing

ing three Ounces and a half: the whole consisting of 28 Pieces, weighing betwixt 2 and 3 Pounds. They were found all in a Bundle with the largest Ends one way, and the smallest the other; the small End of one of the large Nails was so bent, that it would have made a perfect Circle, had not the tip of that same Nail been bent back again; this End was forked and wonderfully sharp, as were likewise the Ends of the Compasses. None of the Pieces were found polish'd, neither cou'd I find the Brass nor the Lead any ways impair'd or endamag'd; but the Iron pieces were extreamly corroded, especially one of the Sides of the Knife, which had lain in the Stomach about 8 Months, was eaten quite through in two or three places, towards the Blade's End; and three or four Nails mightily indamaged did appear as if some particular Menstruum or Dissolvent had been poured upon them, capable only to dissolve that Metal, as *Aq. regalis* has the Property to dissolve Gold, *Sp. Nitri* Silver, Vinegar Lead, leaving those other Metals joyned and alienated with them untouch'd: the Lead had lain in the Stomach about eight Months, and the Brass Pin above-mentioned above twelve. It was very easy to guess at the time those different pieces of Iron had been in the Stomach, in considering how much one piece had suffered more than the other. This Observation is like to give a check to the Notion of those who believ'd that Ostridges did dissolve Brass and Iron by Friction only; for if so, I see little reason why the Iron Branches of the Compasses should have been found so very much worn out, and the Brass Branches not in the least impair'd. Mr. *Ricks*'s Son, who open'd him, told me, That the Stomach had been no ways wounded or indamag'd; which does not appear to me probable, when the Patient wns known to have vomited and evacuated Blood by Stole for six Weeks before he dy'd, as I have already mention'd It's necessary to remark, That this Fellow, from his Youth, had accustomed himself to swallow large Morsels, Glutton like, and without Chewing; which, no doubt, made the Passage of the *Oesophagus* wider, and disposed it to give Entrance to all those Extraneous Bodies. This Idiot, and sometimes Mad Fellow, was never known to sleep a Wink, tho' he was often compell'd to go to Bed, and had, to incline him to Sleep, been very much harrass'd and fatigu'd before: he was always known to eat three times as much as the rest of Mankind, and when furious, to grow quiet upon the approach of Meat.

XVIII. In *April* 1708. I was desired, to see a Woman of *Tiverton*, named Mrs. *Pear*. She is about 30, of a tender Constitution; had an ill habit of Body, and about Candlemas last a Fever; which continuing near three weeks, was at length overcome by Testaceous Powders, *Alexipharmacs*, but chiefly by the *Cortex*. In this Fever she had sowr Vomitings, and a pain in her Stomach; which remain'd long time, and, after the Fever, was accompanyed with a copious Salivation; with Wind, and pains in her side, to a degree extraordinary; under

Hydatides voided by Stool, by Dr. Musgrave *n.* 295. *p.* 1798.

all which ſhe labour'd to the time of my ſeeing her. About three weeks before my Viſit, ſhe was ſeiz'd with a Jaundice, and while taking Medicines (*Pilulas & Decoctum Ictericum Fulleri*) for that Illneſs, ſhe brought off ſeveral *Bladders, by Stool*; and continued ſo to do, ſometimes every day, at other times once in two or three days, ever ſince the firſt diſcharge of this kind, to the time of my Viſit. Theſe Bladders were of various ſizes; the leaſt that came off, was of the bigneſs of a great Pins head; the largeſt, equal to a Pullets Egg: They were alſo of differing Colours; ſome white, others more yellow, from the Liquor contain'd in them, which was a ſort of Gelly, like that of Hartshorn, ting'd more or leſs with Saffron.

Before the Diſcharge of theſe Bladders, there was (beſides the Symptoms already mention'd) a Coldneſs, and Sickneſs at Stomach, almoſt perpetual; with frequent Inclinations to Vomit, and Hyſteric Suffocations. Since that Diſcharge, theſe Symptoms are vaniſh'd, and ſucceeded by a Soreneſs of the ſame part, as if ſomething had been torn there. The Bladders came off without pain; many of them whole and entire; one of which I ſaw about the bigneſs of a large Gall, or Marble Stone. Others were broken, and appear'd not unlike the empty Skins of Corrants, Gooſeberries and Plumbs. Only one Bladder came away by Vomiting, and that broken; but to all appearance, had been as large almoſt as a Gooſe Egg. The Gelly thrown up with this Bladder, and which in all likelyhood had been contain'd in it, (before it broke in coming up) was thicker, and more fœtid, than was found in any of the other Bladders. The number of thoſe which came off by Stool, made ſeveral ſcores.

During the whole courſe of this illneſs, the Patient was rather looſe than Coſtive; had no manner of Appetite, and ſeldom ſlept without an Opiat. I found her much waſted in Fleſh, with a dead pale Look; ſuch as argued her to be very low. She had Stools of an unuſual Smell, no way natural, and had vomited a great deal of cold Phlegm. She was very willing to think theſe Bladders came from her Stomach, and urg'd the following Reaſons for her opinion; Firſt, for that, had they been originally in the Bowels, in all likelyhood the Purges (of which ſhe took many in the Jaundice) would, as ſhe ſaid, have carried them off much ſooner. Secondly, From the (almoſt) conſtant pain of her Stomach, and frequent inclination to Vomit, ever ſince her Fever, to the time of the Bladders being Diſcharged Thirdly, From the Rawneſs and Soreneſs of her Stomach, after the Bladders came off. There was no appearance, in any one of theſe Bladders, of ſuch an Order of Parts, or Organs, as ſhew'd them to be Inſects; nor upon Examination was there any Animal diſcern'd by the naked Eye, in the Liquor contain'd in them: The Medicines given, after I was call'd in, were chiefly of Vulnerary and Digeſtive kinds: That which did her moſt ſervice, (but it was after the Bladders were come off) was Tincture of Myrrh and Gentian, in large and frequent Doſes; and with a proper Vehicle:

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Under the use of this slight, but advantageous Medicine, from a very weak condition, she recover'd an Appetite, *&c.* and is now perfectly well.

XIX. A Gentlewoman between 40 and 50 years of age, in the Autumn of 97, drank some Aluminous Waters for a month or five weeks, and in a months time after the use of these Waters, found a pain in the Renal region, where she never had been afflicted with any before; this pain returned after the first Paroxysm in about a months time, and afterwards more frequently, till about the *Christmas* following it visited her every day; about which time she sent for me; and had when I came to her the Symptoms of a Stone in her Left Kidney, *viz.* a grinding, and sometimes a very acute pain on that side of the *Spina Dorsi*, a Vomiting, her Urine during the Paroxysm tinged with Blood, and in it Bloody *Ramenta*; but, what most surprized me, a dozen at least of *Hydatides*, some of the biggest of them 1 ½ inch long, their circumference equalled that of an ordinary Goose quill; in shape they exactly represented the *Vesiculæ Natatoriæ* in Fish, growing smaller about the middle, as those generally do, and were filled with a Liquor, which my taste and smell made me believe to be Urine; I never discovered any Pus in her Urine, nor had she any pain at the Sphincter of the Bladder, nor in the *Meatus Urinarius*, either before, at, or after making Urine. The Paroxysm generally lasted 3 or 4 hours; as soon as these *Hydatides* came away, (which they did not all at once making water, but at several times) the pain in her Back, *&c.* abated very sensibly, and she continued easie and well the rest of the day, excepting an external soreness, which the pain had caused. I thought these *Vesiculæ* at first to be Membranous, since their consistence was so tough as to bear taking out of the Chamber pot and gentle handling, but afterwards was convinced that they ow'd their origin to a glutinous slimy matter, because upon long standing in Urine or fair Water they quite disappeared and were dissolved, making the Water or Urine to look thick and turbid. By the use of Medicines all these Symptoms disappeared, and she continued well when I last heard from her, which was about two months ago.

Hydatides voided by Urine. n. 273. p. 897. by Dr. Davies.

XX. *Gobsill* (whose case you printed in the Transactions for *June*, 1699. Numb. 253. *) came lately to me, and told me the Stones grew very troublesome to him; that he had of late vomited up two of them, which he shewed me, and I caused them to be weighed. One weighed two Drams, and the other one Dram two Scruples and a half. He complains, that his Strength is of late much impaired; that he voids great quantities of Blood by Stool, which keeps him very weak. His Stomach is much decayed, and will retain but few things. His Hands are palsyed; always extream cold, and his Fingers contracted; he is not able to open them without Help, or keep them so, unless by Force.

One swallowing Stones, by Sir Charles Holt, *n. 275. p. 995.*

* *See Abr. Vol.* III. *p.* 92.

Force. His Legs are very likely in a ſmall time to be as uſeleſs to him as his Hands, for, he ſays, they begin to fail him, and in the ſame manner, grow cold, and have little ſenſation in them. But the moſt remarkable of all his complaints, was, a new Progreſs the Stones had either found or made. Formerly at night in bed, they us'd to get up (as he expreſs'd it) to his Heart, and upon turning to his Knees, or ſtanding upright on his Feet, they would drop one by one ſo diſtinctly, that they might be counted, and in this ſtate they always aroſe ſtraight up on the right ſide of his Breaſt; but now they riſe obliquely, and get under his right Arm, inclining towards the *Scapula*, and when they are in this place, by giving him a Blow with the Fiſt on his right Shoulder, they will all fall down in a Lump together, and may very plainly be heard to claſh on the other Stones, which lye as they did formerly juſt above the *Os Pubis.* I made the experiment before Dr *Fowke* and Dr *Davies*, and the matter of fact proved true as he related it.

Prune ſtones found in a Body. by Mr. Vaughan. *n.* 281. *p.* 1244.

XXI. A Gentleman about a Month before Chriſtmas eat ſome common Prunes, about two pound, or more, and about a Month or five Weeks ago he eat about a pound more: About a Fortnight before he died I found, according to his Complaint, that he had ſome Symptoms of the Stone. He had a violent Pain in the *Veſica*, and about the *Urethra*, with Obſtructions in his Urine, *&c.* I ordered him a Terebinthinate Clyſter, which gave him Eaſe; but ſeeing his Pains encreaſed, I adviſed him to a Phyſician: He made uſe of one of his Acquaintance, he likewiſe preſcribed Clyſters, with Diureticks and Narcoticks, to no Purpoſe. Upon his Death, I obtained Leave of his Relations to diſſect him; accordingly I did with the Help of a Surgeon, Dr. *Weſt* being preſent, with ſeveral others of the deceas'd Patient's Friends. I found upon the Diſſection, the Prune-ſtones paſſed into the *Inteſtinum rectum*, and had there made a Perforation, or Rupture into the *Pelvis*. We tyed one Part of the Gut, and cut out a piece, and emptied it. There was taken out 128 Prune-ſtones in number, beſides what we left behind *in ſtercore*, in the other part of the *inteſtinum Rectum.* There was likewiſe a large *Polypus* taken out of the Left Ventricle of the Heart, *&c.*

A Ball with a Plumb-ſtone in it, extracted by Mr. Yonge. *n.*282.*p.*1279.

XXII. *Sarah Swayn*, of a thin habit and middle ſtature, when but ſix years old, was firſt afflicted with a violent pain, together with a large hard ſwelling on the left ſide of her Belly, which laſted twelve hours, and then went off without uſe of any Remedy or ſenſible evacuation; and at the end of three months returned, laſted and went off, as before. Several years it obſerved that Period, and then changed its intermiſſion from three months to 3 weeks, and ſo continued till ſhe was 35 years old, in which time ſhe married, and bore one Child, the pain of which ſhe averr'd to be much leſs, than what theſe Paroxyſms gave her. During her pregnancy, her Pains nor Intermiſſions had no alteration, and in her whole Life ſhe found no Diet diſturb'd her but Milk and Salt

Meat

Meats. About 9 months before ſhe was cured, the Pain and Tumour encreaſed to the bigneſs of a mans two Fiſts, ſhe endeavoured by many Remedies to get eaſe, but in vain, till the torment and watching had ſo weakned her, that ſhe could not riſe out of her Bed nor lie down in it.

In this deplorable condition ſhe was adviſed by a Woman to take a doſe of Powder'd Jalop but it operated violently, and ſuddenly drove the pain from her ſide down to the *Anus*, where it reſembled a *Teneſmus*, *viz.* a conſtant and violent inclination to ſtools without being able to force off any thing, and after ſhe had been thus tortur'd four days her Urine ſtopt, and two days after that the charitable Neighbours, who had all along given her their beſt aſſiſtance, craved mine. I perceived by their report of the matter, that ſomething obſtructed the paſſage of her Excrements, and ſoon found it ſo by a probe; I then anointed the paſſage with *Popu leum*, and taking hold of the ſubſtance with a pair of large Forceps, made to extract Stones from the Bladder after Lithotomy, I drew it forth. Abundance of Wind and Excrements guſht out, and continued to flow till her Guts were emptied of all the matter, which had been ſo long retained; after which I ordered her an Anodine Clyſter, and a compoſing draught, and ever ſince (being ſeveral years) ſhe continueth well.

The thing extracted was round, ſomewhat oblong, having on it ſome ſuch impreſſions, as mens Fingers make on Wax or Plaiſter. It then weighed 10 Drachms, now ſcarce an Ounce, it was 5 inches in circumference, and altho it felt and otherwiſe appeared a Stone, it ſwam on Water, which made me, ſee the inſide of it, by cutting it in two with a Knife; externally it was black, and ſmooth as if varniſht, and no thicker, which made me before I had conſidered the whole, think it the expanded skin of the Plumb indurated; next to this thin blackneſs was a cruſt of matter like Brick, the thickneſs of an half Crown, within that appeared a ſubſtance reſembling Paſte-board or chewed Paper, and within that lay a Prune or withered Plumb, with the Stone and Kernel cut aſunder by my Knife. See Figure 3. *Plate* 5. *Fig.* 4, 5, 6.

Inſtances of other perſons who were hurt by ſwallowing of Plumb-ſtones, by Sir Hans Sloane. *n.*282. *p.* 1283.

XXIII. A Man in *Lancaſhire*, being for many Years ill of the Cholic, and receiving Relief from no Medicine, deſired he might be diſſected after his Death, to ſee what might be the Cauſe of his Diſeaſe. This was accordingly done, and they took out of one of his Guts a large Ball 6 Inches about, of an Ounce and half Weight made up of a Spungy Matter which ſwims in Water, and viewed by a Microſcope, appear'd to be made up of very ſmall, tranſparent Hairs or Fibres, wrought together, after the manner of the *Tophus Bovinus* taken out of the Maws of Oxen. In the Middle or Center of it was a common Prune or Plumb-ſtone, which had been ſwallowed, and ſticking ſomewhere in the Guts, had gathered that Subſtance about it which reſembled the ſmall Hairs on the Skins of ſeveral Creatures or Fibres of Plants we eat,

eat. Dr *Charles Leigh* in his natural Hiftory of *Lancafhire*, &c. in his firft Table has figured this Fig. 4. He fhewing me the fame, I was defirous to fee what it was made of, and had it cut for that purpofe, and found it of a hairy or fibrous fubftance, layer upon layer, or coat upon coat, over a Plumb ftone. This Ball with its Plumb-ftone and feveral Coats, my very good Friend Dr *Leigh* was pleafed to give me, and it is now in my poffeffion, and feems to be of the fame fubftance with that mentioned by Mr *Yonge*.

The fecond inftance I faw of thefe Balls was by the means of Dr. *Will. Cole*, who did me the favour to fhew me a Letter he had from the Country, and fome fmaller Balls than the 2 before mentioned, which had in their centers Plumb-ftones. The perfon he was confulted for, had, I think, the Colick to a great degree, and had voided feveral of them, they were not fo Sphærical, but of a compres'd figure, fmooth on the outfide and glaz'd as fome of the *Tophi Bovini* are; and feemed within of the fame fubftance with the former *ftratum fuper ftratum* upon a Plumb-ftone.

A third inftance like thefe I faw, through the favour of the late Dr. *William Stokeham*, who fhewed me a Ball about the largenefs of that I had of Dr *Leigh*, which had been been voided by a perfon after great Sicknefs, and preferved by the Patients Phyfician, who was one famous for practifing Chymically fome years ago, *viz.* Dr *George Thompfon*. I had this laft Ball in my Poffeffion fome time, and in appearance it was of the fame fubftance, but what was contained in it I could not tell, not being permitted to open it, but that Author tells us it had feveral Plumb and Cherry-ftones, in it. Thefe Balls feem to be form'd fomething after the manner of *Bezoars*, which generally have fome Seed for their Center or *Nucleus*, on which Coats of another fubftance are gathered.

I once faw as ftrange a Diftemper almoft as obftinate and long, as I ever met with, proceed from a great quantity of Strawberry feeds, which had lodged in the Guts, and after their Difcharge, the Perfon was eas'd.

Of fwallowing Bullace and Sloe-ftones. by Mr. Derham. *n. 349. p. 484.*

XXIV. The Man-fervant of a neighbouring Clergyman, complained of exceffive Pains in and about his Stomach; that he lay under a Dejection of appetite; and whenever he eat, he could not retain it, but in a little time vomited it up. By which means he was in a fhort time brought to a very low and languifhing Condition, infomuch as they begun to defpair of his Life. Upon this, he applyed himfelf to fome Practitioners in Phyfick; one of which ply'd him with ftrong Vomits eight Days together, with very little Signs of Succefs. But fome time after, having Occafion to ride fomewhat more than ordinary, he found himfelf very fore in his Stomach, and Sick; which ending in violent Vomiting and Straining, brought up the firft Stones he ever perceived to come from him, which were about Twenty in number,

ber. After this he had frequent returns of the Vomiting up of *Bullace* and *Sloe-Stones*, especially upon strong Exercises; particularly moving and stooping much in Weeding in the Garden; in Riding also, although it was only to water his Master's Horse. Upon these Occasions he would be seized with acute Pains in his Stomach, and soon after Vomit up more of those Stones. He hath counted above One hundred and twenty *Bullace* and *Sloe-Stones* that have been discharged; and many others he could not number, by reason they came up when he was in Riding or in his Business. He is not yet free of them, but is in Pain oftentimes, and Vomits them up, especially in Riding; but after he hath discharged them, he is much easier for a while. He commonly brings up a slimy Matter with them, mixed with Blood or something very like Blood.

The Cause of all this Disaster the Man assures himself was, that being in his Youth a great lover of Fruit, he used greedily to devour all sorts he could come at, and *Bullace* and *Sloes* being the easiest to be gotten, he used to ingurgitate great quantities of them, without evacuating many of the Stones by Stool, as he well remembers, and as he observed others did. These Stones he thinks have lain in his Stomach (some of them at least) above ten Years; but he felt no Pains till about four Years ago. And those at first were not so violent, nor attended with such severe Fits of vomiting, and loss of Appetite, as they by degrees came to be afterwards.

XXV. In agro Cestriensi, loco non longè a vico Malbano dissito, quidam Thomas Olton annum jam septuagesimum octavum, & quod excurrit, agens, homo sane probus, & tàm re quam famâ spectabilis per multos retrò annos vixit: qui quidem Vir cum vesicæ calculo graviter cruciatus esset, opem a me aliisq; Medicis circumdegentibus non semel petiit: eratque a nobis variis medicamentis pro rei exigentiâ, sed inutiliter tractatus. Tandem verò in uno paroxysmorum præ omnibus aliis atrocissimo, calculos duos, benedicente Numine, per Urinæ ductum excrevit: quorum formam exhibet *Fig.* 7. *Large Stones voided per Urethram by* Dr. Bullen. *n.* 295. *p.* 1804. *com. by Mr.* Lhwyd, *Plate* 5.

Calculum primò egestum dolor ferè intolerabilis comitavit, secundum verò vix ullus, qui molestiæ sensum crearet; nec mirum, si Urethra a lapide priùs emanante adeo disrupta, & lacerata, alteri jam transituro facilem & patentem præberet exitum. Duo autem hi calculi unum aliquando in vesicâ constituebant; quòd ex fracturæ, si committantur, convenientiâ, palam est. Ista via, ex neglectâ Chirurgiâ, jam usque patet; cui quidem incommodo Cornu peramplo, radici penis aptato, quoties Urinam mittit consulere coactus est; ne secùs tam sua, quam adstantium vestimenta inter mingendum, inquinaret.

XXVI. *Joshua* the Son of *Thomas Spurrit*, upon the *Quarry-Hill* near *Leeds*, having been for a long time sadly afflicted with the Stone, was the last Year in an extraordinary manner tormented. I have three Stones that *Another Instance. by Mr.* Thoresby. *n.* 336. *p.* 539.

that he voided, which are of a great bigneſs to paſs the *Penis*, and five more that he could not get rid of without the kind aſſiſtance of Mr. *S. Pollard*, a Surgeon, who by an Inciſion made way for them, as they came ſeverally near the *Glans*. When ever one of theſe great Stones broke out, there was a Crack within his Body, as if the Sphincter Muſcle, or Bladder itſelf, was rent. The Youth being diſſected, there were found in the top of his Bladder (which was contracted like a Purſe) two prodigious large Stones; one eſpecially which I meaſured, and it was rather more than 5 Inches and a half one way, and 4 the other; it weighed two Ounces, wanting 3 Drams: The other ſeems lighter, and weighs but one Dram above an Ounce. There were two very odd Stones taken out of the Right Kidney; the Left was wholly degenerated into a kind of Mucilage: And betwixt the Neck of the Bladder and the end of the *Penis* (which was mortified thereby) were lodged no leſs than half a dozen ſuch Stones as this herewith ſent you. There was little Moiſture left in the Bladder; the Ureters being broke off, and almoſt wholly conſumed.

A Stone in the Ductus biliarius, *cauſing the Jaundice. by Dr.* Muſgrave. *n.* 306. *p.* 2233.

XXVII. I ſaw at *Clifton* in *Dorſetſhire*, a Stone voided, ſome years ſince, by Stool; which was repreſented to me by Mr *Harvey*, as having come from the *Ductus communis bilarius*: But the Largeneſs of it is ſuch, as made the latter part of the account ſeem, at firſt hearing, ſomewhat dubious. Its Figure is Oval; the Length almoſt an Inch; the Breadth, (or ſhorteſt Diameter) $\frac{7}{10}$ of an Inch: It weighed 59 Grains, when I ſaw it; but, at its coming off, was (as I am inform'd) above a Dram in weight; Some part of it being, by frequent handling, rubb'd away. The Surface rough, unequal, divided into ſeveral little Riſings, each about the ſize of half a Vetch, or ſomewhat leſs. *Y. Z.* ſhew the Proportions exactly drawn. The many ſtrong annular Fibres, which appear not only at the Orifice, where the *Ductus communis* opens into the *Duodenum*; but alſo all along the oblique paſſage, of that *Ductus*, between the Coats of the Inteſtine, (which paſſage is, according to Dr. *Gliſſon*'s meaſure, about half an inch in length) do, by way of Sphincter, keep this end of the *Ductus communis* very ſtrait and cloſe. And beſides this ſtraitneſs of the *Ductus*, the two Oblique Inſertions, it makes at ſome diſtance from one another, thro' the two outer Coats of the *Duodenum*, render it yet more difficult, for a ſubſtance of any Bulk, to paſs this way. So that, however great Stones may be generated in the Gall-Bladder, *Ductus Cyſticus*, *Hepaticus*, or *Communis*, it is not eaſy to conceive, How a Stone of the Magnitude here deſcrib'd, could poſſibly, through a paſſage of itſelf ſo very narrow, ſtrait, and difficult be conveyed into the *Duodenum*.

Plate 5. *Fig.* 8.

To prove, That this Stone was not form'd in the *Fiſtula alimentaris*, but (large, as now it is) came this way into it, the Gentleman was pleas'd to let me know, That, before the Diſcharge of this Stone, He had the *Jaundice*; which came ſuddenly on him, and continued

ſeveral

ſeveral months, in a ſevere, and moſt exceſſive manner. That this *Jaundice*, beſide the diſcolouring of his Urine and Skin, to a very great degree; beſide Loſs of Appetite, Faintneſs, and many other Symptoms, uſual in this Diſtemper; was alſo accompanyed with a Pain (in or) near the Stomach.

That, during this *Jaundice*, his Stools were of a white colour, as having very little, or no Mixture of Choler in them. That, Travelling under theſe circumſtances, more eſpecially with a conſtant Pain, (as before mention'd,) in his Coach from *London* to *Clifton*; and, after a little time, to *Bath*; he found, a little after his Arrival at *Bath*, this Stone come off by Stool; and, together with it, almoſt a Spoonful of *Gravelly Matter*; and a conſiderable quantity of Choler, as appear'd from the yellowneſs of the Stool: All which happen'd ſo ſoon after he came to *Bath*, as evidently to prove, the Diſcharge of both [Choler and Stone] to proceed from the motion of the Coach. That his deliverance, from the *Jaundice*, commenc'd from the Expulſion of this Stone: For, ſoon after that, the Colour of the Skin and Urine, indeed all the ill Symptoms vaniſh'd; and, in a very little time, (Weakneſs only excepted) He recovered.

Theſe Propoſitions, put together, make a conſiderable Argument, That the Orifice of the *Ductus communis* (how ſtrait, and how ſtrong ſoever) was, in this Gentleman, ſo far dilated, as to give way to the Stone, here deſcribed; that is, dilated to a Circle, in Diameter $\frac{7}{10}$ of an Inch, in Circumference one whole Inch and $\frac{3}{4}$.

The *Jaundice* is often obſerv'd to be a moſt ſtubborn Diſtemper, not eaſily yielding to our moſt probable Methods; and many times to none at all. *Riverius* poſitively affirms, That, when it proceeds from a Stone, obſtructing the Current of the Choler, it is incurable: Urging this reaſon for his opinion; **Calculus, cum diſſolvi non poſſit, morbum facit incurabilem.* When the *Jaundice* is thus difficult of Cure, eſpecially when there is a probability (whether from a Pain fixt in, or near the Region of the Liver, or from any good Argument whatſoever.) That it ariſes from the Cauſe now mentioned; rather than to beat over the ſame ground to no purpoſe, or other ground equally improbable; it may not be amiſs, to adviſe Exerciſe on Horſeback, in Coach, or any other ſuch way, as ſhall be likely to diſlodge the Stone, and bring it off. But, to make this Exerciſe effectual, it ought to be Violent, as the Patient can well bear it; and in ſuch manner, as may, by much agitation of the Body, be moſt conducing to the Deſign in hand. The Hiſtory, here mentioned, does ſufficiently recommend this *Gymnaſtic Courſe*; as capable of relieving in ſome Caſes of the *Jaundice*, when the beſt methods of Phyſick (for ſuch we ought to ſuppoſe this Gentleman had preſcrib'd himſelf) fail of ſucceſs.

**de Icter.*

Large Stones evacuated, by Mr. Thoresby. *n.* 304. *p.* 2164.

XXVIII. Some large Stones were taken from *William Coldell* of *Green*, *May* the 10th, *Anno Dom.* 1693. They weighed 9 Ounces when firſt

evacuated, and were removed by Diet Drink with an Alkaly Powder, and a Magiſtral Stomach Plaiſter; that the Perſon died 7 years after, of one too large to be evacuated; for upon the griping of it, betwixt the Hypocondrias and Share-Bone, it felt to be as large as a Gooſe Egg.

Several ſolid bodies voided by Urine. com. by Mr. Yonge. *n.* 323. *p.* 414.

XXIX. *Nathaniel Mitchell* of *Loo* in *Cornwall*, aged about 50, was in the Summer 1690, ſeized with violent Colical Pains which he mitigated by Clyſters, but could not perfectly free himſelf of them. About *Michaelmas* 1691. his Pains being very violent, he was relieved by the ſame Remedy; and by the perſuaſion of a skilful Woman, he drank the Powder of Nettle-roots in White Wine; After the firſt or ſecond Doſe he diſcharged a great quantity of Urine, with a very feculent Sediment. About the beginning of *November* 1691. being Coſtive, he eat Mallow roots and Corinths boil'd and mixed with Butter, (his uſual Medicine to render him Laxative.) In a little time after eating it, he was much diſordered, and complained of an Oppreſſion by Wind; at length the Wind (as he termed it) ſettled at the bottom of his Belly, and in a very little time with his Urine he emitted ſome of the Herbs, with above 40 Corinths: A few Days after he piſs'd off ſeveral Parſley-Leaves, which he had a little before eaten. I was called to him about the 12th of *November*, when his Urine being ſhewn me, I thought that part of his Excrements had been evacuated that way, and that ſome Latent Ulcer had made a Paſſage through the *Inteſtinum Rectum* into the Bladder, but found it otherwiſe; for there was no Fætor in the Urine, he had no *Teneſmus*, nor bloody, nor purulent Dejections; but to ſatisfy my ſelf further in this Particular, I ordered him a Clyſter tinctured with Indigo, which he retained above half an hour, but his Urine was not at all diſcoloured with it. I preſcribed Pills of——, two of which came off in his Urine *November* 18. in an oblong form, about the bigneſs of the end of the firſt Quill in a Gooſe's Wing. The Pills I have by me, except the half of one, which I rubb'd abroad with my Fingers. Some time after he piſs'd off a piece of a Raiſon. He lived 'till Midſummer 1692. in which time he ejected at divers times parts of Roots, and other things he eat.

His Wife reſiſted all the Importunity that could be made to have his Body diſſected: So that a great Secret was buried with him.

Stones voided by Stool. by Dr. Holbrooke. *n.* 325. *p.* 28.

XXX. 1. One *Crumbleholm* came to me ſometime ago, and complain'd of a great loſs of Appetite, with Scorbutick Itch, and ever and anon ſevere Convulſive Cholicks below his Navel, all along the *Hypogaſtrium.* They laſt not above a quarter of an Hour, but often return, and raiſe Tumours the bigneſs of a large Walnut, which diſappear and remove as the Pain ſhifts. He has been troubled with it ſome Years, and took Phyſick of almoſt every one he met with; but, as far as I can perceive, not in any regular Method, which gave me ſome hopes, that I might relieve him. Accordingly I began with mild Emollient and Carminative

native Clysters; purged with *Decoct. Sen. Gereon. Syr. de Spin. Cervin. & Tinct. Sacr.* In the Intervals of the Purges I gave *Æthiops Mineral*, with bitter Decoctions Alterative, made more Carminative with *Rad. Zedoar.* and *Castor.* He was relieved for that purpose; his Appetite and Complection mended, but presently was as Ill as ever. Then he shew'd me the Stones voided by Stool, upon a slight Mercurial Purge, which he took last *Easter.* Upon opening one of them, I found he had swallowed either some Plumb or Apricock Stones, which by their stay in the Intestines were inclos'd in the Excrements, as I take it; and, by the Purge being dislodged from their *Sinus*, sent forth, as you find. Hoping then that by stronger Evacuations, if I could remove any other that might remain, it might tend to his Cure, I order'd stronger Medicines. However, I could not get any more from him; and he being out of hopes, and uneasy to be kept any longer from his Business, has left off taking any thing. Last Week I saw him, and found him much in the same Condition, tho' somewhat weaker, and sunk more in his Flesh.

2. I look upon these Stones to be not formed of *adhering Excrements*, as you seem to suppose, but to be made thus. When the Plumb-Stones happen to be included in a fit Glandulous Receptacle, I conceive they may come to be thus coated over by the viscous Liquor secreted out of the Secretory Ducts of those Glandules, which by long lying there may come to acquire so great Bulk, by the continual appulse of the same Liquor. This Receptacle I guess to be the *Intestinum cæcum*, which tho' small naturally, may be, as other Membranous and Glandulous Parts are, capable of a considerable Extension: So that, when by reason of the *Peristaltick* Motion of the *Intestines* above, one of the Plumb-Stones may happen to be, by its pointed Extremity, intruded; the whole may, by the same repeated, tho' slow Motion, dilate the Cavity so, that the whole Body of the Stone may by the same Method be still farther and farther protruded, till it come to the further Extremity; which being closed must be presumed to detain it there, since 'tis hard to conceive it can quickly get out again, that *Peristaltick* Motion being always forward. One of these Stones being thus enter'd, 'tis easy to conceive more may be admitted, since the first cannot but dilate the Passage for another that follows, and so on till the Cavity be full. Whilst these Stones lye there, they must be conceived to offend the part, as having extended it beyond its Natural State: So that the Secretory Ducts of the Gland, of which the inner Coat of that, as well as the rest of the Intestines, is constituted, must be proportionally dilated; whereby an easier way is made for the Liquor they separate, to be excreted. This being of a viscid and concrescible Nature, must, since it cannot get forth, be presum'd to adhere to the *Substratum*, the Stones, and so by degrees Incrust them; which Crust by the long confinement must grow so much thicker, for the same reason as it began, the Ducts being kept constantly open, and the Cavity more and more dilated

An Answer. by Dr. Cole. *ib. p.* 30.

dilated, the greater the Incruſtation is. So that I conceive the Symptoms are eaſily accountable for, from the offence given to the part, which being ſenſible, as all Membranous and Fibrous parts are, the Pain muſt grow greater, the greater the Extenſion is; and the change of the Poſture of the Tumor may very well be conceived to proceed from the different Poſtures the Inteſtines put on, by the Chyle or Excrements paſſing along them, and ſometimes filling one part, ſometimes another, as they are protruded further and further, their Lubricity on the Surface, Length, and Confinement obviouſly favouring that *Phænomenon*. I am of opinion, the true *Bezoar Stones* are form'd in the Beaſts, that yield them, in the ſame manner; but whether their Stomachs or Inteſtines have other Cavities capable of receiving and retaining them to their full growth, is to be determined by Anatomy. This I think is certain, that all of them have either a Straw, Stick, or other Subſtance different from the Exterior Matter, which we call the Stone, in the middle of them; and thence I conclude the manner of their Formation to be the ſame. From the continuance of his Symptoms, I believe there may be more behind; and cannot think any other Method more likely to extrude them, than by having his *Abdomen* well anointed with ſome Emollient Oyls or Liniments, and very well agitated backward and forward as much and as long as he can bear, and this both Morning and Evening: After a little while, that the Stones may be preſum'd by this agitation to be ſomewhat diſlodged, ſome gentle Purgatives I conceive may be of uſe to be now and then given to carry them downwards, and with all Emollient Clyſters to ſollicit it gently.

A Ball voided by Stool, by Mr. Thoresby. n. 291. p. 1595.

XXXI. A Girl about 14 years of Age, having been tormented with Colical, and as was ſuppos'd, Nephritick Pains for ſome time; at length voided a roundiſh Ball *per Anum*, as hard to feel upon as a Stone. After a while, the Pains returning with greater violence, ſo as to make her roul upon the ground, ſhe voided another as hard, and much bigger. Upon which, one Mrs. *Ward*, a neighbouring Gentlewoman, who had been much afflicted with Gravel, gave her ſome of thoſe Medicines which ſhe us'd to take herſelf. Whereupon the Girl voided a third Ball, alſo *per Anum*, with leſs pain, yet the greateſt of the three. The firſt of theſe Balls is ſmooth and gloſſy, of the colour of a right Hazel nut, 3 inches about, and ſomewhat compreſſed. The other two rough and gritty, and in like manner a little compreſſed into a kind of obtuſely triangular figure. The 2d, is 4 inches and a half round about; the laſt, 5 inches and a half. Conſidering their bulk, all 3 are very light, eſpecially the 2 latter and greater ones, of which the laſt weighs but 5 Drachms 36 Grains; and both of them ſwim in water. This lightneſs proceeds from the Matter whereof they conſiſt; which, in ſome places is purely Downy or Fuzzy; in others, mixed with a Gritty ſubſtance, yet not confuſedly, but regularly mixed.

The

The Fuzzy parts posseſt the central part of the Ball, with a ſmall particle of blackiſh Glaſs or other Vitrify'd ſubſtance in the very Centre it ſelf. Over which are ſeveral Coats, gritty and fuzzy, alternately ending in the circumference with a grit, much reſembling the Ground work and Superſtructure of the Oriental Bezoar ſtone. The Powder of one of theſe Balls ſcraped off with a Knife, is no way mov'd or affected with any ſort either of Alcaline or Acid Liquor dropp'd thereupon. Neither being burn'd doth it ſtink, it conſiſteth therefore of no Animal ſubſtance; but the Girl being of the Green ſickneſs age, the gritty parts (with the glaſſy particle in the Centre, as the moſt ponderous and leaſt moveable) ſeems to be broken off of Tobacco-Pipes, and grown'd ſmall between her Teeth; the downy or fuzzy to be lick'd or ſcrap'd off the Lean of Mutton, or the Rind of Peaches, or ſome other Part or Plant. Her Stomach kneading the Matter into a Coat, as her changeable Appetite ſupply'd it alternately with one or the other ſort.

Balls of Hair taken from the Uterus *and* Ovaria *of Two Women; by* Mr. Yonge, *n.* 309 *p.* 2387.

XXXII. 1. In *November* 1705, I was call'd to deliver a Woman 30 Years old, who had 4 Days laboured in vain to bring forth her firſt Child: The Head, being too big for the Paſſage, ſtuck immoveable at the *Os pubis*; ſo that I could neither faſten a Crochet, nor draw it out by a Cupping-Glaſs fixt to the Scalp with an Air Pump. In this Extremity I directed my Son to open the Childs Head, and take out all the Brains, with ſo much of the Scull as he could; and then by a Cord faſtned round the Neck with a Nooze, to pull it out, which was ſoon and eaſily done. The Child was Corrupted and ſtunk much, ſo did the *Lochia*, which flowed three Weeks; ſoon after they ceaſed, the *Menſtrua* appeared, and the Woman went abroad: About ſix Weeks after her Delivery, ſhe was ſeized with violent Convulſions, and Hyſterick Fits, which laſted near three Days; when a painful Tumor aroſe in the left ſide of her Belly, which ended in an Eruption of white thick Matter near a Pint, with ſmall Knobs of a Subſtance like the Yolk of boiled Eggs: All Symptoms immediatly vaniſht, only ſhe complained of the great Hollowneſs where the Tumor had been.

Four Days after this, the like Swelling appear'd on the right ſide of her Belly, which continued with a ſmall Flux of Matter about five or ſix Months, in deſpight of the many Remedies I uſed to cure her. About that time there appeared in the *Pudenda* a Bunch of ſomething like greaſy Wool, which being drawn forth, proved a Ball, or Wad of Hair, the bigneſs of a Turkeys Egg, immerſed in an Unctuous Slime; adhering on one ſide to a Membrane ſo large as the Palm of a Mans Hand: And in the midſt of it a ſmall Pyramidal Bone reſembling a ſplit Tooth. The Tumor ſunk upon this, and the Fluor ceaſed immediately, and her Lunary Flux (which all this while had not appeared) flowed as uſual, and ſhe continueth in perfect Health ever ſince, full nine Months. The Bone and Hair are fine, ſoft, and

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indifferently ſtrong, of no great length, of a light brown Colour, intangled like a parcel of Combings. *

* *Dr.* Hook's *Phil Coll. n.* 2. *Abr.Vol.*III. *p.* 15.

2. About ten Years ſince, Sir *Andrew Leak* gave me a ſmall Bunch of Hair, being part of what had been found in the Belly of a young Woman at *Deal*, by Mr. *Joſ. Nichols* a Surgeon there.

A. D. 1696. At 30 Years of age ſhe fell into a Periodical Fever, and afterward a total ſuppreſſion of her *Menſtrua*; which was ſoon followed with a Pain and Tumor in the right ſide of her Belly, which grew and encreaſed, maugre all the Remedies adviſed by the Neighbouring Phyſicians, till it became bigger and harder than that of a Woman in her laſt Month. When it had grown a full Year, it began to ſoften, and then the Cenſorious People who ſuſpected her Honeſty thought her in a Dropſie. At fifteen Months end, the Belly was ſo diſtended, that it ſeemed ready to Burſt; which made the Patient deſire the Phyſicians to adviſe Mr. *Nichols* to make the *Paracenteſis*; but all were ſurprized, when inſtead of Water there ruſhed out a pint and half of ſweet well digeſted Matter: The next Day he let out as much more, and then perceived Hair four or five Inches long iſſue forth with the Matter, but ſo faſtned in the Inſide, that he could not pull them out, the Woman complaining he would draw out a piece of her Belly.

She lived but four Days after the Operation; and on Diſſection of her Belly there was found ten Quarts of the ſame Matter which flowed through the Tap hole, and in it floating a Lump of Hair ſo big as an Halfpenny Loaf, wrapt up in a Fatty Matter, from which being cleanſed, it weighed full half an Ounce. On the Right ſide of the Womb he found a Protuberance bigger than a large Walnut, from which the Hair grew eight Inches long; that Tumor, or rather the *Ovary* being ſeparated from the *Matrix*, there was found in it a perfect Dog-Tooth, ſocketed in a Bone of a triangular Figure, in which another Tooth was growing; the Bone had a *Perioſtium* on it ſurrounded with Fleſh, faſtned at the *Calvaria* to the Scull. She was found on a nice and ſtrict Scrutiny, to die a Virgin, and Intact.

A Bunch of Hair voided by Urine, by the ſame, n. 323. *p.* 414.

XXXIII. 1. A Plethorick Woman about Fifty Years old, that uſed often to be afflicted with Nephritick Pains, employ'd me to relieve her *May* the 9th, 1707. I found by the Purulency and Stench of her Urine, that ſhe had not only Stones and Gravel, but an Ulcer in one or both her Kidneys; and therefore gave her a Doſe of *Cantharides* with *Camphire* made into Pills, and followed it with plentiful Draughts of a ſlippery Emulſion. This made her piſs off abundance of blackiſh Gravel, and white thick Matter like Bird-Lime, without any Pain or ill Symptoms, and ſhe continued eaſie for a Week; then her Pains returned, and went off by the ſame Remedy. About eighteen Days afterwards her Pain ſeeming to threaten a return, I repeated the Medicine; but that Night it gave her very great Pain in the ſide of her Belly, and at laſt threw her into Convulſions, which went off upon

upon the Discharge of Urine, of a great deal of Matter, and in it a Bunch of short Hair almost rotten: For some time after she used a Nephritick Course, which hath hitherto preserved her from the Return of Pain, Matter, Stones, and Impediment of Urine.

I herewith send a third part of that Bunch.

2. I viewed Part of the hairy Substance thro' a Microscope, and judged it to be the Hair or white Wool of a Sheep; which Wool was broken into such small or short Particles, that some of 'em were no longer than six Diameters of the Breadth of a Hair; which I suppose could not proceed from the Body of a Man, but that it was rather found in the Heel of ones Stocking. And the oftner I repeated my Observations, the more I was confirm'd in my Opinion; for I could not only discover the short broken woolly Particles, but I saw also a great number of the Ends grinded to pieces as it were; insomuch that not only the Bark (if I may so call it) or outside of the woolly Particles were rubb'd off, but the inward little Hairs, of which the Wooll is composed, were so divided from one another, that they appeared with their Ends like little Brushes. Moreover under the said Stuff or white woolly Parts, there lay very small Particles composed of exceeding slender Tubes or Pipes, which I look'd upon to be small Bits of Straw, and they were so small, that one Grain of Sand could cover 'em; there were likewise other small Particles of the same Figure, but I did not take them to be Straw, but rather the outmost Husk or Skin of a Grain of Wheat or Rye; and under those I saw one Particle cover'd all over with small Hairs, such as we see at the Top of Wheat or Rye; as likewise some few little Bits of Wood, somewhat thicker than a Hair of ones Head: there was also a small Particle of the outmost Skin of a Man, for I could see the little Scales of which our outmost Skin is composed very plain; Now these Particles that were not Wooll, might be very easily brought into the Stocking, in case one sets ones bare Foot upon the Floor before one puts it on.

Microscopical Observations on it. by Mr. Lewenhoeck, ib. p. 416.

There lay moreover in the said Matter an unspeakably great Number of exceeding slender long Particles, which I imagine to be those hairy Particles, of which a little Fibre of Wool (setting aside the Bark or Skin of it) is composed; as also several earthy Particles, which I took to be Part of the Dirt of the Floor or of the Foot itself. There also lay a great many particular little Figures, which I could not discover what they were; and these last mention'd Particles were so strongly joyned to some little Hairs or Wool by the perspired viscous Matter from the Foot, as I suppose, that I could not separate 'em but by the help of some Water. I observed amongst 'em one small Particle, not of a single Feather, such as it appears to our naked Eye upon the Body of a Bird, but rather of the finest Down; and the more I unravell'd or separated the Particles of Wooll from one another, still the greater reason had I to judge, that the Person who had worn the Stock-

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ing had been used to go often bare-footed upon the Floor. Now supposing that these woolly Particles might have fallen into any Spoon-Meat thicker than ordinary, the Person might swallow it down without being aware of it. Now my reasons for guessing these woolly Particles to come out of a Stocking, and to have been occasion'd by the motion of the Foot, are because I my self always wear heavy white woollen Under-stockings, and I lye in the same; insomuch that I can wear 'em three Weeks together, because I am not inclin'd to sweat in my Feet; Now having several times view'd the broken woollen Particles which lye in a heap as it were cleaving together under the Heel, and having also singled out of them several Fibres or Threads of Wooll, to prove that they are composed of little Hairs; and these woolly Particles exactly agreeing with those that were sent to me; I could no longer doubt that the said woolly Particles that were so sent to me, were any ways different from those Particles that were found in the Heel of the Stocking; 'tis true that amongst the woolly Particles of my Stockings I never met with any Wood or Straw, but the reason of that was, that I have not touched the Ground with my naked Feet for some Years, being unable to bear any Cold in my Feet; nay, so far that in the Nights, even in the Summer time, I put a Tin or Pewter Bottle filled with warm Water to the bottom of my Feet, by which means I preserve my self, as I fancy, from that Plague call'd the Gout.

Dissection of a Woman dying in Child Birth. Communicat. by Dr. Silvestre. *n.* 269. *p.* 787.

XXXIV. M. Duchesne quadraginta circiter annos nata, plethorica, & quæ sæpius etiam gravida menstruum patiebatur fluxum, & cæterum optimè valens, & obesa jamque vigesimum prægnans, urgente legitimo partus tempore, circa quartam pomeridianam die Novembris 12. Anno 1697. horrenda Uteri Hæmorragia corripitur. Vocatus post horam nonam *Jacobus Arnaudin* Obstetriciæ non minus Artis quam Chirurgicæ peritus, & gravissimo illo sanguinis profluvio certior factus, nullo prorsus remedio urgenti adeo sanguinis jacturæ occurri posse putabat, nisi quamprimum sarcina sua Uterus levaretur. Sed, ut plerumque fit, adstantes nihil temere moliendum, & rem totam naturæ permittendam clamabant. Exinde viribus prorsus exhaustis, circa mediam noctem iterum accersitur prædictus *Arnaudin*, qui nulla interposita mora operi se accingens intra dimidium horæ Puellam mortuam extrahit, secundis integris & illæsis absque labore subsequentibus. Uteri tamen Hæmorragia usque perseveravit; hancque summa jactatio, Pulsus debilitas, Lipothymiæ frequentes, sudor frigidus, motus convulsivi subsequebantur. Tandem circa sextam horam matutinam vitam cum morte commutavit.

Die sequenti cadaver secuimus: Aperto Abdomine, & remotis tum pingui admodum Omento, tum intestinis tenuibus, uterus in conspectum veni, instar majoris Cucurbitulæ, crassus ac satis distentus. In dextro ligamento lato Ecchymosis ingens erat, quæ ex latere uteri, illic ubi Vasa

Vasa *Spermatica* & *Hypogastrica* per Anastomosin uniuntur, descendebat, & in inferiori parte Colli Matricis, ubi scilicet intestino recto incumbit, maxime conspicua erat. Ut omnia facilius examini subjicerentur, Uterum cum annexis resectum, & e pelvi extractum tabulæ imposuimus. *Vagina* admodum laxa erat: Orificium internum adeo apertum, ut dimidium pugnum facile excepisset. Utero secundum longitudinem secto, hæc conspiciebantur: fundum grumis aliquot sanguinis scatens, cæterum integrum & sanum: in hoc autem *Placentæ Uterinæ* adhæsio nobis facile conspicua; quo loco etiam interna Uteri substantia inæqualis, crassior, & quasi carnosior observabatur. In inferiori parte colli uteri causa mortis apparuit; Laceratio nempe duorum digitorum capax, quæ etiam extrinsecus, ubi scilicet ea Ecchymosis, conspici poterat. An a causa aliqua externa uterus disruptus fuerit, an potius a vehementiori Calcitratione Infantis, ut aliquando accidisse memorant observationum Medicarum scriptores, ignoro: Sed exinde aperta fuisse majora vasa, quæ enormem illam sanguinis copiam effudere, certum est. Diductis tunicis, & tubulo inter duplicaturam immisso, flatus ex plurimis angustis foraminibus, cum ex ea fundi parte cui *Placenta* jungebatur tum præcipue ex lacera Colli parte, undique erumpebat. In dextro *Ovario* Vestigium seu Cicatriculam manifeste notavi, ex quo *Ovulum* deciduum, a tubæ cavitate exceptum, ejusdemque Motu Peristaltico ad uterum delatum fuisse mihi dubium non erat. *Ovarium* sinistrum flaccidum & quasi exuccum: *Tuba* ex utroque latere aperta, ligamenta rotunda, valida; reliqua omnia sana observabantur. Licet Uterus subito post Puerperium contrahatur & coarctetur, adeo ut, quod modo dixi, in hac fœmina sex circiter horis a partu mortua, Cucurbitulæ majoris capacitatem vix superaret; attamen illum instar marsupii, vel sacci digitis extendere facile potui. Et revera fibras carneas in graviditate sensim adeo elongatas, & tanto intervallo a se mutuo discretas, non nisi post aliquod tempus pristinum tonum & robur acquirere posse, quis non videt?

A Hydrops Ovarii, *in a Puerpera com. by* Dr Douglas *n.*308. *p.*2317

XXXV. I lately opened the Body of a Woman, aged 27, who dyed the third Day after Delivery, on which I made the following Remarks. She measured round the Waste a Yard and three quarters, and from the *Scrobiculus Cordis* to the *Os Pubis* a Yard and a quarter. All the cutaneous Veins of the *Abdomen* were of a very unusual and extraordinary Bigness, and very much distended with Blood. From the largest of them, being opened, I extracted several polypous Concretions. The *Cuticula*, from the *Umbilicus* downwards, was rough and scaly to the naked Eye. In several Parts it appeared gangreened, occasioned probably by the Sharpness of the *Serum* that always ouzed out of it, when she scratched the little Pimples or Wheals that arose on its Surface; these for some time used to go off without any Scar, but as her Strength decayed they became mortified.

Upon all the *Regio Epigastrica* the outward Integuments were very thin, little or no fat being visible: But from the upper part of the *Regio Umbilicalis*, down to the *Os Pubis*, the Skin was almost half an Inch thick, of a whitish Colour and hard, some of it appearing as if it were granulated, caused by some obstructions in the Milliary cutaneous Glands. The Fat under this part of the Skin did exceed the thickness of an Inch, being distinguished into several Lobules of an irregular Figure, and lodged in so many Cells adhering to the *Membrana adiposa*, which here also was much thicker than it usually is in a natural state. Her Thighs, Legs and Feet were all *Anasarcous*, being extremely big and swelled, easily retaining any Impression made by the Fingers: And her Nurse told me, that she used to wet a great deal of Linnen in drying up the Water, that would always issue out from these parts on the least rubbing, yet all her superiour parts were extreamly lean and emaciated.

The fleshy part of the *Abdominal* Muscles was much extenuated by the great distension, yet their Tendons were as thick as usual; and being very easily separable one from another, I could plainly observe that the Tendon of the *Obliquus Internus* adhered firmly to that of the *Transversalis*, along the edge of the *Musculus rectus*, and was not double, as *Realdus Columbus*, and all Anatomists after him, down to *Diemerbroeck*, who was first aware of this mistake, have maintained: However, this streight Muscle derives the same Benefit from this situation, being as it were hemm'd in on one side by this firm adhæsion, and on the other by what they call the *Linea Alba*, as if it had indeed been inclosed between the two supposed Tendons of the *Obliquus ascendens*; that is, 'tis much strengthned thereby in time of acting. I observed also, that the Tendons of the two oblique Muscles, and the fleshy part of the *Transversalis*, between the anteriour Spine of the *Os Ilium* and the *Pubis*, near its commissure, did inseparably join and unite with one another, forming as it were a thick and hard border, from the outside of which there was continued over the Blood Vessels, Nerves and Muscles, on the Fore part of the Thigh, a large *Aponeurosis*, which braced them down: The two *Laminæ* of the Membrane of the *Abdomen* being expanded on its inside. Now this border is what Authors call the *Ligamentum Pubis*, and what I have in another place supposed to be the firm union of the Tendons of these three *Abdominal* Muscles, with the *Peritonæum*.

Vid. Myograph. comparat. Specim. pag. 5.

Having perforated the *Abdomen* in the most convenient depending part, for it would have been endless labour, considering the great bulk of the Tumor, to have laid it bare, by freeing it carefully from the Muscles and *Peritonæum*; there issued out with great Impetuosity in a rising stream a vast quantity of slimy Viscid Water, in colour and consistence very much resembling a brown, thick and ropy Syrup. This Water measured between 16 and 17 Gallons, besides what was lost on the Floor, and imbibed in Sponges and Linnen made use of in drying

ing it up. When the Water was quite emptyed, I fancyed it had been all contained in a duplicature of the *Peritonæum*, and had made a Dropsy in that Membrane, because none of the *Viscera* appeared; for in such a Case I have more than once observed, that the inner *Lamella* of that Membrane of the *Abdomen* being separated from the outer, is forced inward by the weight of the Water upon the Bowels, to which it closely adheres, contracting the Guts and Mesentery into a very small volume. But upon a narrower view I perceived that the thick Membrane, including the Water, could be easily separated from the *Viscera*, having freed it from its adhesions by membranous filaments to the *Peritonæum*, and by Blood Vessels to the *Omentum*. Now this Bag reach'd from the *Pubis* to the Midriff, and from the Left Region of the Loins to the Right; in a word, it filled up the whole cavity of the *Abdomen*, distending her Belly so far, that a Plate could easily lye upon it, when alive. Having gradually freed it from all the neighbouring parts, and rolled it up, I found it adher'd inseparably to the Left *Tuba fallopiana*, the Spermatick Vessels being ramified upon it; and observing no *Ovarium*, which in the other side was naturally disposed, I concluded that the Bag was nothing but the Membrane of the *Ovarium* covering the *Ova*, preternaturally thickned and distended by the collection of the above mentioned humor, and that the Distemper was a true *Hydrops Ovarii* inasmuch as all this vast quantity of Water was included in one Bag, being all of the same colour and consistence.

All the other *Viscera* in the *Abdomen* were sound, and in their natural state. In both Cavities of the Breast there was contain'd a great quantity of reddish Water. The Liquor in the *Pericardium* was very abundant, and of a greenish hue. The Right Lobe of the Lungs was ty'd to the *Thorax*, covering the upper part of that Cavity, but the Left was free from any adhesion. In the left Ventricle I found a large Polypus or Serous Concretion, of a round figure, a white colour, and of a pretty hard Consistence, with several long Roots of a Red colour, which extended thro the Auricle and Bulb of the Pulmonary Vein into its nearest divarications in the Lungs.

Having carried home this large Bag, with the *Uterus* appendent, cut off below the Orifice of the *Meatus Urinarius*, and viewed it at leisure: I observed, that the Right Spermatick Vein, which opens into the *Cava* a little below the *Emulgent*, was three times larger than the Left; and from a little above the *Ovarium* it was continued, without any division to its termination. The Right *Ovarium* was in a very natural state. The *Cicatrix* or *Caruncula*, whence the fæcundated *Ovulum* had dropt, was yet remaining, and the Blood Vessels were ramified upon this *Testis*, in a very pleasant and beautiful manner. The *Tuba Fallopiana*, with its *Fimbria*, were all well disposed.

The Diameter of the Left Spermatick Vein, which opens into the *Emulgent* of that side, was much less than ordinary. And from the extraordinary narrowness of the bore of this Vessel we may draw a not

very improbable Reason of some Cause of this Watery Swelling; for the Blood being hereby hindred in its Reflux to the Heart, a great deal of *Serum* or *Lympha*, thro its slow return, must needs be thrown off upon the *Ovarium*, already indisposed, whence the gradual Increase of the Tumor did proceed. The two Spermatick Arteries were contorted, and full of turnings and windings, from their meeting with the Veins to the *Ovaria* and *Tubæ*. A little below the Kidneys each Artery sent out a Branch, which was lost on the *Peritonæum*, and fatty Membrane of the Kidney: And from the same places the Veins received two considerable Branches. One of the Arteries went off by a narrow Orifice from the side of the *Aorta*, the other rose up from its middle, a little below the first.

Between the Bag and the *Uterus* all these Vessels were much dilated, making several Turnings and Circumvolutions upon the *Peritonæum*, called in this place the *Ligamentum Uteri latum*. The Left *Tuba Fallopiana* was only remarkable in its being much longer and larger than usual. In the Bag, which was nothing but the Membrane called *Dartos*, which covers all the *Vesicular* Glands of which the *Ovarium* is compos'd I observ'd several little Bladders of several sizes, distinct from one another, which contain'd a limpid or clear slimy *Serum*, in Colour and Consistence like a Mucilage of the *Semen Cydoniorum*, these were either Hydatidal Tumours only, or the Eggs themselves distended. This Liquor hardned by a slow heat into the Consistence and Colour of the White of an Egg.

All the *Fundus Uteri* was about an Inch and a half thick, but near the *Collum minus* it grew something thinner, which did proceed from the distention of its Spongy and Vesicular Substance, by the Blood in the Vessels running thro it in variety of turnings and windings; so that when it was cut, it very much resembled the substance of the Lungs. Upon the inner Membrane of the *Uterus* I observed, upon wiping it with a Sponge, several little Eminencies, which I took to be the Glands mentioned by *Malphighius*, which separate a Humour, to lubricate and moisten its cavity. On the upper part of the *Fundus Uteri* I took notice of a great number of small Vessels, like slender Filaments or Threads, running off from its Membrane, and terminating into a reddish and soft Spongy sort of substance, not unlike the *Uvula*, bating its colour, which hung down from that side of the Womb in form of a Nipple. These perhaps are the Vessels, which, in the opinion of some, do separate and excern the Matter of the *Lochia* and the *Menses*, they being only visible at those times.

Near the beginning of the *Tubes*, I perceived two *Tubercles*, or little Bunchings, about the bigness of a Nut, to which perhaps the *Placenta* was fastned, and to these adhered Glandules of a Blackish Colour, of different sizes. The *Collum minus* was composed, as it were, of two *Labia*, the uppermost was most protuberating, and upon it I observed several small Glands, out of which, upon compression, issued a viscid

cid clear Liquor, which is faid to feal and clofe up this part, in time of Pregnancy. The lower *Labium* was longer and thinner, its Edges being cut or indented in feveral places. The *Rugæ* in the lower part of the *Vagina* run as they are reprefented in Books, but thofe in the upper part had a quite different Courfe, as they are exactly delineated in the annext figure. Near the Orifice of the *Meatus Urinarius* there were obfervable two very large Caruncles, in Shape like a Mulberry.

This is what I obferved in the opening of this Woman, I come in the next place to relate, as far as I was informed, the Symptoms that accompanyed her big Belly, and the Method made ufe of for her Recovery.

About three years ago, not long after fhe had lain in of her firft Child, fhe had a violent blow upon the Left fide of her Belly, very painful for the prefent, but in two or three days, upon keeping herfelf quiet in Bed, the pain and anguifh went off. About two months after this, fhe began to feel fome fmall pain in the Left Hypogaftrick Region, where fhe had lately received the Blow; and fhe obferved that fide of her Belly to grow abundantly bigger than the other: Thefe pains increafed more and more, till they grew very violent, but upon the Conception, which was three months after fhe was firft afflicted with them, they went off, and her Belly fwelled gradually, as is ufual in Pregnancy, having no other Symptoms but what is incident to that ftate, only fhe was much bigger than ordinary; and on that account fhe forbore the ufe of Medicines, which poffibly might have been effectual in the beginning of her Diftemper, had fhe been aware of her Danger. After Delivery, the fwelling and bulk of her Belly continued much the fame as before the Birth, only upon a plentiful evacuation of the *Lochia* it decreafed a little. When her month was up, fhe ufed Emeticks, ftrong Catharticks, Diuretick Dyet-drinks, and all the train of Medicines commonly ufed in a Dropfy, her fuppofed cafe. All the effect they had, was to prevent the farther Increafe of the Swelling while fhe ufed them; but upon leaving them off, the Tumor increafed very remarkably.

Thus fhe continued about one year, and then fhe conceived again, which fhe fufpected by the ftoppage of her *Catamenia*, having always been very regular but at fuch a time. Her Stomach was always good, fhe never was very thirfty, fo drank but little, made Water freely and in great quantity, and was attended with none of the Symptoms of an *Afcites*, except the Swelling of her Belly: Only when fhe was half gone, her Legs began to fwell and pit, growing very big all of a fudden; from thefe, and likewife from her Belly, there would often iffue out a great deal of watery Humour upon rubbing, as I have mentioned already, efpecially if fhe fcratched the little Pimples, that would often arife in thefe parts. About this time fhe began to be afflicted with a difficulty in breathing, with a violent Trembling and Palpitati-

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on of her Heart, and to be often ſubject to great and involuntary Sighings. She was not able to lye down, but was ſtill obliged to ſleep in a ſitting poſture, for fear of being choak'd. After ſhe was brought to Bed of a live Child ſhe became exceeding weak, being unable to fetch her Breath, and complained much of a heavy Load and Oppreſſion on her Breaſt, and the third day ſhe expired.

The Explanation of the Figures. Plate 6.

Fig. 1. repreſenteth the *Glandulæ Renales,* the *Uterus,* with the Parts belonging to it, and the large Bag or Membrane of the *Ovarium* præternaturally diſtended.

a The *Glandula Renalis* on the Right-ſide. *b* An Eminence, or riſing in its middle. *c* A Vein that runs from it to the *Cava.* *d* The *Glandula Renalis* on the Left Side. *e* A Sulcus or Furrow in its middle. *f* A Vein running from it to the Emulgent. *g* A ſmall Vein that comes from the Diaphragm, and opens into this Vein before it leaves the Gland. *h h* Two ſmall Arteries from the *Aorta.* *i i* Two nervous Twigs from one of the Intercoſtal *Plexus's.* *A A* The Kidneys. *B B* The *Uterus* cut off. *C* The *Cava* cut off. *D* Its Diviſion into the *Rami Iliaci.* *E E* The Internal Branches into which the *Hypogaſtricks* open. *F F* The Emulgent Veins. *G* The *Aorta* cut off. *H* Its Diviſion into the Iliacks. *I I* Its internal Branches, which are ſpread upon the *Uterus.* *KK* The external Iliacks of both Veſſels. *L L* The Emulgent Arteries. *M M* The Spermatick Veins. *N N N N* The Spermatick Arteries, very much contorted in their Progreſs, that on the Right ſide being cut off. *O* The Union of the Branches of the Spermatick Vein on the Right ſide. *P* The Right *Ovarium,* with Blood Veſſels ramified on its outer Membrane. *Q* The Right Tube. *q* Its *Fimbria.* *R* The Tube on the Left ſide, its *Fimbria* adhering to the large Bag. *S S S* The Membrane of the Left *Ovarium,* diſtended to a vaſt Bigneſs, with the Blood Veſſels ramified upon it. *T* Some of the *Ovula* grown big. *W* Some Hydatidal Tumours on the Inſide of the great Bag. *V V V* The *Ligamenta lata.* *U* The *Fundus Uteri.* *X X* The *Ligamenta rotunda* ; the Membrane that covers them being laid open, that the Veſſels of which they are compos'd may be view'd. *Y* The *Vagina* cut off. *Z* The *Veſica Urinaria.* ** A ſmall Artery and Vein on each ſide, the firſt going off from the Spermatick is ſpread upon the *Membrana Adipoſa* and *Peritonæum* under the Kidney ; the latter bringing back the Blood from theſe Parts, opens into the Spermatick Vein.

Fig. 2. Sheweth the *Vagina* and *Uterus* cut open.

A A A The *Fundus Uteri* laid open, and its ſides folded back. 1 1 1 1 The inner ſpongy Subſtance, with the Orifices of the Hyſterick Veſſels. 2 2 The Glands appearing on the inner Membrane of the *Uterus.* 3 3 The ſmall Veſſels, by which the *Lochia,* &c. are ſeparated. 4. A ſoft Subſtance, depending from the upper Part of the *Uterus,* into which the foreſaid Veſſels terminate. 5 5 Two Tubercles, ſeated near the Beginnings of the *Tubæ,* to which the *Placenta* adher'd. *B B* The

Fig. 1.

Fig. 3.

Fig. 4.

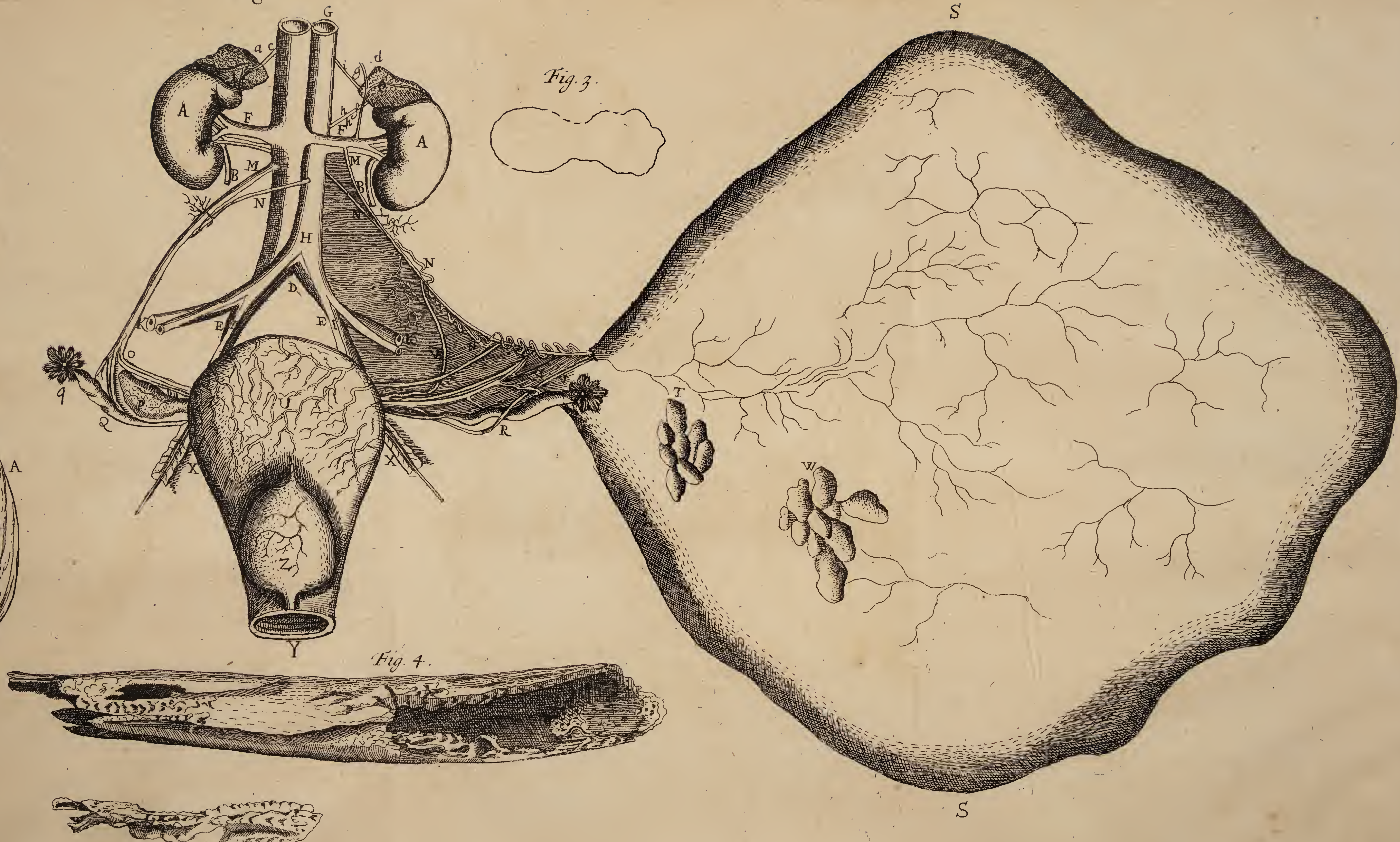


Fig. 2.

Vagina laid open. 6. 6. The two *Labia* of the *Collum minus.* 7. 7. Several small Glands plac'd on the upper *Labium.* 8. The Course of the *Rugæ* on the upper side of the *Vagina.* 9. 9. Their Direction on the under side of that Part. 10. 10. Two Orbicular Substances, near the Orifice of the *Meatus Urinarius.*

XXXVI. One *Raper*'s Wife at *Coxwold*, 12 Miles from *York*, falling in Labour of Child Birth, the Midwife using her Endeavour, extracted the Secundine, it offering first, and could not, by her best Skill, perceive any thing else remaining: The Woman's Body falling, and for some Days being pretty easy, and the Womb contracted, the Midwife took the Secundine to be a Mola, or false Conception, but in about a Week's time she began to discharge plenty of fœtid Matter by the *Vagina*, which continued, and in time she felt a troublesome Hardness of the *Hypogastrium*, which encreased daily for above six Weeks, by which the Woman was brought so low, that they despaired of her Life. This Lump, Hardness and Soreness wrought up to the *Regio Umbilicalis*, and I suppose continued there fixt for about a Month, at length being exceeding painful, the Neighbouring Gentlewomen took it for a great Boil or Apostheme, and apply'd what they usually do in such Cases, to assist its Suppuration and Breaking, which had its Effect, and it broke upon (or rather under) the Navel, discharging then (and afterwards) a great quantity of a thin Fætid and discoloured Liquor. The Part about it mortified, and the Ulcer inlarged to that Bigness, that a Man's Hand might be introduced therein; it continued exceeding painful, and emitted such an exceeding Stench (that neither her self (being extreamly weak and faint) nor any Body else could endure to look on it; a small time after they found some little Bones wrought out of it, which they shewed to me, I found them to be the Bones of a Child's Finger, which made me curious of going to see her. When I examined it, I perceived the Fœtus in a confus'd Heap or mortified Lump, for with my Probe I felt several Bones, and at that time extracted (after I had separated and dilated the Mortification about it) above half the Ribs, some Vertebres of the Back and other Bones, and cut out above a pound of the Child's mortified Substance, as black as Ink, with an extream nauseous Smell. Every second or third Day for a Month, I extracted what I could, being forced to do it very deliberately, by reason of the exceeding Weakness of the Woman, who had certainly dyed in the operation, had I forcibly extracted it, and not given her time; for we were obliged every Moment to support her with Cordials, and after every Operation she found her self lightsomer, and by degrees sweeter, which gave me Hopes of her Recovery, which before I had no Thoughts of. For not only the *Linea Alba* and Muscles of the *Abdomen*, but the *Peritonæum* and *Omentum* was mortified to a great Breadth, and the Intestines lay fairly in view, and exposed to the Air a long time. When I had extracted Part, and had

A Fœtus voided by the Navel. by Mr. Birbeck, *n.* 275. *p.* 1000.

a plentiful discharge of thin Fætid Matter, the other discharge downwards began to lessen and abate; so that I endeavoured to assist it by bandage and compresses, with deterging and drying injections up the *Vagina*, by which means in a little time I had no discharge that way, and those parts became shortly perfectly well, and in some time after the Ulcer separated (with the assistance of Fomentations, good Digestives and Mundificatives,) from its Putrifaction, contracted and united wonderfully, and hath now been quite cicatrized near 3 months ago, all the whole Abdomen being soft, easie and well condition'd. The Woman laboured all this Season at Hay and Harvest.

The Bones of an human Fœtus, voided through an Imposthume in the Groin. by Sir Phil. Skippon. *n.* 302 *p.* 2077.

XXXVII. *Jan.* 21. 1662. I visited a Woman 66 Years old in *Drury Lane*, who had a Child consum'd in her *Uterus* about 28 years ago; She bore two Children after this, one lived 11 Years, the other 16. About 8 Years ago, an Imposthume broke out in the right Inguen; and then several Bones of a dead Child were expell'd. She hath a great Swelling now in the Groin, where she feels somewhat very hard, which she suspects are Bones.

Two extrauterine Fœtus's by Mr. Yonge. *n.* 323. *p.* 426.

XXXVIII. 1. I receiv'd an Account from a very Learned Divine of *Devonshire* in these Words——; 'A Gentleman's Servant having kill'd an 'Ewe, which was thought fat, and taken out the Bowels, found a very 'unusual and monstrous lump of Fat, proceeding like a Wen from the 'middle of the *Omentum*. I was call'd to see this Wonder; and having 'cut it open, found inclosed a Lamb of the same Parts, Feature, and 'Dimensions with others of that kind. How it came there? And how 'nourished? are Questions I would have resolved.

I soon apprehended what it was that seem'd so very strange and unaccountable to my Friend, having thirty Years since been shewn the like, found in a Bitch, by an expert and ingenious Surgeon in *Oxford*; and from that time observed, and considered all of that Nature which have occured to me in Books, or otherwise; and so was ready to tell him that that Lamb was not conceived in the Womb, but in one of the *Fallopian Tubes*; wherein growing too big to be contained, it either broke out into the place where it was found, or slipt back toward the upper Orifice, and thorough it into the Belly: That afterward assisted by the prone and inclining Posture of the Sheep's Body, it slipt forward to the *Omentum*, and was there nourished the usual way, *viz.* by the *Placenta*, which was doubtless fixed in the *Tube*, and the *Pedunculus* being kept whole, will easily extend from thence to the *Fœtus*, where it lay.

2. I am told by a Gentleman Hunter, that he lately found in the Paunch of an Hare, two full grown young Ones among the Bowels, but almost rotten: and three immature *Embryos* in the *Uterus*. The former were certainly *Fœtus*'s broke out of the Womb.

XXXIX.

XXXIX. 1. On *Saturday* last I gave my self the Satisfaction of visiting a Woman brought to Bed *Nov.* 6. 1709. of a Boy, that had Cry'd in her Womb, at times, for five Weeks wanting one Day. The Child appears to be Lusty and Strong, and is, since its Birth, a very quiet Child. She told me, the first time the Child cry'd was in the Night, as she lay in Bed, after a great Pain which forced her out of Bed, and gave her Apprehensions of her Labour being nearer than her Reckoning. And every time after, whenever the Child cry'd, she had violent Pains like those of Labour. From the very first time of its Crying, the Child settled it self on the Mother's Left-side, and she never perceived it to stir in the least, 'till its Birth approached. Scarce a Day in all the five Weeks escaped without Crying little or much. But the Women observed, that every other Day it cry'd the most, and most certainly. The Midwife told me, she heard it cry seventeen times in half an Hour. Its Crying might be heard into the next Room; and sometimes it seemed to be so Hearty, that the Child would sob again. Both the Mother and Midwife told me, they found no great Difference between her in her Case, and other Women in the same Condition.

A Child crying in the Womb. by Mr. Derham. n. 314. p. 485.

2. *Etmuller* having in his Dissertation *de abstruso Respirationis humanæ negotio*, Chap. 9. together with the learned *Diemerbroeck*, doubted the Truth of Accounts of that nature, may in my Opinion receive an Answer, in some measure, from this Case into which I have made some farther Enquiries. Concerning which I must needs say, that notwithstanding I should be as much enclined as any Man to doubt of the Fact, being clearly of Opinion that the *Fœtus* doth not live in the Womb by Breathing, yet the Evidence is so clear to me in the present Case, that I am fully satisfy'd it was really *Crying of the Fœtus*, and not Groaking of the Guts, or Womb, or the Effect of any Feminine Imagination. For here we have a thing happening not once, or twice only, but a great many times; almost every Day, and divers times in the Day; and that for near five Weeks together. Enough to have discovered any Mistake, or to have undeceived even a fanciful Person. In the next Place, we have the Child heard to Cry aloud, so as to be distinctly heard by Persons in another Room. Consequently the Hearers could more easily and certainly distinguish whether the Noise was *Crying* or *Croaking*. The Description the Mother and others gave me thereof was, "That "the Noise the Child made, was as if a Born-Infant had Cry'd eagerly, "shut up close in a Tub. In the third Place, The Crying seemed to be so eager and hearty, as to end in Sobbing, like what is observable oftentimes in Born Infants. In the fourth Place, It was heard not alone by the Father and Mother, or one or two besides, but by many, or most of the Neighbourhood, both near and farther off, and many of them Persons long used to Children; who do all with the greatest Assurance affirm it to have been as manifest Crying, as ever they heard from a Born-Infant, and nothing like any Noise of Wind, or the Guts,

Remarks, by the same. ib. p. 487.

as on Enquiry they all particularly told me. And in the laſt Place, The Midwife told me, that laying her Hand on the left ſide of the Woman's Belly, where the Child lay when it Cryed, ſhe could plainly feel a Motion under her Hand, like that of Reſpiration, every Blaſt of the Child's Crying ſenſibly touching upon her Hand.

Ibid. As to the *Peeping of Chickens in the Egg*, about which *Etmuller* hath the ſame doubt, as of the *Vagitus Uterinus*, I have my ſelf divers times heard that, both from Chickens and Ducks. And a Perſon more Converſant in ſuch Matters than my ſelf aſſures me, That a little before the Hatching, ſhe hath often, and can at any time cauſe ſome Chickens, and Ducklings to peep in the Egg. She ſaith, that ſometimes whole Neſts of Eggs will yield a Cry, ſometimes only ſome particular Eggs: But that ſuch Eggs as have once afforded a Peeping, may be made to Peep and Cry at any time, by ſhaking the Egg, and putting the Youngling into a diſorder. And ſometimes where there hath not been any Noiſe before heard, the Bird hath been made to Cry, by ſhaking the Egg in which it was encloſed. The Cauſe of this Peeping in the Shell, I take to be from ſome Uneaſineſs the young Bird may find there. It being arrived to its perfect State in the Egg, is either weary of its Confinement therein, and deſireth more liberty; or elſe it lies uneaſily, or is offended with ſhaking, and therefore Peepeth and Cryeth, as when uneaſy out of the Shell. And after ſome ſuch manner I take it to be with an Humane *Fœtus*; that it is in ſome Diſorder, and uneaſy in the Womb, and therefore Cryeth as well in, as out of it. Thus I am apt to think it befel the *Fœtus* I have ſo often ſpoken of, *viz.* That it lay very uneaſily in the Womb all the while it Cry'd there, the Mother being in great Pain before, and during the time of the Child's Crying, and the Child it ſelf being cloſely confined, and pent up on the Left-ſide the Mother's Belly, *all the time of Crying only, and not all the 5 Weeks*, as by miſtake I told in my former Letter. Perhaps alſo the Child might find ſome Uneaſineſs from a Bone the Midwife told me ſhe found to ſtick out ſomewhat farther than ordinary: Which, upon Examination, I take to be one of the Vertebræ of the Back-bone. And if this Bone cauſed Uneaſineſs to the Child, it might alſo by that means occaſion perhaps the Woman's Pains I ſpake of.

A Woman who had her Menſes *regularly to 70 Years of Age. by Mr.* Yonge. *n.* 337. *p.* 236.

XL. At *Lamerton*, fifteen Miles from *Plymouth*, dy'd lately a Woman of Eighty ſix Years old; who to the Age of ſeventy had her *Menſes* plentiful and regular. At that time they ceaſed, and ſoon after followed the like Efflux from the *Hæmorrhoids*, which continued till ſhe got on the wrong ſide of Fourſcore. She was till then healthful and ſtrong, of a vigorous Aſpect, ſmooth, plump, and florid in Countenance, like one not half ſo Old: Her Appetite was very good; her Intellects clear and ſound; and her ſight ſo perfect, that ſhe could to the laſt thread a Needle, and read ſmall Print without Glaſſes. When that Flux ceaſed, ſhe became Gouty; and about one Year before ſhe

dy'd,

dy'd, there arose an Apostumation on one of her Wrists, which open'd and discharg'd much Chalky matter and some Stones. The Day she dy'd, she arose out of her Bed; and after performing some Christian Devotions she expired. She was never Sick before the Hæmorrhoidal Flux stopt, except once at *Exeter*, where she was born, and then lived, she became infected with what they called the Plague: It ended in a critical Abscess in one of the Emunctories; and which is very strange, during all the time of the Sickness, she nurs'd a Male Child, who is yet alive, and one of our Faculty.

XLI. A married Woman, about 3 Miles from *Shrewsbury*, about 40 Years old had the common Reasons to believe herself with Child, At the time of her account, she had the usual Signs of Labour, and a good Midwife assur'd her, it was so, but she could not be deliver'd without bringing away the Child in pieces. She not consenting, her pains went soon off, and she continu'd without any Disorder 9 Months longer, when she had again the Pains of Labour, and the same Midwife assur'd her as before, and she persisting in her former Resolution, her Pains after a Day or two went off. Soon after her Belly swell'd to a surprizing Size, by which she got Subsistence for her Family by being seen in a Shew. I saw her first above Twenty Years since, when her Belly was almost even with her Chin, the Weight of it so great that she was oblig'd to support it with a Stool. She could not stand without the help of a Rope from the Cieling, which assisted her in changing her Posture of sitting. She slept commonly with her Arms folded on her Belly, and her Head rested between them. She had no swelling in her Legs: every other Part emaciated as usual in the like Cases. Thus this Poor Creature liv'd without any other considerable Complaint above Thirty Years, the most remarkable Circumstance, I think, in her Case. She dyed in *May* 1715, when this appear'd to be an *Ascites*.

Dissection of a big Belly'd Woman, suppos'd to have continued many Years with Child. com. by Dr. Hollings. *n.* 348. *p.* 452.

I need not mention the State the common Teguments must necessarily be in from so great a Distention, which had distorted many of her Ribs, and forc'd the Diaphragm so high, that it was surprizing to find her breathing could be so long continu'd. The Water was all contain'd in the Duplicature of the *Peritonæum*, 13 Gallons besides a Quart that was spilt: It was Saltish, with some little fat upon it, and towards the latter Running ting'd with Blood as usual. There was not any Water in the Cavity of the *Abdomen*, except what was contain'd in a kind of Bladder of the Shape I have sent, which lay a cross the *Fundus Uteri*. This was divided by a Cartilaginous Substance into two Cavities; in one there was about a Pint and a half, in the other three Parts of a Pint of Water. I believe it was this (I know not how) impos'd on the Midwife: The *Uterus* was of the natural Size without Alteration, except that the *Os Tineæ* and *Collum minus* were fill'd with a gritty Substance, hard as Stone, which I take to be the Humor sepa- *Plate* 6, *Fig.* 3.

 rated

rated there, and coagulated by Time. Mr. *Cowper Tab.* 15. *Fig.* 4. says he found the same Parts fill'd with a glutinous Matter, which he thinks is useful to prevent Abortion: which if vitiated, Impregnation is hinder'd.

The Liver and other parts contain'd in the *Abdomen*, were forc'd into an incredible small Compass (and by that Pressure a little chang'd in Shape) to perform their Office so long; to which the Muscles of the *Abdomen*, distended so as to be scarce discernible, could give but little, if any, Assistance.

Of the maculæ maternæ. *by* Dr. Mather. *n.* 339. *p.* 65.

XLII. A Woman in *New England*, longing for Peas, but refusing to gratify her Desire, by reason of a sort of Bug, wherewith at that time most of their Peas were infested, had a Child born with an Excrescence on the Forehead, resembling one of those Peas, with a black Speck, as the buggy Peas had, which after some time dryed away, and shell'd out, they fancied as the Bugs are observed to leave the Husk of the Pea.

XLIII. *A Paper omitted.*

n. 270. *p* 819.

A Letter from Dr. *Martin Lister* to Dr. *Tancred Robinson*, commending the Doctor's Book of Fevers, and hinting at an Experiment made by him and Dr. *Musgrave*, concerning Powder'd Blues passing the Lacteal Veins. *This Experiment is related at large by Dr.* Musgrave, p. 75.

XLIV. *An Account of a Book*, viz.

ib. p. 829.

Profluvia Ventris: or the Nature and Causes of Loosnesses plainly discover'd, their Symptoms and Sorts evidently settled, the Maxims for curing them fully demonstrated, and all illustrated with the most remarkable Methods and Medicines in all Ages; with some Practical Observations concluding every sort. By *William Cockburn*, M. D. late Physician of his Majesty's Fleet, *F. R. S.* and of the College of Physicians, 8vo. *Lond.* 1701.

CHAP. V.

Of the Veins, Arteries, and Blood.

1. *Schemes of Veins and Arteries presented to the R. S. by* Mr. Evelyn *n.* 280. *p.* 1177.

BEing some Years since in *Italy*, and curious of seeing the many repeated Dissections at the Anatomical Theatre at *Padua*, *Cavalier Veslingius* being then Professor, and reading on divers Bodies several Days, during the Lent; Dr. *Johanno Athelsteinus Leoncenæ*, who was then

then Operator, by Extracting the Veins, and other Veſſels which contain the Blood, Spirits, *&c.* out of Humane Bodies, (which the many Hoſpitals and Infirmaries of that City plentifully afford) began to apply and diſtend them on Tables, according to their natural Proportion and Poſition, as an Improvement which might be of uſe in Anatomy: Some of theſe Tables being finiſh'd, with the Direction and public Approbation of the Profeſſor and ſeveral other Learned Phyſicians and Anatomiſts, preſent at thoſe Lectures and Operations; and underſtanding that *Leoncena* was going ſhortly (I think) into *Poland*, and willing to diſpoſe of his Tables, before he took his Journey; I deſir'd the late Dr. *George Rogers*, (Conſul then at *Padua* for the Students of our Nation in that Univerſity) to purchaſe and procure them for me; which he did, for as I remember 150 *Scudi*; with condition, that he ſhould add a Table more, namely, that of the Liver, Gaſtrick Nerves and other Veſſels, to compleat the Fourth: When theſe were perfected, I immediately ſent them to *Venice*, from whence they were ſhipp'd for *England*: But, upon what Accident or Occaſion I know not, the Veſſel was carried into *Holland*, and lay there a Year or two, (without any Tydings what was become of my Concerns, being then my ſelf at *Paris*) till coming at laſt to be unladen, Sir *Richard Ford* (afterwards Lord Mayor) took care to have them all ſafely convey'd to me at *London*, to my no ſmall Charges.

2. That the Arteries are the Veſſels which convey Blood from the Heart to all Parts of the Body, is well known; and we ſee by Fig. the 1ſt, that the common Practice of Nature in diſtributing theſe Veſſels, to ſupply the parts with Blood, is from the next adjacent Trunk, till their Aſcending and Deſcending Trunks become Conical, as well as their collateral Branches: Not that all the Trunks and Ramifications of *Arteries* are Uniform, and become Conical in the ſame manner; nor do all of Them paſs directly to the parts to which They convey Blood; nor do all parts receive *Arteries* from their neighbouring Trunks. The Trunks of the *Carotid*, *Vertebral* and *Splenick Arteries* are not only Contorted in their progreſs, in the Adult; but the Diameters of their bores are variouſly dilated in divers parts of them, eſpecially where they are contorted; but as theſe dilatations of their Trunks are caus'd by the reſiſtance the Blood meets with at thoſe Angles of Inflection, ſo thoſe Enlargements of them afterwards contribute to retard the protruſion of the Blood to the Extremities of thoſe *Arteries*: Hence it is, That as the *Arteries* of the *Fœtus* are not Contorted in ſuch acute Angles as in full grown Bodies, ſo their Trunks are more Conical, and not here and there dilated in divers parts of them, as in the Adult. The Trunk of the Splenick *Artery* has a ſtrait Progreſs in the *Fœtus* and in Infants; but in the Adult I have hitherto conſtantly found it very much Contorted, as expreſt in Fig. 1. 23. The peculiar Contrivances of the Spermatick *Arteries* of *Quadrupeds* as well as Men, ſhew a conſtant deſign in Nature of taking off the Velocity with which the Blood would

Anatomical Remarks on them, by Mr. Cowper *ib. p.* 1179. *Plate* 7.

would otherwise pass thro the Glands of the *Testes*: It seems to be for this end that the *Testes* of most Animals (especially Men and *Quadrupeds*) hang out of the Cavities of their *Abdomens*, that the Canals of their Blood Vessels may be lengthened: for the Spermatick *Arteries* (contrary to all others) arise from their Great Trunk, at a far greater distance from the *Testes* than the *Arteries* of any other part of the Body. Nor would the *Testes* (which are such necessary Organs) have been thus exposed to external Injuries, if the end of Nature in lengthening their Blood Vessels had not been very considerable. Besides this lengthening of the Spermatick *Arteries*, we find Nature still contriving other Impediments to check the Current of the Blood in those Parts; it seems for this end that the Spermatick *Arteries* are lessen'd at their Original from the Trunk of *Arteria Magna* in Men, and that the Spermatick *Arteries* of *Quadrupeds* are so much Contorted before they reach their *Testes*. The principal Inducement of Nature in making use of these different Contrivances in the Spermatick *Arteries* of Men and *Quadrupeds* seems to be, That if the Humane Spermatick *Arteries* were Contorted, as in *Quadrupeds*, before they reach their *Testes*, the Apertures in the *Abdominal* Muscles of Man must be much larger than they now are, and would frequently let the *Intestines* descend into the *Scrotum*; which we know neverthelefs often happens: such Ruptures (as they are call'd) are not so Incident to *Quadrupeds*, tho the Passages for their Spermatick Vessels (through their *Abdominal* Muscles) are much wider than in Men, because the position of the Trunks of their Bodies is Horizontal, and their Intestines therefore cannot press on the processes of the *Peritonæum*, as in Men, who are Erect. I shall, at present, pursue the Thread, and describe the Extremities of the *Arteries*, with their Communications with the Veins, and afterwards produce some Instances of the Art of Nature in conveying the Refluent Blood to the Heart.

After the Circulation of the Blood through the Heart, Lungs, and large Blood Vessels, was demonstrated by Dr. *Harvey*, it was only guest how the extremities of the *Arteries* transmitted the Blood to the Veins, till Mr. *Lewenhoeck*'s Microscopes had discovered the continuation of the Extremities of those Vessels in Fish, Frogs, *&c.* which is now commonly shewn by Microscopes made by other Hands: Yet there are not wanting those who doubt of the like Continuations of the Extremities of *Arteries* and Veins in Human Bodies and *Quadrupeds*; since those Animals it has hitherto been seen in (to any satisfaction, as Mr. *Lewenhoeck* confesses) have been either such Fish, or of the Amphibious kind, that have but One Ventricle in their Hearts, and their Blood actually cold, except in Bats, in which it appears very obscurely: Add to this, that the Blood in those Creatures does not Circulate with such Rapidity as in Animals whose Hearts have Two Ventricles. For in all Animals, that have Biventrous Hearts, the Vessels of the rest of the Body return their Blood to the Heart in equal time and quantity with those of the Lungs,

Lungs, notwithſtanding the Inæquality of their Courſe. * This Difference in the principal Organs of the Circulation of the Blood in thoſe Creatures, (on which only theſe Experiments have been hitherto made) mov'd me to make ſome on Animals whoſe Organs differ only from the Human in their groſs Figure, and not in their Intimate Structure: For this end I took a young Cat, about ten or twelve Days old, and faſtened it to a Board as in Viveſection; and making an Inciſion through the *Linea Alba*, the *Omentum* and Inteſtines were extruded; then cauſing the Creature to be ſo held (on the Board) under a large Double Microſcope, where a flat Glaſs for receiving of Objects was placed Horizontally, on which I expanded the *Omentum* or Caul, (a Light being placed underneath) I ſaw the Globules of the Blood move very ſwiftly in the ſmall Veſſels, which are only to be ſeen in the moſt Tranſparent parts of the Membranes of its *Omentum*; but the motion of the Blood ſoon abated, and its Globules were withdrawn from the Extremities of its Blood Veſſels; and in a little time became ſtagnant in their larger Branches. This appearance of the continuation of the Extremities of the *Arteries* and Veins, while the Blood was moving in them, in the *Omentum* or Caul, is expreſt by *Fig.* 4. *A A* ſhews the Trunks of the *Arteries*. *B B* the Veins, which were diſtinguiſhable by contrary currents of the Globules of the Blood in each Veſſel. *C C C* ſhews the Branching of the Extremities of the *Arteries* and Veins, that no longer Aſſociate with each other, but are United, as here expreſt. After I had ſeen this, I attempted to ſhew the like to ſeveral Friends, but did not always ſucceed ſo well as when Mr. *Chambers* and Mr. *Buckeridge* favoured me with their Preſence, at a time when I happened to have a young lean Dog, that was not large; in whoſe *Omentum* we ſaw it very well; but by the Aſſiſtance of an Inſtrument I had prepar'd to expand the Meſentery, we all ſaw it there much better; that part having not only larger and clearer Spaces than the *Omentum*, but its Blood Veſſels are diſtributed more regular, as appears by *Fig.* the 5th, where the ſame Letters of reference ſerve as above.

* Mr. Wotton *in his Reflections upon Antient and Modern Learning. Chap. xviii.*

Thoſe who will entertain themſelves in viewing the tranſparent parts of Living Creatures with Microſcopes, will find that the extremities of their *Arteries* and Veins are not all equally leſſen'd, tho united. In the Tail of the *Lacerta Aquatica*, *Tadpoles*, and in moſt Fiſh I have examin'd, I have frequently obſerv'd ſeveral Communications between the *Arteries* and Veins; in which more than two Globules of Blood have paſt abreaſt: And in the ſame *Area* I have ſeen ſome of thoſe Communications ſo ſmall, as that but one Globule could paſs, and that very ſlowly before the other. In young Fiſh, particularly Grigs, I have frequently obſerv'd a Communicant Branch, ſo very ſmall as that one Globule of Blood only has paſt it in two or three Seconds of a minute: at other times I have found conſiderable Intervals in paſſing of one Globule in ſuch a Communicant Branch; even half a minute, a whole minute, and once in two or three minutes I have ſeen one Glo-

bule

bule of Blood only pafs in a particular tract The prompt paffing of Liquors, injected by the *Splenick Arteries,* to the *Veins,* fhews the Communications between thofe Veffels are more open than the *Arteries* and *Veins* of other parts, of which I have elfewhere fpoken. Liquors alfo Injected into the *Pulmonick Arteries* pafs to their *Veins,* though not altogether fo freely as in the *Spleen.* On viewing the Extremities of the *Pulmonick* Blood-Veffels in a living Frog with my *Microfcope,* I found their *Communications* much larger than *thofe* that I had before feen in the Membrane between the Toes and in the Feet of the fame Creature. Nor can we reafonably doubt of the like patent Communications of the *Arteries* and *Veins* of *Humane Lungs* and thofe of *Quadrupeds,* when we confider the *Blood* of their Lungs muft return to the Heart in equal *Time* and *Quantity,* with *that* of all the parts of the Body befides, as before noted. Hence it appears that the Bronchial Blood Veffels (firft taken notice of by the Accurate *Ruyfch*) are abfolutely neceffary, elfe the parts of the Lungs could not receive nourifhment; nor could the Glands of the *Bronchiæ* feparate their Liquor, if they were fupplied with *Blood* from the Pulmonick Blood Veffels *which* is fo quickly difpatched through the Lungs.

On viewing the Membrane that is between the Toes of one of the hinder Feet of a living Frog, after I had frequently taken hold of the fame Leg of that Creature, to apply it to the Microfcope, I found that Membrane very tranfparent, and without any motion of the Globules of the Blood in it, as if the part had been dead; but while I was looking on it, it was, I confefs, not a little entertaining to fee the Globules creep into it by degrees, and at length the Blood move in all the Branches of its *Veins* and *Arteries* as before, when no violence had been offered to the part: While the Blood is thus leifurely creeping through the Veffels, you may plainly fee its Globules compreft into Oval Figures, which are made more or lefs oblong, by the refiftance thofe Globules meet with by the contraction of the fides of the Veffels they pafs through; and this I have more than once obferved in the Tails of the Water *Newts* or *Lizzards*: But on examining the Blood of thefe Creatures with a Microfcope, and comparing it with the Humane Blood, I found the Globules of the *Lizzards* Blood more incline to an oval Figure, and were as big again as the Globules of Humane Blood, and that of a fmall Fifh: which I in like manner viewed at the fame time. It is not unlikely a fudden retroceffion of Blood from the extremities of its Veffels often happens, and its *Circulation* in the fame Veffels is afterwards carried on without any impediment; as on fome Paffions of the Mind, Deliquiums by the effufion of Blood, or otherwife. But if the Blood is once become ftagnant in its Veffels (efpecially the *Arteries*) the part is in no fmall danger of a Mortification, unlefs its neighbouring Veffels, which enjoy the motion of the Blood, drive on the ftagnant Blood, and it efcape by the fides of the Veffels that retain'd it. Experience affures us, that in Bruifes when the Blood is extravafated

it,

it goes off either by *Transcolation* or elſe cauſes an *Abſceſs*; for there's little reaſon (in my opinion) to ſuſpect any of the ſtagnant Globules of the Blood will be fit to re-unite with the *Circulating* Maſs. But that the Blood after ſtagnation in its Veſſels will ſometimes paſs their ſides, appear'd to me from the following Experiment. On viewing the *Meſentery* of a Dog when living, in which I had before ſeen the Blood paſſing the extremities of the *Arteries* and *Veins*, I conſider'd how to preſerve the Blood in its Veſſels, that I might afterwards at any time ſee it in their Extremities when ſtagnant: For this end I caus'd ſeveral parts of the *Meſentery* to be tyed on as many pieces of ſmall round *Pill-Boxes*, cut tranſverſely like little hoops; on which *Portions* of the *Meſentery* were extended like the head of a *Drum*; and on viewing *them* afterwards with my *Microſcope*, I found the Extremities and Branches of the Blood Veſſels charged with Blood, which before appeared in Motion; ſome of which *parts* of the *Meſentery* I ſtill keep by me. On laying one of theſe *Parts* of the *Meſentery* (thus expanded) in Water, the ſtagnant Blood in its Veſſels diſappear'd; but on juſt immerſing another of thoſe *Pieces* in Water, I could with my Naked Eye ſee the ſtagnant Blood diffuſed in the Interſtices of the Blood Veſſels, and between the Membranes of the *Meſentery*: Hence it's evident, the Blood may paſs the ſides of its *Veſſels* after ſtagnation in *them*; but whether its *Globules* are broken, or what figure renders *them* fit to paſs thoſe pores that are in the ſides of the Veſſels, I leave to the Inquiſitive; but we muſt return to our *Tables*, and firſt of that of the *Syſtem* of the *Vena Cava*.

As the *Arteries* are known to export the Blood, ſo the *Veins* carry it back again to the *Heart*; but having already deſcribed their Extremities, we come next to the large Trunks of the Veins; and here, as in the *Arteries*, we find the common practice of *Nature*, in diſpoſing the Branches of *Veins* to diſcharge the Refluent Blood into the next adjacent Trunk, and ſo on to the Heart. As the *Arteries* afford abundance of Inſtances of *Checks* given to the Velocity of the Current of the *Blood* through ſeveral parts, ſo the *Veins* ſupply us with as many *Artifices* to aſſiſt its regular return to the *Heart*, as well as favour thoſe *Contrivances* in the *Arteries*. The Trunks of the *Carotid*, *Vertebral* and *Splenick Arteries* are not only variouſly Contorted, but are alſo here and there *Dilated*; ſo the *Veins* that correſpond to thoſe *Arteries* are alſo variouſly *Dilated*. The beginnings of the Internal *Jugulars* have a *Bulbous* cavity (Fig. 7. H, H,) which are *Diverticuli* to the Refluent Blood in the *Sinus*'s of the *Dura Mater*, leſt it ſhould deſcend too faſt into the Jugulars. The like has been alſo taken notice of by Dr. *Lower* in the *Vertebral Sinus's*. The *Splenick Vein* has divers *Cells* opening into it near its Extremities in Human Bodies; but in Quadrupeds the *Cells* open into the Trunks of their *Splenick* Veins. The *Spermatick* Veins do more than equal the Length of the *Arteries* of the *Teſtes* in *Men*; their various *Diviſions* and ſeveral *Inoſculations* and their Valves, are admirably contriv'd to ſuſpend the Weight of the Blood, in order to diſcharge it into

the larger Trunks of the *Veins*; and were it not that the Refluent Blood from the *Teſtes* is a *Pondus* to the Influent Blood from the *Arteries*, and ſtill leſſens its current in the *Teſtes*; theſe Spermatick Veins, like *thoſe* of other parts, might have diſcharged their Blood into the next adjacent Trunk. Who can avoid ſurprize at the Art of Nature, in contriving the *Veins* that bring part of the Refluent Blood from the lower parts of the Body? when they conſider the neceſſity of placing the Human Heart, as well as that of moſt Quadrupeds, ſo far from the *Center* of the Body towards its upper part? It is for that end neceſſary the large Trunks of the *Veins* and *Arteries* ſhould not aſſociate each other; for if all the Blood ſent to the lower parts, by the Deſcending Trunk of the *Aorta*, ſhould return to the Heart again by one ſingle Trunk (as it is ſent out from thence) the *Weight* of ſo much Blood in the Aſcending Trunk of the *Vena Cava*, (Fig. 6. C, C, A) (for ſo its lower Trunk is call'd) would oppoſe the force the Heart could give it from the Arteries, and hinder its aſcent: For this reaſon the *Vena Azygos* (Fig. 6, b) or *ſine pari*, is contriv'd to convey the Blood ſent to the Muſcles of the Back and *Thorax* into the Deſcending Trunk of the *Vena Cava* (ib. B, A.) above the Heart: Hence it's evident, more Blood comes into the Heart by the Deſcending, or upper Trunk of the *Vena Cava*, Fig. ib. B, A. than paſſes out by the Aſcending Trunks of the *Aorta*. Nor does this quantity of Blood convey'd to the Heart by the *Superior Trunk* of the *Cava*, ſeem without ſome other deſign in Nature, beſides Tranſporting it thither to free the *Inferiour Trunk* from its Weight: But perhaps it was neceſſary ſo much Blood ſhould be ready there to joyn with the Chyle, (Fig. 6. †) for its better Mixture, before it reaches the Right Auricle of the Heart.

Vid. Lower de Corde, cap. 1. pag. 1.

Plate 7.

Fig. 1. Repreſents the Trunks and large Branches of the *Arteries*, diſſected from an *Adult Human Body*, when diſplayed and dryed; as they are now to be ſeen in the *Repoſitory* of the *Royal Society*. 1. The Trunk of the *Aorta* cut from the *Baſis* of the Heart. 2. That part of it, whence the Coronary Artery of the Heart does ariſe. 3. That part of the *Arteria Magna*, where the *Canalis Arterioſus* of the *Fœtus* terminates; which in an *Adult* becomes a Ligament. Vid. Fig. 2, 3. 4. 4. That part of the *Axillary-Arteries*, by ſome called the *Subclavian Arteries*. 5. The Left *Carotid Artery* (in this ſubject it ſeems) ariſing from a Common Trunk with the Right *Carotid* and *Axillary Arteries*, as in ſome Quadrupeds. 6. The Left *Cervical* Artery in this ſubject ariſing from the Trunk of the *Arteria Magna*, as expreſt in a Figure given by *Bergerus* in the *Acta Eruditorum An.* 1698. pag. 295. But in all the Human Bodies in which I have hitherto examined theſe *Arteries*, I have conſtantly found them as expreſt Fig. 2. 6. 6. 7. The *Arteries* that carry Blood to the lower parts of the Face, Tongue, Adjacent Muſcles and Glands. 8. The Trunk of the *Temporal Artery*, ſpringing from the Carotid, and parting with branches to the Parotid Gland 9, and Temples 10, and parts adjacent. 11. The *Occiputal Arteries*. 12. The *Arteries* that con-

vey-

Fig: 2.

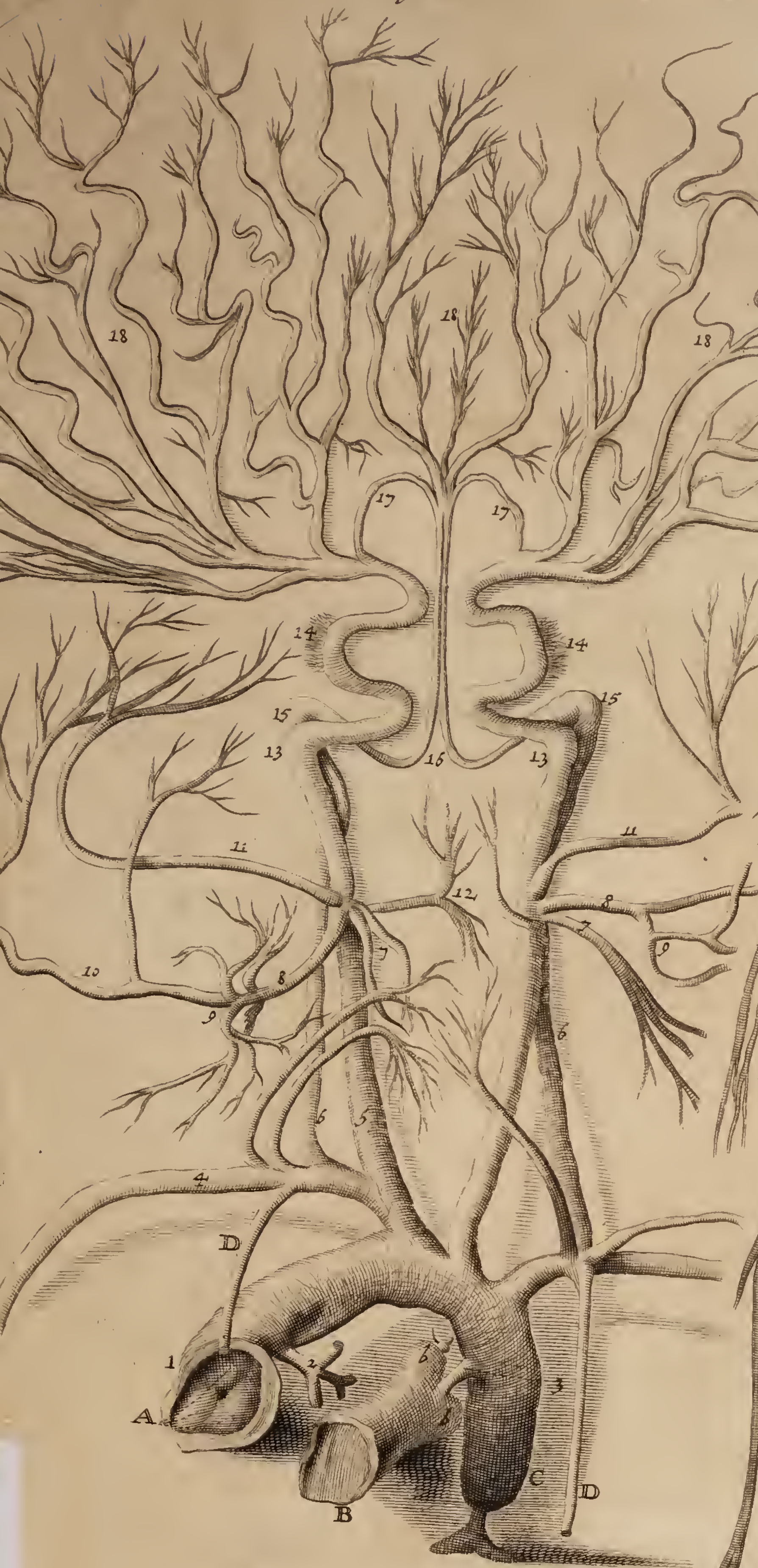


Fig 1.

Fig: 3.

Fig: 4.

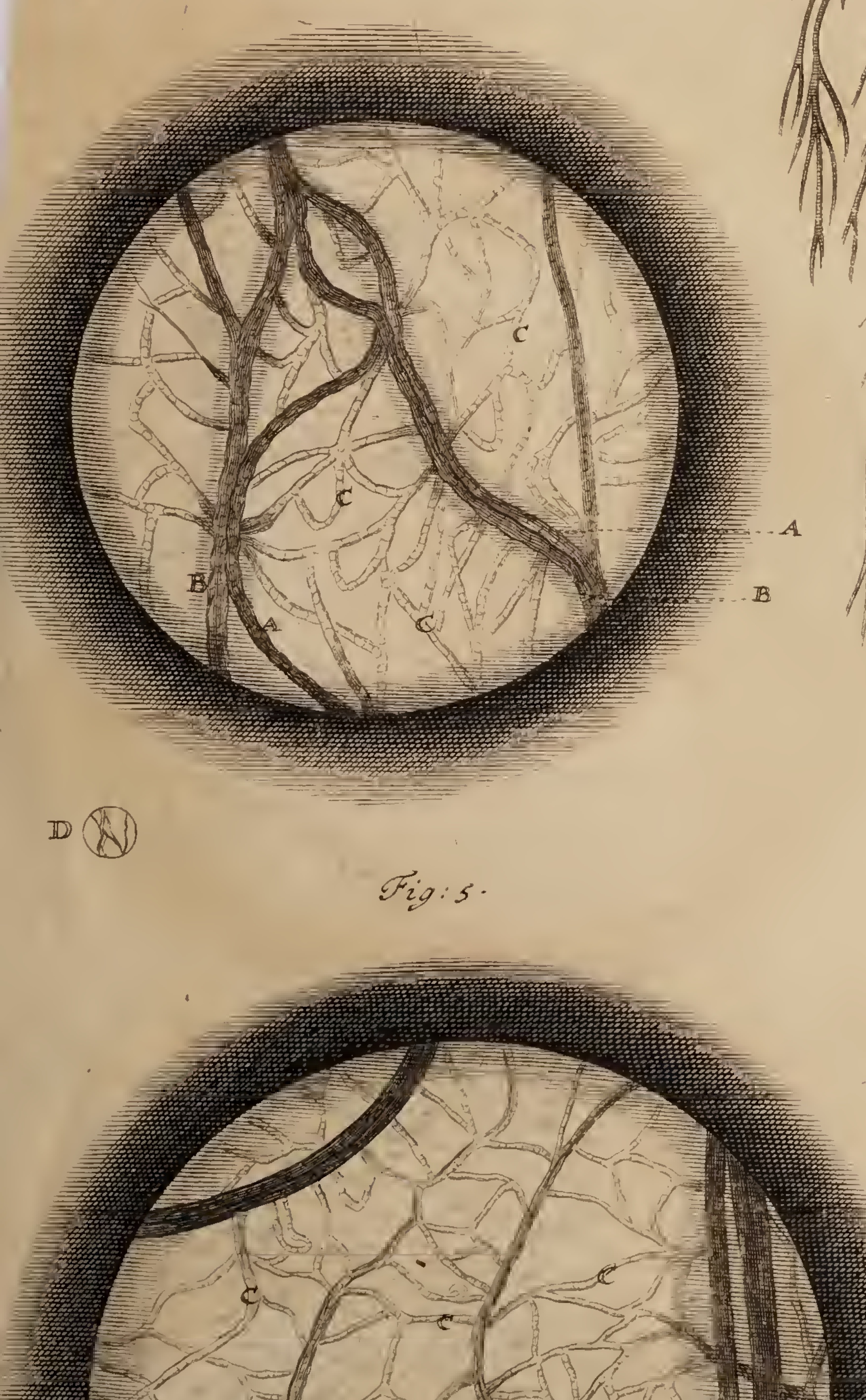


Fig: 5.

M V^d Gucht Sculp.

vey Blood to the *Fauces Gargareon* and adjacent Muſcles. 13. The Trunk of the *Carotid Artery* cut off, before it is *Contorted* in paſſing the Skull. 14. The Trunk of the *Artery* of the Arm parting with Branches to the adjacent Muſcles and Parts.

* That part of this *Artery* which is ſometimes prickt in Letting Blood, and makes an *Aneuriſma*, in which caſe this Trunk of the Artery muſt be bared and *firmly tyed above* the *Aneuriſma*; and if it afterwards happens (as it has been frequently known) that the flux of Blood to the *Aneuriſma* in the *Artery* is not very much abated, though the *Artery* has been *tyed above*; the *Operator* in that caſe muſt make another Ligature on the Trunk of the *Artery below* its *Aneuriſma*: Theſe Collateral Communications of the Trunk of the Artery at the bending of the Cubit, preſerve the Circulation of the Blood in the Cubit and Hand, though the *Trunk* is totally compreſt both *above* and *below*; and the ſame *Trunk* afterwards divided between thoſe Ligatures. Hence it is, if one Ligature made above the wound in the Artery is not ſufficient, but the Blood ſtill pours out from below, the *Patient* will ſooner recover the Action and Strength of the Muſcles of the Cubit, than Thoſe in whom the upper Ligature proves ſufficient; the reaſon of which is obvious to any who conſider that the *Communicant Branches* muſt be larger where the lower Ligature is required, than when the ſuperior Ligature only is ſufficient: Theſe *Communicant Branches* (as I have ſeen them in ſome ſubjects) are here markt out in prickt Lines, *vid.* the Fig.

While theſe Papers were lying by me, the two following Inſtances happen'd, in which the *Communications* of the large Trunks of the *Arteries* of the Cubit and Arm were remarkable. The firſt was,

A *Boy* of thirteen years, who, about three weeks before I ſaw him, receiv'd a Wound near the middle of the *Cubit* in which the *Trunk* of the *Artery* (markt in the Fig. †.) was divided. The *Surgeon* who was firſt call'd had frequently bound up the Wound, and put a ſtop to the ſeveral diſcharges of Blood (which they told me did not amount to leſs than 6 or 7 quarts at times) but not without a Compreſs on the Trunk of the *Artery* above the Wound. On another impetuous Flux I was called; but ſeeing no ſmall quantity of Blood diſcharged, I was contented to let the Wound be bound up, in the ſame manner as it had been done before; omitting the Compreſs on the Trunk of the *Artery* above, and adding a piece of Deal-board, on which the Hand and Cubit were faſtened, to prevent any Motions of thoſe parts, as well as the Fingers: Three days after, the applications were taken off, and little or no Blood appear'd; but two or three hours were ſcarce elapſed e're I was alarm'd with notice of a freſh Flux. The *By-ſtanders* being inſtructed in that caſe, to compreſs the *Trunk* of the *Artery* above the *Cubit*, *they* had thereby prevented no ſmall effuſion of Blood, which muſt otherwiſe have happen'd: His *Surgeon* being out of the way, I laid the Trunk of the Artery bare above the Wound as expeditiouſly as I could, being forced more than once to let looſe the Compreſs above to diſcover its

Orifice by the Flux of Blood. I passed a *Needle* with strong Waxed Thread under the Artery, and make a ligature on its Trunk, (which lay concealed in the Interstice of the *Musculus Flexor Digitorum*, and the *Musculus Ulnaris Flexor Carpi*;) but notwithstanding this Ligature on the Trunk of the Artery above the Wound, the Blood still flow'd from the Lower Trunk of the divided *Artery*, yet the velocity of its Current was so much abated, that it seem'd like Blood flowing from a Vein. I left the Wound with a digestive, and the part without hard bandage. It being now five Weeks since, I hear the Wound is almost Cicatriz'd.

A *Boy* about eight years of Age, came to Town with an *Aneurisma* of the Left Arm, upon Bleeding six Weeks before. The Tumour was indeed very large in proportion to so small an Arm. After laying the *Aneurisma* or Tumour bare, and making a Ligature on the Superior Trunk of the *Artery* (in the annext Fig.* . . . . .) I found, on loosening the Compress on the superior Trunk of the Artery, very little abatement of the Pulsation of the *Aneurisma*; I then passed a Ligature in like manner on the Trunk of the Artery below the Tumour; but notwithstanding the Pulsation continued, tho much abated. I then discovered another Trunk of the *Artery*, arising from the lower part of the Tumour, on which I also made another Ligature, and the Pulsation was then taken off. However on cutting off the surface of the *Cystis* or dilated *Artery*, and clearing it of the coagulated Blood, there still poured out some fresh Blood, which was soon stopt with a common astringent; I left the part without any other Ligature or hard Bandage. Is is now eighteen Days since the Operation, the Ligatures on the *Arteries* are all come off, and the Pulsation of the *Artery* of the Wrist begins to be very manifest, nor does any Symptom appear that threatens
N. 285. p. 1391. success. He has since recover'd the intire use of his Arm, and is in perfect Health.

15. The division of the Trunk of the Artery of the Arm below the Flexure at the Cubit. 16. The external Artery of the Cubit, which makes the Pulse, that is commonly felt near the *Carpus*. 17. The Arteries of the Hands and Fingers. 18. The Mammary Artery. 19. 19. The descending Trunk of the *Arteria Magna*. 20. 20. The Intercostal Arteries. 21. The *Arteria Cæliaca*. 22. The *Arteria Hepatica*. 23. The Trunk of the *Arteria Splenica*. 24. The *Arteria Epiploica Sinistra*. 25. A Branch of an Artery which passes to the bottom of the Stomach. 26. The superior Coronary branch of the Stomach. 27. 27. The superior Mesenterick Artery. 28. 28. The emulgent Arteries. 29. The inferior Mesenterick Artery. 30. 30. The Lumbal Arteries. 31. 31. The two Spermatick Arteries, which in this subject seem to arise at a greater distance from each other than commonly. 32. The Iliack Artery. 33. The *Arteria Sacra*. 34. The Internal Iliack Branch. 35. The External. 36. The Epigastrick Artery. 37. Branches of the External Iliack Artery, passing to the Oblique Muscles of the *Abdomen*. 38. 38. The Arteries that

pass

paſs to the Muſcles of the Thigh and *Tibia*. 39. The Crural Artery. 40. The Umbilical Artery, with thoſe of the *Penis*. 41. That part of the Crural Trunk that paſſes the Ham. 42. The three Trunks of the Arteries of the Leg. 43. The Arteries of the Foot and Toes.

Fig. 2. The Trunks and ſome of the Ramifications of the *Arteries* of an adult Human Body fill'd with Wax, to ſhew the Variety in Nature, and ſupply the defects of the former Figure. 1. The *Aorta* cut off at the *Baſis* of the Heart. *A.* The three Semilunary Valves as they appear when the Heart is in Diaſtole, and hinder the Blood coming back from the *Arteries* into the Left Ventricle of the Heart. *B.* A Portion of the Trunk of the *Arteria Pulmonalis*. *b b* its diviſion before it paſſes to the right and left Lobes of the Lungs. C. The deſcending Trunk of the *Arteria Magna*. *D D* The Internal Mammary *Arteries*. 2. The Trunk of the oronary , cut off. 3. The *Ligamentum Arterioſum*, which in the *Fœtus* is the *Canalis Arterioſus*, and conveys Blood from the Pulmonick Artery to the great Artery. 4. The Trunk of the Subclavian Artery. 5. 5. The Carotids. 6. 6. The Vertebrals. 7. 7. The Arteries which paſs to the lower parts of the Face, Tongue, adjacent Muſcles and Glands. 8. 8. The Trunks of the Temporal Arteries ariſing from the Carotids, giving Branches to the Parotid Glands (9. 9.) and the Temples (10. 10.) *&c.* 11. 11. The Occipital Arteries. 12. The Arteries of the *Fauces, Gargareon*, &c. 13. 13. The Contortions of the Carotid Arteries, as they paſs the *Baſis* of the Skull: Theſe Trunks of the Carotid Arteries in Dogs (like thoſe I gueſs of moſt Quadrupeds) are very much Contorted before they reach the *Baſis* of the Skull: on filling theſe Veſſels of that Animal with Wax, I found thoſe Branches of them which paſs to the Brain, firſt clipping the hinder parts of the lower Jaw, immediately under its Condiloide Proceſſes; where thoſe Arteries are received in two *Sinus*'s of that Bone, which *Sinus*'s may alſo be ſeen in the Jaw-bones of other Quadrupeds, but not in Human Bodies. 14. 14. Thoſe parts of their Trunks that paſs by each ſide of the *Sella Turcica*, whence divers ſmall Branches ariſe, and help to compoſe the *Rete Mirabile*; which is more conſpicuous in Quadrupeds than in Human Bodies. 15. 15. The Contortions of the Vertebral Arteries, where we find their Trunks conſiderably dilated. 16. The Vertebral Arteries, as they aſcend on the *Medulla Oblongata* towards the annular Protuberance or *Pons Varoli*. 17. 17. The Communicant Branches of the Vertebral and Carotid Arteries. 18. 18. The Arteries of the Brain diſplayed.

Fig. 3. I choose to place this Figure on the Copper Plate of one of the Trunks of the Arteries of the *Tibia*, (diſſected from the Leg after Amputation) rather than the following Diſtich, which I find written on the Original Table of this Scheme of the *Arteries*.

Pulſificus Sanguis, de Cordis Ventre ſiniſtro,
Funditur, ut Corpus nutriat hiſce viis.

Before

Before I explain the Letters of Reference of this 3d Figure, it will be neceſſary to let you know that Mr *Stringer* was in his ſixty ſeventh Year when this Artery was taken from him, and near twenty Years before loſt the uſe of both his Legs; and in that time he had been ſo perſecuted with Convulſions in them, that neither Leg was free a quarter of an hour together, whether Sleeping or Waking. At length one of his little-Toes mortified, which was taken off by Mr *Goldwyer*, an expert Surgeon of *Saliſbury*; not long after more Toes of the ſame Foot followed the like fate: The Convulſions following that Leg ſtronger and quicker: That part of the Foot next the Toes became tumid and inflam'd, the Tumor extending itſelf above the *Maleoli*: A Sinuous Ulcer paſſed by the ſide of one of the Metatarſal Bones; the extremity of which Bone (whence the Toe was taken off) lying bare. In this condition I found the Left Foot and Leg of this Gentleman, when I had the Honour to wait on him by Command of the Right Honourable the preſent Earl of *Shaftsbury*, he living in the Neighbourhood of that Noble Peer in *Wiltſhire*; where I met with Mr *Goldwyer* above-mention'd; and finding the Leg very chilly, the neceſſity of parting with it was too evident; which Mr *Stringer* ſuffer'd with extraordinary Fortitude, he not ſo much as expreſſing the leaſt outcry during the Operation, tho the part did not want the moſt exquiſite ſenſe of feeling: On the abſciſſion (which was about five or ſix Inches below the Knee) it was unexpected, by me, I muſt confeſs, to to ſee ſo little Blood ſpouting from the *Arteries*. The Stump being bound up, and committed to the Hands of two or three Servants, a leſs number not being ſufficient to hold it, by reaſon ſuch ſtrong Convulſive motions purſued the part on the Operation. I was very deſirous to examine the *Arteries* of the amputated Leg, having before diſcovered the Cauſe of a Mortification of the Arm of a Young Gentlewoman who dy'd not long after an Amputation of the part, tho the Gangreen did not appear to reach near the place where the Abſciſſion was made, (*i. e.* below the ending of the *Muſculus Deltoides*); In which Caſe, I found the ſides of the Trunk of the *Artery* of the Arm ſo thickn'd, that the Diameter of its Bore was contracted to leſs than a third part, and would ſcarce admit a common Probe to paſs it, *Fig. G. H. I.* When I had found the ends of the *Arteries* in the Leg above-mention'd, I endeavour'd to paſs my Probe into one of them, but meeting with ſome oppoſition, I ſuſpected I had miſtaken the Vein for the Artery, and that the Valves oppoſed the paſſing of the Probe that way; but on further diſſection I clear'd the Trunks of both thoſe Blood Veſſels, and found the Veins in their Natural ſtate; but the ſides of the *Arteries* were grown Bony or Stony; having clear'd two of their Trunks, I left one of them at *Saliſbury*, the other I brought to Town, and is here Figur'd.

A The Upper part of the Artery cut off in Amputation of the Leg; from *A* to *B* The Trunk of the Artery diſtended and dry'd to ſhew its

Fig: 7.

Fig: 6.

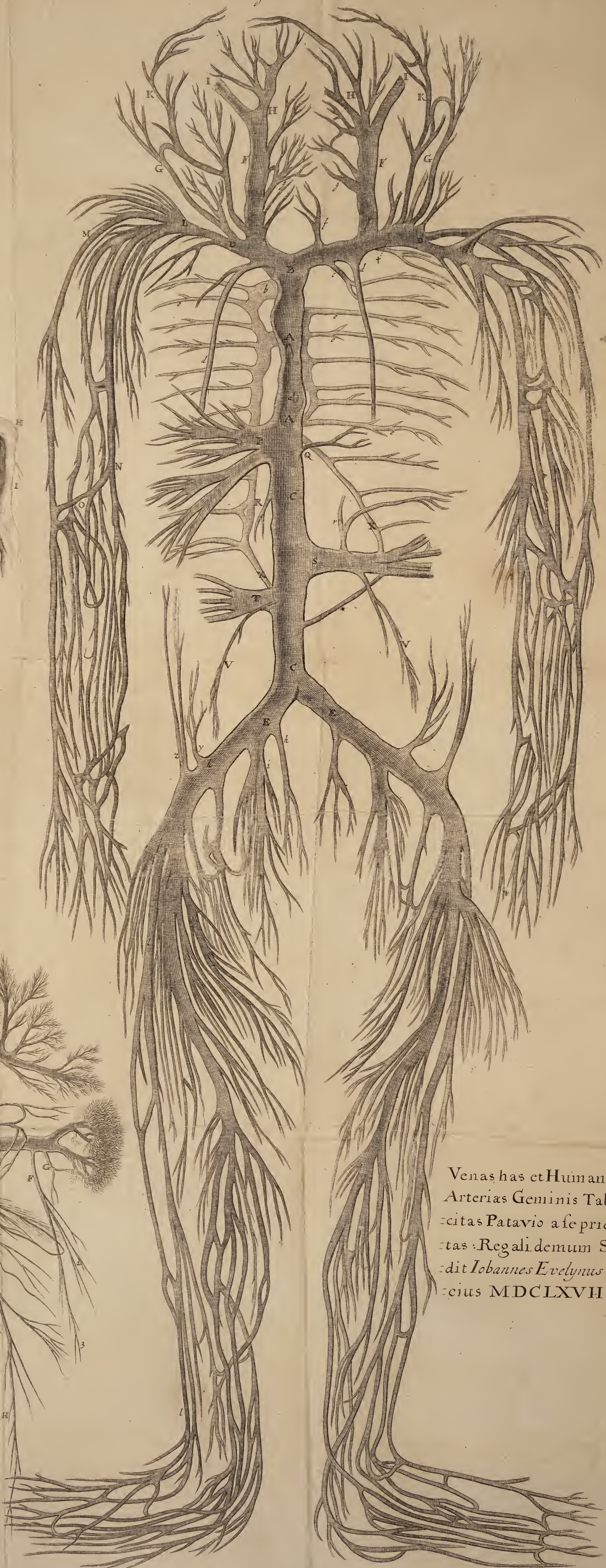
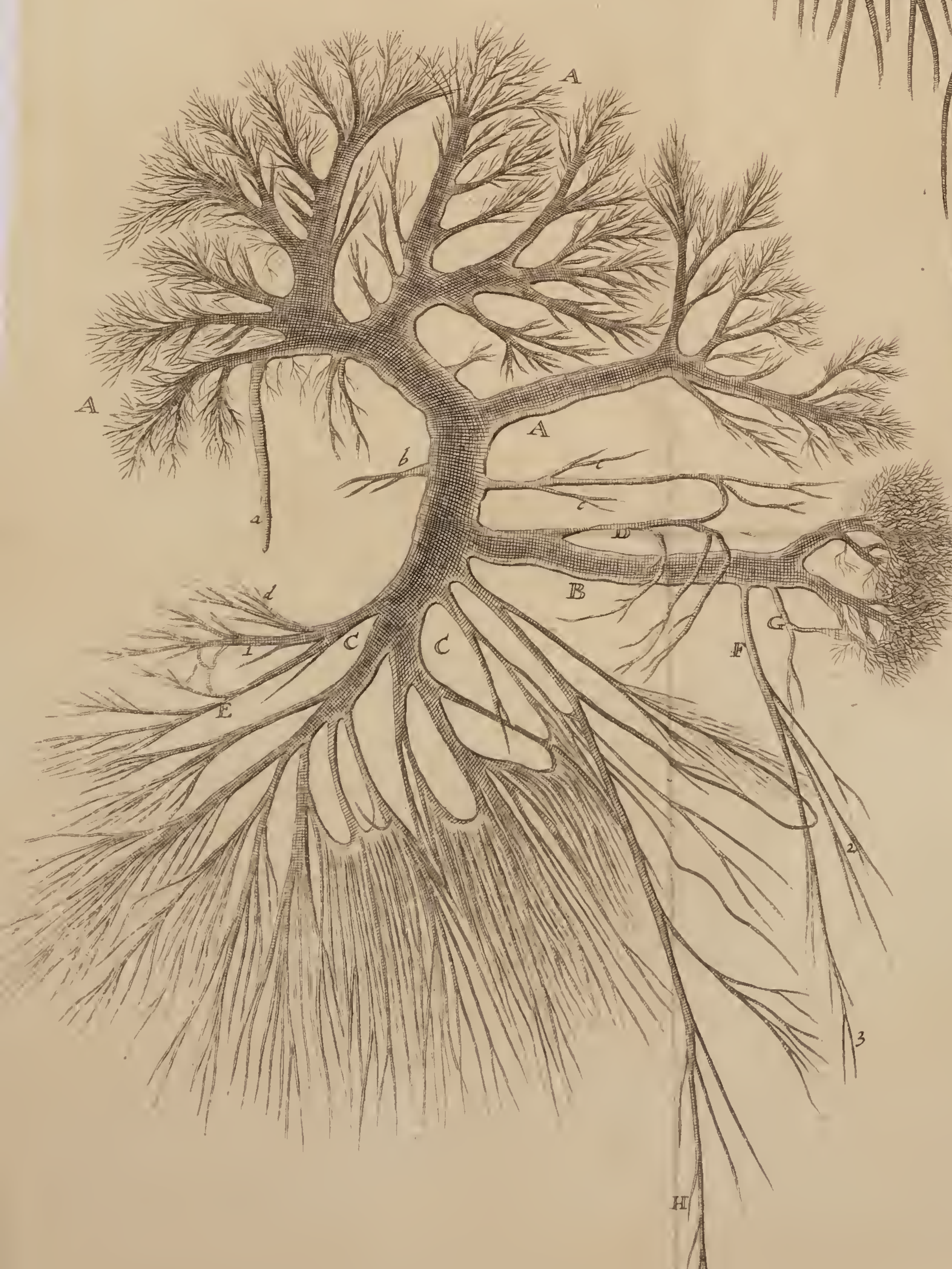


Fig: 8.



Venas has et Humani Corpori
Arterias Geminis Tabulis adpl
citas Patavio a se pridem dedu
tas : Regali demum Societati d
dit *Iohannes Evelynus* ejusdem S.
cius MDCLXVII.

M. v d. Gucht Sculp.

its Canal. *C* That part of the Trunk of the Artery which was so contracted by the Petrifaction or Ossification, that a Probe would not pass its Canal; From *C* to *D* The Trunk of the Artery opened and expanded. *E E* The Petrifactions or Ossifications in the sides of the Artery. *F F* Their specks in the lower part of the Artery, not so large as in the upper part, and placed at greater distances. *a a* &c. The Branches arising from the Trunk of the Artery. *G* A Portion of the Trunk of the Artery of the arm above-mentioned. *H* The sides of the Artery very much thickned, whereby the Diameter of its *Canalis* was so diminished that the Probe *I* would not pass it.

The Ossifications in the Coats of *Arteries* have been frequently observ'd, especially in their large Trunks within the Cavities of the Thorax and *Abdomen*, but I don't remember the like has been taken notice of in the Limbs; or that such impediments in their Canals have been found the cause of Mortifications of particular parts, as in the Instance above-mentioned; tho I doubt not, but the like has often happen'd in aged People, especially where we find the progress of the Gangreen not very swift, and its beginning from no external Cause; the consequences of which are commonly found Fatal. When the Arteries of one Leg (or of any other Limb) are so affected, we may well suspect the like in those of other parts; which probably happened in the Instance I now mention'd; for tho no Gangreen came on the stump, yet the other Foot and Toes began to Mortifie about 6 weeks after the Amputation, as did the parts about the Hips, which were compreſt in Laying or Sitting, before he expir'd.

FIG. 4. Represents the Extremities of the Blood Vessels, as they appear while the Blood is passing them in the *Omentum* of a live Dog, view'd with a Microscope. *A A* The Branches of Arteries, and *B*, *B*. the Veins which Associate. *C C* Their lesser Branches where they pass from each other, and are United at their extremities.

Fig. 5 The like appearing in the *Mesentery* of a Dog when living. *D, D*, The *Areæ*, that are here viewed with the Microscope, as they appear to the naked Eye.

Fig. 6. The Trunks of the *Vena Cava*, with their Branches dissected from an adult humane Body, done from the Original Scheme in the Repository of the *Royal Society*. *A A* The Orifice of the *Vena Cava*, as it appears when cut from the Right Auricle of the Heart. *a* The Orifice of the Coronary Vein of the Heart. *B*. *A*. The Superior, or Descending Trunk of the *Vena Cava*. *C, C, A*, The Inferior or Ascending Trunk; so distinguished from the Motion of the Blood in these Trunks, which is contrary to their Position. *D D* The Subclavian Veins. † That part of the Left Subclavian Vein, where the Thoracick Duct enters it, and discharges it self of its *Chyle* and *Lympha*. *b*. The *Vena Azygos* with its Branches going to the Ribs, *e*, *e*. *c*. The superior Intercostal Veins. *d, d* The internal Mammary Veins. *E, E*, The Right and Left Iliack Branches. *F, F*, The Internal Jugular Veins. *G, G*, The External *Plate* 8.

ternal

ternal Jugulars. *H, H,* The Veins which bring Blood from the lower Jaw, and its Muſcles. *I, I,* The Trunks of the internal Jugulars cut off at the *Baſis* of the Skull. *f,* The Veins of the *Thymus* and *Mediaſtinum*. *g, g,* The Veins of the Thyroid Glands. *h,* The *Vena Sacra*. *i,* The Internal Iliack Branch. *k,* The External —— *K, K,* The Occiputal Veins. *L,* The Right Axillary Vein. *M,* The Cephalick. *N,* The Baſilick. *O,* The Median Vein. *P,* The Trunk of the Veins of the Liver. *Q.* The Phrenick Vein of the Left ſide. *R,* The Right Phrenick Vein. *r* A large Vein from the left *Glandula Renalis* and parts adjacent. *S,* The Left emulgent Vein. *T,* The Right Emulgent, in this ſubject very much lower than the Left, which is not uſual. *VV,* The two Spermatick Veins. *XX* Two Communicant Branches between the Aſcending Trunk of the *Vena Cava* and *Vena Azygos*, by which the Wind paſſes into the Deſcending Trunk of the *Cava*, when we blow into the Aſcending at *A.P.C.* tho' the Trunk at *A. P.* and *C.* is firmly tyed on the Blow-pipe. * An uncommon Branch between the lower Trunk of the *Vena Cava* and the Left Emulgent Vein. *y* A Vein which brings Blood from the Muſcles of the *Abdomen* into the external Iliack Branch. *z* The Epigaſtrick Vein on the Right ſide. *l* The *Vena Saphena*.

The reſt of the Branches here diſplayed commonly differ ſo much in various ſubjects, that the particular Deſcriptions of them (which none but the Operator who diſſected them could pretend to be Maſter of) would be perhaps as uſeleſs, as tedious to repeat: Wherefore I paſs to thoſe conſiderable various Trunks which are wanting in this Scheme.

Fig. 7. Some of the large Trunks of the Veins and their *Sinus's* within the Skull, with the Beginnings of the Internal Jugular Veins, filled with Wax and dryed together with the *Falx, &c.* *A* The extremity of the *Falx* cut from the *Criſta Galli*. *a* Its lower *Limbus* that touched the *Corpus Calloſum*, as it divides the Right Hæmiſphere of the Brain from the left; where the Fifth *Sinus* paſſes, which is here dryed and diſappears. *B. B.* The ſecond proceſs of the *Dura Mater*, which ſupported the hindmoſt parts of the Lobes of the Brain, and defended the *Cerebellum* from being preſt by thoſe parts of the *Cerebrum*. *C.* A portion of the *Dura Mater* remaining to the Longitudinal *Sinus*. *D. D.* Several Trunks of the Veins of the Brain cut off before they enter the Longitudinal *Sinus*. *E E* The Longitudinal *Sinus*'s. *F F* The two lateral *Sinus*'s. *G* The Fourth *Sinus*. *g* The Veins from the *Plexus Choroides*. *HH* The *Bulbi* or *Diverticuli* at the beginnings of the internal Jugular Veins. *II* The internal Jugular Veins. *KK* The Trunks of Veins which bring Blood from the lower Jaw and parts adjacent.

Fig. 8. The Trunks of the *Vena Portæ* diſſected and diſplayed; done from the Original Scheme in the Repoſitory of the Royal Society. *A A* The Branches of the *Vena Portæ* freed from the Liver. *a* The Umbilical Vein. *B* The Splenick Branch. *C C* The Meſenterick Bran-

Branches which are continued from the Inteſtines. *b* The Trunk of the *Vena Pancreatica*, which receives Branches alſo from the *Duodenum*. *c c.* The *Vena Gaſtrica dextra Coronaria Superior.* *D* The Superior Coronary Vein of the Stomach of the Left ſide. *E* The inferior Coronary Branch of the Stomach of the Right ſide, and *F* The ſame Coronary Vein of the Left ſide removed from their proper Situations; from theſe two laſt are continued the *Vena Epiploica Superior dextra* 1, and the *Siniſtra* 2, with the *Media* 3. *G* The Vein call'd *Vas breve*. *d* The *Vena Duodeni*. *H* The *Vena Hæmorrhoidalis* ariſing from the *Rectum* and *Anus*, in this ſubject emptying itſelf into the left Meſenterick Branch; but in other Bodies (and particularly in a Preparation of theſe Veins, which I have now by me) I find this Trunk of the *Hæmorrhoide* Veins ending in the *Ramus Splenicus*.

The length of the Trunk of this Hæmorrhoide Vein, and its progreſs under the Inteſtines, renders it liable to be compreſs'd, and its Refluent Blood retarded; whence its Branches in the *Inteſtinum Rectum* and *Anus* become diſtended with Blood, and cauſe the *Hæmorrhoides Cæcæ* and *Apertæ*; which are frequently attended with Apoſthumations in the *Anus* and Parts adjacent; which Diſorders are the more incident, not only becauſe theſe Hæmorrhoide Veins (like the reſt of the Branches of the *Vena Portæ*) are without *Valves*, and the Blood has an aſcending progreſs in them, together that the long Trunk *(H)* is not only expoſed to the Compreſſions made by the Inteſtines in both Sexes: But particularly the *Uterus* in Women in time of Geſtation, eſpecially near the Birth, ſo compreſſes this Trunk, that it's no wonder we find Women more afflicted with the Hæmorrhoides at that time than any other. Nor are the Iliac Veins, and the Lymphaducts that accompany them without being expoſed to the like Incumbrance in Women with Child, whence the Veins of the Legs and Thighs become Varicoſe, and thoſe Limbs are ſo frequently ſwollen; which in a late Inſtance I was acquainted with, when the Intumeſcence proved ſo great, that at length the Abdominal Teguments were vaſtly extended; but the Gentlewoman recovered, beyond the Expectation of ſome, on the happy Delivery of two large Children.

II. 'Tis well known from the Obſervations of Mr. *Leewenhoeck* and others, that human Blood conſiſts of red Globular Particles ſwimming in a pellucid Lympha or Serum; which two different Subſtances, tho' of unequal Specifick Gravities, yet ſo long as they continue to circulate in the Veins and Arteries, are prevented from ſeparating by their Motion and Warmth. But when the Blood comes to ſtagnate and cool in a Porringer, the globular Particles uniting together by their attractive Power, and ſinking by their Weight, which is greater than that of the *Serum*, form the *Coagulum* or *Craſſamentum* at the bottom of the Porringer, the *Serum* ſwimming above it. Things always happen in this manner, when the *Craſſamentum* is at liberty to ſubſide; but it often

Experiments relating to the Specifick Gravity of Humane Blood. by Dr. Jurin, *n.* 361. *p.* 1000.

falls out, that either by its Adhesion to the sides of the Veins, or by the Bubbles of Air, which the Blood gathers upon falling into the Porringer, and which stick to its Surface, the *Crassamentum* is kept from sinking, and seems to float upon the top of the *Serum*. These Accidents seem to have given the first occasion to the Opinion of those who have writ upon this, that the globular Part of the Blood is specifically lighter than the *Serum*, in which it swims. But that which has so fully establisht this persuasion, is the Authority of the late excellent Mr. *Boyle*, who has left the following Experiments upon this Subject.

The Specifick Gravity of *Serum* of Human Blood was found by weighing a piece of Sealing-wax first in *Serum*, and afterwards in Water, to be to the specifick Gravity of Water, as 1024 to 1000. In a second Experiment, which for accuracy was made with an Instrument contriv'd on purpose, the specifick Gravity of *Serum* was found to be to that of Water, as 1194, to 1000. In a third Experiment made by the same Instrument, and with *Serum* from the Blood of another Person, its specifick Gravity appear'd to be 1186. The Medium between these two last Experiments is 1190, which has since been universally receiv'd for the specifick Gravity of *Serum* of Human Blood, the first Experiment being declar'd by Mr. *Boyle* himself to be less exactly made than the other. The specifick Gravity of Human Blood was found by Mr. *Boyle*, to be to that of Water, as 1040 to 1000; though on account of Difficulties by him mentioned, he was far from being satisfy'd with this Experiment, and recommended the thing to farther tryals.

These Experiments however having hitherto past uncontroverted, and it appearing from them, that the specifick Gravity of *Serum* was greater than that of Blood in the proportion of 1190 to 1040, or of 8 to 7 nearly; it was a necessary consequence of this, that the Blood Globules were specifically lighter than the *Serum*, and that in a very great degree, considering the small proportion that the bulk of the *Crassamentum* was found to bear to that of the *Serum*, from other Experiments. From this it was not improbably conjectured, that these Globules were thin Vesicles fill'd with an Aereal Substance: and this Opinion seem'd to receive a great Confirmation, upon its being observ'd, in viewing the Circulation by a Microscope, that a Blood Globule, in passing through a very narrow Vessel, would change its shape from a Globular to an Oval Form, and would recover its former Figure, as soon as it was got through its narrow passage; which appearance seem'd to be naturally accounted for from the Elasticity of the included *Aura*. Upon this Conjecture have been built a great many Solutions of the Phænomena observable in the Animal Oeconomy, and the Disorders of it; particularly a late ingenious account of Muscular Motion. I hope I shall be easily pardon'd for enquiring into the soundness of the Foundation, when the Superstructure erected thereupon is so considerable; and the following Experiments, however trivial in themselves, will not appear unworthy the Consideration of

of the *Royal Society,* if it be found, that they may prevent us from running into Errors of the greateſt Conſequence.

Exp. I. I have ſeveral times cut off a ſmall part of the *Craſſamentum,* when by its adheſion to the ſides of the Porringer it has ſeem'd to ſwim upon the Surface of the *Serum,* and have put it into another Veſſel fill'd with *Serum*: upon which it has immediately ſunk to the Bottom.

Exp. II. When the *Coagulum* has been buoy'd up in the *Serum* by the bubbles of Air adhering to its Surface, I have ſeparated a ſmall part of it, where thoſe Bubbles have been thickeſt, and put it into a Glaſs of *Serum,* in which it has ſwom as before. Then ſetting the Glaſs upon the Air-pump, thoſe Bubbles burſt after one another, as the Receiver was exhauſting, and the Air being again let into the Receiver, the Lump of the *Craſſamentum* ſunk to the bottom of the Glaſs.

Exp. III. I have often placed a Drop of *Serum* upon a clean Glaſs before a Microſcope, in which I had diſſolv'd a very ſmall Quantity of Blood, and obſerv'd, that when the Glaſs was held in a perpendicular Poſture, the Blood-Globules ſubſided to the bottom of the Drop; and inverting the Glaſs, the Globules again deſcended through the *Serum* to the bottom. I had the ſame Succeſs with a ſmall Quantity of *Serum* and Blood in a Capillary Tube. And the ſame thing has been long ſince obſerv'd by the famous Mr. *Leewenhoeck.*

Theſe Experiments undeniably demonſtrate, that the *Craſſamentum,* or globular part of the Blood, is ſpecifically heavier than the *Serum*; and conſequently it is by no means probable, that the Blood Globules are Veſicles fill'd with Air, or any other Fluid lighter than *Serum.* And that they are not fill'd with any ſort of Elaſtick Fluid, will appear from the following Experiment.

Exp. IV. In a ſmall Quantity of *Serum* of Human Blood, I diſſolv'd ſo much Blood, as that the Globules might not lye too thick together, to hinder their being ſeen diſtinctly. Then having lodged a ſmall Drop of this Liquor on the inſide of a thin Glaſs Tube, I fitted the Tube on to the Air-Pump, and placed a Microſcope by it, ſo that I could ſee the Blood Globules through the Tube. This being done, I caus'd the Tube to be exhauſted, keeping my Eye upon the Globules all the time, in order to obſerve whether they dilated themſelves, as the Air was withdrawn; but could not perceive the leaſt Alteration, they appearing exactly of the ſame bigneſs in the *Vacuum,* as they had done before. Whereas if they had been fill'd with an Elaſtick Fluid, they would either have burſt, or have been dilated to at leaſt ſeventy or eighty times their former Magnitude. The Stop-cock being afterwards turn'd, and the Air ſuffer'd to re-enter the Tube, the Blood-Globules ſtill retain'd the ſame bigneſs, as in *Vacuo.* I ſhall proceed to demonſtrate the ſame thing by Hydroſtatical Experiments.

Exp. V. *Nov.* 13. 1713. Having ſuffer'd a quantity of my own Blood to ſtand about 24 Hours in the Porringer, and then drawing off the *Serum* carefully with a ſmall Siphon into a convenient Glaſs, I found by the Hydroſtatical Balance its Specifick Gravity to be to that of Water, as 1029,8 to 1000.

Exp. VI. *February* 21. 1716-7. I examin'd the *Serum* from the Blood of another Perſon in the ſame manner, and found its Specifick Gravity to be 1028,6.

Exp. VII. VIII. and IX. *April* 8th 1717. I obtain'd 3 ſeveral quantities of *Serum* from the Blood of different Perſons. The firſt of theſe was of a deep colour, inclining ſomething to red, and a little Turbid. It's Specifick Gravity was 1029,7. The ſecond was likewiſe a little Turbid, and of a pale whitiſh Colour. The Specifick Gravity of this was 1030,2. The third quantity of *Serum* was perfectly clear, and of the colour of Canary. The Specifick Gravity was found to be 1030.

Exp. X. *January* 15th 1718-9. I drew off all the *Serum* from five or ſix ſeveral Porringers, containing the Blood of different Perſons. This I found to be a little ting'd with Blood, which was occaſion'd by my being oblig'd to draw it off pretty near to the bottom of the Porringers, in order to obtain a quantity ſufficient for my purpoſe. For this reaſon I ſuffer'd it to ſtand about two Days, in which time the Globular part of the Blood was entirely precipitated to the Bottom, and the *Serum* was become perfectly fine and tranſparent. I then drew it off with a Siphon into a Glaſs Vial with a narrow Neck, which I fill'd to a certain mark made in the Neck for that purpoſe. This done, I plac'd my Vial in a nice pair of Scales, in which I had a Counterpoiſe for the weight of the Vial, and found that quantity of *Serum* to weigh 2284$\frac{11}{4}$ Grains. Then pouring out the *Serum*, I fill'd the Vial with common Water to the ſame mark, and found the weight of the Water to be 2219 Grains. From which it follows that the Specifick Gravity of this *Serum* was 1029,4.

Exp. XI. *July* 14. 1719. I procur'd a quantity of Blood taken from the temporal Artery, from which I drew off the *Serum* the next Day, and weighing it in the ſame manner found it's Specifick Gravity to be 1028,8.

The little difference between theſe Experiments may very well be attributed to that which is between the *Serum* of different Perſons; or to the variations occaſion'd by Heat and Cold in the ſeveral Seaſons of the Year, in which they were made. So that from them we may ſafely determine the Specifick Gravity of *Serum* of Human Blood at a Medium to be 1029,5, or in a round number 1030. From which the greateſt Variation in any of theſe Experiments is little more than one in 1000; whereas the difference between Mr. *Boyle*'s Experiments and mine amounts to 160 in 1000.

Exp. XII. *April* 6. 1717. In order to find the Specifick Gravity of Human Blood, which by reaſon of it's tenacity, and ſudden alterations upon ſtanding, cannot be determin'd by the Hydroſtatical Balance; I took

took a narrow neck'd-Vial, and fill'd it to a Mark, with Blood pour'd immediately out of the Porringer, as ſoon as the Perſon was blooded. This I weigh'd, as I had done the *Serum* before, and found it's Specifick Gravity to be 1051.

Exp. XIII. *Auguſt* 5th 1717. Having fill'd the ſame Vial with the Blood of another Perſon, running immediately out of the Vein through a Funnel, it's Specifick Gravity was determin'd at 1053. Suffering this to ſtand till it was cold, I found the Blood was ſunk a ſmall matter below the Mark in the neck of the Vial. This being fill'd up with the Water, which in ſo ſmall a quantity could make no ſenſible difference from Blood, I found the Specifick Gravity of cold Blood to be 1055.

Exp. XIV. *Auguſt* 6th 1718. The laſt Experiment being repeated in the ſame manner as the Year before, the Specifick Gravity of cold Blood was again found to be 1055.

Exp. XV. *July* 14th 1719. The Arterial Blood, from which the *Serum* was afterwards drawn off for the 11th Experiment, being weigh'd in the ſame manner, it's Specifick Gravity was 1052,5.

As this Arterial Blood and it's *Serum*, differ no more in Specifick Gravity from Venal Blood and *Serum*, than the ſeveral Portions of theſe do from one another, it's plain, that the difference in this reſpect between Arterial and Venal Blood is wholly inconſiderable. The Animal Oeconomy indeed teaches us, that the Serous Liquor is perpetually drawing off from the Arterial Blood by the ſeveral Secretions, but as the quantity ſeparated in one Circulation is very ſmall, the Blood muſt arrive in the Veins nearly of the ſame denſity, as when it runs through the Arteries.

In the 13th Experiment we obſerv'd, that the Blood alter'd it's Specifick Gravity upon cooling from 1053 to 1055; from which we may infer, that if the Blood made uſe of in the 12th Experiment had been ſuffer'd to ſtand till it was cold, it's Specifick Gravity would have been 1053; wherefore, taking a Medium between the four laſt Experiments, we may allow the Specifick Gravity of cold Human Blood to be 1054. The difference of 14 Parts in a 1000, between this and the Specifick Gravity determined by Mr. *Boyle*, is eaſily accounted for, if we conſider, that that Gentleman did not make uſe of a Veſſel with a narrow Neck, as plainly appears from the Circumſtances mentioned in his Experiment; and conſequently a ſmall error in the height of the Liquor would make a conſiderable alteration in the Specifick Gravity.

Since therefore the Specifick Gravity of Human Blood is 1054, and that of its *Serum* 1030, it is plain, that Blood is heavier than *Serum* by about one part in 43. From which it manifeſtly follows, that the Globular part of the Blood is ſpecifically heavier than the *Serum*, ſince the Globular part being ſeparated from the Blood leaves the remainder, or the *Serum*, ſpecifically lighter than the intire Maſs. But in order to determine the exact Specifick Gravity of the Blood Globules, it is firſt neceſſary to know the Proportion, which the whole quantity of

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the *Craſſamentum* contained in Blood bears to the *Serum*. To this end Mr. *Boyle* has given us two ſeveral Obſervations of the weights of the *Craſſamentum* and *Serum*, after they have ſeparated one from another in the Porringer. But beſides the difficulty of making this Experiment with any tolerable exactneſs, it is to be conſider'd, that there is a great deal of *Serum* contain'd in the interſtices of the Globules, that compoſe the *Craſſamentum*. This difficulty however is in ſome meaſure anſwer'd by two other Experiments, which Mr. *Boyle* made for this purpoſe, after the following manner. He put a quantity of the *Craſſamentum*, already ſeparated from the *Serum*, into an Alembick, and diſtill'd off the remaining *Serum* to dryneſs, but without drawing off the Oil, or Volatile Salt; after which he weigh'd the diſtill'd Liquor, and the dry Maſs left behind. By comparing theſe Experiments with the two former, it will be found that the entire weight of *Serum* contain'd in Blood is nearly $\frac{13}{15}$ of the whole, and conſequently the weight of the dry'd *Craſſamentum* is only two Fifteenths of the Blood.

But for farther ſatisfaction, an Analyſis was made at my deſire with a large quantity of Blood, amounting to Four Pounds Fourteen Ounces, by that ingenious and skilful Chymiſt, Mr. *John Brown*. From this was obtain'd, with a very gentle heat, two Pounds, Fourteen Ounces, and ſix Drachms of a Phlegmatick Liquor, that had ſcarce any thing of the fœtid Scent, which is uſual in the diſtillation of Animal Subſtances; and it's Specifick Gravity was nearly the ſame with that of common Water, being but 1000,8. This being mixt with a ſtrong ſolution of Alum, ſcarce afforded any *Coagulum*; but exhibited a conſiderable one upon mixture with a ſolution of Roman Vitriol. The diſtillation being continued with the ſame Heat, we had ſeven Ounces more of Phlegm conſiderably impregnated with Volatile Salt, as was manifeſt from the Smell. The Specifick Gravity of this was 1007, and having mix'd it with *Tinctura Martis optima*, Solution of Alum, and of Roman Vitriol, a large *Coagulum* was precipitated. In diſtilling theſe there was loſt by Evaporation, two Ounces and two Drachms.

The third portion of Liquor, being rais'd with a ſtronger Fire, amounted to ſeven Ounces ſix Drachms. This was reddiſh, and Turbid, and ſo ſtrongly charg'd with Volatile Salts, that it might very well deſerve the name of Spirit. It's Specifick Gravity was 1080,1. Beſides theſe we had ſeven Drachms of Volatile Salt, an Ounce of Oil, and eight Ounces Four Drachms of *Caput Mortuum*, which ſtill retain'd ſome ſmall remainder of the Oil, as was manifeſt from its taking Fire at the flame of a Candle. In this latter part of the Operation was loſt three Ounces ſeven Drachms.

Upon making due allowance for the difference between the Specifick Gravities of the three firſt Portions of Liquor and that of *Serum*, as likewiſe for what was loſt in the two ſeveral parts of the Operation, which we may reaſonably conclude to have been of a Specifick Gravity

vity nearly the ſame with that of the Liquor drawn off, it will be found, that the quantity of *Serum* contain'd in this Maſs of Blood was about $\frac{15}{17}$ of the whole Weight, and conſequently that the quantity of *Craſſamentum* was $\frac{2}{17}$ of the ſame Weight. If we calculate therefore upon this Suppoſition, that the weight of the Globular part of the Blood is $\frac{2}{17}$ of the whole, we ſhall find the Specifick Gravity of a Blood Globule to be to that of Water as 1277 to 1000. If we follow the proportion of $\frac{2}{13}$ which reſults from Mr *Boyle*'s Experiments, the Specifick Gravity of a Blood Globule will be 1242.

But this computation is in all appearance a great deal too large; for we cannot be aſſur'd, that our whole quantity of aqueous Liquor was rais'd from the *Serum* of the Blood. On the contrary it is more than probable, that a conſiderable part of it was afforded by the Blood Globules themſelves, eſpecially in the latter part of the Operation, when their texture muſt of neceſſity have been broken and diſſolv'd by the ſtrong Fire that was made uſe of. To prove this, we need only conſider the condition of the dry'd *Craſſamentum*, after the Phlegm is drawn off, that being now a hard and brittle Subſtance: whereas the Globules in their natural State are ſoft and yielding. For which reaſons it may perhaps be more ſatisfactory, if we attempt to find the quantity of the Globular part of the Blood after another manner. It appears therefore from Mr *Boyle*'s Obſervation, that the quantity of *Serum*, which may be pour'd off from the *Craſſamentum*, is about one half of the whole Maſs. The remaining *Craſſamentum* conſiſts of the Blood Globules; and a quantity of *Serum* filling up the Interſtices between them; which, if the Globules keep their Spherical Form, may eaſily be found by the principles of common Geometry, to be nearly one half of the bulk of the *Craſſamentum*: but if the Globules by their preſſure againſt one another change their Figure, the quantity of *Serum* will be ſomething leſs. If this quantity of *Serum* lying between the Blood Globules be added to that pour'd off, it appears, that the *Serum* contain'd in Blood is about $\frac{3}{4}$ of the whole bulk, and conſequently that the Blood Globules make about $\frac{1}{4}$ of the whole. From which we ſhall find the Specifick Gravity of the Blood Globules to be to that of Water as 1126 to 1000. If we ſuppoſe the Blood Globules to make $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{3}$, or $\frac{1}{2}$ of the whole bulk, their Specifick Gravity will be reſpectively 1174, 1150, 1102, or 1078. So that upon any of theſe Suppoſitions, the Specifick Gravity of the Blood Globules will be conſiderably greater than that of the *Serum*, and conſequently they cannot be ſuppos'd to be Veſicles fill'd with an Aereal Subſtance.

It will therefore perhaps be askt, What do they really conſiſt of?

In order to come to a Solution of this Queſtion, it may be proper to take notice, That Blood is compos'd of Phlegm, Oil, Volatile and Fixt Salts, and Earth. For as to the Spirit, we look upon it with Mr *Boyle*, to conſiſt of the Phlegm and Volatile Salt united together. That the *Serum*, upon a Chymical Analyſis, exhibits a great deal of the

the first of these, and the others in a very small quantity. That on the contrary the *Crassamentum* yields much less Phlegm, but the other Principles much more copiously than the *Serum*.

From which *Data*, I think, we may safely conclude, that the *Crassamentum*, or Globular part of the Blood, consists of some Phlegm united with the Oil and Salts, and a small quantity of Earth. But what is the exact proportion of these several Principles to one another; what alterations are produced in the Body by a change of this proportion; how, and in what part these Globules are form'd; by what means they preserve their Figure, without dissolving in the *Serum*, or uniting with one another; what variations are made in their Specifick Gravities by Heat and Cold; and what are the effects of those Variations, are Questions not very easy to be solv'd, and yet of so much importance to the Animal Oeconomy, that it were greatly to be wisht, we had a number of *Data* sufficient to determine them.

The following Experiments, serving to confirm the Method last made use of, for finding the Specifick Gravity of the Blood Globules, it may not be improper to relate them.

August 6. 1719. I took a lump of the *Crassamentum* and wash'd it gently in fair Water, to free it from the loose Globules, which precipitating out of the *Serum*, after the *Coagulum* is form'd, do not unite into one Body with it. This done, I laid it on a Spungy brown Paper, in order to drain off the superfluous Moisture. After which, weighing it first in Air, and then in Water, I found its Specifick Gravity to be $1083\frac{1}{7}$. Another lump of the same *Crassamentum* being weigh'd in the same manner, its Specifick Gravity was 1082,9.

September 18. 1719. I found the Specifick Gravity of another piece of *Crassamentum* to be 1082,1. A second piece from the Blood of a different Person gave me 1086,1. A third from the same Person gave 1086,6. From this it follows that the Specifick Gravity of the Blood Globules is at least 1084, which is the Medium between these five Experiments. But if we allow one half of the bulk of the *Crassamentum* to consist of *Serum*, filling up the Spaces between the Blood Globules, we shall find their Specifick Gravity to be 1138. From this we must make a small abatement, because some part of the *Serum* must have been squeez'd out from between the Globules, by their yielding to one anothers Pressure, when the lump of *Crassamentum* lay upon the Paper: and this will reduce their Specifick Gravity sufficiently near to 1126, as we had before determin'd it.

Of Human Blood, by Dr. Arch. Adams. *n.* 325. *p.* 26.

III. Human Blood view'd in a Microscope, is so far from shewing any red Globules swimming in *Serum*, that immediately after its Emission, it appears to be a Body of infinite Branches, running in no certain Order, variously colour'd; where it lies thickest on the Glass, 'tis of a dull red; where thin, inclining to yellow; but the whole so blended, as to represent very near the Top of an Yew-tree, in a Landskip,

Landskip, having its ſuppos'd Branches of a red and yellow confuſedly intermixt. Not ſatisfy'd with this Appearance, though try'd eleven different Ways, I view'd by diluting one third of thick in the *Serum* of Blood, and laying it on my Glaſs, I could ſee the red Branches, as before, and the Tranſparent fill'd with Particles of great Variety of Figures, which I took to be the Salts of the Blood, but feweſt Globular, and they were pellucid.

If the Fluids moving in an evaneſcent Artery appear Globular, I ſuppoſe, 'tis becauſe the Canal is round, which alters the Caſe much.

At the ſame time, I ſaw ſome Pleuritick Blood, and thought that its Branches ſpread in a different method from the ſound, and more ſtrongly perplext with overthwart Branches, which appear'd black like Blood that had ſtood two or three Days. Whether the Attraction of Particles ariſing from this difference of Figure, may not render the Blood incapable of paſſing thro' the Capillary Artery of the *Pleura* in that Caſe I ſubmit; but ſhould think, ſince the propellent Force of the Heart is leaſt at the Capillary Arteries, there the attractive Force of the Blood ſhould be greateſt; and ſince Spherical Bodies are moſt attractive of any, reſpect being had to their Solidities, were the Blood ſo plentifully ſtock'd with Globules as ſome ſay, we ſhould never be free from Obſtructions, the neceſſary Conſequence of this attractive Force. If in any thing I have err'd I am ready to retract.

An unuſual Hæmorrhage in a Child, by Dr. Monginot, *n.* 268. *p.* 756.

IV. Madam *R*——'s Girl fell into violent Convulſion Fits; and while ſhe was in them voided a large quantity of Blood by the Mouth, the Noſe, the Ears, and the Eyes. The Mother ſhew'd me ſome bloody Tears ſhe had gather'd. All theſe Symptoms were over in half an hours time, and the young Girl, that ſome days before had been taken with a violent Head-ach, attended with a Fever, and a mighty oppreſſion, was very well preſently after that *Hæmorrhagia.* The ſame accidents as I hear, have happened ſeveral times.

She was between two and three Years old, when on a ſudden ſhe complain'd of a violent Head-ach: She was alſo obſerv'd to be feaveriſh, and extremely reſtleſs. At the ſame time her Eye-lids did very much ſwell, and they were ſo heavy, that one could hardly open her Eyes but with a great deal of pain. Theſe ſymptoms did continue for three or four Months, though ſometimes more or leſs violent. At laſt, ſhe fell into Convulſive Motions of her Arms, Legs, and other parts, and theſe were very fierce for two days, till ſhe began to bleed by the Noſe, the Mouth, the Ears and the Eyes. The Linnen Cloaths, that were then applied to the Eye-lids, were all wet and bloody. This *Hæmorrhagia* laſted above a day; and what is more remarkable, When it was expected that the Child ſhould be extremely weakned, ſhe found herſelf ſo well freed from her Illneſs, that ſhe came to her Gaiety again, and ask'd for ſome Victuals. Within twelve Months after ſhe

had four of theſe *Attacks*, but not ſo great: in particular, the Convulſion-fits were very inconſiderable, in compariſon of the firſt. The ſame Symptoms did not return again above two or three times every year, and the Head-ach did not continue above eight days before the *Bleeding*: and even then it was much more ſupportable. About two months ago I was ſent for to ſee the Girl, who is now ſeven years old. I found her in Bed, complaining extremely of a Head-ach, attended with a Feaver, a great *Catarrh*, and ſuch a ſhortneſs of breath, as if ſhe had had a *Peripneumonia*. She had been three days in that condition. I told the Mother, my opinion was ſhe ſhould be blooded preſently; but it being very late, it was put off till next morning, and then indeed it was needleſs; for very early in the morning, after ſome Convulſion fits, ſhe began to bleed from the Noſe, the Mouth, *&c.* When I came it was almoſt over, ſo that I could only ſee a little bleeding from the Mouth; but all the people in the houſe told me they had ſeen the Blood drop from all theſe parts. The Girl was then pretty well, without any Feaver or *Catarrh*: She could breath freely, was in good humour, and had a good Stomach; and ever ſince has continued in perfect health.

A very extraordinary Periodical Hæmorrhage, *by Dr.* Muſgrave, *n.* 272. *p.* 854.

V. Mr. *H.* had (from his Infancy, as he has been inform'd, but to his certain knowledge) from as far back as he can remember, up to the twenty fourth year of his age, a *Periodical Hæmorrhage* in one of his Thumbs. The *Time* of the Eruption was about the full of the Moon, ſeldom more than a day before or after it. The Orifice (to ſpeak more exactly) on the Right ſide of the Nail of the Left Thumb. He has not known the Blood to be leſs in weight at any one Turn, or Periodical Diſcharge, than four ounces; and is poſitive, that when he came to be ſixteen years of age, the quantity was then encreas'd to half a pound at each eruption; he and his friends having often view'd the quantity, and found it to be thus much. The *Manner* of the Flux was alſo remarkable; for, without any Pain of Head, Streightneſs of Breath, or any ſigns of Fulneſs, or other Symptoms whatever obſerved by him, excepting only a ſtiffneſs, on the utmoſt Joynt of the ſaid (Left) Thumb; the Blood uſed to ſpin out, with a conſiderable force, on a ſudden, in ſeveral little ſtreams, and continue ſo to do, until the greater part of the quantity was diſcharged. Under this diſcharge, however copious, he was ſtrong and vigorous to the age of twenty four, from his moſt early and tender years. At that age (of 24) finding this Evacuation troubleſome, and being uneaſie under it, he ſear'd with a hot Iron the part, which uſed to open and give vent to the flux of Blood. I ſaw that part, it was hard and callous to the diameter of $\frac{1}{2}$ of an inch. The ſearing had ſtopt the Hæmorrhage to the day I took this account, (which was on *Dec.* 11. 1697,) that is, about twenty years.

This ſtoppage was in its effects very dangerous, and of ill conſe-

quence;

quence; for within one quarter of a year after it he fell into a *Sputum Sanguinis*; bringing up from his Lungs vaſt quantities of Blood. This new Complaint, together with a Cough attending it, reduced him very low; ſo that his Phyſician, old Dr. *Dike* of *Somerſetſhire*, (a perſon of great Learning and Experience in our Profeſſion) thought him utterly loſt in a Conſumption; but by frequent Bleeding, *&c.* deliver'd him from this *Hæmoptoe*: but in a very little time the Patient fell into a moſt violent Colic; from no other occaſion (that he could diſcover) than his late illneſs, putting on a new form, and the matter ſettling on the Bowels. This Colic was in good meaſure overcome by Purging Medicines; but a diſpoſition to it ſtill remains; for he has ever ſince been often troubled with it (as alſo with a Spitting of Blood) on the leaſt exceſs of Cold or Motion. He has, ever ſince the ſtoppage of that firſt Hæmorrhage, been weak, ſickly, of a ſallow, faint look; much impair'd as to health, in compariſon to what he enjoy'd during the time of its Periodical Returns; and gives an Argument, *That when Nature has choſen, and for ſome length of time, exerciſed new and extraordinary methods of Oeconomy, ſhe ſeems to be as fond of their Continuance, as at other times, and in her moſt regular ſtate, ſhe is of that which is her moſt uſual and ordinary courſe.*

An Eruption of Blood from ſeveral parts of the Body, by Dr. Meſaporitus. *n.* 303. *p.* 2144.

VI. Honeſtiſſima viſendæque ſpeciei Juvenis, cui natura ad 18 annum, integerrimam veletudinem hilaremque genium dederat, per multiplices nunc ducta morborum modos incidit in fluxum ſanguinis ex inuſitatis partibus, & tandem in deterrimam partialis ſudoris ſpeciem ex ambitu corporis. Quamprimum in eâ ætate concipiendæ ſpei aptiſſimâ gravi ſenſu correpta fuit, poſt diuturnum ſtomachi laborem cœpit ſanguinem tuſſi expuere occaſione ementitæ pleuritidis, quâ die nonâ *Aprilis* febricitabat cum dolore lateris, & difficili reſpiratione: *Quarto* inſultûs *die* thoracis morbo evaneſcente, capitis dolore, quo & antea laboraverat, gravari, & ſine levamine ſanguinem fundere è naribus obſervabatur; quocirca hujus copiam è pede detraxit *Chirurgus Familiaris*, qui bis eductum primis diebus ex brachio gelatinoſum inſpexerat. Verum haud decreſceret morbus, qui conceptum virus in promptu habebat movere ad ulteriora; hinc ſuperaddita eſt afflictæ Juveni Cardialgia, unde vomitus primum viſcidi, & viridis humoris, mox ſanguinis, nec proinde capitis ægritudine allevabatur, quin circulo quodam ordinato recurrentibus purpureus liquor ex utraque parte, & ad multos dies prodibat. Malorum importuna viciſſitudo nimis adhuc increbuit, cum circa initium *Maii* erumperent Menſes, & prout ipſi mos erat in ſufficienti menſurâ. Chirurgus *Mannæ* potionem exhibuit poſt ſeptimum diem, & ab ea licet non parum perturbata in pejus ægra ruere videretur, attamen paulo poſtea febris in totum ceſſavit, quæ antea plus vel minus una cum vigiliis ingraveſcens fuerat continua. Vix per integram hebdomadam habuit otium morbus, cum febris rurſus invaſit comitata dolore artuum, & potiſſimum ventris quibus licet ſeſe cius

expediverit absumpto nempe *Oleo Amygd. dulcium rect.* sine igne extracto, nihilominus constans vomitionis molestia, & sanguinis jactura summopere urgebant. Diro Symptomati ut occurrerent Clarissimi Medici, consuluerunt in minori dosi, & per vices sanguinem ex *Salvatella* detrahere, anodina, adstringentia, & alia quam plurima adhibere; quod etsi executioni mandatum fuerit, inconstanti methodo procedens effectus, frequentiores excretionum vias fere omnes cum attigisset, cæpit per inusitatas errare.

Cum igitur circa initium *Junii* thesaurus vitalis ex auribus effluxisset, paulo post curiosiori spectaculo ex acumine digitorum primo manuum, deinde pedum se fundere visus est: Mox ex umbilico, & ex angulo oculi; hinc pluries per sudorem inde tandem ex medio pectore, postea ex pede, eoq; loci, ubi tunditur Saphena, tandem ex utraque volâ manuum, & ex opposita parte; post biduum ex mento, noctu ex acumine linguæ, ita ut quatuordecim dierum spatio discrimen hoc, veluti fabulosum maxima ex parte absolverit. Motus novitate rei, & Amicorum impulsu veni ad Juvenem, quam miratus viribus mediocriter constare, & jucundo vultu singula mihi enarrare, prout ab aliis audiveram: Adhuc cernere erat superstitem cicatriculam in volâ sinistræ manûs ad instar inæqualis puncturæ leviter impressam, unde sanguis pridie fluxerat, retulitq; id non sine sensatione tristi factum fuisse, uti semper contigerat, cum ex speciali loco prodiret: ubi vero ex pectore, aut aliis partibus sudoris specie emanaverat, nullum apertum pori reliquerat vestigium; id tantum ex indusiis patebat, quorum duo vidi ad latitudinem scuti aurei cruore roseo sparsim colorata. Die 14 *Junii* apparentibus menstruis justò copiosioribus, inde sublatum morbum nonnulli crediderunt, potissimum quòd viginti dierum inducias fecisset. Verum inchoato altero sudoris exordio, & ad multum tempus procedente misella debilitari sentiebatur, & sanguis discolor apparere. Hinc recreandi spiritus, mutandæq; fortunæ gratiâ in alterum aerem sese transtulit. At qualis secesserat, rediit ad Urbem die quinta *Augusti*, imò infirmior, quod præter morbi acerbitatem, artis quoq; methodo exhaustâ ulteriorem jacturam fuisset experta. Hinc non dissimili modo progrediente ægritudine, & paucis tantum habitis intervallis effundendo sanguinem modò ex unâ parte, modò ex alterâ, sæpius per sudorem, quod reliquum erat ejusdem mensis, & medium *Septembris* confecerat. Petii ægræ domum, & accidit, quod forte expetiveram, ex dolore lancinanti in extremo unius digiti manûs admonitam fuisse sanguinem inde extillaturum, quem notavi, veluti ex profundiori puncturâ, mox ex altera parum distanti guttatim decidere, & indusium supra Pectus recenti maculâ signari videram priusquam recederam. Post triduum mirabiliorem sanguinei sudoris copiam exhibuit. Vel nullum, aut exigui momenti sudorem deinceps passa est. Attamen reticere non debeo ex levi excoriatione supra maleolum casu torturæ pedis factâ uberiorem inde penum serositatis effluxisse, ut stipticis remediis, & compressis fasciis non facile sisteretur. Omnium Senior Medicus *Fabricius*

bricius Ardisonus, qui nonagesimum attingit annum, retulit mulierem annorum triginta se olim curasse, cui suppressæ erant menses, & sanguis ad multum tempus ex oculi angulo fluebat, revocatis nimirum Menstruis ad consueta naturæ.

CHAP. VI.

The Bones, Joynts and Muscles.

I. *A Callus supplying the Place of the* Os humeri, *by Mr.* Fawler, *n.*312. *p.* 2466.

JOhn Marsh of the Parish of *Denton* in *Kent*, when about sixteen years old, was troubled with a Tumor on his Arm, in the end of a continual Fever, which seem'd to be a Critical Discharge of the Humour of the Feaver on his Arm; he was managed by a Surgeon of the Parish for two Years together for this Tumor; at length there being no appearance of a Cure, he was sent to me: At first Dressing I found two Sinuous Ulcers in his right Arm, one upwards about the Deltoid Muscle, the other under the upper part of his Arm, within an Inch and half of the Juncture of the *Cubitus*; the *Sinus* above passing upwards within an Inch and half of the Juncture, and downwards to the *Cubitus*. The *Sinus* of the lower part pass'd downwards to the *Cubitus*, and upwards about an Inch and half. When both these *Sinus* were laid open, the Bone soon shewed itself carious and loose, so that I easily took it out, and was about five Inches long. Three Weeks after came off another Spelt of Bone of the inner side, about two Inches long, having the Channel of the Marrow, Fig. 5. These Ulcers with much Care and Diligence, were cur'd very well in nine Months, and the place of the Bone is so well supply'd with a strong *Callus*, that he is not only very strong, but can lift Fifty pound Weight with that Arm.

Plate 6. Fig. 4.

Fg. 5.

A Callus supplying the Loss of part of the Os femoris, *by Mr.* Sherman. *n.* 323. *p.* 451.

II. A Compound Fracture happening in the Thigh of a young Man about seventeen, I was obliged to take out the whole Substance of the *Os femoris* about two Inches; and yet by keeping a due Extension, Nature did in four Months supply such a *Callus*, that the Part is not a quarter of an Inch shorter than on the other Side; and the Person is as strong as ever, and walks without any Lameness.

A Woman that lost her Leg and greatest part of her Thigh by a Gangrene, by Mr. Calep, *n.* 313. *p.* 41

III. 1. The Wife of *Tho*. *Steven* of *Maidenhead* in the County of *Berks*, (aged about 62 Years) was seized with a Fever about the latter end of *November* 1697. Her Physician used various Remedies to remove her Fever, which in about 14 Days terminated in a Tumor and Numness in her left Foot, both which did by degrees creep up her Leg, and half way up her Thigh. A Fomentation was ordered by her Physician made

made of *Centaur. Absinth. Hyperic. &c.* boyl'd in a ſtrong *Lixivium*; and after fomenting, he order'd them to anoint her Foot and Leg with *Ol. Terebinth.* wherein *Galbanum* was diſſolved. This Method they had uſed daily for a Month before I ſaw her. Coming *Jan.* 3. 1678. from *Henley* upon *Thames* (where I then lived) to ſee ſome Friends and Relations I had at *Maidenhead*, they deſired me to ſee this poor Woman; which I did, and found her in the following Condition, (*viz.*) Her Foot and Leg cold, inſenſible, wither'd, hard as if dry'd in a Chimney, and of a dark Tawney Colour. Her Knee was ſwell'd, and had ſeveral large black Spots upon it, which pitted when preſs'd with my Finger. There was ſeveral Diſcolourations in the Skin half way up her Thigh. She complained of great Pains, eſpecially at Nights in her Knee and Thigh, yet could not feel me when I touched thoſe Parts. Her Fever was now encreaſed again, and ſhe was delirious ſometimes. She begg'd heartily of me for Help; but alaſs! what could I propoſe to relieve her? Nothing but the taking off her Thigh, which ſhe would not conſent to. I was not ſorry for her not admitting of that Operation, becauſe I could not expect any Succeſs in the performing it, by reaſon of her Age, Weakneſs, *&c.* So I took leave of her, ſuppoſing I ſhould never ſee her more. I adviſed her Friends to continue the uſe of the Fomentation, which they did almoſt Night and Day. About a Month after, I coming to *Maidenhead*, was ſurprized to find this poor Woman alive. There was now a Diſcharge of a Black Fætid Matter, at a ſmall Orifice about the middle of the Inſide of her Thigh, which Orifice I enlarged to make a better Diſcharge for the Matter. I likewiſe cut into a Tumour that appeared upon her Knee, but found nothing in it but Wind. I then took leave of her, (as before) adviſing to continue fomenting daily. About a Month or 5 Weeks afterwards, I came to *Maidenhead* again, and found her alive, and to my admiration ſaw how Nature had made a perfect Separation of the mortify'd Fleſh from the ſound, quite round the Thigh, the Bone of the Thigh lying wholly bare, above the breadth of four Fingers, and deprived of its *Perioſteum.* The Fleſh above was freſh and florid, and had good white Matter upon it. I now perſwaded her to let me take off her Thigh, which I did about 2 Fingers breadth in the ſound Fleſh, (becauſe the Fleſh ran tapering down to the Bone,) by which I made the Stump pretty even. The Bleeding was little, by reaſon that the Veins and Arteries (which were eaten aſunder by the mortifying Matter) Nature had cloſed again. I dreſt the Stump with *Pul. Reſtring.* mixt with *Album Ovor.* ſpread upon Pledgets, and dipt in *Ol. Terebinth.* hot. The next Dreſſings I uſed Digeſtives, and performed the reſt of the Cure according to the Rules of our Art.

A Remark by Mr. Cowper. *ib. p.* 43.

2. About the beginning of *October* laſt I hapned to be at *Maidenhead*, where I ſaw the Woman whoſe Caſe is here related; ſhe appeared to be very decrepid, and would have ſhewn me the Stump of her Thigh, but the Coldneſs of the Weather, ſhe ſaid, would make it uneaſie to

her

her. I felt it through her Cloaths, and the end of the Stump seem'd to be not above four or five Inches below the Trunk of her Body.

Since I have so frequently found the large Trunks of the Arteries of the Thighs and Legs of Aged People petrified, as I have mention'd in the * *Transactions, Numb.* 280. and most commonly in those who have had Gangreens in the Legs, *&c.* I am apt to suspect the like happen'd in the Crural Artery of this Woman; which, like a Ligature, did at length put a total stop to the Influent Blood below that Stricture. ** v. supr. p.* 134.

IV. The Tumour is of a Schirrous nature, springing from the Thigh Bone, somewhat tending to that of a Cancerous. It first took its rise about 2 years ago in a Child 10 years old, just above the Patella, without any evident cause, and hath, notwithstanding all possible Care, expanded itself so, that it now occupies the whole Thigh, to the very Groin, and has extended it to above a Dutch Yard, in Circumference. As it encreases very much daily, it must soon exhaust the Patients Strength. *A Schirrous Tumor in the Thigh by Mr.* Amyand. *n.* 317. *p.* 173.

V. The *Os femoris* belong'd to an old Woman turn'd of Fourscore, who only fell from her Chair whereon she was sitting, and thereby suffered this breach of continuity in the Substance of the Bone. She lived three Weeks after it; and tho' it never was reduc'd, yet she complained of very little or no pain, which may seem very extraordinary. It is observable that the Fracture is not only Oblique, near the Neck of the Bone; but that each Trochanter, *i. e.* the two processes near its Cervix, are likewise broke short off; and that they were both drawn up almost as high as the Head of the Bone it self, by the strong contraction of the *Glutei* and other Muscles. *A Fracture in the Thigh Bone, by Dr.* Douglas. *n.* 349. *p.* 501.

VI. *A Paper omitted.*

An Account of several Observations concerning the Frame and Texture of the Muscles, by Mr. *Muys* of *Francquer*. From the *Journal Litteraire* for *Jan.* and *Feb.* 1714. *p.* 238. *n.* 339. *p.* 59.

VII. *Accounts of Books omitted.*

1. De Arthritide symptomatica Dissertatio. Auctore Gulielmo Musgrave, M. D. Coll. Med. Lond. & *R. S. S.* 8vo. Lond. 1704. *n.* 291. *p.* 1597.
2. De Arthritide anomala sive Interna Dissertatio. Auctore eodem. Lond. 1707. *n.* 310. *p.* 2435.

CHAP.

CHAP. VII.

Monſters, Longævity.

I. *A Monſtrous Birth, by Mr.* Robert Taylor. *n.* 308 *p.* 2345.

A Poor Woman in a Village near *Hitchin*, being in ſtrong Labour, the Midwife finding the Birth coming very awkwardly, and more Legs than uſual, after a tedious time delivered the poor Woman of Twins (deſign'd by nature doubtleſs,) but joyned together; there being but one Trunk of a Body with 2 Necks, on each a Head, 4 Arms 2 backwards, and 2 forwards, thoſe backwards croſſing each others Shoulder, like two perſons ſide to ſide. There is but one Navel, two Matrix's, two Fundaments, two pair of Hips, 4 Legs: They had gone the full Time, having Hair on their Heads, and Nails on their Fingers and Toes. The Midwife tells me they were alive within half an hour before they were delivered. They look very clear and well. The Children are near * * Inches long, and by reaſon of their being joyned, are about 7 Inches over.

Monſtrous Births, by Mr. Neve. *com. by Mr.* Derham, *n.* 320. *p.* 309.

II. *Dec.* 6. 1706. A Woman at *London Derry* had a Monſtrous human Birth, *viz.* with 2 Heads, 4 Arms, and but one Body at the Navel. It was of both Sexes, Female on the Right-ſide and Male on the Left. The Right hand of the Male was behind the Females Back, and the Left hand of the Female behind the Male's Back, holding each other as in a loving manner. It liv'd but a little while.

Death and Diſſection of John Bayles, *at* 130 *years old. by Dr.* James Keill. *n.* 306. *p.* 2247.

III. *John Bayles*, the old Button maker of *Northampton*, is commonly reputed to have been 130 years of age when he dyed. There is no Regiſter ſo old in the Pariſh where he was Chriſtened; but the oldeſt people, of which ſome are 100, others 90, and others above 80 years, remember him to have been old when they were young. Their accounts indeed differ much from one another, but all agree that he was at leaſt 120 years. He himſelf did always affirm that he was at *Tilbury* Camp, and told ſeveral particulars about it; and if we allow him to have been but 12 years old then, he muſt have been 130 when he dyed. He uſed conſtantly to walk to the Neighbouring Markets with his Buttons within theſe 12 years, but of late he has been decrepid, and carryed abroad. His Dyet was any thing he could get. I never heard he was more fond of one ſort of Food than another, unleſs it was that about half a year before he dyed he longed for ſome Veniſon Paſty, but had it not. He dyed the 4th of *April* 1706. He lived in 3 Centuries, and in 7 Reigns.

His Body was extremely emaciated, and his Fleſh feeling hard, the ſhape of all the External Muſcles was plainly to be ſeen through the Skin.

Skin. The *Abdomen* being laid open, the whole *Viſcera* appeared in good order, but more pale than they are commonly. The *Omentum* was very ſmall. The Stomach was very much diſtended with Wind, and the Bottom of it wore extreamly thin in that part which is next the Spleen, being hardly thicker than thin Writing Paper. In the inner Membrane there were no *Plicæ*. The Liver was pale, but upon cutting was found perfectly ſound. The Gall Bladder was of a larger ſize. The Spleen was not ſo big as one of his Kidneys. His Kidneys were firm and ſound, as were all the Urinary Paſſages. In the Right Kidney were a few ſmall yellow grains of Gravel. The Inteſtines were all ſound; the Meſentery was covered with Fat. The *Cartilages* of the *Sternum* were not harder than uſually they are. The Ribs were brittle, for by leaning gently upon one of them it broke. The Lungs were attacked even to the *Pleura*: They were ſpongious, whitiſh, with many ſmall black ſpots of Blood. The Cavity of the *Thorax* was large and clear. The Heart was large, thick and fat; and though he was always a little Man, yet the Diameter of the *Aorta*, before the *Carotidales* go off, was above two inches, which is conſiderably bigger than ever I remember to have ſeen. The *Aorta* in the *Abdomen*, and *Iliacks*, was for the greateſt part Cartilaginous. The Bones of the Skull were ſound and good. On the inſide of the *Dura Mater*, by the *Falx*, was a ſmall oſſification. The Brain was more firm and ſolid than uſual, and in cutting, hardly moiſtened the ſides of the Knife. The *Ventricles* were full of *Serum*. He had loſt the uſe of his Eyes for ſome years; but his Hearing was good till he dyed. His Genitals, both Teſticles and *Penis*, were of a large ſize.

There is no doubt but that the weakneſs of his Stomach, and the hardneſs of the *Aorta*, were the Cauſes of his Death. The Coats of the Stomach were ſo thin, that they had not ſtrength enough to keep out the Air, and conſequently his Digeſtion muſt have been ſpoiled. He had not eat Meat for ſome years, and of late he lived only on Small Beer, Bread and Butter, and Sugars. And it was impoſſible that his Blood could circulate duly, whilſt the great Artery, having loſt its Elaſticity, by being become Cartilaginous, could give no motion to the Blood. It is very probable that this was the Cauſe of his irregular and intermitting Pulſe, which I have felt ſome years before he dyed. It is obſervable, that the greateſt part of his Blood (which was in greater quantity than I expected) was contained in the Arteries, whereas generally in all dead Bodies the Veins are full, and the Arteries almoſt empty; for the Arteries being diſtended by the Blood, which they receive upon the laſt Syſtole of the Heart, by their natural Elaſticity contract again, and empty themſelves into the Veins, from whence it returns no more; but in this Man, the Great Artery having loſt the power of contracting itſelf, it retained the Blood it received by the laſt Syſtole of the Heart. This account agrees with that given of old *Parre* by the famous *Harvey* in moſt particulars, except in the Cauſes

of their Deaths. But in both nothing seems more remarkably the effects of old age than the smallness of their Spleens, which undoubtedly was owing to the contraction of their Fibres in such a lax and spongious Bowel. The whiteness of Bowels in both must be likewise either from the same contraction or closeness of the Coats of the Blood Vessels, or for want of Blood. *Harvey* says nothing of the quantity of Blood he found in old *Parre*; but if we may guess from his Body being fleshy, from the goodness of his Stomach and Appetite, and from the Disease he dyed of, there could be no want of Blood in him. In this Man there seemed to be more Blood than in several others I have seen, whose Bowels appeared more Red. And it can hardly be conceived, that the *Aorta* could be so large, without a large quantity of Blood, unless there had been some Stricture upon some other parts of it, which I did not perceive: And therefore it seems not improbable, that this whiteness of the Bowels was owing to the closeness of the Blood Vessels in both. It is no small confirmation of this opinion, that the Flesh and Skin felt hard, and the Brain firm and solid. I might add that it is highly probable, that the same disposition might give a closeness or hardness to the Vessels every where else. It is true, this was a Distemper, but then it is as true that it is a Disease of old Age, and may justly be reckoned one of the effects of it. And for a farther Proof of what I have said, I cannot but take notice, that in preparing a piece of the small Gut for an Injection, the *Tunica Villosa* felt more like a fine File than the softest Velvet; and that I could use more violence in injecting the Vessels than these parts will usually bear. Whoever considers how soft a Substance an Animal Body is at its first beginning, and how from time to time it acquires a firmness and solidity, will easily be induced to believe, that Old Age brings a more than ordinary hardness to all the Fibres and Vessels.

The necessary consequence of this hardness, and contraction of the Fibres and Vessels of old people, is a diminution of their Secretions, which *cæteris paribus* are always proportional to the Orifices of the Glands. Hence it is that we find the Skin of old people always dry, their perspiration being very little. They are likewise generally bound, old *Bayles* went to Stool but once in ten or twelve days for some years; and old people are always complaining of a want of moisture, not that the Radical Moisture is dryed up, but because the natural Secretions, by reason of the contraction of the Glands, are diminished. I have already observed, that we found in this old Man more Blood than could have been expected in such an emaciated Body, and without doubt it had been larger, if his Stomach and Appetite had been as good as old *Parre*'s. The fullness of the Vessels, and the frequent Rheums and Catarrhs of old People, evince this necessary consequence of the closeness of the Coats of the Vessels. From this retention of the excrementitious parts of the Blood, we may expect all the ill consequences of a vitiated *Plethora*, and languid motion of the Blood; for the Fibres

of

of the Arteries being now become hard, instead of assisting, they obstruct the Heart in circulating the Blood; and the quantity of Animal Spirits separated in the Gland of the Brain must likewise be less, not only because of the retention of the Excrementitious Humours, but also because of the closeness and firmness of the Brain itself, so that the contractions of the Heart and all the Muscles must be weak, and consequently the motion of the Blood languid. A due conformation of all the Vital parts is most certainly necessary to bring a Man to a full old Age; but above all the rest, there are two which to me seem to have had the greatest share in procuring a Longævity to old *Parre* and *Bayles*, by retarding the ill effects just now mentioned. The first is the Heart, which in both was strong and fibrous; for that being left alone to labour the circulation of a large quantity of sluggish Blood, a great strength is absolutely requisite to propel the Blood through unactive Vessels to the extremities of the Body, and back again. No doubt this is more easily done in Men of a low stature (as old *Bayles* was) which I apt to think was a qualification of old Age. The second was the largeness of their Chests, and goodness of their Lungs, by which the Air had its full effort upon every Particle of the Blood, in rendring it florid and attenuating it, that it might easily move through the contracted Channels of an old Body.

IV. *Martha Waterhouse* and *Hester Jager* Sisters in *North Bierley*. *Martha* died in 1711. in the 104th Year of her age: *Hester* in 1713. in her 107th Year. They had both relief from the Parish nigh fifty Years. There are other Instances here of Longævity; but joyntly I don't remember any that have come up to them.

Two very aged Persons. by Dr Richardson. *n.* 337. *p.* 167.

V. Dr. *Mather* says, 'tis no rare thing with them to have a Gentlewoman see many more than 100 of her offspring. He mentions one Woman that had 23 Children, of which 19 liv'd to Man's Estate. Another that had 27; another 26, of which 21 were Sons, one whereof was Sir *William Phipps*; another 39 Children. Here he gives several Instances of Persons living, with them, to above 100 Years of Age. One *Clement Weaver* lived to 110, his Wife being upwards of 100. This Man, to the last Year, could carry a Bushel of Wheat to the Mill, above 2 Miles. He relates the Case of an old Man, above 100, that lost the memory of several of the latter Years of his Life, but very well retain'd the Remembrance of what past in his younger days.

Longævity and Fertility in New England. *by* Dr. Mather. *n.* 339. *p.* 71.

VI. *A Paper omitted.*

De Monstris & quasi Monstris & Monstrosis Philippensibus, by *F. Camelli*, com. by Mr. *Petiver*. Those Particulars in this account *n.* 307. *p.* 2266.

 which

which were moſt conſiderable are deſcribed by Mr. *Petiver* in his *Gazophylacium Naturæ & Artis.*

CHAP. VIII.

General Affections of the Body, and remarkable Caſes and Obſervations not reducible to the foregoing Heads.

1. *Of the Worms in the Itch. by Dr.* Bononio. *n.283. p.1296.* I Found an Itchy Perſon, and asking him where he felt the greateſt and moſt acute Itching, he pointed to a great many Puſtules not yet ſcabbed over, of which picking out one with a very fine Needle, and ſqueezing from it a thin Water, I took out a very ſmall white Globule, ſcarcely diſcernible. Obſerving that with a Microſcope, I found it to be a very minute Living Creature, in ſhape reſembling a Tortoiſe, of whitiſh Color, a little dark upon the Back, with thin and long Hairs, of nimble motion, with 6 Feet, a ſharp Head, with 2 little Horns at the end of the Snout.

Plate 4. *Fig.* 2. *and* 4.

Not ſatisfied with the firſt Diſcovery, I repeated the Search in ſeveral Itchy Perſons, of different Age, Complexion and Sex, and at differing Seaſons of the Year, and found in all the ſame Animals; and that in moſt of the watry Puſtules, for now and then in ſome few, I could not ſee any.

And tho by reaſon of their minuteneſs, and Colour the ſame with the Skin, 'tis hard to diſcern theſe Creatures upon the Surface of the Body, nevertheleſs I have ſometimes ſeen them upon the Joynts of the Fingers in the little Furrows of the *Cuticula*, where with their ſharp Head they firſt begin to enter, and by this Gnawing and Working in with their Body, they cauſe a troubleſome Itching, till they are got quite under the *Cuticula*; and then 'tis eaſy to ſee how they make ways from place to place by their biting and eating, one ſingle one happening ſometimes to make ſeveral Puſtules, of which I have often found two or three together, and for the moſt part very near to one another.

I examin'd whether or no theſe Animalcules laid Eggs, and at laſt from the hinder part I ſaw drop a very ſmall and ſcarcely viſible white Egg, almoſt Tranſparent, and oblong, like to the Seed of a Pine-apple. I oftentimes found theſe Eggs afterwards, from which no doubt theſe Creatures are generated.

Fig. 3. *and* 5.

From this Diſcovery it may be no difficult matter to give a more Rational account of the Itch, than Authors have hitherto delivered. It being very probable that this contagious Diſeaſe is no other than the continual biting of theſe Animalcules in the Skin, by means of which ſome portion of the *Serum* ouzing out thro the ſmall apertures of the

Cutis,

Cutis, little watery Bladders are made, within which the Infects continuing to gnaw, the Infected are forced to fcratch, and by fcratching increafe the Mifchief, and thus renew the troublefome work, breaking not only the little Puftules, but the Skin too, and fome little Blood Veffels, and fo making Scabs, Crufty Sores, and fuch like filthy Symptoms.

From hence we come to underftand how the Itch proves to be a Diftemper fo very catching; fince thefe Creatures by fimple contact can eafily pafs from one Body to another, their motion being wonderfully fwift, and they as well crawling upon the Surface of the Body as under the *Cuticula*, being very apt to ftick to every thing that touches 'em, and a very few of them being once lodged, they multiply apace by the Eggs which they lay. Neither is it any wonder if this infection be propagated by the means of Sheets, Towels, Handkerchiefs, Gloves, *&c.* ufed by Itchy Perfons; it being eafy enough for fome of thefe Creepers to be lodged in fuch things as thofe; and indeed I have obferved that they will live out of the Body 2 or 3 days.

Nor fhall we be at a lofs to know the reafon of the cure of this Malady by Lixivial Wafhes, Baths, and Ointments made up with Salts, Sulphurs, Vitriols, Mercury's Simple, Præcipitate or Sublimate, and fuch fort of corrofive and penetrating Medicines. Thefe being infallibly powerful to kill the Vermin lodged in the Cavities of the Skin; which fcratching will never do, partly by reafon of their hardnefs, and partly becaufe they are fo minute as fcarcely to be found by the Nails. Neither do inward Medicines perform any real fervice in this Cafe. And if in Practice we oftentimes experience, that this Difeafe, when we think it is quite Cured by Unction, does neverthelefs in a fhort time return again, this is not ftrange, fince tho the Oyntment may have killed all the Living Creatures, yet it may not probably have deftroyed all their Eggs, laid as it were in the Nefts of the Skin, from which they may afterwards breed again and renew the Diftemper. And upon this account, 'tis very advifeable after the Cure is once performed, ftill to continue the Anointing for a day or two more; which it is the eafier to do, becaufe thefe Liniments may be be made agreable enough, and of a good fmell, as particularly is that compounded of the Ointment of Orange Flowers or Rofes, and a fmall quantity of red Præcipitate.

Of Secretions in an Animal Body, by Dr. Morland. *n.* 283. *p.* 1293.

II. It feems to me, that the whole Bufinefs may be reduced to this double enquiry. 1ft, How a thin Fluid (fuch as is the Urine) may be feparated from the Mafs of Blood, and the remaining parts of the Blood circulate back to the Heart. 2dly, How a thick Fluid (fuch as is the *Bile* or *Semen* for example) may be feparated from the Mafs of Blood and the other Fluids both thinner and thicker than this particular Fluid to be feparated, circulate back to the Heart.

And

And that I may be the more plain, I ſhall give a general Idea of the Structure of the Glands.

A Gland I conceive to be compoſed, 1ſt, Of the Ramifications of the Blood Veſſels incloſed in a common Membrane, which ſends off ſeveral Fibres, by which theſe minute Veſſels are tied together; and that the Veins are a continuation of the Arteries. Of this Dr. *Areskin* has fully convinced us, by an injection of Wax in an Humane Body, ſo dexterouſly performed, that the Wax being injected by the Arteries filled the Veins at the ſame time; and afterwards by a nice diſſection of the part, where the continuation of the ſmall Ramifications of the Arteries and Veins appeared to the naked Eye. 2dly, I conceive, that when the Branches of the Arteries begin to grow very ſmall, they ſend off ſeveral Ducts, whoſe Orifices are of different Dimenſions. Theſe Ducts are of two ſorts. The Firſt of theſe, which in the ſame Artery are always ſmaller than the ſecond, paſs immediately from the Artery, and open the Veins. The Second which paſs off nearer to the Extremity of the Arteries, unite and carry off a Liquor from the Maſs of Blood for particular ends in ſeveral parts of the Body. It is to be obſerved, that in one caſe the ſecond ſort are only to be found. I imagine, that a thin Fluid may be ſecern'd from a thick one, when the Orifices of the ſecretory Ducts are ſo ſmall, as to admit no other but that thin Fluid, and that at the ſame time the remaining parts of the Blood which are thicker, continue their Courſe in the Veſſel. Again, I imagine, that a thick Fluid may be ſecern'd, when the thinner parts are carried off ſome other way, ſo that the Liquor to be ſecerned will be the thinneſt of the remaining Maſs.

Upon theſe principles I think it will be eaſie to explain the Doctrine of Secretions.

Let us firſt examine how the Secretions are performed: As for Inſtance, the Urine. When the Blood by the Contraction of the Heart is puſh'd into the Arteries, they are dilated, which contracting themſelves puſh it forward into all the parts of the Body, and amongſt the reſt into the Ramifications of the Arteries of which the Glands of the Kidneys are compos'd. By this means the Blood paſſes by the Orifices of the Secretory Ducts; when theſe Arteries contract themſelves they preſs the Blood, and force the thinner parts into the Orifices of thoſe Ducts (which will admit no thicker Fluid) and carry it toward the *Pelvis* and the remaining part of the Blood into the Veins, by them to be carry'd back to the Heart. Thus a thin Liquor may be ſeparated from the Maſs of Blood.

Let us next examine how a thick Liquor may be ſeparated from the Maſs of Blood where thinner Liquors are mixt with it. For Inſtance, let us take the *Gall* or *Semen*. When the Blood is puſht into the *Celiack* or *Meſenterick* Arteries,'tis forced to paſs into the Glands of the Stomach, *Pancreas*, *Spleen* and *Inteſtines*,&c. where the *Liquor Gaſtricus*, *Succus Pancreaticus*, *Liquor inteſtinalis*, are ſeparated by the abovemention'd Method.

Method. The Blood thus robb'd of various thin Liquors is push'd on into the Veins, which answer to these Arteries, which Veins unite, and form a large Trunk called the *Vena Porta*, which entring into the Substance of the Liver, by its small Ramifications chiefly forms the Glands of which the Liver is compos'd. Here again all the Fluids contain'd in the *Vena Porta*, which are thinner than the Bile, are separated from this Mass of Blood by the first sort of Secretory Ducts (which we said opened into the Veins) and there are discharged and mixt with the Blood, which is passing toward the Heart. At the same time the *Bile*, with the rest of the Blood which is thicker, continues its Course in the Artery: Now all the thin Liquors being separated, the Bile is the thinnest part of this Mass of Blood, and so may be received by the Excretory Ducts, which are capable to receive it, and no other. The *Semen* being a very thick Liquor is separated much after the same manner, *viz.* The Blood being pusht into the Spermatick Arteries, passes into the Substance of the *Testicles*, where all the Liquors, that are thinner than that out of which the *Semen* is to be taken, are separated by the first sort of Secretory Ducts, and carried back to the Mass of Blood. Then this *Liquor Seminalis* being the thinnest part of the remaining Mass is separated by Excretory Ducts capable to receive it and no other. After the *Liquor Seminalis* is separated from this Mass of Blood by the aforesaid method, it is pusht forward into the Excretory Ducts, where there are other Ducts, which take their origin all along from them, which Ducts are capable to receive the thinnest parts of the *Liquor Seminalis*, and convey them to the Mass of Blood; and thus the *Semen* is left behind to pass into *Vas deferens.* And 'tis worth remarking, that as the *Semen* grows thicker and thicker by continual separation, the Canal in which it is to run, grows larger and larger, as appears by the structure of the Testicles, *Epididymis* and *Vas Deferens.* Hence we may give a true account, why the Canals of which the Testicles are composed, are so long, *viz.* That there might be time enough to separate all the thin Fluids. By this method we see, how the thickest and thinnest Fluids may be separated from the Mass of Blood. And how intermediate Liquors may be separated after the same manner by Canals of intermediate Dimensions. Thus in a word the whole Doctrine of Secretions may be reduc'd to this. To separate a Liquor of any determin'd thickness, all the Fluids, which are thinner, must be carry'd off by small Canals, and the Liquor to be separated, being the thinnest of the remaining Mass, is secerned because the Ducts are capable to receive it and no other.

Corollaries. 1. Hence the use of the Spleen is evident.

2. Hence appears the origin and use of the Lymphaticks.

3. Hence the Texture and use of many minute parts of the Body may be discovered, which hitherto has been unknown.

III.

Three Cases of the Hydrophobia. *by Dr.* Mead, *n.* 323. *p.* 433.

III. The firſt was of a Lad of about the Age of nine Years, a ſturdy and bold Boy. A mad Bitch of the Mungrel kind was hunted in the Street, he ſtruck at her with a Stick, and ſhe flying in his Face, bit him in the right Cheek, which was torn with a large Wound to the middle of the Noſe. This was on the 20th of *April.* A Surgeon cured the Wound in about 14 days time, by applying for the firſt three Days, *Theriac. Andromach.* in *Sp. Vin.* and afterwards dreſſing it with *Liniment. Arcæi* and *Balſam. Terebinthin.* No other Care was taken, only a *Bolus* of *Theriac. Andromach.* was given him every Night while under Cure, and quickly after he was bit, he was perſuaded to eat the whole Liver of the Bitch fry'd. He continued very brisk and well to the 22d of *May*; upon that day he ſeem'd dull and ſick, would eat no Dinner, except a little boil'd Spinnage, walk'd out in the Afternoon, and in the Evening complained of his Stomach and Head; his Mother gave him a ſmall Glaſs of Brandy, for he would drink nothing elſe In the Night he was very bad, ſtartled often, and ſcreamed out as in an Agony, eſpecially when deſired to drink, and complained miſerably whenever he made Urine, ſaying it hurt him. The next Morning he vomited up the Herbs he had eat the day before, unalter'd. I was ſent for that Day in the Afternoon, and found him in a perfect Agony, all in a ſweat, trembling, toſſing himſelf up and down, talking continually, looking very wild; his Pulſe low, and ſometimes quicker, then ſlower: His Urine made the Night before as well Colour'd as ordinary. I deſired him to Drink; he took a little in his Mouth, but as it was going down, he threw it out with Violence, ſaying it hurt him; and praying that he might take no more. We over perſuaded him to hold a little in his Mouth and ſwallow it by degrees and gently; he did ſo with a little more eaſe, but was glad when 'twas over. We bid him ſuck the Drink through a Quill; he try'd, but could not get it down by continual Gulps, but ſtopt as ſoon as a very little was paſs'd, ſtill crying out that it hurt him to ſwallow it. I preſently declared the Caſe to be deſperate. However for the ſatisfaction of the Relations, Bliſtering Plaiſters were apply'd to the Back and on each ſide of the Neck; and a Diuretic *Bolus* of *Sal. Succin. Camphor.* and *Conſerv. Lujul.* was given every ſix Hours; for he ſeemed from the firſt of his Complaint to have a difficulty of Urine.

The next day, the 24th at Noon, I found him much worſe, he had raved all Night; could not bear the ſight of any thing white, and ſaid, that if all the Women in the Room who had white Aprons would go out, he ſhould be well preſently. He ſaid he would drink if we would give him it in a black Cup; but when brought made many Excuſes and could not, though at the ſame time complain'd he was dry, and pleaſed himſelf with talking of full Pots. He eat ſome Bread and Butter heartily, but vomited it up quickly together with a frothy Slime. We dipp'd him in a Tub of warm Water; he ſaid he was not

afraid

afraid of Water, and was quiet in it for a little while, but ſoon fell into a Convulſion Fit, which obliged us to take him out. I obſerved his Eyes to grow more ſtaring, and the Pupil to be prodigiouſly inlarged. He was thrown continually with ſuch Violence from Place to Place, that it was very hard to keep him in Bed; and quite tired and ſpent, fell into cold Sweats, and dy'd this day at Four in the Afternoon. The next day I obtain'd leave to open the Body. We examin'd the Brain, Throat, Breaſt and Stomach, but met with no extraordinary appearance any where, excepting that there was a great quantity of greeniſh viſcid Bile in the Stomach.

The other Patient was a very luſty vigorous Man of 45 Years. He had ten Weeks before been bit in one of the Fore-fingers near the Nail, by a little Naked Dog of the *Guinea* Breed. On the 8th of *November* in the Morning he complained of a great Sickneſs at Stomach, and vomited green and yellow Choler. The next Morning he took a Doſe of *Rad. Ipecacuanh.* Whilſt he was vomiting, he complained of a difficulty of ſwallowing; and when preſs'd to drink to work off the Medicine, contrived himſelf a way of ſucking the Gruel given him, through a piece of a Tobacco Pipe, but could not get down above one Pint; and tho' he afterwards often try'd this Trick, yet it did not ſucceed. On the 10th he had eight Ounces of Blood taken away at the Arm, and took a Bolus of *Theriac. Andromach.* with *Lap. Contrayerv.* I came to him on the 11th; found him ty'd in his Bed, raving Mad, biting and ſpitting at the By ſtanders, crying out Murder, making an odd Noiſe as if he cough'd up ſomething from the Throat; this motion I had alſo took notice of in the Boy, and I ſuppoſe this is what ſome Authors have call'd Barking. He ſay'd he would drink if we would unbind him, and give him Water; but as ſoon as it came to his Mouth, he threw away the Cup with the greateſt Fury imaginable, and grew ſo unruly, that he was with much ado ty'd down again. I obſerved that he had a Palſie of his right Arm, for he moved this only by the help of the other; and thoſe who attended him, had taken notice that this Symptom began the Day before, and that at the ſame time he had endeavour'd to Read, but could not, complaining of a Miſt before his Eyes. As he ſeem'd afraid of every Body, ſo he ſhewed the greateſt Enmity to thoſe, for whom at other times he uſed to have the moſt Love and Reſpect. I order'd a Surgeon to take away 20 Ounces of Blood at his Arm: And obſerved it to be very thick and black. He was very tame after this for a few Minutes, but fell again into his outragious Fit, in which he ſoon laid himſelf down quite ſpent and dy'd.

Since theſe Accidents I have had an Account ſent me by a Surgeon from *Stamford* in *Lincolnſhire*, of a young Man of about 18 Years of Age, who dy'd Hydrophobus by the Bite of a Mad Fox, that had been bit by a Mad Dog. The Symptoms diſcovered themſelves three Months after the Wound, which was upon the back of

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the Hand, and being healed by the Application of *Theriac. Andromach.* had left a small black Scab behind.

Three days before his Death he was seized with a Fever, for which he was Blooded, Vomited and Blistered; he bit to pieces the Glass in which Drink was given him. When Dissected, the *Fauces* were found very much inflamed; The left Lobe of the Lungs black, with the Vesicles full of black Blood; The surface in some places, which the blackness had not cover'd, appearing Blistered, as if raised by *Cantharides.* The Liver was hard and of a yellow bilious Colour.

During the whole Violence of the Distemper, the *Penis* was observed to be continually erected, and as hard as a Bone. This Symptom is particularly taken notice of by *Cælius Aurelianus.*

The Surgeon who opened the Body, with his Knife slightly wounded his Fore-finger, and was surpriz'd to find that it festered, and gave him much more Pain than a greater Cut had at other times done. This I the rather take notice of, because something of the same nature happen'd to the Surgeon who Dissected my Patient. His Hand the following Night was taken with an *Erisipelas,* attended with great Tension and Pain: This was owing to a little Wound made in one of his Fingers a Day or two before, from which, in turning over the Parts, he had rubb'd off the Plaister; and it went not off without the continued Application of Cooling and Discutient Medicines.

Vid. *Essays on Poison.*

From all these Histories, it may not perhaps be wrong to conclude, that the *Hydrophobia,* (a Name not very proper for the Distemper) is the Effect of a particular kind of an Inflamation in the Blood, accompany'd with so great a Tension and Dryness of the Nervous Membranes, and such an Elasticity and Force of the Fluid with which they are filled, that the most common Representations are made to the Mind with too great Effect, and the usual Impressions of Objects upon the Organs cannot be suffered: Hence proceed the Timorousness, unaccountable Anxiety and Inquietude, which are always the forerunners of the Dread of Liquids; as also did the Pain in making Water, and the strange Aversion observed in the Boy at the sight of any thing White; the *Retina* being really hurt and grieved by the striking of the Rays of Light upon it. Nor is it hard to conceive that when the *Salival* Liquor is hot, and the Throat inflam'd and dry, the swallowing of Drink should cause such an intolerable Agony; no more than it is that, when things are wrought up to this wretched Condition, the dismal Tragedy should not last above three or four days at most, in which the Patient is perfectly fatigued and torn to Death by the Violence of his Actions and Efforts.

Of the Bite of a Mad Dog, by Mr. Dela Pryme. *n.*277. *p.* 1075.

IV. In 1695. my Brother had a pretty Grey-hound Bitch that had Whelps; soon after came a Mad Dog and bit this Bitch unknown to the Family, upon which about 3 Weeks after she run Mad, and they were forced to kill her; but saving her Whelps, because that no sign of

of Madneſs appeared in them, in about 3 Weeks more they all pull'd out one another's Throats except one, which eſcaping, my Brother's men valued and nouriſh'd, made much of it, and ſtroaked it: at length, perceiving it could not lap, nor ſwallow any liquid thing, they put their Fingers in its Mouth, and felt its Tongue and Throat, but finding nothing wrong therein as far as they could diſcover, they let it alone a day or two longer, and then it run mad and dy'd. They being thus dead were ſoon forgot, until that about 3 Weeks after, my Brother's Servant, a moſt ſtrong laborious Man, that had frequently put his Fingers into the Whelps Mouth, begun to be troubled now and then with an exceeding acute Pain in the Head, ſometimes once, ſometimes twice a day, ſo very vehement that he was forc'd to hold his Head with both his hands, to hinder it from riving in two, which fits commonly held him about an hour at a time, in which his Throat would contract, as he ſaid, and his Pulſe tremble, and his Eyes behold every thing of fiery red Colour. Thus he was tormented for a whole Week together, but being of a ſtrong Conſtitution, and returning to his Labour in every Interval, he ſweat and wrought it off, without any Phyſick.

But it went worſe with one of his fellow Servants, a young Apprentice of about 14 Years of age, who had made as much of the Whelp as he, but was not of ſo ſtrong a Conſtitution; he was ſeiz'd alſo with a pain in his Head, was ſomewhat Feveriſh, ſometimes better, and ſometimes worſe, cough'd much, had a good ſtomach, eat heartily, but could drink nothing. I know not what I ail, ſays he, I cannot ſwallow Beer, *&c.* and ſo laugh'd at it. When he went out of doors tho there was but a little North Wind, yet he always ran as if it had been for his life; when they ask'd him why he did ſo, he told them he could not tell——but that the Wind would needs ſtop his breath. A day or two after this he was worſe, and vomited a ſtrange naſty ſort of Matter, like black Blood, which ſtunk like Sallet Oyl, but much ſtronger; which he did ſeveral times; after which he would be pretty well, and walk about, but moſt commonly ran as faſt as ever he could, firſt out of one corner, then into another, then up ſtairs, then down again, as if it was for his life. But upon the third day of his confinement within doors he grew perfectly Mad, would ſtart, and leap, and twiſt his Hands and Arms together, point at people, and laugh and talk any thing that came in his mind. In ſome of his fits he was ſo ſtrong that he was too hard for four young Men to hold him down in the Chair where he ſat: but as ſoon as they were over he was lightſom, and laugh'd, and talk'd, but all his diſcourſe was of fighting, and how if that they would but let him alone, he would leap upon them, and bite, and tear them to pieces: And when one ſaid unto him, that he was ſure that he would not hurt him, he'd been always his Friend; he anſwer'd ſharply, that Friends and Foes were all alike to him, he'd tear them all in pieces, *&c.* About an hour after his fit

came again, which ſoon made him ſpeechleſs, ſeiz'd wholly upon his Brain, and then he dy'd juſt before the Phyſician came.

An extraordinary Spaſmus in two Families. by Dr. J. Freind. n. 270 p. 799.

V. Æſtate præteritâ magnus apud nos percrebuit rumor puellas quaſdam apud Blackthorn in agro Oxonienſi latratibus crebris more canum correptas eſſe. Duas ſcilicet familias peſtis hæc invaſit: quarum unam fortè adiit, ut de re tam inauditâ ſibi certò conſtaret, Vir omni rerum Medicarum laude præcellens D. Williſius; ille, quâ eſt ſemper in me benevolentiâ, caſum, qualem oculis accepit, mihi impertitus eſt, quem ipſius verbis, hoc eſt optimis, ſic accipe.

Funeſtum hoc & inamænum liberorum ſpectaculum reverſurum ex itinere, ut obiter inviſerem, tum rei novitas, tum amicorum deſideria paulo morabantur. Tunc enim non procul aberam a Villâ dictâ Blackthorn, quæ paſſim hodie celebratur non tam loci antiquitate, quam ſonorâ lue circumquaque nota ac infamis. Ubi primum aures meas infeſtabat terribilis e longinquo ejulantium concentus; domum verò ingreſſum ſtatim ſalutabat vociferantium puellarum horror propinquior, cuilibet harum reſpondente prorſus æquis vicibus violentâ capitis motatione, quaſi ad ejus nutum, pagi nequam tibicinem æmulantes, ingratam pariter modularentur harmoniam. Vultu erant ab omnibus ſpaſmis immuni, præter motus oris frequenter oſcillantes; & pulſus iis ſanorum ſimilis, niſi ſub finem paulo debilior. Sonus, ut mihi videbatur, non tam canum latratum referebat, quam eorum erat ejulatibus ſimilis, niſi quod crebrior fieret, ſingultibus illum viciſſim reciprocantibus. Morbus hic novus Cynicus nulli harum quinque quas videre erat, ætati pepercit; cæterum omnes pariter invaſit, ab annum ſextum aut circiter agente uſque decimum quintum. Sæpe inter has vociferationes colloquiis pariter & ſenſibus integris fruebantur: Interdum renovans, more canum venaticorum, cæteras omnes quaſi ad mali ſocietatem vocaret. Tandem deficientibus ſpiritus ſingulæ cedebant paroxyſmo velut Epileptico, quas quidem certo certius lapſuras ſtratus in domo media lectulus expectabat: pauliſper alta quies, & qualem ſororum eſſe deceat, concors ac juxta decubitus. Actutum vero novus Spirituum orgaſmus; quæque harum pectus aut alia membra ferire & proximæ cuivis facere moleſtiam. Par puellarum ætate minorum, dum ibi commoratus fui, expergefactæ & ſoporem morboſum & ſorores in lecto deſeruerant: Cæterum iis rediit eadem vociferatio, nec diſſimilis capitis motantis ſpaſmus. Nota & Viſa dico, quæ ſecus mihi fabulæ viderentur; ſed neque liberorum ætas hæc ſuſpectæ eſſe fidei, neque lucri ſtudium procul habitum aut parentum ſinit conditio.

Hiſtoria adeo notabili permotus; 12. Junii 1700. cum Blackthorniam peterem, alteram ibi familiam affectu laborantem inviſi; atque hic quidem filium unum & tres ejuſdem matris filiolas motibus convulſivis ante decem hebdomadas laceſſitos reperi, præeunte nec morbo aliquo nec evidenti cauſâ. Initio quidem puellarum altera tantum correpta eſt; eam prima acceſſio ad duas horas exercuit, reliquæ cum fratre,

fratre, uti mater retulit, sororis ægritudine ita turbatæ & perculsæ sunt, ut intra paucos dies ipsæ etiam terrore victæ spasmodicis insultibus succumberent. Sub adventum meum omnes Spasmo immunes præ foribus ludos agebant, quo quidem otio ultra semihoram lætæ fruebantur, satis longum scilicet iis videbatur hoc tantillum intervalli, quod a multis retro hebdomadis frustra optârant. Vultus iis colore satis probabilis, sermo alacris, indoles perquam vivida, membra vegeta & ad motum quemvis idonea; nec ulla ab hoc affectu accrevit mali nota, nisi quædam eaque perexigua virium imminutio & languor, pulsus tam impetu quam intervallo rite compositus. Tandem puella natu major annum agens decimum quartum, pro more spasmis afficitur; unicum accessionis prænuntium sensit quasi Ventriculi intumescentiam, quæ gradatim tanquam Globus ad fauces perreptans musculos Laryngis & capitis monuit, ut solitas contractionis tragædias inchoarent; ascensus iste in omnibus notabatur paroxysmi certus & perpetuus præcursor, quem si sistere aut compescere conarentur, vehementiori impetu accessit insultus sæviitque diutius. Sonus, quem nullâ interpositâ morâ satis injucunde modulabatur, canum latratum aut ejulatum (aliter ac fama erat) non omnino referebat, videbatur potius inauditum quoddam Cantilenæ genus ex tribus numeris sive tonis iisque bis repetitis compositum, quos suspirium veluti anhelum excepit, & tandem simplici modulo clausum qui reliquis multo vehementior erat simulque acutior. Desunt sane verba, quibus næniam hanc ad vivum depingam, nullâ forte oratione pingendam; ejus scilicet ingenii est, quæ fidelibus tantum auribus potest percipi, descriptioni parum commoda. Quin quomodocumque adumbraram cantilenam continuo fere recinit puella; aliquando enim per intervalla modulos variabat, quandoque cum jam ei spiritus deficeret, velocius increbuit tum motus tum vociferatio, donec tandem pene suffocata modulum unum aut alterum interponeret concussionemque capitis paulum reprimeret; hac arte scilicet resuscitatis viribus eandem cantiunculam statim de novo integravit. Huic semper comes adfuit capitis reciproca nutatio, quod jugi agitatione antrorsum retrorsumque ex æquo ferebatur, motu autem semicirculari aut obliquo minime jactatum. In collo musculi valide tensi & inflati; cætera membra spasmis libera. His tamen cum insultibus conflictanti haud intercidit vel pedes suos pro arbitrio vel prout jussa erat in quamvis partem flectendi: Toto enim paroxysmi tempore sensibus integris atque imperturbatis fruebatur, & ad libitum imperantis vel sedebat vel obambulabat, verbo interim alicui efferendo prorsus impar. In vultu nulla coloris immutatio, oculi tanquam in morte immobiles, nulla usquam nisi in ore distortio, quæ ibi ob Musculos contractos erat omnino Cynica. Durante Paroxysmo vix digitum feriit pulsus. Hisce motibus agitata ultra Semihoram perstitit; in quo quidem statu vere luctuoso eam relinquere nox tum proquinquans me admonuit. Sorores, cum fratre quanquam una astarent, præter morem spasmo intactæ. Noctu somnus iis haud malus, si nempe horâ decubitus ad eum fuerint

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proclives: Secus Paroxyſmi ad mane uſque ingruebant, brevibus æque intervallis ac interdiu.

Puellas ex alterâ familiâ cum novo anno morbus hic invaſit; quo quidem tempore etiam fauces iis male affectæ ſunt & tumidæ. Paulo poſtquam eæ, de quibus jam ſermo fuit, ſpaſmis ſuccubuerant, hæ Epilepticis inſultibus cum ſenſuum abolitione correptæ ſunt; interdum pectora furentium more tundebant, interdum velut œſtro percitæ circumcurſitabant, &c. quorum nihil ſub tribus primis menſibus perceperant, iis tantum Symptomatis quæ a me deſcriptæ ſunt vexatæ.

Hunc naturalem eſſe morbum non prohibet ſymptomatum nova facies aut perennis duratio; Siquidem ſpaſmi hujuſce ratio cum communi omnium convulſionum indole congruit, a ſpiritibus ſcilicet animalibus oriunda, qui turbas intra nervos cient, muſculoſque in varias pro orgaſmi genio contractiones impellunt. Ita ut in hoc caſu nihil præter morem agat natura, neque aliquid monſtri præ ſe magis ferat quam in reliquis Spaſmorum generibus; quippe motibus Organicis non minus Mechanice hic uſa eſt, quam in Choreâ S. Viti aut verſatili illâ affectuum Hyſtericorum ſcenâ, ubi nunc cachinnus, nunc ejulatus, nunc violenta pectoris percuſſio ope muſculorum varie atque invitâ voluntate peragitur. Cum igitur muſculorum, tum qui laryngi & Capiti tum qui manibus & pedibus dicantur, eadem omnino ſit vis, eadem ad ſpaſmos aptitudo, quicquid novi & prodigioſi in puellis hiſce reperiri videatur, id non tam a ſymptomatis ingenio quam a parte affectâ derivandum eſt, cujus licet eo modo quo diximus convulſæ exemplum non facile occurrat, eam tamen ita poſſe corripi minus mirabimur, ſiquid apud nos valet Cl. Williſij obſervatio, In pueris ſcilicet qui affectibus cordis & membrorum exteriorum exercitiis nondum aſſueverunt, materiam ſpaſmodicam in nervos proximos h. e. eos 3tii 5ti & 6ti paris ſæpius incurrere; quare illis faciei & oris partes maxime convelluntur.

Ægri in duabus hiſce Familiis erant Conſobrini, utrum in cauſa fuerit cognatus Sanguis, ut altera familia per menſes aliquot intacta morbum tandem affinem ſuſceperit, horoſcopi & ſympathiæ ſtudioſis conjiciendum libens relinquo.

A deaf and dumb Perſon, who recover'd his Speech and Hearing after a violent Fever, by Mr. Martin Martin, *n.* 312 *p.* 1469

VI. *Daniel Fraſer* a Native of *Strabarig*, ſome ſix Miles from *Inverneſs*, continued Deaf and Dumb from his Birth, till the ſeventeenth Year of his Age. The Counteſs of *Crawford* kept him in her Family for the ſpace of eight or nine Years: After ſeventeen Years he was taken ill of a violent Fever, but being let Blood his Fever abated, and had not its Natural Courſe: About five or ſix Months after, he contracted a Fever again, and had no Blood drawn from him, and this went on with its Natural Courſe. Some Weeks after his recovery he perceived a motion in his Brain, which was very uneaſie to him, and afterwards he began to Hear, and in proceſs of time to underſtand Speech; this naturally diſpos'd him to imitate others, and at-

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tempt to Speak: The Servants were much amaz'd to hear him, and ſome ran away; he was not underſtood diſtinctly for the ſpace of ſome Weeks; he is underſtood now tolerably well, tho he yet retains the *Highland* Accent, as *Highlanders* do who are advanc'd to his Years before they learn the *Engliſh* Tongue: He can ſpeak no *Iriſh*, for it was in the *Low Lands* of *Scotland* that he firſt heard and ſpoke.

VII. A Wound or Scarification croſs the Crown of the Head is us'd in *Scotland* to cure Fluxes and Dyſenteries. The Blood being ſtanched, the Wound is cured as other Wounds commonly are. *A Cure for Fluxes and Dyſenteries. by the ſame, ib. p.* 1470.

VIII. *May* the 13th, *Anno* 1694, one *Samuel Chilton*, of *Tinsbury* near *Bath*, and Labourer, about 25 years of Age, of a robuſt habit of Body, not fat, but Fleſhy, and a dark brown Hair, happen'd, without any viſible cauſe, or evident ſign, to fall into ſome profound Sleep, out of which no Art uſed by thoſe that were near him, cou'd rouze him, till after a months time; then roſe of himſelf, put on his Cloaths, and went about his buſineſs of Husbandry as uſual; ſlept, cou'd eat and drink as before, but ſpake not one word till about a month after. All the time he ſlept, Victuals ſtood by him; his Mother, fearing he wou'd be ſtarv'd in that ſullen humour, as ſhe thought it, put Bread and Cheeſe and ſmall Beer before him, which was ſpent every day, and ſuppoſed by him, tho no one ever ſaw him eat or drink all that time. From this time he remain'd free of any drowſineſs or ſleepineſs till about the 9th of *April* 1696, and then fell into his Sleeping fit again juſt as he did before. After ſome days they were prevail'd with to try what effect Medicines might have on him, and accordingly one Mr. *Gibbs*, a very able Apothecary of *Bath*, went to him, Bled, Bliſtered, Cupp'd and Scarified him, and uſed all the external irritating Medicines he could think on, but all to no purpoſe, nothing of all theſe making any manner of impreſſion on him; and after the firſt fortnight he was never obſerv'd to open his Eyes. Victuals ſtood by him as before, which he eat of now and then, but no body ever ſaw him Eat or Evacuate, tho he did both very regularly, as he had occaſion; and ſometimes they have found him faſt aſleep with the Pot in his Hand in Bed, and ſometimes with his Mouth full of Meat. In this manner he lay for about ten Weeks, and then could eat nothing at all, for his Jaws ſeem'd to be ſet, and his Teeth clincht ſo Cloſe, that with all the Art they had with their Inſtruments they cou'd not open his Mouth, to put any thing into it to ſupport him. At laſt, obſerving a hole made in his Teeth, by holding his Pipe in his Mouth, as moſt great Smoakers uſually have, they through a Quill pour'd ſome Tent into his Throat now and then: And this was all he took for ſix Weeks and four Days, and of that not above three Pints or two Quarts, ſome of which was ſpilt too; he had made water but once, and never had a ſtool all that time. *Auguſt* the 7th, which is ſeventeen Weeks from the 9th of *April*, (when he began to ſleep,) he awaked, *A ſleepy Perſon by Dr.* Oliver. *n.* 304. *p.* 2177.

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put on his Cloaths, and walkt about the Room, not knowing he had slept above a Night, nor cou'd he be perswaded he had lain so long, till going out into the Fields he found every body busy in getting in their Harvest, and he remembred very well, when he fell asleep they were sowing of Barley and Oats, which he then saw ripe and fit to be cut down.

There was one thing observable, That tho his Flesh was somewhat wasted with so long lying in Bed, and fasting for above six Weeks, yet a worthy Gentleman his Neighbour assured me, when he saw him which was the first Day of his coming abroad, he lookt brisker than ever he saw him in his Life before; and asking him whether the Bed had not made him sore, he assured him and every Body, that he neither found that, nor any other Inconveniency at all; and that he had not the least remembrance of any thing that past or was done to him all that while. So he fell again to his Husbandry as he used to do, and remain'd well from that time till *August* the 17th, *Anno* 1697, when in the Morning he complaining of a shivering and coldness in his Back, vomited once or twice, and that same day fell into his Sleeping fit again.

Being then at the *Bath*, and hearing of it, I took Horse on the 23d, to inform my self of a matter of Fact I thought so strange. When I came to the House, I was by the Neighbours (for there was no body at home at that time besides this sick Man,) brought to his Bed-side, where I found him asleep, as I had been told before, with a Cup of Beer and a piece of Bread and Cheese upon a Stool by his Bed within his reach: I took him by the Hand, felt his Pulse, which was at that time very regular; I put my Hand on his Breast, and found his Heart beat very regular too, and his breathing very easie and Free; and all the fault I found was, that I thought his Pulse beat a little too strong: He was in a breathing Sweat, and had an agreeable warmth all over his Body. I then put my Mouth to his Ear, and as loud as I cou'd called him by his Name several times, pull'd him by the Shoulders, pincht his Nose, stopt his Mouth and Nose together, as long as I durst, for fear of Choaking him, but all to no purpose, for in all this time he gave me not the least signal of his being sensible. I lifted up his Eye-lids, and found his Eye-balls drawn up under his Eye-brows, and fixt without any motion at all. Being baffled with all these Tryals, I was resolv'd to see what effects Spirit of *Sal Armoniac* would have, which I had brought with me, to discover the Cheat, if it had been one; so I held my Viol under one Nostril a considerable time, which being drawn from Quick-lime, was a very piercing Spirit, and so strong I could not bear it under my own Nose a moment without making my Eyes water; but he felt it not at all. Then threw it at several times up that same Nostril, it made his Nose run and gleet, and his Eye-lids shiver and tremble a very little, and this was all the effect I found, tho I pour'd up into one Nostril about a half ounce Bot-

tle of this fiery Spirit, which was as ſtrong almoſt as Fire itſelf. Finding no ſucceſs with this neither, I cram'd that Noſtril with Powder of White *Hellebore*, which I had by me, in order to make my farther trials, and I can hardly think any Impoſtor cou'd ever be inſenſible of what I did. I tarried ſometime afterwards in the Room to ſee what effects all together might have upon him; but he never gave any token that he felt what I had done, nor diſcover'd any manner of uneaſineſs, by moving or ſtirring any one part of his Body, that I could obſerve. Having made theſe my Experiments I left him, being pretty well ſatisfied he was really aſleep, and no ſullen Counterfeit, as ſome people thought him. Upon my return to *Bath*, and relating what I had obſerv'd, and what proofs this Fellow had given me of his Sleeping, a great many Gentlemen went out to ſee him, as I had done, to ſatisfie their Curioſity in a Rarity of that Nature, who found him in the ſame condition I had left him in the day before; only his Noſe was inflamed and ſwelled very much, and his Lips and the Inſide of his Right Noſtril bliſter'd and Scabby, with my Spirit and *Hellebore*, which I had plentifully dos'd him with the day before: His Mother upon this for ſome time after wou'd ſuffer no body to come near him, for fear of more Experiments upon her Son. About ten days after I had been with him, Mr *Woolmer*, an Experienc'd Apothecary at *Bath*, called at the Houſe, being near *Tinsbury*, went up into the Room, and finding his Pulſe pretty high, as I had done, takes out his Launcet, lets him Blood about Fourteen ounces in the Arm, tyes his Arm up again, no body being in the Houſe, and leaves him as he found him; and he aſſured me he never made the leaſt motion in the world when he prickt him, nor all the while his Arm was bleeding. Several other Experiments were made by thoſe that went to ſee him every day from the *Bath*, but all to no purpoſe, as they told me on their return: I ſaw him my ſelf again the latter end of *September*, and found him juſt in the ſame poſture, lying in his Bed, but remov'd from the Houſe where he was before about a Furlong or more; and they told me, when they remov'd him, by Accident, carrying him down Stairs which were ſomewhat Narrow, they ſtruck his Head againſt a Stone, and gave him a ſevere knock, which broke his Head, but he never mov'd any more at it than a dead man wou'd. I found now his Pulſe was not quite ſo ſtrong, nor had he any Sweats, as when I ſaw him before. I try'd him again the ſecond time, by ſtopping his Noſe and Mouth, but to no purpoſe; and a Gentleman then with me ran a large Pin into his Arm to the very Bone, but he gave us no manner of tokens of his being ſenſible of any thing we did to him. In all this time they aſſured me no body had ſeen him eat or drink, tho they endeavour'd it all they could, but it always ſtood by him, and they obſerv'd ſometimes once a day, ſometimes once in two days all was gone. 'Tis farther obſervable, he never foul'd his Bed, but did his neceſſary occaſions always in the Pot. In this manner he lay till the 19th of *November*, when

his Mother hearing him make a noiſe, ran immediately up to him, and found him Eating; ſhe askt him how he did? He ſaid, Very well, thank God: She askt him again, Which he lik'd beſt, Bread and Butter, or Bread and Cheeſe? He anſwer'd, Bread and Cheeſe: Upon this, the poor Woman overjoy'd left him to acquaint his Brother with it, and they came ſtrait up into the Chamber to diſcourſe him, but found him as faſt aſleep as ever, and all the Art they had cou'd not wake him. From this time to the end of *January* or the beginning of *February*, (for I cou'd not learn from any body the very day) he ſlept ſo profoundly as before, for when they call'd him by his Name he ſeemed to hear them, and be ſomewhat ſenſible, tho he could not make them any anſwer. His Eyes were not now ſhut ſo cloſe, and he had frequently great tremblings of his Eyelids, upon which they expected every day when he would wake, which happened not till about the time juſt now mention'd, and then he wak'd perfectly well, not remembring any thing that happened all this while. 'Twas obſerv'd he was very little alter'd in his Fleſh, only complained the Cold pincht him more than uſually, and ſo preſently fell to Husbandry as at other times.

Strange Effects of the Scurvy, at Paris. *by Mr* Poupart. *n.* 318. *p.* 223. *From* Hiſt. Acad. Sc.

IX. The Diſeaſe, which I am now going to treat of, was a true Scurvy; for they who were ſick of it, felt, as common Scorbutick Perſons do, pains in their Thighs, the Calves of their Legs, their Belly, and Stomach, and were deprived of the Motion, or Uſe of their Limbs, tho' they ſtill retained their feeling. They were troubled with Head-achs, Convulſions, and ſuch ſtrange itching in the Gums, that the Children pulled off certain Pieces of them with their Nails. The Blood which came from them, was Watery, Salt and Corroſive; and the Stink, which came from their Mouth, was intolerable. They had hard blew Spots on their Legs and Thighs, frequent Hæmorrhages, or Bleedings at the Noſe and Fundament, and alſo ſo great a Weakneſs in their Knees, that they could not go without reeling or ſtaggering. Theſe were the Symptoms which they had in common with other ſcorbutick Perſons.

When we removed theſe ſick Perſons, we heard a ſmall clattering of their Bones.

I obſerved at the opening of all thoſe Bodies or Cadavers in which we heard the aforeſaid little noiſe, that the *Epiphyſes* were intirely ſeparated from the Bones, which by rubbing againſt each other occaſioned this clattering. We have opened ſeveral young Perſons, in whom we alſo perceived a ſmall low noiſe when they breathed. In all theſe ſort of Bodies we found, that the Griſtles of the *Sternum* were ſeparated from the Bony part of the Ribs; and as the Griſtles are of a ſofter Subſtance than the *Epiphyſes*, the noiſe, which their rubbing produced, was greater than that of thoſe Bones which rubbed againſt the *Epiphyſes*. They, in whom we heard this noiſe at the time when they breathed,

ed, are all dead, except one young Man, whose Ribs were visibly reunited to the Griftles, for after his Cure, we heard no more of this noise. All those, in whose Breasts any Matter or Serosity were found, had their Ribs separated from their Griftles, and that bony Part of those Ribs, which were over against the *Sternum*, was rotted for the length of four Fingers; which is an Evidence, that the *Lympha* of these Bodies was extreamly caustick. The greatest part of those Bodies, which were opened, had their Bones black, worm-eaten, and rotten.

Most of the Sick went staggering: This is an Accident common or usual to scorbutick Persons, and very well known to most Physicians; but the reason of it, which you have here, is not so well known. It is certain, that the support of the Joints proceedeth from the Force and Spring of the Ligaments, which bind the Bones close to each other; the Ligaments of these sick Persons, were corroded, loose, and the Bones were separated from each other; which proceeded from this, that instead of finding in their Joynts that sweet Oily *Lympha* (which commonly aboundeth there in order to make the Joynts supple, and give them an easie free Motion) there was nothing but a greenish Liquor, which by its over caustick Quality had corroded the Ligaments, and consequently destroyed the Force of their Spring. All the young Persons under eighteen, had in some degree, their *Epiphyses* separated from the Body of their Bones, and by the least endeavour or force we separated them entirely. The reason of it is this, that young Persons have not yet their *Epiphyses* so strongly fasten'd to the Bones, so that when they are never so little soak'd with that Corrosive *Lympha* which is in the Joynts, that caustick Liquor may easily separate them entirely from the Bones. All the Bones, which we found entirely separated from their *Epiphyses*, were more than twice as big as they should be in their Natural State, because these *Epiphyses* were separated in them only, whose Bones were well soak'd with a Water which had penetrated into their very Substance and made it swell.

The Bones of those which recovered, or were recovering, remained swell'd, without giving them any pain: They might grow less in time, as it happens to Children, which are troubled with the Rickets, whose Bones grow dry by little and little as they grow up. All they who had any Difficulty of Breathing, or had their Breasts stuff'd or stopp'd up, had there good store of *Lympha*, or Matter; and we often found more or less of them in their Lungs, according as they were oppressed. We have seen some sick Persons, whose Breasts have been so oppressed, that they died all on a sudden; in the mean while we found no Serosity neither in their Breasts nor in their Lungs: But the *Pericardium* was entirely fasten'd to the Lungs, and the Lungs were glued to the *Pleura* and *Diaphragma*; and all the Parts were so mixed and blended together with each other, that they all made up but one Mass or Lump,

ſo confounded, that one could ſcarce diſtinguiſh the one from the other: Now as the Lungs were ſqueez'd together in the midſt of this Maſs, they were deprived of their Motion, and the ſick Perſon was choak'd for want of Breath. The cloſe Adheſion, and Confuſion of theſe parts one with another, proceeded from this, that being ulcered as they were, they muſt needs ſtick to each other. The ordinary or common Scorbutick Perſons have the Glands of their Meſentery much obſtructed and ſwell'd; thoſe we treat of, have theirs partly corrupted, and Impoſthumes on the Subſtance of it. In the Liver of ſome few, the Matter or Corruption was hardned, and as it were petrified; their Spleen was three times bigger than it ſhould be, and fell to pieces as if it had been compoſed of coagulated Blood; and ſometimes the Kidnies and the Breaſt were full of Impoſthumes.

There were ſome Bodies or Cadavers of thoſe of Fifteen, in which, if we ſqueez'd betwixt two Fingers the end of the Ribs, which began to be ſeparated from the Griſtles, there came abundance of corrupted Matter, which was the ſpungy part of the Bone; ſo that after the ſqueezing of it together, there remain'd nothing of the Rib, but two bony Plates. We have ſeen ſome certain Perſons, who had no other Token of the Scurvy, but ſome ſlight Ulcerations in the Gums: They had afterwards ſome ſmall red, hard Tumours on their Hands, their Inſteps, and in ſome other parts of the Body. After that, there appeared large Impoſthumes on their Groin and under their Arm-pits, attended with ſeveral blue Spots all over their Body, which were the certain Fore runners of Death. We found that the Glandules under their Arm pits were very big, and ſurrounded with Matter or Corruption; as well as the Muſcles of their Arms and Thighs, whoſe Intervals were all filled with them. We obſerved ſome whole Arms, Legs, and Thighs, were of a reddiſh Black, and as it were burnt; which proceeded from that black and coagulated Blood, which we always found under the Skin of thoſe Perſons. We alſo found their Muſcles ſwelled, and as hard as Wood, which proceeded from the Blood which was fix d in the Body of the Muſcles, which were ſometimes ſo full of it, that their Legs remained bent without being able to extend or ſtretch them out. We obſerved that the blue, red, yellow, and black Spots, which appear in their Bodies who have the common Scurvy, proceed purely from extravaſated Blood under the Skin. As long as the Blood kept its red Colour, the Spot was red; if the Blood is black or coagulated, the Spot is alſo black; when there is ſome Bile mix'd with it, the Spot is of a yellowiſh black; in ſhort, according as the Blood is mixed with the Humours of different Colours, ſo alſo the Spots appear of a different Colour. We ſometimes ſaw on the Bodies of theſe Perſons certain ſmall Tumours which grew bigger every Day: we applied Emollient Ointments to ſoften them, and thoſe Tumours on their breaking, formed a Scorbutick Ulcer, which proceeded from the Blood with which the Tumour was filled;

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for as often as we took off the Plaifter, we ftill found under it a great deal of coagulated Blood; we put on a frefh Plaifter, and fome time after we ftill found under it coagulated Blood: We continued dreffing of them after this manner, and by thus taking away the Blood, we dried up the Tumour, and the Perfon was cured. Some old perfons had fuch large Bleedings at the Nofe and Mouth, that they died of it, it being impoffible to ftop it, becaufe the *Lympha* of thefe perfons was fo fharp and corrofive (as I faid before) that it corroded and eat through the Coats of the Veins. And this kind of Hæmorrhage was fo much the harder to ftop, becaufe the Blood of old Perfons is more fluid and watery than that of young Perfons, who are feldom fubject to this Accident.

Old Perfons, as well Women as Men, were troubled with fuch mighty Fluxes, that the weakeft of them died under them; but if they had ftrength enough to withftand them, they were foon cured. There were fome of thefe Sick Perfons, who were fo Coftive in their Body, that they never could go to Stool without taking fome Glifters. Several of them had fuch large Swellings over all their Bodies, their Hands, Arms and Feet, that they feemed to have been blown up. We cured feveral of them by proper Medicines, Glifters, and fweetning Juleps. A Youth of Ten Years old, had his Gums much fwelled and ulcered; his Teeth were eaten up to the Roots of them, and ferved no longer; and his Breath was intolerably ftinking. The Chirurgeon was obliged to pull out all his Teeth, for the better dreffing of his Mouth, tho' they would have fallen out of themfelves: His Gums were healed, but there arofe a Tumour on the fide of his Tongue as big as a Walnut. In the middle of this Tumour there was a bluifh Hole, which degenerated into an Ulcer, which eat up half the Tumour, the other half remained whole and entire. Some fmall time after, there appeared another Tumour in the Cheek, which was very hard: It was Blue in the middle, and turn'd to an Ulcer alfo as the firft. This Youth died all on a fudden, when it was leaft expected, and all the inward Parts of his Body were Corrupted.

All they who died fuddenly, without having any vifible Caufe of their Death, had the Auricles of their Heart as big as one's Fift, and full of coagulated Blood, which, by putting a ftop to the Circulation of the Blood, brought an inevitable Death on them. There came in the Cheeks of feveral a fmall White Ulcer, which was hard all round; unlefs we took care to ftop it prefently, and to take it off with the Spirit of Vitriol, it grew prefently livid or blue black and ftinking, and eat up part of the Cheek, fo that one might fee the Teeth thro' it. We have feen feveral from the Age of Eighteen to the Age of Thirty, who were without pain caft down ftupid and without any Motion. They had their Mouth open, their Eyes funk in, their Looks frightful, and appeared rather like Statues than Men. All thefe Perfons had no apparent Sicknefs, only their Gums were Ulcered;

their Skin was ſmooth and Fair, without any Spots or Hardneſs: Yet we found their Muſcles were Gangreen'd, and all wet with a black corrupted Blood, and in handling of them, they fell into Pieces in our Hands. There was a Man who had a Carbuncle on his Inſtep, his Lips and his Noſtrils were chopped, and a ſtinking Water flow'd gently from his Noſtrils. This Man linger'd out a long time in a dying Condition: His *Cadaver* made me afraid, I durſt not open it. A Young Man, who as to all outward appearance ſeem'd not to be very Ill, died ſuddenly. We found his *Pericardium* was ſo eaten up, that there remain'd but a little of it, and his Heart was Ulcer'd all about very deeply.

Scorbutick Perſons are commonly better in the Summer, than they are in the Winter, which may proceed from their great Tranſpiration. On the other ſide, theſe were indifferently well from the Month of *April*, to the beginning of *June*, the Spots, hardneſs and other Accidents of the Scurvy then diſappearing; but on the coming of the great Heats, all thoſe Accidents returned They, who were ſo well as to be in a readineſs to quit the Hoſpital, relapſed again: Their Legs and Thighs grew all Black, and Death often put a Period to there Miſeries. This Diſorder might arrive from this, that there was ſuch a great quantity of corroſive *Lympha* in them, that it was in a manner impoſſible for it to be carried off by Tranſpiration, ſo that by ſtagnating in their Bodies it grew hot, fermented, ſower, and putrified; from thence aroſe thoſe Corroſions, Ulcers, and great Impoſthumes, Corruptions and other Accidents which we ſpoke of before. All theſe Poor People eat very heartily to the laſt Moment of their Life; this proceeded from a ſharp Humour, with which their Stomach always abounded, which created in them a kind of *Fames Canina*. Nothing is ſo apt to corrupt the Blood as long Want; the uſe of ill Food is ſtill worſe; Cold ſtops the Circulation of the Blood, and makes the Blood remain too long in the Parts, where it ſoureth and ſoon corrupteth; Sadneſs and Grief (which theſe poor Creatures are ſubject to) is worſe than all the reſt; and what all theſe may do when they meet altogether *in* one Perſon, we may eaſily judge. They produced there *Lympha*'s of different Colours, with which the Belly, the Breaſt, and ſeveral other Parts of their Bodies were fill'd. Thoſe *Lympha*'s were ſo Cauſtick, that having put our Hands into their *Cadavers*, the Skin of them came off, and our Faces were thereby Ulcered; ſo that we were obliged to riſe in the Night to waſh our Faces with Freſh Water, to take off the Heat and Inflammation of it. But that which was very ſurpriſing in this great Diſeaſe, was that the Brains of theſe poor Creatures were always ſound and entire.

Diſſection of the Body of Mr. Dove. *by Mr.* Cowper. *n.* 335 *p.* 512.

X. The Body in divers parts appear'd of black, blue, livid, and various Colors, before I made any Inciſion into it, particularly the Back, where the Blood was ſettled, had a cadaverous Blackneſs, where the

Cuticula

Cuticula was here and there veficated, or diftended with *Serum.* Of this there was no Appearance before Death.

The Mufcles of the *Abdomen* had a Mortified Appearance, being a blackifh green Colour. The Liver was intirely fphacelated. The Spleen had large mortified Spots on its Surface: Both thefe Parts were fpecifically lighter than in the Natural State; infomuch,that Portions of each of them fwam on the Surface of the Water, and feemed to have more Air in them than we commonly find in the Lungs in their Natural State. The reft of the *Vifcera* in this lower Cavity were not in fo low a State; tho' the Guts had here and there blackifh Spots on them.

The Pectoral Mufcles were in little better State than thofe of the *Abdomen;* nor were the Intercoftal Mufcles like thofe of the Limbs. I am apt to think all the Mufcles imployed in Refpiration, had more or lefs of this Blackifh Appearance. The Right Lobes of the Lungs were difeafed; and the fame fide of the *Thorax* had a fmall quantity of *Serum* in it. The Lungs on the other fide were in no ill Condition. The Heart was very flaccid and large: The Right Ventricle and *Vena Cava* had no fmall *Polypus* in them. The *Vena Pulmonaris* was exceedingly dilated next the *Bafis* of the Heart. The Left Ventricle of the Heart was furnifhed with a fmall *Polypus,* and a great quantity of Grumous Blood. The Great Artery was very thin, and appeared not a little extended, and had fome Cartilaginous Bodies interfperfed in its Membranes.

In the Head; the *Dura Mater* was found infeparable from the *Cranium* in its upper part. A *Polypus* was drawn out of the upper great Vein of the Brain, called *Sinus Falcis fuperior.*

The Carotide Arteries were very thin, and much larger than they ought to be, before they entered the Subftance of the Brain. In fhort, all the Blood-Veffels which I examin'd were very much dilated, and feemed to be charged with as much Wind as Blood.

A Child deliver'd full of the Small-Pox, by Mr. Derham, *n.* 337. *p.* 165.

XI. There was, in the Parifh next *Upminfter,* a Great-bellied Woman who had the Small-pox,and was pretty well recovered,fo that fhe was able to take fomething to purge her after it: And on *Auguft* the thirtieth laft, fhe took a Purge (I know not what) which did not work; and on *September* the Firft, another Purge, which gave her only a Stool or two. Upon which, *September* the Third, fhe took another ftronger Purge, that work'd fo violently upwards and downwards, that fhe fell into Faintings and Convulfions: about which time I conceive her Child died within her, but of which fhe was not deliver'd till *September* the eighth. The Child was a Female, and in appearance well made, lufty and ftrong. At its Delivery, the Midwife judged it had been dead five or fix Days; fo that the Belly was burft, and the Bowels came out, and the whole Body was inclining to be rotten. The Child was very full of the Small-pox, fo full, that

the

the Midwife said, hardly a Pins head could be put between the Blisters, which were very Plump, and full of Matter, like the Pustules of an Adult, when the Small Pox is at the Height, only a little depressed in the middle. But as full as the Child was, the Mother had as few, and very favourably.

Of procuring the Small Pox by Inoculation. by Dr. Timone. *com. by Dr.* Woodward. *n. 339. p. 72.*

XII. The *Asiaticks* have introduced this Practice of procuring the Small Pox by a sort of Inoculation, for about 40 Years at *Constantinople*; and tho' at first 'twas us'd with Caution, yet the Success it has met with in thousands of Subjects for 8 Years past has put it beyond all Suspicion and Doubt; since none who have had it this way have been found to die of the Small Pox; when at the same time 'twas very mortal, when it seiz'd the Patient the common way, of which half the affected dy'd.

They that have this Inoculation practis'd on them are subject to very slight Symptoms; some being scarce sensible they are ill or sick; and what is valued by the Fair, it never leaves any Scars or Pits in the Face.

The Method of the Operation is thus. Choice being made of a proper Contagion, the Matter of the Pustules is to be communicated to the Person proposed to take the Infection; whence it has, metaphorically, the name of Incision or Inoculation. For this purpose they make choice of a Boy, or young Lad, of a sound healthy Temperament, that is seized with the common *Small-Pox* (of the distinct, not Flux sort) on the Twelfth or Thirteenth day from the beginning of his Sickness; they with a Needle prick the Tubercles (chiefly those on the Shins and Hams) and press out the Matter coming from them into some convenient Vessel of Glass, or the like, to receive it; it is convenient to wash and clean the Vessel first with warm Water: A convenient quantity of this Matter being thus collected, it is to be stop'd close, and kept warm in the Bosom of the Person that carries it, and as soon as may be, brought to the place of the expecting future Patient.

The Patient therefore being in a warm Chamber, the Operator is to make several little Wounds with a Needle, in one, two or more places of the Skin, till some drops of Blood follow, and immediately drop out some drops of the Matter in the Glass, and mix it well with the Blood issuing out; one drop of the Matter is sufficient for each place prick'd. The Punctures are made indifferently in any of the fleshy Parts, but succeed best in the Muscles of the Arm or *Radius.* The Needle is to be a three edg'd Surgeon's Needle; it may likewise be perform'd with a Lancet: The custom is to run the Needle transverse, and rip up the Skin a little, that there may be a convenient dividing of the Part, and mixing of the Matter with the Blood more easily perform'd; which is done, either with a blunt Stile, or an Ear-picker: The Wound is cover'd with half a Walnut-shell, or the like Concave Vessel,

Veſſel, and bound over, that the Matter be not rubb'd off by the Garments; which is all removed in a few Hours. The Patient is to take care of his Diet. In this place the Cuſtom is to abſtain wholly from Fleſh and Broth for 20 or 25 days.

This Operation is perform'd, either in the beginning of the Winter, or in the Spring.

Some, for caution, order the Matter to be brought from the Sick by a third Perſon, leaſt any Infection ſhould be convey'd by the Cloaths of the Operator; but this is not material.

As to the Proceſs of the Matter, in reſpect of the *Idioſyncraſy*; the *Small Pox* begins to appear ſooner in ſome than in others, in ſome with greater, in others with leſſer Symptoms; but with happy Succeſs in all. In this Place the Efflореſcence commonly begins at the end of the ſeventh day. Which ſeems to favour the Doctrine of *Criſes*.

It was obſerv'd, in a Year when the common *Small-Pox* was very mortal, that thoſe by Inciſion were alſo attended with greater Symptoms. Of 50 Perſons, who had the Inciſion made upon them almoſt in the ſame day, four were found in whom the Eruption was too ſudden, the Tubercles more, and Symptoms worſe. There was ſome ſuſpicion, that theſe four had caught the common *Small-Pox* before the Inciſion was made. It is enough for our preſent purpoſe, that there was not one but recovered after the Inciſion: In thoſe four the *Small-Pox* came near the confluent ſort. At other times the inoculated are diſtinct, few and ſcatter'd; commonly 10 or 20 break out; here and there one has but 2 or 3, few have 100: There are ſome in whom no Puſtule riſes, but in the Places where the Inciſion was made, which ſwell up into purulent Tubercles; yet theſe have never had the *Small-Pox* afterwards in their whole Lives; tho' they have cohabited with Perſons having it.

It is to be noted, that a no ſmall quantity of Matter runs for ſeveral days, from the place of the Inciſion.

The Pocks ariſing from this Operation are dry'd up in a ſhort time, and fall off, partly in thin Skins, and partly contrary to the common ſort, vaniſh by an inſenſible waſting.

The Matter is hardly a thick *Pus*, as in the common, but a thinner kind of *Sanies*; whence they rarely pit, except at the place of the Inciſion, where the Cicatrices left are not to be worn out by time, and whoſe Matter comes near the nature of *Pus*.

If any Apoſteme breaks out in any (which Infants are moſt ſubject to) yet there is nothing to be fear'd, for it is ſafely heal'd by Suppuration. If any other Symptom happens, 'tis eaſily cur'd by the common Remedies.

Obſerve, they ſcarce make uſe of the Matter of the Inciſious Pox, for a new Inciſion. If this Inoculation be made on Perſons who have before had the *Small-Pox*, they find no alteration, and the places prick'd preſently dry up; except in an ill Habit of Body, where

poſſibly a ſlight Inflammation and Exulceration may happen for a few days.

To this time, he ſays, I have known but one Boy, on whom the Operation was perform'd, and yet he had not the *Small-Pox*, but without any miſchief; and ſome Months after catching the common ſort, he did very well. It is to be obſerv'd, that the places of the Inciſion did not ſwell. I ſuſpect this Child prevented the inſertion of the Matter, for he ſtruggled very much under the Operation, and there wanted help to hold him ſtill. The Matter to be inſerted will keep in the Glaſs very well for 12 Hours. He goes on.

I have never obſerv'd any miſchievous Accident from this Inciſion hitherto; and altho' ſuch Reports have been ſometimes ſpread among the Vulgar, yet having gone on purpoſe to the Houſes whence ſuch Rumors have ariſen, I have found the whole to be abſolutely falſe.

It is now eight Years ſince I have been an Eye-witneſs of theſe Operations; and to give a greater Proof of the Sedulity I have uſed in this Diſquiſition, I ſhall relate two Hiſtories.

There was in a certain Family, a Boy of 3 Years old, afflicted with the *Falling-Sickneſs*, the *King*'s-*Evil*, an *Hereditary-Pox* and a long *Maraſmus*. The Parents were deſirous to have the Inciſion made upon him; the *Small Pox* were thrown off with eaſe; about the 40th day he dy'd of his *Maraſme*. In another Family, a Girl of 3 Years old, troubled with the like Fits, ſtrumous, attended with an *Hereditary-Lues*, and labouring under a colliquative Looſeneſs for three Months. The Operation was perform'd on this Child; ſhe came off very well of the *Small-Pox*, which was all over the 15th day; on the 32d ſhe dy'd of her Looſneſs, which had never left her the whole time.

Ætiologia. *by the ſame. ib. p.* 76.

2. Contagium Variolarum per puris infuſionem propagari haud equidem mirabitur qui Æſculapii templum vel à primo limine ſalutavit, & fermentationis doctrinam ſubodoratus eſt: Nec obſcurior eſt inſitionis modus quàm panificium aut ars cereviſiaria, in quibus ex admixto fermento maſſæ fermentandæ turgeſcunt; conciliato nimirum motu inteſtino minimarum particularum principiis active pollentium. Si quis quærit interim cur variolæ periculoſæ alioquin & perſæpe lethales, ex inſitione ſine ullo periculo excludantur. Dico: Variolæ communes vel concurrente pravâ aliquâ ſpeciali aeris diatheſi ſuſcitantur, vel ab effluviis à varioloſo corpore emanantibus per contagium propagantur. Primus caſus in paucis individuis accidit, & concurrente quidem vel inſigni cacochymiâ, vel ſaltem varioloſi ſeminii in talibus individuis latitantis accerrimâ exaltatione: Secundus caſus communiſſimus eſt. In primo caſu miaſma malignum aereum, in ſecundo virulenta contagii corpuſcula indolis (probabiliter) ſalino-ſulphureæ ſed ſpecificam fracedinem ſeu ranciditatem nacta, ſtatim ac per reſpirationem hauriuntur, ſpiritus ipſos, & labe quidem teterrima inficiunt; ſubſequenter autem maſſam ſanguineam & lympham vitiari manifeſtum eſt. Spiritus ſta-

tim

tim infici rationi consentaneum est, tum quia in fontes spirituum, cor scilicet & cerebrum, statim ingressum habent virulentum aporriæ, tum ratione analogismi inter miasmata & effluvia ista ipsosque spiritus, cum utraque spirituoso-aereæ texturæ sint. Deducitur etiam cita & prava spirituum (infectio à tot tantisque nervosi systematis symptomatibus, quæ malas plerumque comitantur variolas, & præcipuè à convulsionibus epilepticis quæ infantibus accidunt ipso momento, quo varioloso inficiuntur contagio multo antequam febris illos corripiat. Massam autem sanguineam inquinari præter febrem purulenta tuberculorum exclusio testatur. Lymphæ verò vitiatæ fidem faciunt glandularum in faucibus tumor, screatus, & enormis multoties ptyalismus. Inter hæc circularis etiam sequitur noxa. Sed præcipuè sanguinis particulæ ab indebita spirituum irradiatione in plures ataxias & anomalias perducuntur. Duobus tamen potissimum modis in variolis communibus mortem contingere observavi.

Primus est quando paucis erumpentibus variolis, & tardè ad maturitatem procedentibus, mala alia oboriuntur symptomata; secundus quando nimia tuberculorum copia cadaverosam putredinem inducit. In primo casu malignæ vulgo dicuntur variolæ: causa autem est vel nimia fusio & dissolutio massæ sanguineæ, vel ejusdem coagulatio & grumescentia. Si enim impetus spirituum explosivus justo plus augeatur, particulæ massæ sanguineæ nimium ad invicem atteruntur, comminuuntur, & tenuissimas nanciscuntur acrotitas: sanguis in hoc statu solertis naturæ mechanismum eludit, cumque nil fœculentioris in glandulis secretoriis cribrisque deponat, œconomiæ animalis functionibus requisitas filtrationes & transcolationes celebrari haud patitur: improportionata etenim est figura particularum liquidi ad configurationem pororum in tubulis & colatoriis ratione subtilitatis nimiæ: filtratione enim defæcarentur particulæ sanguinis si naturalem servarent schematismum & molem: hinc dicitur pepsim fieri per incrassationem. Præter hoc celeritas ipsa transitûs sanguinis in causa est ut nihil deponatur in colatoriis. Torrens, ubi nimio impetu & præcipiti cursu fertur, aquas turbidas defæcari haud patitur; quia vis centripeta gravitatem admixti terrei sequens superatur à fortiorum pulsoria virtute aqueorum globulorum rapide ruentium: virtus enim fortis, verbi gratia, ut unum non poterit lineam perpendicularem describere, ubi virtus fortis ut duo ad lineam horizontalem protrudit: sic etiam haud pluit vento flante intensissimo; eadem geometrica proportione (probabiliter loquendo) sanguinis particulæ aucto ab effrænibus spiritibus motu, tubulos colatorios præterfluunt nullâ factâ fæcum depositione. Hæc probabilia fiunt à summa pulsûs celeritate, febre intensissima, sudore nullo, & urina cruda. E contra quandoque contingit ut ab acutis, & scindentibus deleterii fermenti particulis frangatur, corrodatur, vel saltem relaxetur elater spirituum: elanguescente igitur spirituum motu, torpidiores etiam hebetioresque fiunt sanguinis lymphæque particulæ: igitur dum in labyrinthæis tubulorum anfractibus moram indebitam contrahunt,

alias turmatim in vicem complicari, alias autem, congestione factâ, super alias incidere, & diverso ad invicem superficierum suarum contactu à naturali configuratione desciscere, & novas induere angulorum dimensiones necesse est. Sic igitur diversa ab illa, quam superius narravimus, figurarum ad tubulorum meatus improportione, pari tamen calamitatis eventu, dædaleæ naturæ machinationes irritas fieri contingit. Hæc probabilia fiunt à pulsu tardo & raro, ac febris carentia, quandoque in summa malignitate observatis, paucis & tardè erumpentibus variolarum pustulis. Ulterius à trepidatoria, seu subsultoria ac tumultuosa furentium spirituum irradiatione, inæqualis eodem tempore in diversis partibus massæ sanguineæ, & arteriarum etiam venarumque contingere potest impulsus. Sive igitur fibrillæ aliquæ (ut quidam volunt) reperiantur in sanguine, seu chili nondum bene assimilati sint portiones usibus peculiaribus dicatæ; probabiliter istarum motum turbari contingit: has enim in circulatorio motu secundum longitudinem suam naturaliter moveri necesse est: ab inæquali autem pressione dicta rectilineam figuram perdere, & in spiras ac semicirculos crispari coguntur: has igitur sic contortas transversaliter postmodum in circulatione raptari, ad invicem implicatas convolvi, &, ramosis schematibus obortis, racematim adeo conglobari necesse est, ut in majusculos tandem grumos coalescant; sive demum fibrillæ illæ non dentur, certè cujuscumque figuræ sint massæ sanguineæ particulæ, illas à naturali desciscere situatione ex hac motûs inæqualitate contingit: Confusæ igitur particulæ istæ & ad invicem implicatæ statim vehiculi sui, seri scilicet globulis per expressionem à suo contubernio explosis, majorem ratione molis auctæ gravitatem nanciscuntur, ideoque impulsivæ circulatoriæ facultatis vim superant: Has igitur hîc illic restitare ac stagnare necesse est, prout in hoc vel illo loco prima mutua cohæsio forte contigerit: Hinc livida stigmata, & simul (quod sæpe observavi in variolis cum petechiis erumpentibus) frequens sequitur mictus, quo limpidissimum serum in magna copia excluditur. En fusio, & coagulatio. Hinc mirum non est cur moriantur aliqui in variolis cum petechiis, convulsionibus, syncope, vigiliis nimiis, hæmorrhagiis, delirio, vomitibus enormibus, dysenteriis, *&c.* quamvis haud multa pustularum putrilagine perfundantur: In stygium enim veluti characterismum variolarum fermentum multoties evehitur, ita ut quamvis haud magnam crassi puris copiam progignere aptum sit, spiritibus tamen, liquidis & solidis, suprarecensita mala modis vel explicatis vel aliis consimilibus communicare possit, sicque mortem inferre; & hoc ante undecimum plerumque. Veniamus nunc ad secundum modum. Diversa enim aliquando contingit pernicies & longè alterius generis tragœdia: quamvis enim absint illa symptomata, nimia tamen puris, materiæ scilicet cadaverisatæ, copia corpus obruitur. Pus autem generari probabile est. quando sulphureis oleosisque massæ sanguineæ particulis in fracedine & fusione constitutis acido-salinarum particularum coaffusio contingit. Huic asserto facem accendunt innumera chymica ex-

experimenta quibus manifestè edocemur, solutionibus pinguium sulphureorum per alkalia factis acido quolibet coaffuso statim massam albicantis coloris emergere. Multoties igitur miasma seu fermentum variolarum per respirationem haustum ratione indolis propriæ acerrimæ & fortassis septicæ tales in massam sanguineam particularum acido-salinarum & oleoso-sulphurearum producere potest combinationes, ut non seminia solum variolarum, quæ omnibus individuis (mole tamen minima) à nativitate indita sunt, agitentur, actuentur, & in purulentam abeant putrilaginem, sed massa ipsa sanguinea tota acorem contrahat, & motu quodam corruptorio putrescat & cadaverisetur. Sic igitur, incendio veluti coorto, ulterius furere fermentescentes particulas contingit, quam variolosis seminiis per despumationem eliminandis opus sit: hic motus non est depuratorius heterogeneis secernendis inserviens, sed destructivus & corruptorius, fermento nempe massam totam superante & invertente; fracidis scilicet rebellibusque particulis victoria potitis, & omnes alias in sua castra migrare cogentibus. Hoc manifestè observamus in variis potulentis, in quibus, fermentatione aliquando excitatâ, motus succedit corruptivus liquores totaliter vitians: hinc videmus aliquos, quamvis suprarecensitis symptomatibus immunes, immenso tamen, ut ita dicam, putredinis oceano suffocatos: Et hoc periculum usque ad vigesimum secundum protrahitur. Ultimo loco considerandum solida etiam & nobiliores partes in hisce casibus pessimè affici, & in spasmos inordinatos fieri: variis horum distortionibus tubulorum meatus vitiari, at functionum munera depravari necesse est. Ecce igitur continentia, contenta, & impetum facientia, quorum triumviratu humani corporis respublica regitur, una eademque ruina ut plurimum involuta: mirabiturne quispiam malorum inde Iliadem in hominis perniciem pullulare? Observandum ulterius multis, qui peste laboraverint, communibus variolis etiam post annum correptis, bubones eosdem intumuisse, qui antea in peste eruperant: nonne hoc etiam summam malignitatem testatur? Insitionem modo ad rationis trutinam revocemus. At hercule longe aliter in hoc contagionis modo rem procedere quis est qui non fateatur? Primum enim Spiritus nullatenus infici manifestum est: deinde non lymphæ, non sanguini labes illa teterrima inuritur, non solidis vitium aliquod communicatur. Hinc symptomata omnia levia, nulla pessima, nulli infantibus epileptici insultus. Contagionis enim hujusce fermentum non spirituale, non aereum & acutum est, sed humorale, iners ac pigrum: venena autem quo subtiliora eo pejora: Ratione igitur improportionis nulla inter fermentum hoc & spiritus esse poterit lucta. Pus equidem variolarum in ipsa substantia sanguini immediatè infusum statim in largum veluti pelagus exceptum diluitur, involvitur, absorbetur, obtunditur: sic illud mitescit, sic in mansuetiorem indolem circuratur. Contagiosæ istæ particulæ sanguinem ingressæ statim sibi congeneres variolosi seminii particulas sanguini à nativitate inditas inveniunt; iis igitur confermentescunt, sed invicem combinatæ ac complexæ haud

amplius

amplius sui juris sunt, ut ulteriores excitent turbas, regiam vitæ petant, spirituum thesauros diripiant; nam mutuis compedibus constrictæ fixantur, præcipitantur, crassioresque & hebetiores fiunt, quam antea fuerint. Statim igitur volubilioribus aquearum particularum globulis tamquam aptis vehiculis superincumbentes, sanguinis motu à centro ad peripheriam tendente, secundo veluti amne, ad ambitum corporis protruduntur, eliminantur. Nonne manifestè videmus haud pus generari in insititiis variolis, sed saniosam, dilutiorem videlicet aqueamque magis materiam? Nonne ex hoc phænomeno palam est acido-salinas fermenti contagiosi particulas haud oleosas passim sanguinis particulas in cadaverosam purulentiam pervertere, sed blandioribus potius lævioribusque aqueis particulis easdem dilutas & saturatas foras asportari? Ex negatione fovearum & cicatricum nonne manifestum est acres, aculeatas, pungentes & corrosivas salini fermenti particulas à balsamicis statim sanguinis globulis obtundi, spiculis suis orbari, & hebetiori figura modificatas, vi veluti mochlica, extra propelli? Integra interim servatur massæ sanguineæ textura, inviolata consistentia. Nullam hîc vides fusionem, nullam grumescentiam, nullum corruptorium aut destructivum motum. In insitione enim tantum solummodo sanguis fermentescit, quantum impuro à puri consortio separando, ac per despumationem extrudendo satis est. In hoc fermentationis motu solum per undulationem quandam leviter aliquando afficiuntur spiritus, lympha, & solidæ partes; & siquæ ad ista contagii particulæ perveniunt, certè (quod insitionis adumbrat metaphora) non nisi silvestri acrimonia privatæ, ac veluti dulsificatæ pervenire possunt.

On the same Subject at Venice, by Dr. Pylarinus. n. 347. p. 393.

3. In Byzantio operatio hæc Medica latuit initio per aliquot annos, rarò quoque, & inter humiliores dumtaxat recepta: Immaniter autem grassante nuper Variolarum epidemia, latiùs innotescere cœpit; Nunquam tamen sublimiores ausa est ingredi aulas; donec Nobilis quidam nec obscurus inter præstantiores Græcos, & ex antiquo Caryophyllorum stipite clarus, mihi verò intimiori amicitiæ titulo notus, anno salutis 1701. sub hyemis finem, serio me quidnam de hac insitione sentirem, consuluit; & an ad eandem in quatuor propriis filiis celebrandam præstarem assensum: Nam tùm temporis lethaliter totam ferè civitatem morbus hic invaserat; qui, summum ei de natorum salute metum incutiendo, anxium valdè reddiderat. Verùm quid ipse super ignota re decernerem, nulla præcedente novissimæ hujusce methodi notitia, penitus ignorare me dixi; ac simul operatoris conveniendi copiam petii. Triduo peracto, cum ad amicum denuò accessissem, & de eadem materia sermo iteratò inter nos esset initus; Ecce paulò post mulier Græca ad decentiam satis composita cubiculum intravit; quæ totam operationis seriem, modum, locum, tempus, cæterasque circumstantias, ut ego deinceps aperiam, clarè satis lateque nobis exposuit; quamquàm ipsa veram ex insitione excitationis variolarum causam haud intellexerit: His experimenta & casus innumeros

tutò

tutò ſemper & ad ſalutarem eventum perductos annexit; è quibus aliquot (omnes enim quis potuiſſet in urbe ampliſſima exquirere?) veriſſimos eſſe ex aſſertoribus fide dignis deprehendi: Idcircò re bene penſitata, rationi & naturæ haud abſonam omnino comperi: præſertim autem caſibus jam dictis permotus, amico jam fervidius poſt aliquot dies conſilium iterùm expetenti, me haud alienum, ſub levi tamen hæſitantia, præbui; Quâ, tamquam dato aſſenſu, arreptâ, & de ſervando regimine toto ægrotationis tempore ſatis edoctus, inſitionem per mulierem Græcam in quatuor filiis audacter inſtituit: quorum tres natu minores (quinquennes, & vix ſeptennes) leviter ægrotarunt; paucíſque apparentibus puſtulis poſt hebdomadam, febre penitus & periculo evaſerunt: Ætate verò major, octavum ſuper decimum agens annum, graviter decubuit: Nam continenti febre & malignante correptus, ſuperveniente difficilium ſymptomatum ſyndrome, pluſculiſque quamquàm non copioſis exanthematibus obrutus, vix poſt decimum quartum diem morbum eluſit: Quod ego atrabilari ejus temperamento, ſucciſque pravis, ut & neglectæ prius (contra datam admonitionem) corporis expiationi tribuendum velim. Felix operationis eventus mirum quam multas nobiliorum familias ad imitationem traxit; Ut hodie ſine hæſitantia, præter timidiuſculos aliquos, unuſquiſque tranſplantationis emolumentum ſentire velit. Soli Turcæ, utpote Fati decretis addicti minuſque dociles, hanc neglexerunt huculque.

Naturalis eſt penitus hæc operatio, nulloque obducta ſuperſtitionis fuco; quamvis ipſum tranſplantationis nomen prima facie hæſitantiam pariat. Differt toto cœlo à curationibus Sympatheticis; eo magis à magnetiſmo illo tranſplantatorio, per quem morbi ex uno ſubjecto (mediante imaginata quadam, gratis efficta, & imperceptibili Mumia) in aliud traduci dicuntur; qua de re Tenzelius, Bartholinus, Maxuellus, Etmullerus, aliique ex recentioribus, docti alias viri, agunt ſedulo; qui vetuſtas antiquorum in medicina quiſquilias expurgandas eſſe cum jactitent; inter quiſquilias ipſi quandoque ſe volutant, noviſſima vanitatis amurca venerandam hactenus puriſſimamque Scientiam deturpantes. Quare, ut verum fatear, quemadmodum operationes hæ Magneticæ, vel Sympatheticæ ſuperſtitioſæ vanitatis ſuſpicione non carent (ut in unguento armario, pulvere ſympathetico & ſimilibus) utpote extra activitatis ſphæram in diſtans agentes; ita variolarum tranſplantatio, vera, mera, pura Phyſica eſt; quia puris mediis Phyſicis, & ad oculum patentibus, nec non ipſo ſenſibili contactu completur, ut è mox dicendis clarius eluceſcet.

Hujuſmodi igitur variolarum exercitatio fit per metaphoricè ſic dictam inſitionem ſive tranſplantationem; quæ nihil aliud eſt, quam fermenti morbifici ſeu puris ex variolis extracti in corpus ſanum, per vulnuſcula ad hoc facta, inſertio.

Phyſicus excitationis modus hoc pacto ſuccedit. Intruſum in vulnuſcula pus veri fermenti ſuſcipit indolem; Hinc beneficio circulationis per vaſa & canales proprios ad ſanguinis maſſam delatum,

dispo-

disposítas ad hoc particulas, & delitescente vitio turgidas statim aggreditur, inficit, venenumque communicando latitans in illis seminarium fermentativum excitat, agitat, actuat, inque motum ciet; Unde ebullitio universalis, seu fermentatio oritur; Vi cujus impuriores & heterogeneæ partes separatæ criticè protruduntur ad cutem; natura interim placidè virtute talis operationis totum opus moderante.

Sed ad Transplantationem ipsam, & ejus celebrandæ formam properandum; modum quò eadem mulier transplantatrix tutissimè operabatur fideliter describendo, ordinem, cæteraque omnia; ex quibus regula quædam institui poterit pro operatione hac obeunda. Cæterùm quamvis de omnibus oculatus Testis, ut ingenuè fatear, esse non possim; ex operatricis tamen ore multa, plura ex fideli relatione transumpsi; plurima & potiora ipsemet observavi. Omitto quæ ad majorem confirmationem universalis fama canit. Quibus omnibus præstandam esse fidem candidè ac strenuè assevero.

Ergo primo tempus ad celebrandam insitionem opportunum seligendum est; Quod secundùm operatricem hybernum desideratur; & non nisi tali tempore ipsa insitionem instituebat: Vernum ego pariter idoneum crediderim, propter clementiorem aeris temperiem.

Secundo selectissimum adhibet fermentum; Pus scilicet inferendum non ex quolibet subjecto recipit; sed variolis epidemicè grassantibus è pustulis jam maturis decumbentis alicujus pueruli alias ἐυσάρκȣ, iisque benignis, punctione illud extrahit exprimitque; & in conchulam aliquam vel vitreum vasculum mundissimum, nec nimis actu frigidum reponit reconditque; quod vasculum bene sartum tectum in pedissequi sinum fovendum intrudit; mox sine mora ad operationem properat: Pus ex insititiis rejicit, ut inefficax. Quod tamen ego benignioris indolis, nec minoris interim energiæ fecerim: qua in re experientia consulenda.

Tertio, temperatissimum vult patientis inhabitandum cubiculum quoad aeris modificationes.

Quarto, ad operationem jam celebrandam accedens mulier, frontem in confinio capillorum & quidem medio in loco; mentum & utrasque genas acu ferrea vel aurea pungit; non rectà, sed obliquè impingendo, cutemque acuta cuspide à subjecta carne aliquantulùm separando, hinc eadem acu pus jam præparatum in vulnusculum instillat è vasculo, intruditque; superinducta per fasciam ligatura: Manus item ambas in metacarpis, pedes in metatarsis eodem modo ferit, pusque inserit, fasciasque leniter stringit; seriò imponendo patienti ne partes illas scalpat madefaciatve. Potiùs carnosiora pungerem loca, quatenus inflammationibus dolorique minus obnoxia, nec tendinibus intertexta.

Præter hanc operandi formam cæteri omnes rejiciuntur modi, utpotè absoni, inusitati, malè succedentes, infelicisque exitus.

Interim lectulo moderatè manendum, neque plus quàm opus fuerit jacendum.

Quinto

Quinto bonum in ſex rebus nonnaturalibus regimen, præcipue in victu, injungit; Non ſolum enim vino & carnibus, verum etiam earumdem juſculo rigorosè patientes ad quadrageſimum diem interdicit: Pluribuſque monitum non curantibus ſæpius malè ſucceſſit; nam ad oculos, pœna erroris, novas erupiſſe puſtulas, aliaque non parvi diſcriminis ſymptomata ſuperveniſſe, viſum eſt.

Sic ritè peracta tranſplantatione, non omnibus eodem tempore intervallo ſuſcitari ſolent variolarum ſymptomata; Variè enim fermentum agit; ſerius vel citius, prout unicuique proprium favet temperamentum, ætas, robur: Quamquàm variolæ ipſæ, in ſeptimo ferè ſemper apparere incidant, qui dies vere criticus eſt. Nec defuere, quod raro contingere ſolet, quibus ſtatim primo die effloruerint.

Symptomata ægrotantibus evenientia variant ſecundùm temperamentorum diverſitatem, ſuccorum in maſſa ſanguinea habitudinem, & particularem in ſingulis naturæ diſpoſitionem: remiſſiora nempè vel intenſiora ingruunt; At communiter graſſantibus non diſſimilia, quamvis clementiori ut plurimum facie: Plures vix alterationem læſionemve ſentiunt aliquam.

Excitatæ variolæ ferè ſemper ſunt de genere diſtinctarum; nec numero multæ; Decem, ut plurimum viginti, triginta, raro ad centum, rariſſime ad ducentas erumpunt.

Notandum primò nonnullos, unico dumtaxat vulnuſculo ad brachium inflicto contentos, excitaſſe variolas; paucisque apparentibus puſtulis præſervatos tamen impoſterum fuiſſe à contagio.

Notandum ſecundò accidiſſe interdum, ut ex inſitione nullæ penitus excitatæ fuerint Variolæ, vel ob non præexiſtentem ullam prorſus variolicam in corpore diſpoſitionem, vel ob enervatum infractumque fermenti contagium: At poſtea graſſante populariter morbo, correpta ſunt promiſcuè ejuſmodi corpora jam inſitionem paſſa, communi cæterorum ſorte.

Tertiò, Inſertionis loca ſeu vulnuſcula in puſtulas ſemper evadere ſolent: Quibuſdam verò excreſcunt in purulenta tubercula, nullis interim apparentibus puſtulis; Nonnullis in apoſtemata quidem majora degenerant magnam puris copiam effundentia: Non ſemel eadem loca, in pedibus præcipuè manibuſque, ſummo cum dolore intumeſcunt; pureque effuſo ſubſident, iterumque in tumorem attolluntur. Quibuſdam, rariſſimè tamen, ad glanduloſas partes & emunctoria, poſt aliquod tempus, abſceſſus emergunt, ac ſuppurantur paulatim: ludente in diverſiformi corporum craſi natura.

Poſtremò, nunquam fere ex tranſplantatione funeſti quid accidiſſe, obſervatum fuit hactenus; etiamſi in quocumque ſexu, temperamento, ætate celebrata fuerit; quinimò, ritè recteque tractata, & in corporibus per peritum medicum aptè præparatis, certiſſimam promittit ſalutem. Variolæ enim hoc modo excitatæ benignioris ſunt indolis, quam ſunt illæ quæ populariter graſſantur; Utpotè ex fermento ſeu contagio, omni malignitate carente permotæ: Ebullitio, per quam

maſſa ſanguinis agitatur ac totum opus perficitur, blandè non violenter moderante natura, conamina ſua molitur: Sed præter hæc, tempus ad operationem atque tempeſtas magis idonea pro tranſplantatione ad libitum eligi, ut & corpus inciſioni ſubjiciendum congruis adminiculis ad recipiendam illam ex arte preparari diſponique poterit; Quod revera maxime ad ſalutarem fauſtumque morbi ſucceſſum momenti cenſeri debet.

A way us'd in Scotland *to prevent the Small-Pox. by Mr.* Martin. *n.*312.*p.*2470

4. When the Small-pox is Epidemical in the main Land over-againſt *Sky Iſle*, on the S.E. and Eaſt, and likewiſe in *Sky Iſle*; the Natives bath their Children in the Infuſion of Juniper Wood, and they generally eſcape; whereas thoſe who neglect this Precaution, are often obſerv'd to die.

A Plant us'd in drawing a Felon, ibid.

XIII. The Plant Water Lilly being apply'd to the pained part of the Body where the Felon is fixed, it is obſerved that it forces its Paſſage quickly in that place thro' the Skin.

A Woman that lay ſix days covered with Snow. by Mr. Bowditch. *n.* 337.*p.* 265

XIV. *Joanna Crippen* of *Chardſtock* in *Dorſet*, *January* 24. 1708-9. coming homeward, it ſnowing very hard, having no manner of Suſtenance with her only a quarter of Tobacco, a pound of W orſted Yarn, and three Pence in Copper, not ſo much as a Bit of Bread, Bisket, or the like, was conſtrain'd to travel, as well as a poor tired Creature could, towards her own home; but going not far from thence was met by a Man of the Pariſh. He ſeeing her tumbling in the Snow a diſtance off, as he was going to his home, and finding her lying in a Ditch, help'd her up, and bid her obſerve to go in his Track, which he obſerv'd ſhe did indifferently well. But ſhe had not gone a quarter of a Mile, before ſhe was forc'd to lye down under a Hedge, having loſt one of her Shoes; and her Cloaths, which were very mean, were with the Brambles and Thorns torn almoſt quite off her Back: In which place ſhe lay from *Monday* Evening about ſix a Clock, until *Sunday* following about 4 in the Afternoon, and then was diſcovered by ſome Neighbours, who went to ſearch for her; and after ſome time, found her buried in four Foot of Snow or thereabouts, it being more than ſo much higher before the Thaw. One of the Men with his Pole thruſting at her, cry'd out, She was there; then the reſt opening the Snow, found ſhe was Alive. She immediately ſpoke, and begg'd he would not poot her too hard (as ſhe expreſs'd it) for ſhe was almoſt naked; and deſired that ſome of he Women would come to her and take her forth, which accordingly was done; they finding her without Stockings or Shoes, an old Whittle about her Shoulders, with a large Hole in it, which ſhe had eat through; the Snow melting down on her, which ſhe drank to quench her Thirſt. From thence ſhe was brought near my Habitation, where the beſt Care has been taken of her. She had a Mor-

Mortification on one of her great Toes, which is now in a good way of Recovery; and ſhe now is very hearty, and in a fair way of a perfect Recovery.

She was very ſenſible at the firſt taking her out, and ſtill continued ſo; ſhe knowing every Body perfectly well. Her Tobacco and Three Pence were in her Pocket. You may aſſure your ſelf, ſhe had no manner of Food with her, as Bread, or any Eatable whatſoever.

XV. In *Auguſt* 1693, I was called about 4 in the Morning, to ſee a Sailor on board the *Albemarle* Man of War in the Bay of *Biſcay*, in a violent Calenture. He was between thirty and forty Years of Age, brown Hair'd, pretty tall, but thin, and had not much Fleſh about his Bones. When I ſaw him firſt, I found him in the Hands of three or four of his Comrades, who were hardly able to manage him, becauſe of his ſtrugglings, and conſtant endeavours to get from them. I obſerved, he very often cryed out, he would go into the green Fields; his looks were as furious and wild as thoſe of a Lyon, and every now and then he would heartily curſe thoſe that held him. The firſt thing I did was to examine his Pulſe, I felt, 'tis true, a diſorderly motion of the Blood in the Artery, and a burning fiery heat all over his habit of Body, but could perceive no diſtinction, or vibration of Pulſe at all. The Surgeon of the Ship, a good experienc'd Man in that way, had, before I came, attempted to bleed him, but though the Vein of the Arm was fairly open'd, yet could he not procure an ounce of Blood from thence. Upon that I order'd him to open the frontal Vein, which ſucceeded no better, for that ſoon ſtopt too. This put me upon trying a third time, what effects the opening the Jugular Vein might have. From this Vein, though our Orifice was pretty large, we had about two Ounces of florid thick Blood, and then it quite ſtopt there too. I was, I muſt confeſs, not a little ſurprized at this; and ordered the Surgeon to unbind his Arm, and try whether he could make him bleed again at that Orifice, which I remember he did in a ſmall quantity, and then ſtopt as before. However, having three Orifices open at that time, we drew Blood ſometimes from one, and ſometimes the other, where we ſaw it run moſt freely. After ſeveral Eſſays of this kind, I always obſerved as the Veſſels emptied, he bled more freely, and as faſt as I deſired. Not long after this, for he bled well enough now, I obſerv'd his Strugglings were not ſo ſtrong, his Ravings and crying after green Fields left off, his wild Looks much abated, and not only his Pulſe had recovered its due and regular Vibrations, but his heats were moderated too, and the Fury of his Spirits lay'd to that degree, that he that juſt now was as furious as a Lyon, was grown ſo tame that one Man was able eaſily to manage him as he pleaſed. In this half hour, as near as we could gueſs, we took from him about Fifty Ounces of Blood from the three Orifices mention'd. By this time I thought we had enough, ſo I order'd him

A Calenture, by Dr. Oliver *n.*290.*p.*1562.

him to his Hammack, as foon as we had fecured the Orifices from bleeding again, and directed the Surgeon to give him an Ounce of Diacodium in a Draught of Barley Water, as he went into it: Upon this he flept till about Noon, when he awaked with no other Complaint, but of weaknefs from his lofs of Blood, and a foren-efsall over his Body, occafion'd, I prefume, from his violent Convulfions and Endeavours to get loofe.

'Tis very probable, that when they are feiz'd with this violent Heat and Diforder, which for the moft part happens in the Night, they fteal privately over-board into the Sea, imagining they're going into the green Fields. And this I take to be the reafon we fee fo few; tho I have heard frequently in the *Mediterranean* in Summer time, and very hot Weather, of Seamen loft in the night, which the Sailers took for granted were gone off upon fuch like occafions unobferved. And I remember very well, this Perfon was actually going over-board, when one of his Brethren, who fufpected his defign, as he told me, caught hold of him juft as he was going to leap off, call'd for help, and fecur'd him by this Accident. And laftly, Calentures happen oftner by Night than by Day, becaufe our Ships are moft clofely fhut up by Night, and are lefs airy than they are in the day time.

Of ftrange Epileptick Fits, by Dr. Cha. Leigh. n. 280. p. 1174.

XVI. We have this Year had an epidemical Fever, attended with very furprizing Symptoms, in the beginning the Patient was frequently attacked with the *Colica Ventriculi*; Convulfions in various parts, fometimes violent Vomitings and a Dyfentery; the Jaundice, and in many of them a fuppreffion of Urine; and what Urine was made, was highly faturated with Choler: About the ftate of the Diftemper, large Purple Spots appear'd, and on each fide of 'em two large Blifters, which continued 3 or 4 days; thefe Blifters were fo placed about the Spots, that they might in fome meafure be term'd Satellites or Tenders; of thefe, there were in many four different eruptions; but the moft remarkable inftance I faw in this Fever, was in a poor Boy of *Lymm* in *Chefhire*, one *John Pownel*, about 13 years of Age, who was afflicted with the following Symptoms; upon the Crifis or turn of the Fever, he was feized with an *Aphonia*, and was fpeechlefs 6 weeks with the following Convulfions; the Diftemper infefted the Nerves of both Arms and Legs, which produc'd the *Chorea Sancti Viti*, or *St Vitus*'s Dance; the Legs fometimes were both fo contracted, that no Perfon could reduce them to their natural pofition; befides thefe, he had moft terrible Symptoms, which begun in the following manner; he cou'd perceive the Fits to come on about the *Os Sacrum* or extremity of the Back-bone, and the Region of the Navel, and then the diforder, as he imagined, united about the top of his Head; he immediately afterwards fell into violent Convulfions in the *Abdomen* or loweft Cavity, with that violence, that fometimes two or three Perfons were forced to lye upon him to keep him in Bed, his Body being frequently rais'd

from

from it; after this, the Nerves of the Lungs were immediately affected, and then he barked in all the usual notes of a Dog, sometimes Snarling, Barking, and at the last howling like an Hound; after this, the Nerves of the Mandibles were convuls'd, and then the Jaws clash'd together with that violence, that several of his Teeth were beaten out, and then at several times there came a great foam from his Mouth; afterwards he had an extream wild look, snatching at any thing near him, and would have tore off his Flesh, had he not been prevented by the Persons about him; this made me conjecture he might formerly have been bit by a Mad dog, which had introduced the *Hydrophobia*; but I was convinced to the contrary, for I put a Bason of Water by him, and he was not in the least afraid of it, nor attempted to lap it. I saw him in three of these Fits; but at other times in these Convulsions, he roar'd like a Bull, made a noise like a Hog, and sometimes like that of a Gosling; all which different sounds, (I take it) proceed from the different contractions of the Lungs, variously forcing out the Air, and consequently as they were differently convuls'd, form various sounds; In a weeks time I recover'd the Boy his Speech, his Senses return'd, his Convulsions vanish'd, and the Boy is now very cheerful.

XVII. This Disease began to show itself first in this City, about the beginning of *July*, 1711. It increased till the beginning of *September*; after which it diminish'd by little and little to the End of the Year, at which time it totally ceased. It appears, that before this Distemper there were about Sixty Thousand Souls in *Copenhagen*: From whence they infer, that there is Born every Year about Two Thousand, and that there Dies nearly the same Number; which being Multiplied by Thirty makes Sixty Thousand. In the Six Months which this Distemper continued, it is thought it carried off about 25000 Souls. It is true, the Publick Lists reckon but 22535; but it is agreed by all, that in the last Week of *August*, and the two first Weeks of *September*, each of which carried off above 2300 Souls, there died a great many, of which there was no Notice taken. Almost the very same happened two Years before at *Dantzick*; where before the Plague broke out, there died Weekly from 45 to 50; but the Number of the Dead increased by degrees to the beginning of *September*; so that in the first Week of that Month there died 2205 Souls, in the second Week 2070, and in the third 2075. After which the Mortality decreased to the End of the Year.

Of the Plague at Copenhagen *in* 1711. *com. by* John Chamberlayne, *Esq. n°* 337 *p.* 279.

It is observable, 1. That there were some Houses which escaped the Infection; but that there were few where it did not carry off more than one or two Persons; and that there were many in which it did not leave a Soul alive. 2. That generally speaking, this Distemper was most fatal to the meaner sort of People; there scarce dying any Person of Note; but on the contrary a great number of the Poor.

Which

Which may be attributed to feveral Caufes: The firft, and moft general of which, is their nafty manner of Living. The fecond is, that this fort of People live very clofe together, and as it were heaped one upon another; fo that fometimes there are four Families in one Room. The third is, the foolifh Curiofity they have of feeing the Dead Bodies. And fourthly a great many of them are fo bigotted to the *Turkifh* Notion of Predeftination, that they fay, if it pleafes God that I fhould die of this Difeafe, I fhall not efcape it; and if it be his pleafure that I fhall live I can't die: And upon this Notion they go abroad every where, and fo catch the Infection. There are fome of 'em alfo, which make no fcruple of lying in the fame Beds, where others have dy'd. The 3 Sorts of Trades of which there died moft were Coffin makers, (who took meafure of the dead Bodies) Surgeons, and Shoomakers. The Care that was taken and Medicines us'd did great Service, I was told that *Theriaca* did little Good; and they obferv'd the fame alfo at *Dantzick*.

An emaciated Child diffected, by Dr. Blair. *n.* 353. *p.* 631.

XVIII. This Child was five Months old, and was fo emaciated, that he appear'd rather to have decreafed, than to have increafed in Bulk, from the time of his Birth; his whole Body not weighing above five Pounds. The Skin and Mufcles of the *Abdomen* were very thin, but the Peritoneum was preternaturally thick. The Ventriculus was more like to an Inteftin than to a Stomach, its length being five Inches, and its breadth but one Inch. The Coats of it were thick and flefhy, and the Cavity very inconfiderable. The Pylorus, and almoft half of the Duodenum were Cartilaginous, and fomething inclin'd to an Offification, fo that no Nourifhment could have paffed into the Inteftins, tho' the Stomach had been capable of containing it, which makes it no Wonder that the Body was fo emaciated. There were fcarce any foot-fteps of the Omentum to be feen, even at the Bottom of the Stomach, to which it ufually adheres. The right Lobe of the Lungs adhered firmly to the Ribs and had three Exulcerations, which contain'd purulent Matter. It was fo very thin and compact, that it feem'd as if that Lobe had never been of ufe in Refpiration. The left Lobe was of a more florid Red; fpongy, and free from any Adhefion.

Upon enquiring after the Symptoms this Child had been affected with, his Mother told me, he feem'd to be healthy till he was about a Month old, when he was feized with a violent Vomiting, and a Stoppage of Urine and Stool. Some time after, both thefe became more regular, but the Vomiting ftill continued. He feem'd to have a great Appetite, taking what Suck, Drink, or other Food was offer'd him, with a kind of eagernefs; but he immediately threw it all up again. He had all along breathed freely, and had no Cough, notwithftanding the Exulcerations above mention'd. This confirm'd me in the Opinion that he had never breath'd by the right Lobe of the Lungs.

Lungs. There could be nothing more emaciated than this Child was.

XIX. A Gentlewoman in *New England* ſwoons upon the ſeeing any one cut their Nails with a Knife; which if done with a pair of Sciſſars has no Effect upon her. *Of Antipathies. by Dr.* Mather. *n.* 339. *p.* 64.

XX. The Wife of a Perſon vomited, upon ſeeing her Husband take a Vomit; the Perſon that took it being not moved, but forced to take a freſh Emetick. *Of the Force of Imagination, by the ſame.* ib.

XXI. *Nathaniel Hulme,* aged about 17, had the Small-Pox about 8 Years of Age: Soon after which he had a great Itch, almoſt to the degree of a Leproſie, with which his Finger-nails and Thumb-nails began to grow thick, and by degrees hardned into Horns; which grew in 7 or 8 months to the length of an Inch, and ſome almoſt 2 Inches, and ſome much longer. It began in the Fore-finger of his Left-Hand, and ſo to all the reſt of that Hand, which had as many Horns as Fingers and Thumb. All which Horns about the end of 12 Months fell off by degrees; that which grew firſt falling off firſt, without any pain, unleſs when cut off; as they were at firſt, there appearing great quicks (as they call them) or Roots under the Nails. By degrees they came on the Thumb, and then on the Fingers of the Right Hand; which grew to the ſame length in about a years time, and then fell off, he having ſhed them 5 or 6 ſeveral times. One of the Horns that grew on the Ring Finger of the Left Hand, was a quarter long. They are now at preſent all come off his Left Hand, but growing again: That on his Little Finger is two Inches long. I have now by me one or two of them. This Account I took in 1702. *A Boy with Horn-like Excreſcences growing on the Fingers. n.* 297. *p.* 1899.

He had in 1704 all the Fingers of both Hands armed with ſuch like, and ſome as long as theſe I ſend. He has on every Toe alſo, but keeps them cut, that he may be able to wear Shoes. I think he cannot live long, being miſerably overſpread with his Leproſy.

One of the Horns above-mentioned is now in the Repoſitory of the Royal Society.

XXII. Mrs. *Lovelock* had been ſeized ill of a Fever, which affected her ſo, as to make her Light-headed to a great degree, convulſed and reſtleſs; upon ſhe took great quantities of Opiats, in order to compoſe her; but could never procure any thing like Sleep; but ſtill as ſhe took them they ſeem'd to refreſh her, and make her ſenſible, but cauſed nothing of reſt. The quantity ſhe took from *Tueſday* Night twelve a Clock to *Friday* Night twelve a Clock was as follows, *Jan.* 29. four Bolus's with two gr. each of *Laud. Lond.* made up in Venice Treacle. Six Pills, with two grains each of the ſame. A Bolus with *A Perſon who took a great quantity of Opium, without cauſing Sleep. by* ——— *n.* 275. *p.* 999.

eight

eight grains of the fame in Venice Treacle. *Jan.* 30. Twelve Pills with two grains each, and one Bolus with ten grains of the fame in Venice Treacle. *Jan.* 31. four Draughts with tén grains of the faid *Laud. Lond.* one ounce of *Syr. de Mecon.* in each Draught. So that in all fhe took in the time above-mentioned 102 grains of *Laud. Lond.* and ℥iij. of Venice Treacle, and ℥iiij. of *Syr. de Meconio.* She died *Feb.* 1.

XXIII. *Accounts of Books omitted.*

n. 270. *p.* 832. 1. Sanctorii Sanctorii de Statica Medicina Aphorifmorum Sectiones feptem, cum Comentario Martini Lifter. 8vo. *Lond.* 1701.

n. 273. *p.* 914. 2. Jac. Gaveti Acad. Monfpel. Alumni Avenionenfis Doct. Medici, & apud Camberienfes practici, nova Febris Idea, ceu Conjecturæ Phyficæ circa Febris naturam. 8vo. *Genevæ,* 1700.

n. 284. *p.* 1373. 3. Hippocratis Aphorifmi, cum Commentariolo, Authore Martino Lifter, è Medicis Sereniffimæ Majeftati Reginæ Annæ. *Lond.* 1703.

n. 320. *p.* 324 4. An Account of Animal Secretion, the quantity of Blood in the Human Body, and Mufcular Motion. By *James Keill,* M. D. 8vo. *Lond.* 1709.

n. 337. *p.* 101. 5. A Defcription of the Plague, which happen'd at the Royal City of *Dantzick,* in 1709. Written in *High Dutch,* by Dr. *John Chriftoph. Gottwald,* and communicated by Dr. *John Philip Breynius,* as the beft Account of that Diftemper there publifh'd.

n. 303 *p.* 2152. 6. Joh. Conradi Beckeri, Phil. & Med. Doct. Tractus Alsfeld. Medici Ordin. Paradoxum Medico Legale de fubmerforum morte fine pota aqua, aliquot cadaverum fectionibus detectum, & è principiis mechanicis illuftratum. Cui adjicitur Dodecas obfervationum circumftantiis curaque rariffimarum. Gieffæ Hafforum, Anno 1714.

n. 343. *p.* 263. 7. Bibliographiæ Anatomicæ Specimen, five Catalogus omnium pene Auctorum, qui ab Hippocrate ad Harveium rem Anatomicam ex profeffo vel obiter fcriptis illuftrarunt, &c. Cura & Studio Jacobi Douglas, M. D. R. S. S. & in Coll. Chirurg. Lond. Anat. Prælect. 8vo. *Lond.* 1715.

CHAP.

CHAP. IX.

Pharmacy, Chymistry.

QUO rectiùs diversas emeticorum & purgantium Medicamentorum Doses, pro Temperamentorum & ætatum varietate decernamus. Supponendum est, *Primo*, istiusmodi Medicamenta nihil prius posse operari quàm in sanguinis massam appulerint, & eidem penitus immisceantur: Constat enim, ni Nauseam moveant, nullum ab iis effectum sensibilem produci, multo post tempore quo in eandem devehi possint. *Secundo*, generaliorem eorum effectum esse temperamenti sanguinei aliorumque Liquorum gyrantium Alterationem. Ex hisce duobus Postulatis concludimus, quod ubi sanguinis Crasis est eadem, Medicamentorum Doses, ad certum aliquem eliciendum effectum, sanguinis quantitati proportionales esse: Si namque certa quædam Dosis exigatur ad unius sanguinis v. gr. Crasin alterandam ad certum aliquem gradum, duplicem oportebit Dosin adhibere ut Libræ duæ ad gradum alterentur, triplam ut tres, *&c.* Et universaliter si sanguinis quantitas *b* exigat Dosin *d*, quantitas sanguinis *mb* Dosin *md* exiget, & est $b : d :: mb : md$.

I, 1. *Of Purging and Vomiting Medicines. by Dr.* Cockburn. *n.* 303 *p.* 2119.

Coroll. Quum Quantitas sanguinis & cæterorum humorum in gyrum actorum rite, ex Animalis pondere, possit æstimari (partes enim quas solidas nuncupamus sunt tantum Canales qui liquores istos continent) exinde sequitur dosium quantitates, *cæteris paribus*, esse corporis ponderi proportionales; adeoque medicamentorum dosin infanti, recens nato, propinandam esse ad eorundem dosin provectioris ætatis hominibus ut est infantis pondus ab hominis. Exempli causâ, pilularum Rudii gr. xxx unicâ plerumque dose homini exhibentur, & hominum pondus commune est 160 Librarum, & infantum Librarum 12; quapropter, ut sunt 160 Hominum pondus ad 12 infantum, ita sunt gr. xxx Hominum dosis ad gr. $2\frac{1}{4}$ quæ sunt infantum; ac semper in eâdem ratione Medicamentorum doses augendæ veniunt ut crescit infans; dehinc doses exhibendæ manent eædem ad annum quinquagesimum, ex quo tempore sanguinis quantitas & vis minuuntur, quâ quoque ratione minuendæ sunt Medicamentorum doses.

Hâc ratiocinandi methodo, homines omnes eodem gaudere Temperamento simul & Augmentum Ordinemque secretionum ad annorum numerum, esse æqualem supposuimus; Quum vero hominum Temperamenta; seu Constitutiones, sint diversissimæ pro varia sanguinis humorumque gyrantium Crase, Dosium quantitates non semper proportionantur corporis ponderi. Diversum id sanguinis temperamentum in

certâ quadam partium ejusdem cohærendi dispositione consistit, quâ sanguis magis vel minus fluidus evadit; unde provenit quod medicamentorum in sanguinem Operationes fiant diversæ pro variis Cohærentiæ gradibus. Ponamus enim homines duos æquali sanguinis quantitate pollentes, cujus Cohærentiæ gradus sint diversi; manifestum est, medicamenta ista facilius laxioris texturæ sanguini admisceri quam firmius cohærenti, commiscendique Proclivitas quam habent medicamentorum partes particulis sanguinis est semper ut Fluiditas directe vel reciproce ut est sanguinis Tenacitas, & medicamenti in sanguinem vis erit in eâdem similiter ratione: Adeoque ut medicamentum in istis diversi temperamenti hominibus æqualiter operetur, propinandæ doses sanguinis Tenacitati proportionales esse debent, supponendo itidem sanguinem in utroque pari Celeritate circuitum agere. Si vero sanguinis velocitas sit diversa, medicamentorum operationes, *h. e.* Secretionum quantitas ab eis producta, erunt ut sanguinis velocitas: Secretiones enim in qualibet Glandula, in dato tempore, sunt semper ut sanguinis quantitas quæ in eandem Glandulam eodem tempore devehitur, *h. e.* ut ejus Velocitas. Sanguinis etiam velocitas, *cæteris paribus*, semper se habet ut ejus Fluiditas, seu reciproce ut ejus cohærentiæ gradus: Si igitur sanguinis velocitas esset unice consideranda, hoc casu quantitas dosium ad eundem effectum producendum erit directe ut cohærentiæ gradus in partibus Sanguinis.

Prop. I. *In hominibus duobus æqualem sanguinis quantitatem habentibus sed Cohærentiæ gradibus differentis, Medicamentorum Emeticorum & purgantium doses, ad eundem effectum eliciendum necessariæ, sunt in duplicata ratione graduum cohærentiæ sanguineæ.*

Ubi enim Sanguis eâdem velocitate movetur, dosis quantitas ut cohærentiæ gradus sit oportet, & si gradus cohærentiæ essent iidem, dosis quantitas est reciproce ut velocitas; adeoque, ubi nec cohærentia nec velocitas sunt eædem, quantitas dosis est in ratione composita ex directa ratione graduum cohærentiæ in sanguine & reciproca velocitatis: Atqui reciproca velocitatis ratio est directæ Tenacitatis vel cohærentiæ graduum rationi æqualis. Ideoque Dosis quantitas est in ratione composita graduum cohærentiæ & graduum cohærentiæ, *h. e.* Doses propinandæ sunt in duplicata ratione eorundem. Q. E. D.

Prop. II. *Dosium quantitas hominibus diversæ quantitatis sanguinis exhibenda, qui simul diversis pollet Cohærentiæ gradibus, est in ratione composita ex ratione ponderis hominum & duplicata graduum Cohærentiæ.*

Quum enim cohærentiæ gradus sunt iidem, dosium quantitas est ut hominum pondus; & ubi hominum pondus est idem, dosium quantitas est in duplicata ratione graduum cohærentiæ, adeoque quum neuter est idem dosis, Quantitas est in ratione composita ex ratione ponderis hominum & duplicata graduum cohærentiæ. Q. E. D.

Coroll. Hinc, Quantitatem & Qualitatem Sanguinis in quolibet homine scientes, doses ad purgandum & vomendum necessarias non ita arduum est determinare. Hæc qualitas, seu Temperamentum, sanguinis

guinis a perito Medico ex Pulſibus, Urina, & aliis ſecretionibus facile invenitur; adeoque Medicus, obſervando quæ doſes datæ conſtitutionis homines purgant, minimo negotio doſes cuilibet conſtitutioni vel Temperamento idoneas ſtatuet.

Schol. Quæ hactenus in genere demonſtrata ſunt, ex vulgari Medicorum circa Purgationem & Vomitionem Hypotheſi eodem ſequerentur pacto; quoniam Diſpoſitio, quam habent Ventriculus & Inteſtina ut ſtimulentur, eſt ut memorati quantitatis & Cohærentiæ in Sanguine gradus. Ita, vel ex illa Suppoſitione, Veritas hæc maxime conſtaret, licet poſtulati ſimplicitate deſtituitur, atque ob id non adhibetur.

2. From what has been ſaid it is manifeſt in general, 1. that theſe Medicines operate either upon the account of their being mixed with the Blood, or by their ſtimulating the Stomach and Guts: 2. That this their Operation is more or leſs according to the Quantity and Thickneſs of Blood, *h. e.* a greater Quantity, and the thickeſt Blood require the greateſt Doſes: And 3. that when the Quantities of Blood are the ſame, the Doſes of Purging and Vomiting Medicines are in a duplicate proportion of the Bloods thickneſs. As alſo, that in every caſe theſe Doſes muſt be in a proportion compounded of the Quantity of Blood and thoſe Squares of its thickneſs. Now ſince the Operations of Purgative and Vomitive Medicines depend ſo much on the Quantity and Viſcidity of the Blood, which have not been duely conſider'd before; it is no wonder that the Practice of Phyſick in theſe Evacuations has been ſo uncertain, and that the moſt expert Phyſicians, from their moſt accurate Obſervations, could never determine the true Doſes of Medicines, which alter ſo much according to the various Subjects they work upon; they not being acquainted with the true Method of determining either the quantity of the Blood, or the degrees of its thickneſs. We will here ſubmit our Solution to common Obſervations, and try whether every thing propoſed in it, does not exactly anſwer Matters of Fact, and the Viſible Operations of Nature. *n.* 314. 36.

Firſt then, it plainly follows, that theſe Medicines always purge beſt and moſt conſtantly in a liquid form; becauſe they are more eaſily convey'd into the Blood, and can ſtimulate more parts, and that upon the account of this their Fluidity; whatever may be the way that Purges and Vomits work, or whatſoever their Nature may be. This explains very eaſily a very common Obſervation, hitherto very difficult to Phyſicians, about the different Operation of the ſame Medicine in different forms: Why, *viz.* the Infuſion of a due quantity of a Purging Medicine produces its effects ſooner and more conſtantly than a like quantity of the ſame Medicine in a Powder, tho' ſtill more conſtantly in a Powder than a Bolus, tho' ſtill ſooner and more conſtantly in a dry Bolus, than if it be given in Pills made into that form with Gums that do not purge; and this difference in Purging ſhall even be notable,

notable, according to the diſſolubility of the Gums. From whence it follows, that the Evacuation made by ſuch Medicines, is in proportion to the quantity of thoſe Medicines that happens to be diſſolv'd, and not to the quantity adminiſter'd.

Secondly, That purging by Draughts is the moſt excellent form, and will always have the moſt conſtant effect.

The next Conſideration is, that a certain quantity of any purging Medicine affects us after a different manner, according to the different Quantity and Conſtitution of the Blood, or its thickneſs; and it was ſhewn in the Solution, that if its thickneſs were the ſame, the Doſe ſhould always be as its Quantity, but the Blood differing likewiſe in thickneſs, the Doſes of Purging and Vomiting Medicines muſt be augmented on account of its thickneſs. This is perfectly well confirm'd by daily experience; where we find, that People ſick with a manifeſt thickneſs of Blood, as in Dropſies, the Jaundice, *&c.* take far greater Doſes than they did at any other time when they were not ſick, or in that manner. By a further Diſquiſition into this matter, we find that the Doſes muſt not only be greater where the thickneſs of Blood is greater; but that they muſt be encreas'd in a duplicate proportion of their Viſcidity. This is evident by the Tables in *Caſſia. viz.* 9 : 8ʒ :: 4 : 3ʒ, 1℈, 13 $\frac{1}{3}$ *gr.* and therefore *alternando* 9 : 4 :: 8ʒ : 3ʒ, 1℈, 13 $\frac{1}{3}$ *gr.* Therefore the Doſes are as the Squares of the Conſtitutions. So likewiſe 9 : 8ʒ :: 16 : 14ʒ, 13 $\frac{1}{3}$ *gr.* and *alternando* 9 : 16 :: 8ʒ : 14ʒ. 13 $\frac{1}{3}$ *gr. h. e.* the Doſes are as the Squares of the Conſtitutions. The ſame is true in any other Conſtitution beſides the mean: For Example, in the loweſt and higheſt 4 : 16 :: 213 $\frac{1}{3}$ *gr* : 853 $\frac{1}{3}$. So that by this means we are not only led directly to a right uſe of theſe Medicines, and are able to find the true cauſe why the ordinary Doſes produce ſo very different effects in different Conſtitutions; but likewiſe, *The Quantity of Blood in any Perſon being given together with the ordinary and extraordinary effect of a Doſe of a Purging Medicine, the Change of that Perſons Conſtitution, and the Nature of that Change may be determin'd.* It cannot but be a great ſatisfaction to the mind to find a Doctrine founded on a few ſimple Experiences leading us into the cauſe of many more that are very complext, difficult, and obſcure; which is ſufficient to prove its conformity to Nature. But my preſent endeavour being to rectify the common Practice of theſe Medicines by this Doctrine, I ſhall frame, by this Method, Tables of the Purging and Vomiting Medicines in preſent uſe; better adapted to Experience than are hitherto to be found.

The Method of framing ſuch Tables, is by ſetting off the practicable Conſtitutions in the different Ages that I have obſerv'd to take notable Quantities of Purging and Vomiting Medicines; ſo that by comparing theſe Conſtitutions with the Ages, we have the different Doſes in all thoſe caſes, which is all that is requir'd for a better practice; tho' a more proper occaſion may produce a more nice and exact diviſion

of

TABLES,

Shewing the Doses of purging and vomiting Medicins according to the Solution of Dr. COCKBURN'S Problem.

Common Dose	Medicins	Ages	Constitutions	ʒ.	℈.	Gr.
Medicins whose common Dose is ℥i.	*Cassia. Catholicon Diacarthamum. Elect. lenitivum. Succus radic. Irid. ---- Fumariæ. Syrup. de Rhamno. ---- de Pomis magis. ---- Rosar. cum Helleb. Tamarind. Sal cathartic. amar. --- Mirabile. Mann.* ℥ij. Emetica *Vin. emeticum, seu Bened. Suc. Asari. ---- Senecionis.*	16	2	3	1	$13\frac{1}{3}$
			3	8	0	0
			4	14	0	$13\frac{1}{3}$
		9	2	2	2	0
			3	6	0	0
			4	10	2	0
		6	2	1	2	$6\frac{2}{3}$
			3	4	0	0
			4	7	0	$6\frac{2}{3}$
		3	2	0	2	$13\frac{1}{3}$
			3	2	0	0
			4	3	1	$13\frac{1}{3}$

Common Dose	Medicins	Ages	Constitutions	ʒ.	℈.	Gr.
Common Dose ℥ß.	*Confectio Hamech. Elect. Caryocostin. Diaphænicon. E succo Rosarum.* Emet. *Syrupus emeticus.*	16	2	1	2	$6\frac{2}{3}$
			3	4	0	0
			4	7	0	$6\frac{2}{3}$
		9	2	1	1	0
			3	3	0	0
			4	5	1	0
		6	2	0	2	$13\frac{1}{3}$
			3	2	0	0
			4	3	1	$13\frac{1}{3}$
		3	2	0	1	$6\frac{2}{3}$
			3	1	0	0
			4	1	2	$6\frac{2}{3}$

Common Dose	Medicins	Ages	Constitutions	ʒ.	℈.	Gr.
Common Dose ʒi.	*Agaricus. Aloe. Carthamus. Ebuli sem. --- Cortex. Hermodactyli. Mechoacanna. Pil. Aggregativæ. --- Coch. major. --- Fœtidæ --- sine Quibus. Pulv. Diasennæ. Rhabarbarum. Soldanella. Senna. Turbith.*	16	2	0	1	$6\frac{2}{3}$
			3	1	0	0
			4	1	2	$6\frac{2}{3}$
		9	2	0	1	0
			3	0	2	5
			4	1	1	0
		6	2	0	0	$13\frac{1}{3}$
			3	0	1	10
			4	0	2	$13\frac{1}{3}$
		3	2	0	0	$6\frac{2}{3}$
			3	0	0	15
			4	0	1	$6\frac{2}{3}$

Common Dose	Medicins	Ages	Constitutions	ʒ.	℈.	Gr.
Common Dose ʒß.	*Jalappa. Juli juglandis. Lap. Lazuli. --- Armenius. Opoponax. Pil. Coch. minor. --- de Gut. Gamandrâ. ---- Rudii. Pul. Warvicensis. Sagapenum.* Emet. *Rad. Ipecacuana.*	16	2	0	0	$13\frac{1}{3}$
			3	0	1	10
			4	0	2	$13\frac{1}{3}$
		9	2	0	0	10
			3	0	1	$2\frac{1}{2}$
			4	0	2	0
		6	2	0	0	$6\frac{2}{3}$
			3	0	0	15
			4	0	1	$6\frac{2}{3}$
		3	2	0	0	$3\frac{1}{3}$
			3	0	0	$7\frac{1}{2}$
			4	0	0	$13\frac{1}{3}$

Common Dose	Medicins	Ages	Constitutions	ʒ.	℈.	Gr.
Common Dose ℈i.	*Resina jalappæ. Extr. rhabarbari. Pil. de Hermodactylis* ℈ij. *pulv. Cornach.* ℈ij. Emet. *Gilla vitrioli.*	16	2	0	0	$8\frac{8}{9}$
			3	0	1	0
			4	0	1	$15\frac{5}{9}$
		9	2	0	0	$6\frac{2}{3}$
			3	0	0	15
			4	0	1	$6\frac{2}{3}$
		6	2	0	0	$4\frac{4}{9}$
			3	0	0	10
			4	0	0	$17\frac{7}{9}$
		3	2	0	0	$2\frac{2}{9}$
			3	0	0	5
			4	0	0	$8\frac{8}{9}$

Common Dose	Medicins	Ages	Constitutions	ʒ.	℈.	Gr.
Common Dose Gr. VI.	*Colocynthis. Euphorbium. Esulæ Cortex. Elaterium. Gum. Guttæ. Gran. Gnid. Ricini sem. Scammon. Tr. Alhand.* Emet. *Croc. Rulandi. Turbith minerale.*	16	2	0	0	$2\frac{2}{3}$
			3	0	0	6
			4	0	0	$10\frac{2}{3}$
		9	2	0	0	2
			3	0	0	$4\frac{1}{2}$
			4	0	0	8
		6	2	0	0	$1\frac{1}{3}$
			3	0	0	3
			4	0	0	$5\frac{1}{3}$
		3	2	0	0	$\frac{2}{3}$
			3	0	0	$1\frac{1}{2}$
			4	0	0	$2\frac{2}{3}$

Common Dose	Medicins	Ages	Constitutions	ʒ.	℈.	Gr.
Common Dose Gr. III.	Emet. *Hep. Antimonij. vitr. Ant. Merc. vitæ. Tart. emet. Merc. præc. per se ---- solar. ---- ruber. Hercules Bovij.*	16	2	0	0	$1\frac{1}{3}$
			3	0	0	3
			4	0	0	$5\frac{1}{3}$
		9	2	0	0	1
			3	0	0	$2\frac{1}{4}$
			4	0	0	4
		6	2	0	0	$\frac{2}{3}$
			3	0	0	$1\frac{1}{2}$
			4	0	0	$2\frac{2}{3}$
		3	2	0	0	$\frac{1}{3}$
			3	0	0	$\frac{3}{4}$
			4	0	0	$1\frac{1}{3}$

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of Conſtitutions, very much to the advantage of the Practice of Phyſick in all Diſeaſes. The Ages wherein theſe different Doſes are taken, I find to be four; when a Man is about 16 or 20 Years of Age, and weighs about 12 Stone, he then takes the common Doſe: One of nine Years takes three quarters of that; one of ſix the half, and one of three Years a quarter. Moreover, it having already been ſhewn, that the notable healthy Conſtitutions are but three, as alſo the notable Pulſes of each of theſe: Let then theſe Conſtitutions be as 2, 3, 4. That of the moſt fluid Blood as the firſt number, and ſo on; in that caſe, the Doſe of any Perſon will be found by multiplying the common Doſe for his Age into the Square of his Conſtitution and dividing by the Square of the middle Conſtitution. For inſtance, If ℥j. *Caſſiæ*, is the common Doſe, or the Doſe of the middle Conſtitution, ʒiij. ℈j. and *gr.* 13 $\frac{1}{3}$ is the Doſe of the firſt Conſtitution, and ʒxiv. *gr.* 13 $\frac{1}{3}$ that of the groſſeſt or laſt Conſtitution; and ſo proportionably for every Medicine in all the Ages, as appears by the Tables. This Method ſeems to anſwer ſo exactly, that there is not any thing neceſſary beſides, except when a Perſon is more Looſe or Coſtive than ordinary (which may be known from the Patient or otherways) it is to be reputed the ſame, as if he had taken an equivalent quantity of a Medicine proper to produce theſe effects. Any Phyſician, who has conſider'd this caſe in ſome People after Fluxing, will allow the juſtneſs of this Exception. As Vomiting Medicines have the ſame common Doſes with thoſe that Purge, they admit alſo of the like divided Doſes; which therefore may be found by the ſame Tables. Only, as People that are more Coſtive than ordinary require a proportionably greater Doſe of a Purging Medicine: So they require their Doſe of a Vomiting Medicine to be conſiderably leſs, as is very well known in hot Countries. But it muſt be obſerved, that in the Tables, *Age* ſtands inſtead of *Quantity of Blood*; becauſe they encreaſe pretty equally, and it makes the practice more eaſie to ſuch as are not accuſtom'd to Weights and Numbers. The more Skilful are deſir'd to obſerve, that the mean Ages, multiply'd into the mean Conſtitutions, give Doſes more nicely.

OEcon. anim. p. 51. Biblioth. Anat. p. 1124. Tom. 2.

The TABLES.

Some Inſtances ſhewing the Defects of the preſent Practice, and how mended by the foregoing Tables.

1. The Doſes of the foregoing Tables, ariſing from Calculation, agree perfectly well with the common Obſervation of the beſt Authors; tho' their Obſervation is very general and ill made, if we except the very firſt Steps. For inſtance, Authors of all Countries, *Engliſh Dutch*, *German*, *Italian*, and *French*, reckon the Doſes after the ſame manner; whereas

whereas, if they had been obſerv'd, they muſt have been different as are the Conſtitutions of Men in the different Countries.

2. The defect of their Obſervation is manifeſt by the diſproportion'd Doſes of ſome Medicines, their high Doſes being ſometimes double and ſometimes treble and more of their low Doſes, which is not conform to Nature; for, let the low Doſe be what it will, the high Doſe of one Medicine muſt always bear the ſame proportion to the high Doſe of another, as did their low Doſes, *viz.* even in Manna, they reckon it from ℥ij: commonly to ℥iij. and ℥iv. If it is ſaid that the firſt Doſe is the loweſt Doſe that is taken by a Man of a due Age, it is neither true in fact nor conform to their own way of reckoning: For inſtance, Rhubarb is ſaid to be taken from ʒj. to ℥ij. No body will ſay that this is the loweſt Doſe taken by a Man of a due Age as formerly; becauſe it is not in fact true, nor that the high natural Doſe is ℥ſs; for, as I ſaid before, If ℥j. of Manna and ʒj, of Rhubarb are the reſpective low Doſes, then ℥iv and ʒij. cannot be the reſpective high Doſes. As to what concerns ſome extraordinary Doſes given by themſelves, and far exceeding the ordinary Doſe, is eaſily accounted for by the Solution. There are many Examples of this Nature: Turbinth, *viz.* is commonly reckon'd among them from ʒj. to ʒij; yet *Marggravius*, and good Authors, have given it to ʒiv. So *Coloeynthis* from *gr.* vi. to *gr.* xij. and *Fulgin. Fernel. Duncan.* ſay they have given it to ʒß.

3. Authors have been far from being exact; for they have only dos'd theſe Medicines for People of full Age; but have left the Doſes of the different Ages in ſilence; nor have they told us at what time a Man takes his higheſt Doſe, or how that alters in the Growth and Decline of Age, which is ſtill a very great difficulty for the moſt experienc'd Phyſicians to manage.

4. Their general Method is founded in a Miſtake; their loweſt Doſe being really the common Doſe taken by the generality of Men, which produces a multitude of Errours in the Practice. This is manifeſt in their doſing every Medicine.

5. The mentioned Caſe is more manifeſt by theſe Tables, and it is two to one but that a Phyſician over Purges or under Purges any Perſon in Health; and if more Caſes in Sickneſs are ſuppos'd, the odds will encreaſe proportionably. Experience confirms this exactly: For if the middle Doſe is given to one of the loweſt Conſtitution, and the middle Doſe is to purge 7 or 8 times; in that caſe, the Perſon of the loweſt Conſtitution is purg'd near twice as much as he ought to be; and if given to one of the higheſt Conſtitution, he is purg'd but half of what he ſhould be. But if the Doſe of the higheſt Conſtitution is given to one of the middle Conſtitution, he is purg'd twice as much as he ought to be; and if given to one of the loweſt, he is purg'd four times as much, or about thirty times, as we find true by daily Experience. But if the Quantity of Blood, the Age, or Sickneſs contribute to the Er-

Errour, it may prove fatal. If this Confideration were illuftrated by a proper number of Examples, we fhould find fome hundreds of otherways unavoidable, miftakes now prevented by the Practice of thefe Tables.

Laftly, We may eafily account, by thefe Tables, for the Dofes of Children over purging fome People of good Health, and of due Age; a *Phænomenon* fo furprizing, that the fmallnefs of the Dofe is commonly thought a good Excufe for the Miftake.

II. Actorum Lipfienfium Editores, qui neque Experimenta à me prolata, neque modum quo ea ad naturæ Leges perpendere aggreffus fum ne attingunt quidem; principia ipfa, quæ jamdiu pro certiffimis habita funt quibufque innititur mea omnis rerum Chymicarum explicatio, convellere funt conati. Hocque primo impetu faciunt, pofthabita libri ipfius enarratione, ne quis ad legendum non præoccupatus accederet. Et certè minus æque ferenda eft hæc eorum cavillatio, quoniam extra Provinciæ fuæ fines evagati funt; id enim unicè præ fe ferunt ifti Literatores & quafi Indices Librorum, ut quid in quoque Scripto contineatur, compendiariâ quâdam operâ fimpliciter fideliterque recenfeant, legentis interim Judicium relinquant integrum ac liberum. Pro fundamentis Theoriæ *Chymicæ* habui principia, ipfamque argumentandi Methodum, quam Mathematicorum Princeps in Philofophiam intulit *Newtonus*: Qui quidem Vir, admirabili quo eft ingenio, ad res Phyficas promovendas certam patefecit viam, naturalemque Scientiam tanto rationum pondere ftabilivit, tam incredibili rerum inventione locupletavit, ut ad eam illuftrandam plura præftiterit quàm omnes omnium gentium Philofophi. Hoc itaque fagaciffimi Viri inftitutum, quia *Editores* latere vifum eft, paucis aperiam: Oftendamque totum id, quodcunque eft quod jam in hoc cognitionis genere exploratum atque perfpectum habemus, ex hâc ipfâ ratione ac viâ fluxiffe. Porrò etiam Argumenta, quibus ad hanc Phyficæ Doctrinam refutandam ufi funt, ex falfis, quas de hâc re imbiberint, Opinionibus promanâffe planum faciam; plurefque iftiusmodi, quas adducunt, ratiunculas contra illa quæ ipfi amplectuntur principia, quàm contra *Newtoniana* proferri poffe.

Dr. Freind's Defence of his Prælectiones Chymicæ, *n.* 331. *p.* 330.

Cartefiani, iique fere omnes, qui fe magiftros Philofophiæ *Mechanicæ* dici volunt, rationem hanc perpetuò tenuerunt, ut Hypothefin aliquam fumerent feu figmentum, quod nullibi nifi cogitatione fingentium exiftit: Deinde, ut verbis neque perfpicuis neque definitis commifcerentur, quo demum modo omnia ad hujufce Hypothefeos normam efficiat natura. Aliam omnino *Newtonus* infiftit viam: Nihil ille fingit, nihil pro arbitrio fuo affumit; id folum quod Experimento & Obfervatione notatum, fenfibus omnium patet, pro rato habet: Ex his principiis certiffimas Mathematicâ ἀκριβείᾳ elicit conclufiones, quas deinde ad alia Naturæ Phænomena explicanda feliciffimè accommodat. Hanc infiftens viam elegantiffimè demonftravit, Planetas motu Elliptico

tico circa Solem verſari, areaſque temporibus uſque reſpondentes deſcribere: Satellites itidem ad eandem normam circa Planetas, quos ut comites perpetuo conſequuntur, volvi. Hinc extra dubium omne poſuit Planetas ad Solem, Satellites autem ad Planetas primarios ſe inclinare & tendere: Hanc autem inclinationem in ratione Diſtantiarum duplicatâ decreſcere: Ineſſe porrò immutabilem quibuſcunque corporibus vim, quâ itidem in ſeſe mutuò ferantur: Et inde fieri, ut Lunæ in Terram inflectio, idem planè valeat ac gravitatis vis, atque acceſſum receſſumque Maris efficiat. Inclinationem hanc ſive attractionem quidam, ſi ita lubet, *qualitatem occultam* nuncupent, & erit credo ſemper occulta: neque enim adhuc ex *Editoribus* quemquam extitiſſe video ita in penitiore Philoſophiâ perſpicacem, qui docere in ſe ſusceperit, quo modo, quâ vi Mechanicâ Attractionem illam exerceat natura. Sed utcunque hæc Naturæ vis, ſi cauſam ſpectemus, occulta ſit, minimè tamen figmentum, ſive Hypotheſis (quod in eorum principia, ipſis etiam fatentibus, cadit) appellari poteſt; cum eam æquè revera exiſtere ac Solem aut Planetas, luculentiſſimis Argumentis confirmetur. Quòd ſi ſit hujuſmodi principium, quod in materiâ omni perpetuò inſidet, quid vetat quo minus id ad rem ſuam accommodent Philoſophi, explicentque nobis quo modo effectus plurimi, quos quotidianâ animadverſione notamus, vim inde ſuam atque Originem derivent. Pariter obſervatione diuturnâ clariſſimiſque Experimentis varium illum, quo radii Lucis refringi ſolent, modum exploravit idem *Newtonus*; hincque ita feliciter lucis colorumque naturam admirabilem aperuit, ut hanc Optices partem ante eum non niſi tenuiter admodum & nugatoriè pertractatam fuiſſe omnes ultrò agnoſcant. Hanc adeo rectiſſimam eſſe conſtat, quam Philoſophi in Scientiæ perveſtigatione tenere poſſunt, rationem, ut primùm multiplici experimento corporum naturas vireſque perquirant, deinde poſthabitâ omni cauſarum, unde eæ fluxerint, indagatione, Phænomena, quæ cujuſque virtutem ingenitam ſequuntur, enucleent atque exponant. Hâc ipsâ via ingreſſus Divinus ille *Archimedes* leges tum *Mechanicas* tum *Hydroſtaticas* exquiſivit, dum interim neque Gravitatis neque Liquoris cauſam aut ſtatueret aut inveſtigaret; ea ſolummodo quæ ſenſuum cognitione percipiuntur pro principiis habens utriuſque Scientiæ rationem pulcherrimè evolvit. Ita etiam *Galilæus*, quanquàm nullam de Gravitatis cauſâ Hypotheſin commentus eſt, motûs tamen celeritatem, quam gravia corpora cadendo acquirunt, inveſtigavit, projectorum impetum & curſum, pendulorumque reciprocationes primus explicuit: Eaque Scientiæ fundamenta poſuit, quibus celeberrima Phyſicorum inventa hodiè innituntur. Quid? an non in Opticâ illuſtrandâ ampliſſimo cum fructu progreſſi ſunt Mathematici, duobus principiis, altero Refractionis, Reflectionis altero, conceſſis; utcunque alterutrius cauſa pauciſſimis adhuc innotuerit?

Si quid ponderis Editorum authoritas habeat, præclara hæc acutiſſimorum hominum inventa omnino repudianda ſunt, quia ſcilicet ex iis cor-

corporum virtutibus, quarum initia causæque prorsus incognita sunt, ducuntur; *nec sine qualitate illâ occultâ veræ Philosophiæ principia confundente, & in antiquum Chaos reducente, commode explicari possunt.* Video clarif. Wolfium in Aerometriâ, gravitate Aeris, tanquam concesso principio, usum esse; atque eo quidem multa Naturæ Phænomena haud absurdè expedivisse: Qui tamen gravitatis causam Mechanicam ratiocinatione ne attingit quidem; nec credo ullam unquam Hypothesin ad causam hanc explicandam accommodatam fuisse, quam ipse Wolfius à vero alienissimam esse non facillimè probare possit. An huic igitur objicient Editores, quod Scientiæ Physicæ *occultam qualitatem* invexerit? In hâc quidem Gravitate explicandâ, quam sensu percipimus, longissime omnium processit *Newtonus*: Eam quippe à vi attractrice, quæ per omnem se undequaque materiam disseminat, oriri commonstrat. Vim hanc Editores, pro suâ in rebus Philosophicis authoritate, *figmenti* vocabulo appellant; sed quo demum loquendi jure id, quod in rerum natura existere ostenditur, *figmentum* dici queat, ne intelligi quidem potest. Illam certe Attractionis Speciem quemadmodum in toto Planetarum orbe dominatur, luculentissimè exposuit *Newtonus*; neque adhuc videre contigit, quid contra Viri perspicacissimi demonstrationes objectari possit. Alterum hoc Attractionis Genus, quæ in distantiæ ratione magis quàm duplicatâ decrescit, & revera existere, & vim suam in minutissimis corpusculis acriter exercere, plura mihi præsto sunt quæ probent Experimenta, quàm unquam ad demonstrandam Aeris Gravitatem allaturus est Wolfius. Quorsum igitur Principia, quibus ratiocinatio hæc omnis nititur, in altero Argumento pro Commentitiis habere licet, in altero non item? Experientiâ comprobatum est, radios lucis quæ à Sole, stellis inerrantibus, vel etiam ab eo, quo utimur igne, dimanat, versus oras solidorum corporum æqualiter allici; ea autem immutabilis naturæ lex est, ut ubicunque sit Actio, ibi unà non possit non esse Reactio: Itaque verè & jure conclusuri videmur, Principium hoc quod Attractionis nomine vocamus, tum revera existere, tum per universam omnino materiam diffundi. Quod licet in omni materia inhærescat, id tamen in minutissimis corpusculis vim suam ad sensum magis patefacere demonstravit Vir in Physiologiâ acutissimus D. *Keillus*.

At aiunt, *Talibus semel admissis, apertâque fingendi licentiâ, mox erunt qui alias qualitates occultas, seu quas ipsi agnoscunt absolute inexplicabiles comminiscentur, & paulatim ad vetera ignorantiæ asyla redibunt. Si detur vis attrahendi, seu Sympathia, quidni pari jure vis repellendi, seu Antipathia? Ita facile etiam dabitur Antiperistasis, dabuntur qualitates emissæ per modum specierum cum suis Actu potentialibus, dabitur funiculus* Lini *Attractivus, dabitur in materia eadem Variatio* Extensionis, *non apparentis tantum, sed etiam veræ.* Itane incœptant, *si detur vis Attrahendi*; cum eam dari Experientia ipsa apertissimè demonstrat? Non est hoc opinionis commentum ad alia Phænomena explicanda excogitatum sed est per se constitutum à Naturâ Phænomenon; adeoque quanquam sibi plaudant Editores, quod hujusce Sen-

ntiæ fautores ad absurdum quid deduxerint, omnis tamen illa, de qua se ita fidenter jactant, huc tandem redit Argumentatio; nempe si unum aliquod principium, quod in rerum naturâ existere observatione certâ compertum est, concedimus, ideo etiam oportet alia, quæ nusquam extiterunt, approbare; uti verbi gratiâ, si Gravitatem agnoscimus, quam corporibus quibuscunque inesse certò animadvertimus, quanquam illius causam prorsus nescimus, idcirco fabulas Philosophorum omnes & commenta amplecti necesse est, quæ nec Experientiâ ullâ confirmari, nec ratione explicari queunt. Si hoc sit Mathematicorum more ratiocinari, satius est profecto *ad vetera quævis ignorantiæ asyla redire*, quam hanc argumentandi *licentiam aperire*. Sed vim Attractricem in eo maximè oppugnant, quod rationibus Mechanicis minime illustrari possit. An igitur volunt, ut nihil in rem Physicam introduci debeat, nisi cujus ratio & causa perspecta sit? An Editorum aliquis Elaterem aeris, quâ vi Mechanicâ constitutus sit, unquam explicuit? Eum tamen & Philosophi omnes ultrò concedunt, & ad multa Naturæ Phænomena enodanda fæliciſſimè accommodari unâ mente consentiunt. Fabro, utique id libenter damus, ut Horologii artificium intelligat, quanquam interim gravitatis Elaterisque, ex quibus quidem pendet omnis rotarum conversio, rationem penitus ignoret: Hoc idem Physico denegabimus? Qui *vim* illam, quâ universa corpora aguntur, & suo quæque motu atque ordine diriguntur, investigarit, qui hujusce potentiæ motrices leges definire, easque ad præcipua Naturæ Phænomena explicanda adhibere poterit, tametsi cuinam causæ vis illa omnium gubernatrix ortum debet, plane se nescire fateatur, illum de naturæ viribus & Machinatione nihil prorsus scire, nihil animo percipere dicemus? Quod si hoc Attractionis principium ad fontes usque suos persequendi studio teneantur Editores, faciant quod lubet; hanc iis gloriam ultro relinquit *Newtonus*, satis præclarè secum agi ratus, si modo eorum offensionem effugiat, quod involutum longeque difficillimum problema explicandum in se non susceperit.

Non me latet quod Cl. *L.* quem quasi Numen aliquod suspiciunt Editores, in Specimine illo, quod vocabulâ eleganter sonante nuncupat, *Dynamicum*, planiſſimè scripserit, *Vim Activam seu nisum intimum corporum Naturam constituere.* Vis hæc sive Nisus, si quid velit recte intelligo, idem est ac propensio illa mutua, quàm corporibus quibuscunque insitam diximus; quámque multo ante patefecerat *Newtonus*, quanquam eâ materiæ naturam contineri nuspiam asseverarit. Si vera sit *L.* sententia, nobis æquo jure extensionis soliditatisque causa quærenda est, ac attrahentis hujusce, quod omni materiæ inest, Principii ratio excutienda. Hoc autem posito fundamento, effectus omnes quos in hâc universâ mundi machinâ contemplamur, ab ipsâ materiæ constitutione necessariam originem deducunt. At mihi quidem intima corporum natura ita parùm explorata est, ut longiſſime absim, qui affirmem vim hanc æterno rerum fœdere illis intermisceri, & eadem naturali colligatione, ac extensionem soliditatemque inhærescere. Sane ita valde

laborare videtur hæc Sententia, ut argumenta, quæ in contrarium afferre proclive esset, vix recenseri nedum refelli possint. Quod si cum *L'o* sentiant Editores, non video cur amplecti nolint principium, quod ille apprimè necessarium judicat, ut id in intimâ corporum naturâ constitutum esse pronunciet. Cum verò ex motu corporum constet, attractricem hanc potentiam revera existere, si eam neque Materiæ necessario ingenitam, neque rationibus Mechanicis explicandam esse censeant, haud absurdum aliquid credo aut Physico alienum facturi sumus, si eam in voluntatem Dei resolvimus: Legemque universam esse statuimus, quâ omnis hæc mundi moles gubernatur & regitur, corporumque vario utcunque motu labentium convenientia atque concentus servatur: quanquam quidem hæc ipsa potentia, non minus quam naturæ constructio omnis, à divinâ voluntate unicè manaverit. Illi verò, qui nullam hujusmodi legem agnoscunt, sed universum Physices negotium, non modò quoad proximas, sed remotissimas etiam causas, suâpte naturâ & mechanicâ quâdam ratione geri volunt, ita ut nihil sit quod non ab ipsâ materiæ vi immutabilique motûs conditione proficisci putent, quid aliud agunt, nisi ut cum *Epicuro* notionem ex animo hominum evellant cuncta providentis atque moderantis Dei; argumentaque suppeditent, quæ in rem suam traducant impii? Quicquid verò de hâc Attractrice virtute statuendum sit, minime dubium est, rerum naturam sine principio quodam actuoso non posse consistere: Quippe corpora, utcunque in motum semel excitata, si deinde iis suo more uti liceret, vicissitudines suas certo tempore haud ita constanter conficerent. Hoc cum ita necessarium perceperit acutissimus *L.* pereleganter conclusit, quod *agere sit Character Substantiarum.* Ubicunque autem vis hæc omnia ciens atque agitans motibus suis sita sit, in *occultam quandam qualitatem resolvatur* necesse est; aliam enim illius causam, quam divini numinis voluntatem, frustra hactenus quæsivimus. Nonnulli autem, qui sibi in rebus Mechanicis acutiùs cernere videntur, vim hanc in *Æthere* vel in Fluido quodam admodum subtili collocant; quos sane interrogare velim, quid tandem sit quod illum æthera agat, & in motione perenni continuatâque tueatur? Unde fit ut motus omnino contrarii se invicem non extinguant? Quid sit porrò, quod motus hosce eâ facultate instruat, ut suum singuli opus proprium sibique aptum efficiant? Hæc omnia *occultâ qualitate*, quam in æthere sitam esse volunt, oriri necesse est. Etenim si hanc Hypothesin ad naturam revocamus, facilè patebit plures ab iis intromitti *occultas qualitates*, quàm sunt quæ explicanda suscipiunt Phænomena. Quanto rectiùs ille in Philosophiâ suâ *Newtonus*? Qui principium non nisi unum, idque simplex maximè, & observatione confirmatum sibi dari postulat, ——

Et speciosa dehinc miracula promit.

Sed quantâ obscuritate laboret, quamque infirmis rationibus fulta sit tota illa ætheræ cujusdam virtutis, aut subtilis fluidi Hypothesis, nullum clarius, quàm ex iis quæ de hâc re disserunt *Editores*, peti potest argumentum. Aiunt quippe, *Hæc omnia sine qualitate illâ occultâ attractrice, vera Philosophiæ principia confundente, & in antiquum Chaos redigente,*

commode explicari posse, partim etiam à viris doctis explicata esse. Hunc adeo ob finem, *statuunt plurimas materiæ particulas Sphærâ quâdam magneticâ fluidi Subtilioris esse circundatas, cujus motu (ut in Magnetibus nostris fieri videmus) attrahant se invicem, aut repellant, aut ad situm convenientem disponant, quoties scilicet libertatem aliquam sint nactæ.* Quid quæso est *Sphæra quædam magnetica,* nisi aliquid admodum *occultum?* cui utique adsciscitur quid adhuc occultius, scilicet *Magnetismus.* Unde fit, ut hæc materiæ subtilis Sphæra corpori, cui circundata est, perpetuò comitem se præbeat? Res quidem ipsa postulare videtur, ut corpus, cum semel motu impresso locum mutaverit, Sphæram hanc itidem ambientem post se relinquat: quippe si terra novo aliquo impetu acta alium prorsus cursum iniret, ex legibus Mechanicis satis liquet, quod non Atmosphæra modò, sed quicquid à Terrâ liberum solutumque esset, facto diffidio, suâ se in sede contineret. Quid igitur in hoc rerum statu concludendum est? An quòd occultâ quâdam qualitate Atmosphæra hæc motum corporis continuò sequatur? An quòd vi etiam occultâ, altera materiæ subtilis Sphæra de novo gignatur? Uter sit, libenter discere velim, quænam demum ea qualitas sit, quæ Sphæram hanc Magneticam in motum cieat? cujus generis motus ille sit, & quâ potissimùm ratione excitetur, qui efficit, ut *materiæ particulæ attrahant se invicem, aut repellant, aut ad situm convenientem disponant?* Quot tandem *occultas qualitates* ad singula Phænomena explicanda accersere coacti sunt, dum unam illam & simplicem rejiciunt, quæ per universam naturæ fabricam se diffundit, & plurimis Phænomenis solvendis tam præclarè inservit. Id verò in *Editoribus* satis mirari nequeo, quod qui contra vim attractricem ita acriter dimicant, & sine eâ omnia commode explicari posse contendunt, eam tamen in hoc suum de rebus Physicis commentum ipsi transferant: Nec ab *attractionis Voculâ,* quæ quidem iis ita *eleganter sonare* visa est, *abstineant,* ut *ignorantiam suam palliare* possint: Loquuntur enim de Sphærâ quâdam fluidâ, quæ *ATTRAHIT, repellit,* & ad *situm convenientem disponit.* Cum nihil veri sit in hâc Sphæræ subtilis fabulâ, cavendum certè fuit, ne desideraretur ista, quæ rem verisimilem redderet, convenientia. Facillimum profectò esset, naturæ Phænomena omnia ad hunc modum illustrare; mirificè quippe rerum causas expedit *Sphæra Magnetica fluidumque subtile,* atque etiam maxime inter se pugnantia conciliat. Atque hoc quidem quod de *materia subtili* Vique *Magneticâ* excogitarunt figmento (dum agendi ratio ab inventoribus ferè intacta relinquitur) nullum præsentius *ignorantiæ asylum;* etenim omnes illas occultas qualitates, quæ hactenus in Philosophiam irrepserunt, longè multumque superat. Nemo certè non videt, quàm ficta hæc omnia atque commentitia sint, cùm neque quale sit hoc subtile fluidum, neque etiam si ullum omnino sit, aut observatione animadverti, aut ratione colligi possit. Dispiciat itaque Lector, annon ea quæ in veram, h. e. *Newtonianam* Physicen intentant argumentandi tela, in hanc ipsorum *infelicem Philosophandi rationem* fortiùs retorqueri queant. Ea omnia quæ pro certis atque ratis jactanter satis venditant, vana prorsus sunt & fabulis referta, nullâ observatione aut Experimento nitentia;

tia; quæ etiam si pro veris concessa fuerint, eo occultarum reconditissimarumque virtutum agmine stipantur, ut facilius multò sit *Sympathiæ*, *Antipathiæ* & *Antiperistaseos* naturam cogitatione complecti. Hujusmodi nimirum Hypotheses hoc vitio laborare semper comperi, ut obscurius quid magisque difficiles explicatus habeant, quam res ipsæ, quibus eæ explicandis accommodantur. In illâ *vorticum* Hypothesi, quæ iis ante cæteras omnes arridet, rationem nullam afferunt, cur materia fluida curvam semitam affectet, seseque circa centrum torqueat, cùm ea sit corporum omnium natura, ut rectis lineis ferantur: unde tot vorticibus cautum est, ne in cursibus suis se invicem perturbent & impediant; unde per eos transeant Cometæ, motuque prorsus contrario ac ipse vortex, versentur; tantumque absit, ut illius incitatissima conversio eos interpellet, ut in suis, quos circa Solem conficiunt, orbibus, ad eandem ac Planetæ normam dirigantur, seseque versus eum pari modo inflectant. Hâc adeo Vorticum Hypothesi in eos se laqueos inducunt isti *Philosophandi artifices*, à quibus nunquam expedire possunt: in in quâ tamen positum est omne hujusmodi Philosophiæ fundamentum. Quùm ad Phænomenon aliquod explicandum accedunt, ad illorum nutum præsto est subtilis materia, quæ modo motuque admodum ignoto atque inexplicabili rem quam velint efficiat. Num Philosophiam magis sapiunt hæc, quàm si quis dixerit id à *Sympathiâ*, *Antipathiâ*, *vel occultâ aliquâ qualitate proficisci*? Num hæc Philosophandi ratio, non æquè ac illa quam vellicant, in *Asylum ignorantiæ* cessura est? Et si consuetudini fictis hisce fabulis indulgenti obsequimur, quidni cætera etiam, quæ ab hominibus ad comminiscendum ingeniosis fingi possunt, amplectamur?

Quàm longè alia dissimilisque est vera Philosophiæ instituendæ via! in quâ nihil ponitur, nisi quod in ipsâ rerum naturâ constitutum esse observatio evidentissima declarat; & quanquam principii, quo utimur, causa & origo delitescat, ex eo tamen multa, quæ quotidiano usu animadvertimus, fluere & pendere possunt: Itaque ingenui est Philosophi primò corporum virtutes experimentis elicere, deinde, ubi eæ diligenter exploratæ stabilitæque sint, distinctè & perspicuè commonstrare, quinam illas effectus suâ sponte consequantur. Neque ulla credo tanti esse Adversantium argumenta, quæ hanc veri investigandi rationem evertant. Etenim si principia & postulata vim suam omnem in Experimentis positam obtineant; si propositionibus concessis & ritè præmissis, nihil contra Dialectices leges conficiatur, conclusio non potest non esse certissima: ita ut quicquid hâc methodo evolutum explicatúmque habemus, rem Physicam inventis augere atque amplificare meritò censendum sit. Igitur vim hanc Attractricem, utcunque eam labefactare conentur *Editores*, firmam nos stabilemque tenere confidimus.

Est & alterum axioma, quod consensu suo non approbant *Editores*, viz. *Corporum momenta seu quantitates motuum oriri ex ratione quantitatis materiæ & celeritatis compositâ*: quæ qui sic *æstimant in communi errore versantur*,

versantur, uti passim in illorum Actis notatum est. In iis quidem video de hâc re unum aut alterum clarissimi *L.* Commentariolum; qui tamen ita parum rationibus pugnat, ut nihil nisi fallaces conclusiunculas, nihil nisi in verbis captiones sectetur: igitur ea quæ de hoc argumento protulit, unâ fere Mathematicorum voce atque sententiâ improbantur: Quorum nonnulli hoc ipsum Axioma exquisitè confirmârunt. Hos itaque adeat Lector; nam illa Disceptatio, uti per se satis magna est, ita etiam huic instituto nimis aliena.

Of the Principles of Purgatives, by Monsf. Bolduc. *n.*278. *p.* 1099.

III. Monsieur *Bolduc* examined the Principles of Purgatives, and began with *Ipecacuanha,* which he said he had endeavoured to sweeten and qualifie, by trying to take away its too great Emetic Power. He assured, that how violent soever *Ipecacuanha* be, yet it is not so dangerous as *Scamony* or *Colloquintida,* which always leave Gripes and sometimes Dysenteries; whereas *Ipecacuanha* leaves only a gentle astriction after it. He said next, that he having observ'd that the Emetick force of this Root consists in its Resinous parts, he had found out a way to take them from it, and to leave only the Saline parts; that he made use of Spirit of Wine to extract the one, and of Rain-water distilled to draw off the other; that he had afterwards given with very good Success in Dysenteries this *Ipecacuanha* so despoiled of its Resinous parts. From *Ipecacuanha* he passed to Hellebore, which is another violent Emetick; which he distinguish'd into two sorts, the black and the white. He said, that ours was not different from that of the Antients, that the White caus'd mortal Convulsions, for which reason it was not used, and that he had never made any attempts with it. As for the black Hellebore it is to be observed, that that which comes by way of *England* is much weaker than that which grows upon the Mountains of *Switzerland*; which may well have been the reason that Physicians have neglected this Remedy. Then he related his operations on this Root. He said, that having put it in a retort in a Reverbatory Fire, he at first drew off an acid Spirit, next an oily acid Spirit; thirdly a violent *alkali* Spirit came over mixt with Oyl of Tartar, and lastly a fœtid Oyl. That from the *Caput mortuum* he had by a *Lixivium,* a fixt Salt, which fermented with Acids, such as all other Plants give; besides these Operations, he drew an Extract of this Root with Spirit of Wine, to get the Resinous parts, and with distilled Rain Water for the Saline. He got but very few of the former, but a great deal of the other; so that he found that Spirit of Wine was useless in this case. Comparing then the Effects of these Purgatives, he said, that the purely Resinous purge little, and with much irritation; that the purely Saline purge only by Urine, but that both join'd together purge very well. That it is for this reason that Physicians make use of Salt of Tartar, to correct the bad Effects of Resinous Purgatives; but if this Precaution were used, to make the Extract with aqueous

Dis-

Diffolvents, inftead of the Sulphureous, there would be no need of the Corrective.

IV. All watry Plants fhew their Flegm as well to the Taft as in Diftillation; and in all dry woody Tafts we obferve the Earth, as well as we can by the Chymical Analyfis. By the Mucilage and Gumminefs, or oily Tafte, we diftinguifh the Oyl of Plants, as well as by Diftillation. The Aromatical Smell fhews us the Volatility of Oyl and Plants; and by the Fœtidnefs we alfo know that the Oyl and Salt are in a volatile State. By the Acrimony and Pungency we know there is a volatile Salt in Plants, and by the burning Tafte we find there is a Corrofive Salt in them. By a crude rough Acidity we obferve the Tartar or effential Salt of Plants, but if the Acidity be of a Vinous Smell, we may obferve 'tis of middle State of Digeftion, and may be called a Vinous Tartar, and the Crude Tartar an Acerb Tartar, but if the Tartar had a pungent Smell, then 'tis a volatile Tartar or acid acrid Tartar. I will next defcribe the principles obfervable in fweet Taftes, and their feveral Claffes, but muft firft obferve that fweet Taftes fhew their Oyl by their flimy fmoothnefs, and their Tartar is evident in their Extracts, as in the Juice of Liquorifh.

Obfervations on the Clafs of Sweet Taftes, by Sir John Floyer. *n.* 279. *p.* 1160.

1. The Grafs Sweets, as *granum caninum*, have much effential Salt and moderate Oyl, *Juncus equifetum*, *arundo*, *typha*, *nymphæa*, are all of the Rufh kind, fweet and rough, and fome of them have more Oyl, others more acid, and that the moft crude have more Oyl than Tartar.

2. The Corn Sweets, as Barley, Rye, Wheat, Oats, Rice, Millet, have much Oyl and effential Salt, and a little volatile: So Bread yields Oyl and effential and volatile Salt. Note, that Fermentation or the Fire produces the volatile Salt, by exalting the Tartar into a volatile Salt; and the flimy mealinefs in Corn fupplies the Oyl. *Tragopogon* and *Scorzonera* are referable to the Grafs, and contain much Oyl, and effential Salt.

3. The fubacrid Sweets, as *Rampions*, *campanula*, *trachelium*, contain much Oyl and effential Salt, but the Acrimony in thefe Plants fhews a volatile Salt not defcribed by the Chymift.

4. The Ferns contain Oyl and effential Salt, as Polypody; but the acrid principle is not obferved by the Chymift, nor the Fragrancy in Harts Tongue. *Ofmunda* and *Capillaries* have more Oyl than Salt, becaufe more mucilaginous and crude.

5. All the Leguminous flimy Sweets have more Oyl than Tartar, but all much of both, as Broom, *Ononis*, *Aquilegia*, *fumaria*, Afparagus, *rufcus*, *thalyctrum*, *polygonatum Sena*, *galega*, *lathyrus*, *luteola*, *periclymenum*, *glychirriza*, *pfyllium*, much Oyl and volatile or effential Salt. Beans and Peas and Lentils have alfo a volatile Salt and *lens aquatica*. Note, that fince there is no Acrimony in thefe Seeds, the volatile Salt is produced by the Fire. The aromatick Legumens, as *Melilotus* have an ex-

exalted Oyl, and essential Salt. Periclymenum is described to have only essential Salt and Oyl, but since there is an aromatick Odor in the Flavour, and a great Acrimony, there must be some degree of volatile Salt in it. Fenagreke and Meadow Trefoyl have much Oyl and little Salt, and so has *Ophioglossum pinguicula & consolida regalis.*

6. The sweet Nuts, as Almonds, have much Oyl and essential Salt, but the bitter have more Salt than the sweet Almonds, therefore 'tis probable that the Tartar abounds more in all bitters, and that Tartar is the effect of a higher Digestion, and the crudest Tastes, as Stypticks, Sweets and Slimes have least of it. So Chesnuts and Beech-nuts have much Oyl and little Salt. Filberds are described to have essential Salt, and a little Volatile as well as Oyl; but it seems no probability that one may have it, and none in any of the other.

7. The sweet acid or vinous Tastes have much Oyl and essential Salt, as Prunes, Cherries, Strawberries, Rasberries. The Variety of the Tastes of these Fruits shews the different Digestions and Mixtures, tho' the principles are the same. The sweet viscous Fruits, as Sebestens, have little essential Salt, and much Oyl.

8. The sweet aromatic burning Tastes contain a volatile Salt and Oyl, as *Schenanthus*, Ginger, Zed-Oary, Cubebs, Cardamums, *Vaynillas*, *Contrayerva*, *Calamus aromaticus*; but these following are mistaken by the Chymists, who say that *Costus amarus, dulcis, cyperus, galanga*, Orris, have a volatile Oyl, but essential Salt only, for Orris is acrid and aromatic as well as the rest, and therefore there is both Volatile Salt and Oyl in them, and also an essential Salt from their sweetness.

9. The sweet acrid Aromaticks of the Fænil Class have all a volatile Salt and a volatile Oyl, as Angelica, Lovage, Parsly, Meum, Dill. Note, Leaves and Roots of Fænil contain a volatile Oyl and essential Salt, the Seed a volatile Oyl and Salt; but since the Roots and Leaves have a pungent Taste, there is also a volatile Salt in them, tho' the Chymists do not observe it. All the parts of Carraways have the odor of Punaises, except the Seed; from whence I may infer, that the fœtid Plants have the same principles as Aromatics, *viz.* a volatile Oyl and Salt; and this is confirmed by other Fœtids which have them, as Rue and *Assa fœtida*, and *Vulvaria*. *Peucedanum* is described of a bitter acid Taste, with the odor of Pitch, and must have a volatile Salt, tho *L' Emery* describes only its essential Salt and Oyl: So in Smallage he describes only the essential Salt and Oyl, but its acrid Taste manifests the volatile Salt.

10. The sweet Gums, as Manna, Sarcocolla, contain much Oyl and essential Salt; tho Honey and Sugar have more essential Salt than Oyl; by which we may observe how much essential Salt is in all Sweets, and why they are apt to turn sowre, and ferment; and from such sweet Gums all sweet plants have their Acid and Oyl upon Distillation.

11. Citrulls,

11. Citrulls, Melons, Gourds, Cucumbers, which are bitter Sweets, and very mucilaginous, contain much Oyl and little Salt. Since the whole Claffes of Sweet Plants contain an Oyl and Effential Salt, fome more fome lefs of both, the Vertues of the feveral fweet Taftes can never be explained by the Chymical Principles, and no new Virtues by them is difcovered; therefore all the advantage we have obtained by thefe Chymical Diftillations is only to fhew the nature of Sweetnefs in general, by difcovering the principles contained in fweet Plants, and this is a greater advantage to Natural Philofophy than to Phyfick, to which the Tafting of Plants is more ufeful. By the Tafte we diftinguifh the Sweets into their feveral Claffes, and we difcern tempers and digeftion, and mixture of their principles, and thence eafily guefs at their effects in Animals, and by the Tafte we diftinguifh the different ftate of both the Oyl and Acid in Plants of different fweet Taftes; whereas the Chymifts obferve no difference of the Tartar Acid, whether it be Acerb, or Vinous, or Volatile; nor of the fweet Oyls from the bitter and flimy. By he Tafte we diftinguifh the Acrid and the Acid Salts, which mix in diftillation; and they are not well diftiguifhed by the Chymifts. By the Tafte we difcern when the Fire makes new products and mixtures, not naturally found in Plants; for in Corn, Beans, Peas, Bread, Fire produces a Volatile Salt not obfervable in them before: And a Volatile Salt is drawn from the lees of Wine by the Fire; and Leven yields alfo a Volatile Salt; whereas before in Corn only an Oyl and Acid were obferved; and 'tis probable the Tartar is Volatilized, both by the Fermentation and the Fire. Coffee is a Bean by its Tafte and Cods, and acquires a Volatile Salt by roafting; but *L' Emery* only mentions its Oyl and fixed Salt upon Diftillation, but the fixed Salt is the effect of Fire, and its virtue depends on the Volatile.

No more Chymiftry is neceffary for difcovery of the Phyfical Vertues of Plants, than to make Decoctions of them in fair Water, and to obferve the Taftes and other fenfible qualities of thofe Decoctions, and from them to take the natural Hints for the tryal of their Virtues on Animal Bodies. I will confine my felf to the Clafs of Sweet Taftes, and give an account of feveral Decoctions made in all the fpecies of fweet Plants, and add fome Taftes I had not formerly defcribed fully, and fuch reafons as induced me to place them in the Clafs of Sweet Plants. I own it is objected againft Decoctions, that the Volatile parts expire, and the mucilage diffolving in the Water obfcures the Tafte: I therefore do confefs, that Plants are beft tafted in their natural ftate to difcover all their Vertues, but thefe Decoctions do help to confirm our Taftes, and difcover the great variety of Medicines which may be made from Sweet Taftes. Note, that all Decoctions muft be tafted cold.

A Defcription of the Taftes and other fenfible qualities in the Decoctions of Plants of the Sweet clafs.

1. The Grafs Sweets under which are contained all forts of Graffes and Rufhes, Reeds and Corn; I refer *Nymphea Alba* to the Rufhes, both for its figure and tafte, which is crude and ftyptic, with a bitterifhnefs in the Seed; but the flowers are like Lillies of a fweet fmell, and mucilaginous, fweet, ftyptic in Tafte. I boyled Horfe-tayl a handful in a pint of Water, and I found the Decoction to tafte very ftyptic with a bitterifhnefs, and the Decoction looked like Small Beer, the Tafte being like Rufhes: I concluded it to be of the fame Clafs. The ftyptic Virtue is ufeful for all Fluxes. *Plantago aquatica* boyled in Water gave a Cauftic Acrid Tafte, joyned with a crude rufh Tafte; the Decoction was greenifh and pale. This Cauftic Acrimony is ufeful in the Scurvy and Dropfy. It is *a ranunculus*, by its *capitula echinata*, as well as by its Acrimony. *Nymphea lutea* has a fweet Aftringent luftre in the Flowers, with an Acrid fmell like Creffes. A Reed is of a fweet bitter and mucilaginous Tafte, it abfterges by the bitter without Acrimony, and cools by the Mucilage Subacrid ftyptic, by which 'tis a Rufh, *Sagittara* is fweet, Subacrid, and ftyptic, by which it is a Rufh. *Alga Marina* is Subacrid and fweet, and fomething ftyptic. *Fucus* is of a fweet faltifh Tafte. The Palm-Tree is fweet and ftyptic, and, becaufe of this Tafte and its *folia arundinacea*, I refer it to the fweet ftyptic; a fweet Vinous Juice flows from its Bark being wounded, the Immature Fruit is very ftyptic, but the mature fweet, vifcid, vinous, and fubaftringent, proper for Fluxes. Wine, Vinegar and Sugar are made from the fweet Juice. The Infpiffate juice of the Palm Tree is the *Terra Japonica*, whofe fubftance is gummofe, and of a bitterifh ftyptic Tafte, and alfo of a fweet Tafte, and of a grateful odor. Boyl ℥j of *Terra Japonica* in ℔j of Water, and fweeten it for a Cough or Loofenefs. Leven is obferved by *Ætius* to be cool by its acidity, to be hot being putrid, and alfo to have fome Virtue from the Salt and Flower. He obferves that Beer is hotter than Barley, and becaufe of its putrifaction 'tis of an ill juice; and he alfo obferves, its Windinefs depends on the Air included in it; and the Waterifh Beer and the Acid is cooling. All thefe Virtues were difcovered by the fenfible qualities of Leven and Beer, without Chymiftry. And we may obferve that what we call Fermentation in them, was called by *Ætius* a Putrifaction. We have rectified the notion, and given it a new name, but the Antients knew the nature and effects of Fermentation as well as the Moderns; and 'tis that which gives the different ftates of the principles in Plants, and the feveral ftates are beft difcovered by our Taftes and Sences. I boyled *Gramen fpica fecalina*, and the Decoction was of a fweet, mucilaginous and ftyptic Tafte. The Roots of Grafs have fomething of Acrimony and Aftriction, but the Decoction taftes

fmooth and fweetifh. The green Leaves of Grafs are fweet and ftyptic. The Decoction of the Water was of a pale colour. The Seeds of Grafs are more diuretick and binding, I made a Decoction of them, which refembled Water-gruel. *Paronychia foliis rutaceis* muft be referred to the Graffes, becaufe of its fweetnefs.

2. Quere whether *Rapunculus* be not of the Sweet Clafs, becaufe the whole Plant is fweet and Milky, and for that both are given to Nurfes to increafe Milk. The Roots of the rampions are fomething ftyptic and cooling, and proper againft Inflammations. *Scorzonera* and *Tragopogon* are referable to the Graffes, becaufe of their geniculate ftalks, their Graffy Leaves, fweet Taftes and Milky Juice, the Decoction was fmooth and of a pale colour. *Trachelium* is alfo fweet, milky, fubacrid, and bitterifh, of the fame clafs.

3. The fweet mucilaginous and crude *Lychnis*, the decoction of *Alfine*, is fweetifh and mucilaginous, the colour pale like Small Beer. I could obferve no aftringency in *Alfine*. I boyled ℥j in a pint of Water, it hath a cooling and moyftening quality like common *Lychnis*. *Diofcorides* fays, Chickweed being bruifed has the fmell of a Cucumber. All the common *Lychnis*'s are bitter fweets, and them I fhall refer to the clafs of Bitters, tho their joynted Stalks and fweetnefs may juftly place them here, yet their chief Phyfical Virtue is from their Bitternefs.

4. The Sweet Bitterifh ftyptic, and Subacrid Fern. I boyled ℥j of Fern Roots in ℔j of Water to half, the Tafte was very ftiptic and Bitterifh, the Colour Citron. The Mucilage and ftypticity make Fern an excellent Vulnerary, and ftyptic in all Fluxes: the young Buds rubbed in the Fingers fwell fomething like a Kernel, or the Laurel fmell. *Dryopteris* is defcribed Aftringent and Sweet, Acid and Bitterifh. *Hemionitis* has both Aftringency and Bitterifhnefs. *Adianthum album & nigrum* are Sweet and ftyptic, and fince Experience fhews thefe to be good Pectorals, the other Ferns have the fame Pectoral Vertue, as *Vulneraries*. *Herniaria* is defcibed as a ftyptic. In *Adianthum* there is fomething odorate. *L'Emery*. Polypody is Bitterifh, fweet and Aftringent, Naufeous and Subacrid and flimy, it purges by this Tafte both Choler and Flegm. I boyled Harts Tongue ℥j in ℔j of Water to half; it was of the colour of Small Beer; the Tafte was Mucilaginous, fweet, ftyptic, with an Aromatick flavor of Rasberries or Orris, which is its Cordial Vertue, joyned with ftypticity and Mucilage, by which 'tis proper for hot Hypochondriacs. The Roots of *Ofmunda* are of a Subacrid and Bitterifh Tafte, befides the Aftringency, by which they open but the Aftringency much hinders that effect.

5. The fweet, Acrid, Aromatics and Fœtids, the Roots have more fweetnefs and Aromatic Acrimony than the Leaves, which are more crude; the Seeds have moft Aromatic Acrimony. The Seeds of *Siler montanum* have an ungrateful fmell like Cummin, and a Bitterifh Tafte. *Peucedanum* is Bitter and fweet. The green Leaves of Coriander are fœtid like *Punaifes*. *Laferpitium* is Acrid, Aromatic in fmell, and of a

 fweet,

fweet, Acrid, Aromatic Tafte. Sampire is of an Acrid Aromatic Tafte, and the fmell of Smallage. Cumin is Acrid, Aromatic and Bitterifh, and of a difagreeable fmell. *Dracunculus Hortenfis* is fweet like Anis. *Ferula* has an ungrateful Aromatic Odor. The Seed of *Meum* is bitter. *Cachrys* is Bitter, Acrid, Aromatic. *Cancalis* and *Daucus* agree in Virtue and Tafte. *Saxifragia pratenfis* has a great Root, fweet, and Acrid; the Leaves are moft Aromatic, and the Seeds; it refembles the Tafte of Parfly. The Leaves of *Gingidium* are of a difagreeable ftrong fmell, and the Roots bitter. *Tordylium* is fweet, Aromatic, *gravi odore.* *Percipier* is like Chervil, and fo is *Pecten Veneris.* I refer the Umbells to the Graffes, becaufe of their Sweetnefs and Joynted ftalks. Our Botanifts have omitted the fweet Tafte in fome of them, which are bitter. Moft of them are of an Aromatic fmell and Tafte, but fome are fœtid, virofe and fervid in their Tafte. *Smyrnium* Root has the fmell of Myrrh, with a Bitter Acrid Tafte, and it helps the Urine and the Menfes like it. The colour of the Specifick Juice is various; *Thyffelinum* and *Sefeli pratenfe* have a Milk: The Ferula's are Milky, or have a Saffron colour: Their Juices make *Sagapenum* and *Galbanum.* The Juice of the Root of Peucedanum is reddifh, and is called *Opoponax odore piceo vinofo.* The Root of *Syphondilium* has a Saffron colour'd Juice, of a fœtid and bitterifh Tafte. 'Tis obferved that fome feeds grow bitter by the fault of the foyl in which they grow. All of this clafs have a Volatile Oyl and Acrid falt, by which they are diuretic, carminative, and pectoral, alfo by their fweetnefs, Emmenagogue, if fœtid. I boyled Parfley Roots ℥j in ℔j of Water to half, the Tafte was fweet, and Acrid, Aromatic, the Colour pale. The Roots of *Meum* are Acrid, and fmell ftrong; too much of it offends the Head. *Libanotis* has the fmell and Tafte of the feed of *Angelica.* *Sium* or *Apium paluftre fol. oblongis* has an odor of Bitumen on all the Plant. *Thyffelinum* is Bitter, Ingrate and Acrid. The Decoction of Angelica Roots are bitterifh, Aromatic, Acrid, and of a yellowifh Colour. *Imperatoria* Roots decocted fmelled like Angelica, and tafted very Bitter and Acrid; of a green Colour.

6. The fweet, Acrid, Terebinthinate and ftyptic. *Calamus, Nardus, Cyperus,* agree in their Diuretic, Carminative and Emmenagogue Virtue, and their ftypticity, *&c.* *Calamus* refembles a Reed; it is an *Acorus,* the Tafte refembles the Turpentine Plants, as well as its Cones. *Juncus odoratus* is a Rufh, with the fmell of a Rofe, when rubbed of a Burning, Acrid, Aromatic and Bitterifh Tafte. *Cyperus* is of a pleafant Odor, like *Lign. Aloes* whilft it flowers, and of an Acrid, Aromatic, Bitterifh, ftyptic Tafte. *Nardus* refembles the flavor of *Cyperus*; 'tis Bitterifh, Aftringent, Acrid and Aromatic. The Roots of *Cyperus* are ufed for *Nardus.* There is fome fœtor in *Valerian, Afarum, Serpentaria,* whofe Roots have a little Sweetnefs, with a Terebinthinate Bitter Acrid, and all of them refemble *Spikenard odoris gravitate,* and have the fame Diuretic, Carminative, Emmenagogue Vertue,

Vertue, and are proper for Malignant Fevers; and a little ſtyptic. The Red Valerian has a crude Taſte in the ſtalks and Leaves, the Flowers rubbed ſmell like Turpentine; the Leaves ſmell nauſeous, fœtid; the Roots agree with Spikenard. *Spica Celtica* and the Flowers of Valerian agree well. The Leaves of Wild Valerian boyled in Water yield a crude and fœtid Mucilage. The Tincture of the Roots of Valerian, extracted with Spirit of Wine and Sal Armoniac, fines it much. The Roots of Aſarum are Geniculate as Graſſes, and taſte very Acrid, Aromatic and Bitteriſh; 'tis called *Sylveſtre Nardum*, and for its ſimilitude referrible to it. The Antients attributed the ſame Vertue to *Aſarum* as to *Acorus*, but more intenſe and ſtrong. *Galen* obſerved that *Phu* was an Odorate Root, like *Nardum* in virtue. *Pliny* obſerved that Cyperus was a *Gladiolus bulboſa radice*, and like the Odor of *Nardum*. The Decoction of *Aſarum* was greeniſh, with a ſtrong ſmell of Spikenard, and Bitteriſh Acrid. The Decoction of Valerian Roots of the ſame Taſte, but pale colour, and ſmell of Spikenard, but weaker; they ſeem to be of the ſame Claſs.

7. The Sweet, Fervid, Acrid, Bitteriſh, and Aromatic like Orris in Taſte. The Roots of Orris are Geniculate like Graſs. The *Florentine* Roots are bitteriſh and ſweet, and of a burning Taſte. The Water Flag is burning and ſtyptic, but of no ſmell; The Flowers of Common *Orris* have an ill Smell, though the Roots be Aromatic. The whole Herb ſmells like Elder whilſt 'tis freſh bruiſed, but when dry 'tis Odorate; it gives a Rasberry Taſte to Drinks, and purges. The great Galangal is Aromatic, Acrid, Burning and Bitteriſh; the Roots alſo geniculate, odorate; in form like Cyperus. The Roots of Acorus are geniculate, acrid, burning, bitteriſh, and aromatic; it reſembles Orris both in Leaves and Roots. Ginger is acrid, burning and aromatic like Pepper; the Leaves are like *Iris Paluſtris.* *Zedoary* ſmells like Camphire, and is of a ſtrong Taſte, rather than ſweet, 'tis very bitter, and leſs acrid; but reſembles Ginger. Coſtus is very burning, acrid and aromatic, and bitteriſh; it agrees with the vertue of Orris, both in its diſcuſſing quality and deobſtructing. *Coſtus Dulcis* has a ſweet Taſte, and acrid; but the bitter has an ungrateful kind of Taſte; but there is but one ſpecies of *Coſtus*, the freſh is ſweet and white, the old is bitter and blackiſh. *Gladiolus* has a bulboſe Root ſweet, and moderately acrid or burning; and the Leaves are like Orris. *Xyrs* is of an ungrateful fœtid odor, like Cimices; in form and figure like Orris. *Coſtus Arabicus* is acrid, bitter and aromatic in Taſte. The Roots of *Contrayerva* ſmell like Fig leaves, are like Orris Roots in figure; they being chewed taſte ſweet, aromatic and acrid, and have alſo an aſtringent Taſte. *Cardamomum* has the Stalk and Leaves of a Reed, the Seeds in cods of a burning, acrid, bitteriſh, aromatic Taſte. *Amomum* by its external figure and virtue agrees with *Cardamomum*; ſome reckon it like Acorus. *Grana Paradiſi* are of a fœtid and aromatic Taſte, betwixt Cardamums and Pepper, and agree with the ſame in vertue in Paralytic caſes. The Grains of

Para-

Paradice infused in Wine gives the odor of Quinces. *Salmon.* The Roots of our yellow Flag are large and reddish; our Countrymen scrape them and pound them, then mix them with Milk, and give them twice to a Dog, as an infallible Medicine against the bite of a Mad Dog. A person who took them told me they did not purge, but tasted very rough. This Decoction is of a reddish colour and rough.

8. The sweet mucilaginous Pea-tastes, or Legumens. I boyled *Vicia* ℥j in ℔j of Water to ℔ß, which tasted sweet and crude styptic, the colour was pale and greenish. I boyled *Lens Palustris* in the same manner, and found the Decoction of a turbid colour, and reddish like Mum; it is a Legumen by this intense colour, as well as by the Taste, which was sweetish, subacrid and styptic. Note, that Snails and other Insects are usually mixt with it. I boyled *Lagopus* in the same quantity as the other; the Taste of the Decoction was styptic, crude and bituminous. The *Cortex Lentium* is styptic and binding; the Seed of it mucilaginous and loosening; and this breeds an ill, flatulent and crude Juice, obstructing the circulation of the Blood and Spirits, injurious to the Sight and Menses; and by their flatulency producing turbulent Dreams. The Leaves of Liquorish feel gummose, the Roots are sweet and subacid. Tamarinds are of an acid, acrid and sweet mucilaginous Taste; the Stones have the Figure of Lupins, the Leaves of Tamarinds are of an acid agreeable Taste; and they must be referred to the Services and Berberries, tho by the Cods and Seeds I have erroneously class'd them with Cassia or *Lupins*. *Fumaterry* is subacrid, bitterish, mucilaginous and legumen, tho the Flowers be not. *Radix Cava* has bitter sweet Roots like Beans, and a mealy subacrid Taste; the Leaves have a crude smell like Fumiterry, and of the same virtue. The Skin of the Bean is styptick, but the fleshy part bitterish leguminous. Coffee is bitterish and of the Bean kind, in which there is a volatile Salt, which raises the Spirits and produces Urine and the Menses. *Orobus* is of an ingrateful bitterish Taste; all the bitter Legumens open obstructions and promote Urine, as Kidney Beans, *&c.* I boyled Broom ℥j in Water ℔j to ℔ß, the Taste was moderately bitter, sweet and styptic, not nauseous, and without any heat. I boyled of *Genista Spinosa roots* ℥j in ℔j of Water to ℔ß, the Colour was like small Beer, the Taste mucilaginous, styptic, without any great bitterishness or acrimony; the Root tastes bitterish, mucilaginous styptic; it may be proper for the Stone by those Tastes, and for Fluxes. The Leaves of *Bruscus* bruised have the smell of Broom, and its bitterishness, stypticity and sweetness; and are subacrid, and by this Taste a Legumen. *Crato* describes Senna as if it had a *Viscidum quid*, by which it gripes; and by the Taste he discovered the Bitterishness and Astriction. *Matthiolus* observed its viscous Taste and Bitterishness, with a virose nauseous odor, like *Cynoglosse* or *Punica.* The siliquæ, as well as the sweet Taste and Flowers, prove it a Legumen. *Polygala* boyled with the Blue Flowers gave a Colour Blue like Violets, and the Taste was very

very mucilaginous, the Plant is aromatick, acrid. ℥j of Violatri Color boyled in ℔j of Water to half, made a Decoction of a greenish Colour, like Cowslip Wine, and it tasted gratefully and mucilaginous; the Roots of *Polygala* are acrid and aromatic; it purges Bile. The Decoction of the Root of *Periclymenum* is styptic, bitter, and of sweet leguminous Taste. Purple Trefoyle is mucilaginous and styptic, by which it cools Feaverish Heats, and by its burning Acrimony it expels the putrid particles in Perechial Feavers and Fluxes. The Leaves of red Trefoyl rubbed smell acid, and the Flowers bruised smell like Woodbind. The Decoction of ℥j in ℔j of Water to half gave a greenish Colour, pale; and the Taste was of a crude mucilaginous leguminous Taste, with an acrimony, by which 'tis a great Diuretic and Ophthalmic. The Decoction of Purple Trefoyl has a turbid Colour, and mucilaginous Taste. The Leaves of *Periclymenum* are acid and acrid; the Flowers more sweet, the style is of a Bean Taste, and the Twigs of the same: The *Stamina* have the Taste of the Flower, and their Heads are very acrid.

Trifolium Fruticans is Bacciferous, with a blue Juice, and though it wants Cods 'tis a Legumen, as well as Periclymenum, which is bacciferous. The Flowers of Asparagus are hexapetali; and of an Herbaceous Color; the Seed is a Pea, without a Cod, the Root is of a sweet glutinous acrid Taste; the Tops of it boyled resemble Peas Pottage, and are evidently of a Leguminous Taste. In the Legumens these are irregular, in Flowers or Cods, they are notwithstanding certainly of that Class; by which we may infer, that the Taste gives the most certain character of a Class. The *Herba Mimosa* has an Herbaceous odor and a mucilaginous bitterish ingrateful Taste; the Root is of a Taste more grateful, without bitterishness, but of a violent smell like Garlick when first got, offending the Smell and Head, and is accounted a Poyson. *Morison. Mesue* observes that *Psyllium* has contrary Virtues; 'tis of a mucilaginous, acrid and nauseous Taste: The *Medulla* is hot and exulcerating; the *Cortex* is moystening and cooling. *Gossipium* is a Plant like Linum, both in Leaves, Flower and Stalk. There is bitterness in *Linaria. Morson. Linaria* is by its Flower of the leguminous Class, and probably Linum is of the same, tho the Flowers disagree, being like it in Leaves. Linseed has the same Virtue as Fenugreek, which is a Legumen. The Leaves of Flax are gummose and bitterish, subacrid; the Flowers are *Pentapetali*, which differ from the Legumens. I boyled ℥j of *Galega* in ℔j of Water to the half, the Color was pale, the Taste nauseous, bitter, sweet and mucilaginous and the Odor nauseous and fœtid. The fresh Leaves of *Trifolium Asphaltites* smell like Rue, wehn ripe like Bitumen. The Decoction of *Ononis* was mucilaginous, subacrid, the colour was greenish. The Leaves of *Consolida Regalis* are of a crude Smell, the Seeds are in Cods and Taste leguminous. The whole Plant is of a disagreeable Taste. *Glastum* and *Opioglossum* are of the same Virtue, and *Luteola* is like them; place them here or with the Cresses.

Tha-

Thalictrum must be referred to the Docks both by its Taste and Virtue *Lignum Nephriticum* is subacrid and bitterish.

Of the inward use of Cantharides, *by Mr.* Yonge, *n.* 280. *p.* 1210.

V. A Gentlewoman of 54 Years old, who for a long time had been tormented with frequent Fits of the Stone, and usually brought off many, with the Gravel, *&c.* about a Year since grew Dropsical, of which being lately cured, she fell into a total Suppression of Urine, which many days baffled all Remedies. In this condition, about four in the Afternoon, the fifth Day of the Disease I gave her five Cantharides (without Heads, Wings or Legs) weighing four Grains and a half, and with as much Camphire and a little Conserve, made them into 2 Pills or Bolus's. Next Morning, finding no effect good or bad, I repeated the Medicine; after which, about noon the Flood came, and continued above 48 Hours, bringing off in that time much more Urine, than could have been expected from her in the whole time of the Obstruction. Some Gravel and *Sabulous* matter came away, but no Stones, nor did there any thing happen to the Stomach, Bladder or other Bowels, as usual on the internal use of those Insects, but they operated so quietly, as if nothing but two Doses of *Lapis Prunellæ* had been administred.

In several other Cases I have often and successfully given it, and without the *Dysuria* and other painful accidents which attend the internal (oftentimes the external) use of this Remedy; altho' I mixt no Camphire, but I washt it down with large Draughts of Posset, Ptysan, Emulsions or Water-gruel; which in this Ladies Case I forbore to use, because of her Dropsical Dispósition, and used only a Draught or two of middling Ale, impregnated with Broom, Juniper-Berries, Daucus Seeds, *&c.* The form in which I used to administer this fiery Insect, is that of a soft Pill or Bolus, composed of three *Cantharides* prepar. *Troch. e Myrrha* ℈ß *Sem. Amei* gr. vj R. *ob Cynosb.* q. s. This in stubborn Suppressions of the *Lochia* and *Menstrua*, in difficult Child Birth, and Retention of the *Secundine*, does Wonders; what heat or pain it begets in the Neck of the Bladder, is much short of what I have an hundred times seen (and sometimes felt) to proceed from applying an Epispastick to the Back.

About twenty Years since, an enamoured Youth attempted to gain a coy Girl's Love, by giving her a Plumb-Cake, in which powder'd *Cantharides* were mixed; She eat part of it, and gave three others of the Family in which she lived a share: they were soon tormented with burning in the Stomach, bloody and scalding Urine, and great Pain in the Back; I cured them all in a short time, by Powder of *Amoos* and *Lapis Prunellæ*, and Emulsions *aq. Sperm.* R. *&c.* How many *Cantharides* each devoured in the Bread, I could guess by a piece of it which remained, and suppose that viii or ix Grains fell to each share.

VI. In

VI. In *July*, *Anno* 1678. at *Montpelier*, we gave a Dog a piece of Bread steep'd in two Ounces of the Juice of *Dutch* Night-shade [*Solanum Batavicum*] exprefs'd from the green Plant, and mixt with Cheese. As far as we could perceive, he did not seem to receive any manifest Damage from it. The same Dose of the Juice of the Leaves of *Hemlock* [*Cicuta*] had no more effect. We gave also the same Dog a pretty large Root of *Wolfs-Bane* [*Aconitum Pardalianches*] together with the Leaves and Flowers of the same Plant bruised and mixed with Flesh; which did him no Hurt. Two Drachms of *White Hellebore* [*Helleborus albus*] very much disorder'd him, and caused Reachings, Suffocations, Vomiting, and Voiding of Excrements. This Dog (as afterwards we often observed in others that had taken the like Corrosive Medicines) whether because he was not able to endure the Pain, or by reason of any other Uneasiness, often scratched the Ground with his Feet: However he recover'd, and was well again. He swallowed also five Roots of *Meadow-Saffron* [*Colchicum Ephemerum*] dug fresh out of the Earth: With which he was violently tormented, but did not dye.

Effects of several sorts of Poysons on Animals, by W. Courten, *Esq*; *n.* 335. *p.* 485.

At last he took two Drachms of *Opium*, which cast him into a deep sleep; but after vomiting and voiding fœtid Excrements, he recover'd by degrees his former Briskness. So many, and those so notorious Poisons, could not kill this Dog.

Some Weeks after, when the same Dog had recover'd his former Vigour, we try'd on him the Force of a much stronger Poison. We caused him to be bit three or four times on the Belly, a little below the Navel, by an inraged Viper. There arose immediately little black Bladders, containing a liquid blackish sort of Corruption; they were flaccid and tremulous, like the Gall Bladder when it is about half full; and a livid Colour by degrees spread over all the neighbouring Parts. The Venome propagated it self with wonderful Quickness, and weakened all, but more especially the animal Functions: For notwithstanding the Diaphragme did still perform its Office pretty strongly, tho' with some disorder, and the Heart continued beating, tho' faintly and irregular; yet they seemed to fare much better than the Brain, whose Strength was so weakened, that it could not perform the Functions of Sense and Motion but very faintly; insomuch, that the Dog lay without any Strength or Sensation, as if he had been seized with a Lethargy or Apoplexy: Which kind of Stupidity we also observ'd sometimes in a greater, sometimes in a less degree, in all other Dogs bit by a Viper. Being willing to save this Dog, (though we had found by many Experiments, that much slighter Wounds made by a Viper had occasioned Death) we thought fit to have recourse to several Remedies; and therefore cupped and scarified the part that was wounded, and applied Treacle [*Theriaca.*] After this we let him alone for about two Hours: But his Sleepiness encreasing

more and more, and his vital and animal Functions finking, we were forced to have recourfe to another Method of Cure. Wherefore to difpel his Sleepinefs, we forced into his Throat half a Drachm of Volatile Salt of Hartfhorn mixt in Broth ; which we eafily did, by reafon of his Weaknefs. In a little time after, his Eyes, which before looked dead, began to revive, and he was able to ftand on his Feet and walk. Whereupon we repeated the fame Dofe of the Volatile Salt, by which he was freed from his Sleepinefs, and the Strength of his Heart recover'd ; and notwithftanding he remained weak for three days, yet he fenfibly recover'd Strength, though he would eat nothing all that time : But he drank Water very plentifully and greedily ; and on the fecond day did not refufe cold Broth. After the third day he began to eat folid Meats, and feemed out of danger ; only fome large foul Ulcers remain'd on that part of his Belly that was bit, of which he would fcarce have died, had he not been killed by another Dog ; which prevented us from feeing the Event of this Experiment. But to try more fully the force of the abovementioned Poifon, it is neceffary to make feveral Experiments of it : For tho the Bite of a Viper, if it be but flight, may kill fome Dogs, yet in the Month following, a large ftrong Dog, that was bit in the Tongue, which is a very dangerous part, recover'd without any Medicines. His Tongue indeed turned black, and fwell'd fo much, that it could fcarce be contained in his Mouth : He was ftupid, as is ufual from the Venome of a Viper, but not fo much but that he could ftand on his Feet. A few Hours after, his Sleepinefs decreafed ; and the next day he endeavour'd to lap Water, but the bignefs of his Tongue prevented him. On the third day he threatned to bite any Body that difturb'd him, and had recover'd fo much Strength, as to be able to efcape out of the place where he was kept : And two after, was feen in the Streets ; but what became of him afterwards we could not learn.

On the 17th of *October* we gave a Dog fifteen Grains of the dryed Root of *Monks-Hood* [*Napellus*] powder'd, and mixt with Flefh and Broth. He had no fooner taken it, but he was feized with a difficulty of fwallowing, or rather feemed as if he was like to be ftrangled. He immediately grew faint and reftlefs, and dug the Ground with his Feet ; but foon defifted, by reafon of a fainting Fit, as we imagined from the dull Colour of his Eyes, and a Weaknefs of all his Body. This Fainting was prefently fucceeded by a violent Vomiting, in which he threw up the Flefh thathe had eaten, which was very little alter'd. His Fainting foon returning again, he laid himfelf on the Ground ; but being feized with terrible Convulfions of the *Abdomen*, Diaphragme, and of almoft the whole Body, he run from place to place, and vomited fo great a quantity of frothy Matter, that he was like to have been ftrangled. His vomiting encreafed, with a kind of crying and fobbing, like broken Sighs, as if he had endeavour'd to bark at thofe that ftood by. In this manner he was miferably tormented for

the ſpace of an Hour; at which time all his Symptoms remitted, and by degrees he recover'd. In the Summer before, we gave a little Dog a Drachm of the Root of *Monks-Hood* [*Napellus:*] He was ſoon after ſeized with the ſame Symptoms, but they were longer and more violent; and he in like manner recover'd. In both theſe Dogs we particularly took notice of theſe broken and interrupted Sighs, or kind of ſobbing; becauſe we did not obſerve the like to be occaſioned by any other Poiſon that we had made Trial of. An Ounce of the Leaves, Flowers, and Seed of the *Napellus* when green, being bruiſed and given to a Dog, ſcarce diſorder'd him any more than if he had eaten ſo much Graſs. About the ſame time we made tryal of the *Nux Vomica* on another Dog; not that we doubted of its being a Poiſon, but that we might ſee the effect of it on his Body when dead. The Dog accordingly dying in a ſhort time, we found his Stomach and ſmall Guts very red; and judged this Redneſs and Inflammation to be cauſed by the Corroſiveneſs of the Medicine.

On the 20th of *October*, we injected warm into the Jugular Vein of a ſtrong luſty Dog an Ounce of Emetic Wine [*Vinum Emeticum:*] For a quarter of an Hour after the Operation was over and he was let looſe, he continued pretty well, unleſs that he ſeemed ſomewhat dejected; but afterwards he began to grow ill, and an unuſual Agitation was manifeſt about the Diaphragme; this was followed by a continual Vomiting, and a little after by an Evacuation of ſome hard Excrements. By theſe Evacuations he ſeemed to be ſomewhat relieved, but ſoon grew uneaſy, moved from place to place, and vomited again. After this he laid himſelf down on the Ground pretty quietly; but his Vomiting returning again, diſturbed his Reſt, and abated his Strength, which grew weaker and weaker; for in the ſpace of an Hour he vomited 12 times or more, and ſometimes voided ſome liquid Excrements, but in ſmall quantity; having frequent Inclinations to go to Stool, but in vain, as in a *Teneſmus*. An Hour and half after the Operation, he being ſo weak that he could not ſtand, his Eyes dull, and looking as if he were half dead, we gave him ſome warm Broth thro' a Funnel. With this he was wonderfully refreſh'd immediately, and ſoon after could look about, ſtand on his Legs and walk; but by reaſon of his weakneſs, reel'd as if he had been drunk. We left him by himſelf in a warm Room, where he remained cold, and lay as if he had been dying; and in an Hour after, we forced him to take ſome more Broth, which revived him again: But in a little time, after ſome agitation of his Body, he vomited, made Urine very plentifully, howled miſerably, and dyed convulſed. Next Day in viewing his *Viſcera*, we found two things very obſervable (but neither of them occaſioned by the Liquor that was injected;) one of them was in the Heart, the other in the *Æſophagus*. In the Heart there were two *Polypus's*: That which poſſeſs'd the right Ventricle, ſtretched itſelf into the *Vena cava* and Pulmonary Artery; and that which was in the

left Ventricle, sent Branches into the adjoyning Vessels, and was less than that in the Right Ventticle. The Substance of the *Polypus* was pretty firm, of a Flesh colour, somewhat pellucid, and being cut thro' the middle was altogether of the same colour and consistence as on the Surface. To the *Æsophagus* there grew a remarkable Gland, which was hard, callous and foul, and opened with a small, round, fleshy Orifice into the Inside of the Stomach, where, upon pressing it, a little Corruption came forth. Upon opening this Gland or Tubercle, we found in it a great many little Worms, wrapt and entangled together, and moisten'd with a corrupted Matter. Some of these Worms were above 4 Inches long, others less. Afterwards we found the like Glands, full of Worms, in other Dogs, and in most we opened, but not so much corrupted as in this. We observ'd also the like foul Glands in the *Aorta descendens*, but in one only found a Worm like these, which was almost got out of it, thro' an Orifice, into the Cavity of the *Thorax*. After this we likewise observ'd more *Polypus*'s in Dogs.

On the 27th of *October* we injected warm into the Jugular Vein of a Dog a Drachm and half of *Sal Armoniac* dissolved in a Ounce and half of Water. The Liquor had scarce arrived at the Heart, but the Dog presently fell into deadly Convulsions over his whole Body: Wherefore we let him loose, but he dy'd immediately. On the 18th of *November*, we caused a Whelp to be bit in the lower Lip by a Blind-Worm [*Cæcilia*] so that the Blood appear'd in the Wound. The Whelp dy'd indeed the same Day; but because we had committed him to the care of another Person, we could not be certain whether he dy'd of the Wound or not; and what increased our Suspicion, was, that there did not appear on the part that was bit any livid Colour. On the 12th of *December* we injected into the Jugular Vein of a Dog a Drachm of *Salt of Tartar* dissolved in an Ounce of warm Water: He dy'd crying, and in Convulsions, almost immediately. On the 15th of *December* we found a *Polypus* in both the Ventricles of the Heart of a Dog, each *Polypus* stretching itself with a double Root into the Vessels of the Ventricle it possessed. Afterwards we often observed the like *Polypus*'s in other Dogs.

On the 20th of *December* we injected warm into the Jugular Vein of a Dog an Ounce of Urine made by a Man fasting. The Dog was uneasy during the Injection, and while the Liquor passed to the Heart; but was not seized with any Convulsions or other ill Symptoms; and being let loose, eat Breed very greedily. The same Day we made a gentle Decoction of two Drachms of White Hellebore, well powder'd, in Spring Water, and evaporated it away to Nine Drachms and a half; and the next Day injected all the Decoction, strongly pressed out and turbid, into the Jugular Vein of a Dog. At first some few Drops only passed to the Heart, some concreted Blood obstructing the Passage; but those Drops very much affected the Dog, for he was seized immediately with convulsive Motions: But soon after, when the Liquor had

had removed what lay in its way, and had enter'd the Heart, it killed the Dog as ſuddenly as if he had been ſhot thro' the Heart with a Bullet; for having looſen'd him preſently, to ſee if any Life remain'd, he was quite dead and flaccid, and hung like a Fleece in the hand of the Perſon that held him.

On the 2d of *January* 1679. Vinegar was injected warm into the Jugular Vein of a Dog, without doing him any manifeſt harm. The ſame Day we cauſed a Whelp to be ſtung in the Tongue by ſeveral Scorpions; but the Wounds made by the Scorpions, by reaſon of their weakneſs, being but ſlight, and not penetrating deep, we made a ſmall Inciſion on the *Abdomen*, and drawing aſide the Skin, let the Scorpions make ſeveral Wounds on it; but without any effect; tho' we often forced the ſting into the Wounds, and preſſed the Bladder that is ſuppoſed to contain the Venome. In like manner a *Pidgeon*, being ſeveral times ſtung by a Scorpion, remained unhurt. *January* the 3d, two Drachms of Sugar diſſolved in an Ounce of Water, was injected into the Jugular Vein of a Dog: He received no harm from the injection, but continued well for the three Days after, that we kept him. On the 4th of *January*, a Drachm and half of *Spirit* of *Salt*, diluted in an Ounce and half of Water, and injected in the Jugular Vein of a Dog, killed him immediately. In the Right Ventricle of his Heart, we found the Blood partly grumous and concreted into harder Clots than ordinary, and partly frothy. In the ſame Dog that Gland that contains Worms, and is frequently found in the *Æſophagus*, opened with two Orifices into the Cavity of that Part, and in the *Sinus*'s of it there lay ſeveral ſmall Worms. *January* the 5th, we gave a Dog 12 ſmall Caterpillers of the Pine-tree [*Pityocampæ, vel Erucæ Pini*] weighing ½ a Drachm, which we bruiſed alive, and mixed with Fleſh. The Dog, tho' he was but young, received no other hurt, than that now and then he ſeemed as if he endeavour'd to ſwallow ſomething, or was troubled with an Inclination to Vomit; from whence we judged the Stomach and *Æſophagus* to be only lightly affected: But theſe Symptoms vaniſhed in a few Hours, and the Dog continued Briſk, and greedy of Meat, all the reſt of the Day. The ſame Day we included a Rat in a large Glaſs with a Scorpion; but the Scorpion, being dull and benum'd with the extream coldneſs of the Weather, was able to wound the Rat but very weakly; with which however the Rat being provok'd, ſet upon the Scorpion, and knawed off and devour'd part of him keeping his Eyes ſhut all the while, that he might not be hurt by his Claws or Sting. The ſame Fate happened to another Scorpion, which we added to the former; but the Rat notwithſtanding remained unhurt. *January* the 6th, we killed a Dog almoſt in a Moment, by injecting into his Jugular Vein an Ounce of *Spirit* of *Wine*, in which there was diſſolv'd a Drachm of *Camphire*. The ſame Day we injected warm into the Crural Vein of a Cat, 50 Grains of *Opium*, diſſolved in an Ounce of Water. The Cat preſently after the Injection ſeemed very

much dejected, but did not cry; only made a low, interrupted, complaining Noise. After this followed Tremblings of the Limbs, Convulsive Motions of her Eyes, Ears, Lips, and almost of all parts of her Body, with violent Convulsions of her Breast: Sometimes she would raise up her Head, and seem to look about her, but her Eyes were very dull and deadish; and tho' she was let loose, and had nothing tied about her Head or Neck, yet her Mouth was so filled with Foam or Froth, that she was like to be strangled. At last, her Convulsive Motions continuing, and being seized with a stretching of her Limbs, she dy'd within a quarter of an Hour. Upon opening her Body, we did not find the Blood much altered from its Natural State.

February the 7th, we injected into the Crural Vein of a lusty strong Dog a Drachm and half of *Opium*, dissolved in an Ounce and half of Water. The Dog immediately shewed the great Pain he endured, by a violent struggling of his whole Body, a loud Noise that he made, notwithstanding his Jaws were tied, a great difficulty of Breathing and Palpitation of the Heart, with Convulsive Motions of almost all Parts of his Body: In a little time these remitted, and he was seized with a profound Sleep, as if he had been in a Lethargy or Apoplexy. Having let him loose, he lay upon the Ground without moving or making any Noise, in so deep a Sleep, that he would not move with beating. About half an Hour after, if we beat him, he would move a little, but presently lay down again. After an Hour, if we beat him, he would move a little more; and by degrees his Sleepiness a little decreasing, in an Hour and half or two Hours time, when he was beat he would make a Noise and walk a little, but seemed very heavy and stupified, and reeled as he went; but as soon as we left off beating him, as if he had soon forgot every thing that had past, he presently laid himself down again and fell asleep. Next day when we viewed the Place where he lay, we found a great quantity of fœtid Excrements, like a corrupted Blood, or the diluted *Opium* that he had taken: But still his Drowsiness continued, and tho' we beat him with Whips, that he ran crying about the Room, yet he presently forgot it, and immediately fell asleep again. In this sleepy Condition he continued 3 Days, refusing whatsoever was offer'd him to eat, or rather not minding that or any thing else. On the 4th Day we found him dead: But perhaps he would not have dy'd of the stupifying Quality of the *Opium*, if (considering the extream coldness of the Weather) we had put him in a warmer place, and had forced him to have taken some Broth. *February* the 8th, we found in the Bladder of a *Tortoise*, adhering to its Coat, a flat porous Stone, about twice as big as a Lentil. *February* the 9th, a Drachm and half of *Common Salt*, dissolved in an Ounce and half of Water, was injected into the Jugular Vein of a Dog. After the Injection, he was thirsty, and drank Water greedily; but in other Respects he seemed to be pretty well, and the next Day was quite recover'd.

February

February the 20th 1679, we injected into the Crural Vein of a little Dog, half an Ounce of warm *Oil of Olives*, which we did with a great deal of difficulty, and very flowly, by reafon of the fmallnefs of the Vein and thicknefs of the Liquor. For half a quarter of an Hour that we were injecting the Liquor, the Dog did not feem to be uneafy or out of order; but after that, he barked, cryed, looked dejected, and fell prefently into a deep Apoplexy; fo that his Limbs were depriv'd of Senfe and Motion, and were flexible any way at pleafure; his Refpiration ftill continuing very ftrong, with a fnorting and wheezing, and a thick watery Humour flowing in great quantity out of his Mouth, which was fometimes mixt with Blood. He loft all External Senfe: His Eyes tho' they continued open, were not fenfible of any Objects that were put to them; and we touched and rubbed the *Cornea* (as fenfible a part as it is) without any more fign of his being fenfible of it than if he had been dead. His Eye-lids notwithftanding had a Convulfive Motion: His Hearing was quite loft; and his Feeling, tho' at firft he feem'd to have fome fmall Senfe of it when we touched his Wound, yet afterwards it was fo dull, that we pinched his Claws and Flefh with Pincers, and bored Holes thro' his Ears, without his moving or feeming to be the leaft fenfible of it. It is worth obferving, that in the midft of his Sleep, being fometime feized with a Convulfive Motion of his Diaphragme and other Mufcles that help Refpiration, he would bark ftrongly as if he were awake, and in a little time would be quiet again: So that in lefs than a quarter of an Hour his Reft would be difturbed 3 or 4 times with this violent Barking. But confidering this more attentively, we found that at the very time he barked, he was as void of Senfe as before; for we could neither make him Bark, nor leave off Barking, by either beating or pricking him; but in a little time he would leave off of himfelf, and return to it again fome time after. Thus in three Hours after the Injection, fpent in Sleeping and Barking, he dyed; and having opened his Body after he was dead, we found the *Bronchia* of the Lungs filled with a thick Froth. A few days after we injected a larger quantity, *viz.* an Ounce of *Oil of Olives* into the Jugular Vein of a Dog, which fuffocated him the fame Moment. Afterwards the fame quantity of *Oil of Olives*, being injected into the Jugular Vein of a Dog, killed him in an Hours time. He was feized with a great Sleepinefs, Snorting and Wheezing, and a bloody Water run plentifully out of his Mouth. In this Dog, tho' he did not dye immediately, we did not obferve the Barking as in the former: But in all that were fuffocated by Oil, we found their Lungs filled with a very thick Froth. *February* the 27th, we injected 10 Drachms of highly rectified *Spirit of Wine* into the Crural Vein of a Dog. The Dog died in a very little time, very quietly, and as it were with pleafure, licking his Jaws with his Tongue, and breathing quick, but eafily, without barking, crying, or any Convulfive Motion. In the *Vena cava* and right Ventricle of the Heart, the Blood was concreted into a great

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many little hard Clots; which appeared yet more confpicuous and harder in fome Blood that flowed back from the Vein into the Syringe. In this Dog we found the Emulgent Artery of the Left Side to be double.

March the 2d, we injected three drachms of rectified Spirit of Wine into the Crural Vein of a fmall Dog; which made him Apoplectick, and as it were half dead. In a little time he recovered from his Apoplexy, but grew giddy; and when he endeavour'dto go, reeled and fell down. Though his Strength encreafed by degrees, yet his Drunkennefs ftill continued: His Eyes were red and fiery, and his Sight fo dull, that he did not feem to take notice of any thing, and when he was beat would fcarce move. However, in four Hours time he grew better, and would eat Bread when we gave it him. The next day he was brisker, and feemed paft all danger. In diffecting the fame Dog fome time after, we found in the fmall Guts two Flat-Worms; one of them about 6 Spans long, and the other about 5. They had perforated the Gut; and one of them was got half out of it into the Cavity of the *Abdomen.* We found alfo in 2 Dogs a Worm of near a Foot in length,out of the Inteftines, in the Cavity of the *Abdomen*, the Inteftines being no ways perforated, but remaining found and whole. That we might be more certain of this, we feparated them from the Mefentery,and viewed them very carefully. But in both thefe Dogs the *Omentum* was of an ill Colour and putrified; whence we conjectured, that thefe Worms were bred from the Putrefaction of the *Omentum.* We injected into the Crural Vein of a Dog five Ounces of a ftrong White Wine; which made him very drunk, and little different from what a lefs quantity of Spirit of Wine would have done: But in a few Hours his Drunkennefs abated, and he recovered. In the fame Month of *March* we injected into the Vein of a Dog, an Ounce of a ftrong Decoction of Tobacco. He was feized immediately with ftrange Convulfions of his whole Body. At firft his Eyes looked wild and diftorted, his Jaws trembled; and in a little time he died terribly convulfed. This Experiment we repeated feveral times after, and always with the fame fuccefs. Ten drops of diftilled Oil of Sage, mixt with half a Drachm of Sugar, and diffolved in an Ounce of Water, being injected iuto the Crural Vein of a Dog, did him no harm. In a caftrated Dog we obferved the Proceffes of the *Peritoneum* and Spermatick Veffels to be covered with Fat, and fcarce to be feen; and that he did not fmell fo rank and ftrong, as other Dogs that had not been caftrated. A yellow-ftreak'd *Lizard* [*Lacerta Chalcidica*] which had been kept all the Winter in Glafs with Bran, being expofed to the Sun to refrefh it, on the contrary died in a few Hours. We have alfo often found, that Scorpions expofed to the hot Sun, efpecially in the Summer, died in a fhort time. A drachm of purified White Vitriol, injected into the Crural Vein of a Dog, killed him immediately. Fifteen Grains of Salt of Urine, diffolved in an Ounce of Water, and injected into the Crural Vein of a Dog,

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cast him into such violent Convulsions, that we were afraid he would dye under them. When he had recovered himself a little, we repeated the Injection with the same quantity; but the Dog got the better of it, though with a great deal of difficulty, and perfectly recovered.

April the 27th, we made a Decoction of two drachms of Senna in Water, and injected warm three Ounces of it into the Crural Vein of a very fat, and strong Dog. He continued pretty quiet, without any sign of pain or uneasiness, during the Operation; and when it was over we let him loose, expecting the Event of it. He was melancholy and dejected, but easie and without any sensible Commotion, for the space of an Hour. After that, his Respiration grew quicker, he had a murmuring Noise in his Belly, with violent Commotions of his *Abdomen*, Diaphragm, Stomach, and Intestines, and vomited plentifully a Biliose Matter. After his vomiting he grew faint, and in a little time his Vomiting returned again; so that in an Hour and half he vomited four times. His Strength and Appetite were very weak, and he would eat nothing for three days. But on the third day his Appetite, Strength, and former Briskness returned, and he recover'd.

Two Dogs, which had their Recurrent Nerves cut, lost their Barking and Voice. But doubting whether the Wound or Scar might not affect and hurt the Motion of the Muscles, we performed the same Operation on another Dog, but without cutting the Nerves; and when the Wound was healed, he barked freely as before. A Dog that had the Nerves of the *Par vagum* cut asunder, presently grew dejected and faint. He breath'd very freely, and with Sighs; for when he had drawn in his Breath leisurely and insensibly, it came forth again immediately very forcibly and with a Sigh, as if it had been retained a long time in the Lungs. The Muscles of the *Abdomen* and Diaphragm laboured hard, as if they were to supply the defect of the Lungs, which were grown almost useless by being denied an Influx of Spirits by the Pneumonick Nerves. The Dog refused all kind of Meat; sometimes he vomited, or had an Inclination to vomit; and at last, in two days time, he dyed. Another Dog, that had the Nerves of the *Par vagum* only tied, lived 10 days. He vomited frequently and would not eat unless clandestinely: He breathed with Sighs, and was very faint. A Dog, that had the Trunk of the *Aorta descendens* tyed hard a little above the Diaphragm, immediately lost the use of his hind Legs; for when he stood on his fore Legs, he would draw after him his hinder Legs, as if they had been dead: He grew weaker by degrees, and dyed in five Hours.

July the 12th, a Mole being stung in the Side by a *Scorpion*, dyed immediately convulsed. In this we observed, that the *Intestinum cæcum* is wanting in Moles.

The manner of making Styrax liquida, *com. by Mr.* Petiver. *n* 313. *p.* 44.

VII. *Rosa Mallas* grows upon the Island *Cobrosi*, at the upper end of the Red Sea, near *Cadess*, which is three days Journey from *Suez*: It is the Bark of a Tree (taken off every Year, and grows again) boiled in Salt Water, till it comes to a Consistence like Birdlime, then separated and put into a Cask, and brought to *Judda*, and so to *Mocha* in *June* and *July*, where it sells from 60 to 120 Dollars *per* Barrel, according to its Goodness. The best is what is freest from Clay and Dirt, which is commonly mixt with it: and the way to try it is, to wash it in Salt-Water, which will cleanse it: The *Arabs* and *Turks* call it *Cotter Mija*. A Barrel is 420*l.*

Of the strange Effects of the Indian *Varnish, by Dr.* del Papa, *com. by Dr.* Sherrard. *n.* 274 *p.* 947

VIII. The using and handling of the *Indian* Varnish (or Lackar) so far as is necessary to apply or lay it on subjects to be varnisht, having produced such extraordinary effects on Seignior *Ignatio*, and more remarkably on his Maid-servant, *viz.* in great Swellings of their Heads and Eyes, and in their Arms, and indeed almost their whole Body, with an intolerable itching and inflammation, or Heat and Pimples, is so new and extraordinary a Phænomenon in Nature, as exacts the wonder, and presses the Curious to search the Reason. The great difference between this and all things else that we know, and the Ingredients of which its made, being absolutely unknown, renders it impossible to penetrate the Cause of the above-named Effects. Yet whoever would give some satisfaction to the Mind, by proposing some probable Thought, may say that this Varnish contains some Matter which when hot sends into our Body a very subtile thin Vapour, which affects only the Skin, leaving the other parts of the Body untoucht; after the same manner, Cantharides, not only taken inwardly, but also outwardly applied to our Bodies, communicates a venemous quality of a particular Nature, affecting only the Kidneys, Bladder and Urinary Passes, causing there sensible Pains and Excoriations, not in the least touching the other *Viscera.* Some Physicians of Repute are of opinion that this particular disagreement of *Cantharides* with the Urinary Ducts, comes from the Salt and Nitre contained in the Urine, which gives Life and Vigour to the Poyson of the *Cantharides*, without which Salt the Poysonous Quality of the *Cantharides* could have no Power. So after the same manner it may be said that the noxious Fumes of the Varnish becomes hurtful to the Skin, because it mixes there with some Juice it meets with in the Skin itself, especially in the miliary Glands, whereof the whole Skin is full. This is certain that this Varnish exerts all its Malignity against the Skin, the *Viscera* and Blood being untouch'd; besides I observed, that the Maid (at the same time as her whole Skin almost was hard, inflamed, swell'd and full of Pustules) had no Fever, no Pain in the Head, nor any Sickness or Illness; and as to what Disorders she or Signior *Ignatio* felt in their Eyes, this likewise was only a Swelling affecting the Eye-lids only,

which may be reckon'd t as Skins: but 'tis possible the ill Effects of the Varnish was more feble and troublesome in that part, because the Skin there is thinner a more delicate than on the rest of the Body. This Varnish therefore only an enemy to the Skin; and that this mischief should attend it it is not necessary that the Varnish should be heated; for altho it is cd, it sends forth this ill steam, which insinuates itself into the Body especially when touched and handled.

I have several times lead a great deal of this Varnish hot upon the naked skin of Poultry, and they never received any mischief from it, either internal or exteal. I have caused other Fowl to swallow Crumbs of Bread sopt i the Varnish, and they seemed to like it very well. In others I have made several little pricks in their Breasts till the Blood came out, and then anointed it all over with Varnish, which, instead of hurting them proved a Balsam to heal them. It is possible this Varnish on the very thin skin of Fowl does not produce the same effects as on that of Man, because they are very different from one another, in their structures and quality of the humours contained in them. And to say someting of its substance; I have observed that this Varnish is in a great par compofed of a Gummy and unctuous matter, and since it is very ligh swimming upon Brandy and Oyl, and unites neither with Water nor Spirit of Wine, nor any other Liquor but only with Oyl, and urns or takes fire; for I have dipt Cotton in it, which has burnt all away to ashes, tho at first there was some difficulty to make it take fire, perchance some other matter not unctuous being mixt with it. And lastly, since, being observed with the Microscope, its composition shews like that of Oyl or Lard, or the like unctuous matter, it is ver likely from all these, that it is composed of the Gum or Juice of some Resinous Herb or Tree, or of the fat of some Animal, or Hogs Lard. And to make some guess; who knows but the Gall of some Creature may be mixt with it, to make it the more easily receive a smoothness and lustre, as Limners use to put Gall into their Water-colours, to make them run and spread the better; and that the mischief we find in touching and using it may proceed from hence?

I verily believe, there is no Mercury, of what sort soever, in this Varnish; not only becaue it is very light (as was said) but besides, because I have been very diligent in trying whether Gold would discover any sign of Mercury, either in the Body of it, or the smoke, but could never find any; and moreover, Mercury produces very different effects in our Bodies, from those before related of this Varnish. Besides, I have observed that the Varnish, mixt with Spirit of Vitriol, or Juice of Limons or Vinegar, or Spirit of Wine, makes no ebullition nor change of colour; but it readily changes colour, when taken out of the Vessel, it is exposed to the air, becoming at first reddish, and afterwards almost quite black; the outward skin of it, which is next to the air, becoming very hard and black. This Skin is very thin, under

under which the reſt of the Varniſh remains ſt and fluid, of the colour and conſiſtence of Honey; and as often you take off this outward black hard skin, there will another bormed immediately like the former, and this as often as you pleaſe repeat the experiment. So that the whole ſubſtance of the Varniſh wi in time be changed into theſe hard and black skins. Finally, 'tis rth obſerving that this Varniſh has this known power: for havingpread ſome of it on the naked Breaſt, of ſome Fowls, leaving it ſtickg there 3 days, I afterwards found between the dried Varniſh an the Fleſh the place all feſter'd, and full of a yellowiſh *Serum* or Matr, but without any farther Miſchief to the Bodies of the Fowls themves. I have attempted the ſame thing on Dogs and Cats but withouSucceſs, for theſe Animals with their Tongues and Claws, ſoon ke all the Varniſh from their Bodies, and ſo have no hurt by it. Pobly in Horſes and Beaſts the Experiment may ſucceed better, if the rniſh has this Corroſive or Cauſtick Quality on their Bodies, as it has n Poultry,

IX. *Accounts of Books omitte, viz.*

n. 273. *p.* 918. 1. D. Dominici Sanguineti Appuli Diſſertaones Iatrophyſicæ. *Neapoli.* 1699.

n. 283. *p.* 1320. 2. A Mechanical Account of Poyſons, by Dr. *Mead.* 8vo. *Lond.*

n. 306. *p* 2253. 3. Samuelis Dale Pharmacologiæ, ceu Mnuductionis ad materiam Medicam Supplementum; Medicamenta fficinalia ſimplicia priore libro omiſſa complectens: Ut & notas Cnerum Characteriſticas, Specierum Synonyma, Differentias & Vir. Cum duplici Indice, generali altero Nominum & Synonymorum pæcipuorum; altero Anglico Latino, in gratiam Tyronum. 12mo. *Lond.* 1705.

n. 320. *p.* 319. 4. Prælectiones Chymicæ Oxoniæ habitæ a *Johanne Freind,* M. D. Æd. Chriſt. Alumn.

F I N I S